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Anisotropic superfluidity of hadronic matter

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We consider hadronic matter as a new manifestation of anisotropic superfluidity. In order to test the validity of our suggestion, some qualitative features of multiparticle production of hadrons are studied and found to have a natural explanation. A prediction is made following a recent experiment on π^+p collisions.

I. INTRODUCTION

The main problem in high-energy hadron collisions seems to be the recognition, description, and understanding of the particle production mechanisms, especially those responsible for the bulk of particle production at high energies.

However, *prior* to achieving this goal, we must develop a theory for strong interactions; yet, so far, such a theory is still missing.[1] In this respect Feynman and collaborators[2-4] show how, from a simple quark picture, emerges an understanding of the source of high-transverse-momentum mesons as well as an understanding of correlations among particles and jets (collection of particles moving roughly in the same direction) produced with large transverse momenta.

On the other hand, the recent discovery of charmed quarks has complicated the search for fundamental elementary particles (presumably an important step in the construction of a strong-interaction theory). While in the 1950's it was hoped that the building blocks could be the nucleons, π mesons, and a few leptons, in the early 1960's the discovery of hadron families of SU(3) octets shifted the elementarity to the leptons and the three quarks. Thus the combination of the recent discovery of charmed quarks combined with the discovery of massive new leptons may be considered as a suggestion that the quark model does not have the features of elementarity which one originally hoped for.

In the present work we suggest that hadrons may be classified in a way which is not immediately related to internal quantum numbers, but which arises from elementary fermions. In Table I we have shown part of the family of quantum liquids and solids and have included in the gap existing in temperature (from the critical temperature of superfluidity in neutron stars all the way to an "ultimate temperature") the proposed new superfluid state of hadronic matter. It is the purpose of the present work to elaborate on these ideas, with the hope of providing some basis for a theory of strong interactions in which we might try to recognize some of the most important problems in high-energy hadron collisions.

It is very interesting that Eliezer, Galloway, Mann, and Weiner[5-8] have introduced, since 1971, the concept of superfluidity of hadronic matter in terms of the underlying Bose-Einstein condensation of the mesons (which they assumed are contained in the nucleon, in much the same way as the nucleus contains the nucleons). These authors found a meson cloud with a dispersion function, which satisfies Landau's criterion of superfluidity. On the other hand, our new start on the subject gives support to the *Fermi statistics* of the elementary constituents of the hadron (which we call F fermions in Sec. II) and not to *Bose statistics.*

We proceed as follows: In Sec. II we sketch, very briefly, some of the gross properties of the model of hadrons in terms of concepts from the theory of quantum liquids and solids. In Sec. III we describe (subsection A) the hadron as a two-level quantum condensed system, and proceed to discuss (subsection B) the mechanism for multiparticle production of the small and large p_T momenta of secondaries. We conclude the section by discussing the expected single-particle distributions (subsection C), their experimental evidence, and a prediction at higher energies. In Sec. IV we point out some of the data that a quantitative calculation might attempt, thus going beyond the qualitative discussion of Sec. III. We then comment on the implications which the two-level condensed system developed in the previous section might

TABLE I. Classification of quantum liquids and solids.

Temperature range (K)	State of matter
$(0-2)\times 10^{-3}$	Three phases of superfluid ^{3}He
$(0-2)\times 10^{0}$	He II
$(0-2)\times 10$	Metals and alloys (superconductors)
$10^{9}-10^{11}$	Pulsar interiors (neutron-star superfluidity)
$10^{11}-10^{13,14}$	Superfluid hadron drops ?

have in a recent theory of He II. We discuss, briefly, some cosmological applications of the drop model of hadrons. In Sec. V we summarize the aim and limited achievement of the work, and compare it with the only previous suggestion of superfluidity (in terms of bosonic elementary constituents). We point out the way in which our work must be developed in order to face the basic question of the origin of the nuclear force.

II. SUPERFLUID HADRON DROPS

We have been led, heretofore, to view the hadron drop as a condensed system of Cooper pairs of F fermions in an axial-vector state, just as in the case of ^{3}He in its anisotropic superfluid A phase (p-state pairing[9,10]). This assumption will be sharpened later on as we take a closer look at the data (cf. Sec. III B). The important point is that superfluidity is produced up to some critical temperature by the multiple occupation, by the Cooper pairs, of the same quantum mode. This phenomenon has an associated energy gap Δ, which is typical of those quantum liquids whose elementary constituents are *fermions* (e.g., *electrons* in superconductivity produce a gap of the order of 10^{-4} eV, while in *neutron* stars the gap is of the order of a few MeV).

For neutron matter, Ginzburg[11] recalls the well-known result of Bardeen, Cooper, and Schrieffer,[12]

$$k_B T_c = \Delta/1.76 \tag{1}$$

(where k_B is the Boltzmann constant) and evaluates

$$\Delta \sim 1\text{–}20 \text{ MeV} \tag{2}$$

for densities

$$\rho \sim 10^{13}\text{–}10^{15} \text{ g/cm}^3 . \tag{3}$$

In Eq. (1) one obtains

$$T_c \sim 10^{11} K , \tag{4}$$

but $T_c > T_{\text{star}}$; hence, the neutron matter in the "star" (pulsar) is a superfluid.

For hadronic matter ("elementary particles") one expects higher densities than in the Ginzburg estimate; thus

$$\Delta = \Delta_1 10^{1+\delta} \text{ MeV}, \quad \Delta_1, \delta > 0 \tag{5}$$

yielding

$$k_B T_c \geq 10^2 \text{ MeV} \tag{6}$$

or, equivalently,

$$T_c \geq 10^{12} \text{K} . \tag{7}$$

We observe that the statistical bootstrap model of Hagedorn[13] contains a maximum temperature for hadrons with the choice $\Delta_1 = 1.6$ and $\delta = 1$.

III. SOME TESTS OF THE ANISOTROPIC SUPERFLUIDITY OF HADRONIC MATTER

A. Energy classification for particles in collision

We suppose that the ground state of a stationary hadron consists of a large number of pairs in the same quantum mode; namely, there is a large condensed fraction in the ground state. Then, as the hadron is accelerated, the liquid drop will become a *denser system*, since adiabatic acceleration (i.e., changes in the particle position without an abrupt change as, for example, in a collision) will not change appreciably the range of the Yukawa potential, thus confining the increasing particle energy within approximately one Fermi. From this "adiabatic hypothesis" it follows that the constituent boson pairs (of F particles) will gradually become more closely packed, since the presence of the energy gap Δ will be such that it prevents the pairs from being excited into higher states. (Eventually, as the superfluidity ceases, Δ tends to vanish.) Hence, the pair-pair interaction potential $V(\vec{x} - \vec{x}')$ goes from an almost gas regime to a dense regime.

We may infer that in bringing a hadron from rest into a collision with another hadron in a bubble chamber (or, independent of machines, as in the case of cosmic rays), the interparticle potential may go through some of the regimes,

(i) almost free pairs, $V \approx 0$,

(ii) weakly interacting pairs, V small,

(iii) dense superfluid, V large,

according to whether we succeed in accelerating sufficiently the projectile hadron. Therefore, in view of an increasing interparticle interaction, the ground state depletes itself: The He II analog of the drop, for example, is well known to deplete even at absolute zero, forcefully pointing out that this effect is due to *the interactions* as well as to increasing temperatures [recent experiments indicate that the depletion for He II is about 98% (Ref. 14); more recent experiments and theory are discussed in a recent paper[15]].

To summarize, the drop condensation into a zero-momentum mode is dynamically gradually destroyed as we accelerate the hadron adiabatically. Such an effect must be accompanied by a decreasing energy gap Δ, since it must become null with vanishing superfluidity. We are thus led to postulate three energy regions, according to whether the energy gap is large, intermediate, or vanishingly small:

(i) *The Bose-Einstein region*, where most particles are in the state of zero energy momentum.

(ii) *The Planck region*, where there is a fraction of particles in the *same* quantum mode of energy ϵ and small momentum $\vec{p}$, thus constituting a two-level condensed system for the hadron with the "lower" zero-momentum level.

(iii) *The Boltzmann region*, where as the energy increases, the gaps thin down, and the upper level becomes so populated that collisions occur, breaking some of the pairs into their constituting F particles, which then occupy arbitrary quantum levels.

B. Mechanisms for large-p_T production

Suppose a hadron is accelerated and then made to collide in a bubble chamber with a stationary proton. In the c.m. system it is clear that if the projectile is not energetic enough, both colliding particles will find themselves in the Bose-Einstein region and no momentum can be exchanged, hence no scattering occurs. Again viewing the scattering in the c.m. system, both particles enter into the collision in the Planck region. The second levels, in both cases, will be able to transfer momentum; thus the scattering will be close to the forward direction. When scattering occurs in the Boltzmann region there is barely any double condensation (in lower and upper levels), and the F gas of broken pairs together with a poorly populated upper level are no longer able to constrain the final momentum longitudinally.

This simple picture will now be made more accurate and we shall find that the multiparticle production (in the case when the collision does occur in the Planck and Boltzmann regions) may be understood in terms of the energy gap of the colliding hadrons.

To explain how the actual particle production does occur we must look more closely at the two-level system: Essentially, it consists of two condensed systems coupled together. Thus, if we treat either one of them as an interpenetrating quantum fluid, each is expected to have its own energy gap (which, as in other quantum fluids, is temperature dependent). Then, as two hadron drops collide, heat is generated and hence both energy gaps thin down; the upper level tends to disappear faster, since the pairs have $\vec{p}\neq 0$. So at the moment of collision it is these pairs which transfer momentum to the incoming projectiles (this momentum, in the Planck region, is small). However, the collisions generate sufficient heat to thin out the gap of the lower level, which then, from momentum conservation, carries off the balance of the incoming momenta. We return to this picture in Sec. IV. We call the attention of the reader to the fact that no specific mechanism for hadron "leakage" is needed, as in the thermodynamical models.[16]

C. Two-level picture of single-particle distributions

Consider the differential cross section defined through[17,18]

$$d\sigma = W_{(i)}(m_i/p_i)\,|\langle f|U|i\rangle|^2 \times \delta(E-E')d^3p_0\delta(\vec{p}_0-\vec{p}_i)\int d\vec{p}'_f\delta(\vec{p}'-\vec{p}_i)\,, \tag{8}$$

where U is the interaction potential for the scattering of a projectile with a scatterer, producing in the final state a particle of momentum $\vec{p}_0$ and averaging over all the other states; we have allowed for thermal excitation of the scatterer.[18] W_i is the Bose probability that the scatterer (drop) be initially in the state i.

For our two-level system we obtain an (inclusive) cross section

$$d^3\sigma/dp^3 = W_2(m_i/p_i)\,|\langle f|U|i\rangle|^2 \times\delta(E-E')\delta(\vec{p}_0-\vec{p}_i)\int d\vec{p}'_f\delta(\vec{p}'-\vec{p}_i)\,, \tag{9}$$

since the lower level does not scatter.

Now, since in the upper level collisions may break pairs, we find that the particle number is not conserved. Hence the chemical potential is zero.[19] Therefore,

$$W_2 = 1/[\exp(E_2/k_BT)-1]\,, \tag{10}$$

having assumed that the pairs interact weakly, in the problem of finding the probability W_2 that the drop should be in a particular quantum state of energy E_2. However, neglecting the *weak* interactions in the upper level, this problem reduces to the quantum-mechanical problem of determining the energy levels E_n of a gas as a whole. This is equivalent to the determination of the energy levels of a single elementary pair. Denote these levels by $\epsilon_i(k)$, where k denotes the set of quantum numbers specifying the state of the elementary pair. The energy E_2 will then represent the sum of the energies of all the pairs in the upper level where scattering occurs,

$$E_2 = \sum_{i=1}^{N_D} \epsilon_i(k)\,, \tag{11}$$

where N_D is the number of pairs in the upper level (the "depletion"). Hence

$$W_2 = 1/\left[\exp\left(\sum \epsilon_i/k_BT\right)-1\right]\,. \tag{12}$$

However, for energies which excite the drop into the Planck region, the scattering resembles the scattering of nuclei by cold neutrons, and we do not expect the potential to introduce appreciable momentum transfer dependence in $d^3\sigma/dp^3$:

$$U(\vec{r}) = a\delta(\vec{r}). \tag{13}$$

Equations (9), (12), and (13) allow us to write (for a given experiment m_i and p_i are given constants)

$$d^3\sigma/dp^3 \propto \{1/[\exp(\sum \epsilon_i/k_B T) - 1]\} \times a\delta(E - E')\delta(\vec{p}_0 - \vec{p}_i) \int d\vec{p}_f' \delta(\vec{p}' - \vec{p}_i). \tag{14}$$

Thus from Eq. (14) we are led to a "good fit" for the scattering data in terms of the inclusive cross section

$$d^3\sigma/dp^3 \propto 1/[\exp(\sum \epsilon_i/k_B T) - 1]. \tag{15}$$

Then, we appreciate that the upper level acts as a "pilot" for the scattering: in the Planck region, for example, the projectile will be scattered and produce a fast secondary with a large p_L component and its energy will be given by

$$E_c \approx \sum_{i=1}^{N_D} \epsilon_i(k), \tag{16}$$

where the summation represents the total energy of the particles that were in the upper level, and c is the particle detected in the event of the type given in

$$a + b \to c + \text{anything}.$$

In the Boltzmann region, though, large p_T may be produced, which in turn would signify that the p_L component would be small compared with the global energy. Thus we can approximate

$$E_c \cong (m_c^2 + p_T^2)^{1/2} \equiv E_t, \tag{17}$$

where E_t is called the transverse energy. This allows us to write Eq. (15) as

$$\frac{d^3\sigma}{dp^3} \propto \frac{1}{\exp(E_t/k_B T) - 1}. \tag{18}$$

We must emphasize the significance of Eq. (16): It shows that the observed secondary c is understood qualitatively as a collection of F particles of total energy $\sum_{i=1}^{N_D} \epsilon_i(k)$ arising from the upper level. In other words, the c hadron would be a collection of F fermions carrying part of the available charge from the colliding hadrons, thus allowing its detection in the final state.

We remark, in passing, that if individual F particles escape confinement by "leaking out" over the energy gap, they would carry quantum numbers very different from the hypothetical quark. They would be very hard to detect *experimentally*, for it is a whole statistical ensemble of them that make up the mass of a proton; for instance, they all share the total proton charge, so that the individual F charge is very much smaller than $(\frac{1}{3})e$, the quark charge.

D. Experimental evidence and prediction

There is evidence of this type of behavior in the fitting to the data of the experiment of Bartke *et al.*[20]: These authors have used data from a 16-GeV/c $\pi^+ p$ bubble-chamber experiment for the production of pion and meson resonances for

$$|x| < 0.1 \text{ or, equivalently, } |y^*| < 1.6, \tag{19}$$

where the asterisk denotes the c.m. system. The particular slope quoted is[20] $k_B T \approx 120$ MeV.

Our superfluid hadron drop model gives us furthur information on these measurements (and the previous ones quoted by Bartke *et al.*), i.e., the temperature of 120 MeV ($k_B = 1$) *is not universal* but merely a reflection of the particular *energy region* in which the measurements were made: In Ref. 20 the measurements were made at the onset of the Boltzmann region, since we find Eq. (17) is approximately held. Closer to the critical temperature (*deep* Boltzmann region) steeper slopes in $d^3\sigma/dp^3$ ought to be expected. This predicted behavior ought to be evident in cosmic-ray data.

There is also ample evidence of the small-p_T and large-p_T production of hadrons (referred to in Secs. III A and III B), which will be commented upon at the beginning of the next section.

IV. DISCUSSION

We find sufficient evidence to suggest a more detailed consideration of the drop model. This amounts to a field-theoretic study of the pair-pair scattering, in order to be able to face problems such as the explicit power behavior $d\sigma/dp_T^2$ for $1 < p_T < 8$ GeV/c, as considered recently by Feynman, Field, and Fox.[3] We should like to emphasize that in Secs. III A and III B we have achieved a qualitative understanding of $d^2\sigma/dp_T^2$ for *the whole* range $0 < p_T < 8$ GeV/c. We have tried to show that part of the data emerging from CERN ISR and Fermilab does not require such detailed assumptions on the dynamics as, for example, the cross sections that two quarks will scatter one another.

Some support for the two-level system used in this paper may be found in the recent theory of He II, in which a gauge theory[21] is developed with some pleasant properties.[22] In this theory a two-fluid picture of ^{4}He is developed in *configuration* space, one of which is the condensate, while symmetry (i.e., gauge invariance of the second kind) imposes on us the second fluid. If the dynamics is that a two-level system (as in hadrons), the apparent underlying difficulty of the gauge theory of

^{4}He is removed, since, although condensation occurs in momentum space, there are quantum properties (i.e., $\epsilon, \vec{p} \neq 0$) which differentiate a boson in the depletion (upper level) from a boson in the condensate ($\epsilon, \vec{p} = 0$).

As an *illustration* of the future development which these ideas might encounter, we comment on other branches of physics where modifications should follow if detailed quantitative calculations support our qualitative predictions. Such a branch is the cosmogony underlying an evolving Freidmann universe. In such a picture we expect the thermal history in the post-hadronic era to yield a cooling off from the primordial fireball governed by a temperature decrease[23]

$$T \propto R^{-1}, \tag{20}$$

where R is the Robertson-Walker "radius of the universe". The *very* early thermal history is dependent on the particular model of strong interactions one is considering, and the Veneziano model[24] and Hagedorn's statistical bootstrap model[13] lead to a maximum *finite* temperature presumably lower than our critical temperature. However, if a simple extrapolation into the very early times $t \to 0$ follows a similar singular behavior as shown by R^{-1}, at the initial instants of the Friedmann evolution $T > T_c$; as $t \to \infty$, R increases and T decreases until it comes under T_c. Then, the initial statistical ensemble of F hadrons is condensed and goes into a superfluid state (primordial fluid); this is accompanied by an ever increasing R, so that the fluid separates into individual hadrons (drops). Then, after such an event, the temperature has decreased so much that hadron-hadron collisions are unable to raise the temperature over T_c, and subsequently lepton pairs will be created and the hadron era will be followed by the lepton, radiation, matter dominated era, as is well known.[23]

V. CONCLUSION

In the present work we have not attempted to replace the very successful statistical treatment for multiple-hadron production,[25,16,13] but to point out one of the very important features of hadrons, namely, their superfluidity. In this respect we coincide with the notable work of Eliezer, Galloway, Mann, and Weiner,[5-8] but we were led to the underlying Fermi ("quark") statistics, unlike the earlier work mentioned, in which Bose statistics of the pion "substrate" is a key feature.

In our Secs. II and III we have shown explicitly, the difference such Fermi statistics make, i.e., the presence of the energy gap, which was shown to be the key element in our discussions.

We believe that in order to develop a theory of strong interactions, the internal structure of the protons and neutrons must be sorted out. *Current theory is not in terms of the internal structure of the nucleons*.[26] Thus one would expect that learning about the anisotropic superfluid nature of hadrons might, with some further effort, lead us to an understanding of the nuclear force, in much the same way as we were able to derive the chemical force, as a consequence of the quantum structure of the atom, according to Heitler and London.[27] However, to achieve some progress in that direction we would have to develop the detailed field equations of the elementary pairs before undertaking quantitative calculations, which we did not attempt to cover in this work.

After we had concluded this paper, it was pointed out to us that Ghose[28] has come to somewhat similar conclusions about the superfluid nature of hadrons from the point of view of the quark model; he also pointed out an earlier work of Barrois.[29]

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