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Quantum chromodynamics and the soliton model of hadrons

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By starting from quantum chromodynamics (QCD) in a finite volume and then taking the infinite-volume limit, we suggest that there is a "phase-transition" phenomenon, which implies the existence of a long-range order in the vacuum for an infinite volume. This long-range order is represented by Lorentz scalars, because of relativistic invariance; such Lorentz scalars can in turn be identified with the phenomenological scalar fields used in a soliton (or bag) model of hadrons. In the phenomenological approach, a permanent quark confinement can be simply viewed as the vacuum of an infinite volume being a perfect "dia-electric" substance, with its dielectric constant $\kappa \rightarrow 0$, while the "vacuum" inside a hadron is normal ($\kappa = 1$), which may be identified as that of QCD for a finite volume. Inside the hadron, exchanges of gauge quanta between quarks give the QCD corrections to the soliton (or bag) model. Spectroscopy of light-quark hadrons is examined by expanding the hadron masses M in powers of the "fine-structure constant" α of QCD: $M = M_0 + \alpha M_1 + \alpha^2 M_2 + \ldots$. The near-zero mass of the pion is correlated with the existence of a critical value α_c in the mass formula, and the η-η' anomaly is associated with a large enhancement factor in the $O(\alpha^2)$ quark-antiquark annihilation diagrams, due to coherence in the various color and flavor degrees of freedom.

Recently, we have applied the classical nontopological soliton solutions to the light hadrons,[1] and derived in various limiting cases the MIT bag[2,3] and the SLAC bag.[4] In the soliton picture, one assumes a scalar field σ whose average value inside a hadron is very different from that outside the hadron. Thus, the effective mass of the quark can be very light inside, but extremely heavy (maybe infinite) outside, which gives rise to the quark confinement mechanism. In this note, we shall first try to reconcile this simple phenomenological description with quantum chromodynamics[5] (QCD) in which there is no scalar field, but only color quark fields and a set of non-Abelian color-gauge fields. Next, we shall incorporate the QCD corrections into the soliton model. As we shall see, this may lead to an approximate understanding of the spectroscopy of light hadrons, including the near-zero mass of the pion and the relatively large mass of $\eta'(958)$.

I. VACUUM STATE IN QCD

To give QCD a well-defined meaning, we first contain the whole system within a volume of size L^3. For a finite L, the usual perturbation series expansion can be obtained. Let g, V_μ^a, and $V_{\mu\nu}^a$ be, respectively, the appropriately defined renormalized coupling constant, renormalized color-gauge field, and its covariant field derivative, where the superscript a denotes the color index ($a = 1, 2, \ldots, 8$). For definiteness, we consider the vacuum expectation value of the normal product of any *Lorentz-scalar* and color-singlet function $\phi(V_{\mu\nu}^a)$, with $\phi(0)$ set to be zero:

$$\langle \mathrm{vac} | : \phi(V_{\mu\nu}^a) : | \mathrm{vac} \rangle , \tag{1}$$

e.g., ϕ can be either $f^{abc} V_{\mu\nu}^a V_{\nu\lambda}^b V_{\lambda\mu}^c$ where f^{abc} is the usual antisymmetric SU_3 tensor, or $(V_{\mu\nu}^a V_{\mu\nu}^a)^2$, or other combinations. So long as L remains finite, then such a vacuum expectation value is 0 when $g \rightarrow 0$. Thus we have

$$\lim_{L\to\infty} \lim_{g\to 0} \langle \mathrm{vac} | : \phi(V_{\mu\nu}^a) : | \mathrm{vac} \rangle = 0 . \tag{2}$$

For our discussion, we shall assume that the limit $L \rightarrow \infty$ of $\langle \mathrm{vac} | : \phi(V_{\mu\nu}^a) : | \mathrm{vac} \rangle$ exists. In addition, (most of) these functions $\phi(V_{\mu\nu}^a)$ satisfy the property

$$\lim_{g\to 0} \lim_{L\to\infty} \langle \mathrm{vac} | : \phi(V_{\mu\nu}^a) : | \mathrm{vac} \rangle \neq \lim_{L\to\infty} \lim_{g\to 0} \langle \mathrm{vac} | : \phi(V_{\mu\nu}^a) : | \mathrm{vac} \rangle . \tag{3}$$

This assumption is a reasonable one, since the power series expansion of (1) in the limit $L \rightarrow \infty$ is, in general, term-by-term divergent. We note that the noncommutivity of such double limits is a familiar criterion for a phase-transition phenomenon.[6] Equation (3) implies that there is a long-range order in the vacuum state when L is infinite. This long-range order can be characterized by the vacuum expectation value of any one of these scalar functions $\phi(V_{\mu\nu}^a)$; the same function can also be used as the interpolating field for the scalar σ field in the soliton model.

Of course, once the concept that the vacuum for an infinite volume can be very different from that of a finite one is accepted, there is no reason why it should remain invariant under a local color-gauge transformation.[7] Thus, one may also consider in (1) gauge-dependent functions ϕ. However, because of Lorentz invariance, ϕ must be a Lorentz scalar. Consequently, $\langle \mathrm{vac}|\phi(V^a_{\mu\nu})|\mathrm{vac}\rangle \neq \phi$ of $\langle \mathrm{vac}|V^a_{\mu\nu}|\mathrm{vac}\rangle$ which must vanish. Thus (3) is a pure quantum phenomenon.

II. QUARK CONFINEMENT

In terms of these vacuum expectation values, it may be possible to formulate the confinement mechanism by a simple heuristic argument, without a detailed understanding of the underlying dynamics. Let us first take the infinite-volume limit, and introduce into the vacuum a small external slowly varying c-number field distribution $(V^a_\mu)_{\mathrm{ext}}$. We assume that the corresponding change in the action integral Z can be evaluated by using an "effective" Lagrangian density[8] $\mathcal{L}_{\mathrm{eff}}(V^a_\mu, V^a_{\mu\nu})$, at least for fields at their long-wavelength limits. To second order in $(V^a_\mu)_{\mathrm{ext}}$, we may write

$$\delta Z = \frac{1}{2}\int \left\langle \mathrm{vac}\left|\frac{\delta^2\mathcal{L}_{\mathrm{eff}}}{\delta V^a_\mu\,\delta V^b_\nu}\right|\mathrm{vac}\right\rangle (V^a_\mu)_{\mathrm{ext}}(V^b_\nu)_{\mathrm{ext}}d^4x,$$

or

$$\delta Z = -\kappa\int \left[\tfrac{1}{4}(V^a_{\mu\nu})_{\mathrm{ext}}{}^2 + \tfrac{1}{2}m_V{}^2(V^a_\mu)_{\mathrm{ext}}{}^2\right]d^4x, \tag{4}$$

where κ and m_V refer to the appropriate vacuum expectation values. If $m_V \neq 0$, then the vacuum for an infinite volume is not invariant under a local color-gauge transformation. Here we assume that (4) remains invariant under a global color-gauge transformation.

Now, let us imagine that the space is divided into two regions: one is a finite volume L^3, and the other refers to the infinite domain outside that volume. The vacuum inside L^3 is assumed to be that of the finite volume, while the vacuum outside, that of the infinite volume. Suppose that an external c-number current distribution $(j^a_\mu)_{\mathrm{ext}}$ is set within a small sphere of radius R inside L^3, with

$$R \ll L. \tag{5}$$

The field $(V^a_\mu)_{\mathrm{ext}}$ generated can be obtained by taking the extremum of the action integral. For the volume inside L^3, we assume that the renormalized coupling constant g is sufficiently small so that we can neglect all loop diagrams due to the (inside) Lagrangian density

$$\mathcal{L}_{\mathrm{in}} = -\tfrac{1}{4}(V^a_{\mu\nu})^2 - g(j^a_\mu)_{\mathrm{ext}}V^a_\mu. \tag{6}$$

For the volume outside L^3, because of (5) we can take the long wavelength limit of $\mathcal{L}_{\mathrm{eff}}(V^a_\mu, V^a_{\mu\nu})$. Thus, for a weak $g(j^a_\mu)_{\mathrm{ext}}$, (4) should be adequate. We can combine (4) and the effect of (6) into a single action integral

$$-\int d^4x\{K[\tfrac{1}{4}(V^a_{\mu\nu})^2 + \tfrac{1}{2}M^2(V^a_\mu)^2] + g(j^a_\mu)_{\mathrm{ext}}V^a_\mu\}, \tag{7}$$

where $K=1$ and $M=0$ in the inside region, while $K=\kappa$ and $M=m_V$ outside. Consequently, $(V^a_\mu)_{\mathrm{ext}}$ satisfies

$$\frac{\partial}{\partial x_\mu}(KV^a_{\mu\nu}) - KM^2V^a_\nu = g(j^a_\nu)_{\mathrm{ext}} - gf^{abc}V^b_\mu V^c_{\mu\nu}. \tag{8}$$

Let us define the "electric" and the "magnetic" fields $\vec{\mathrm{E}}^a$ and $\vec{\mathrm{H}}^a$ by the familiar expressions

$$V^a_{ij} \equiv \epsilon_{ijk}H^a_k \text{ and } V^a_{4j} \equiv iE^a_j. \tag{9}$$

Equation (8) implies that across the boundary the normal component of $K\vec{\mathrm{E}}^a$ and the tangential component of $K\vec{\mathrm{H}}^a$ are continuous. Thus, at the boundary

$$\begin{aligned}(E^a_n)_{\mathrm{out}} &= (E^a_n)_{\mathrm{in}}/\kappa,\\ (H^a_t)_{\mathrm{out}} &= (H^a_t)_{\mathrm{in}}/\kappa,\end{aligned} \tag{10}$$

where the subscripts n, t, in, and out refer, respectively, to the normal component, tangential component, inside region, and outside region.

With our approximation, the limit $\kappa\to 0$ corresponds to a permanent quark confinement. This can be seen by assuming the total color of $(j^a_\mu)_{\mathrm{ext}}$ to be nonzero. By Gauss's theorem, $(E^a_n)_{\mathrm{in}} \neq 0$ at the boundary; hence, as $\kappa\to 0$, $(E^a_n)_{\mathrm{out}}\to\infty$ which makes the action integral to be $O(\kappa^{-1})\to\infty$. On the other hand, if the total color of $(j^a_\mu)_{\mathrm{ext}}$ is zero, then in order to have a finite action integral $(\vec{\mathrm{E}}^a)_{\mathrm{out}}$ and $(\vec{\mathrm{H}}^a)_{\mathrm{out}}$ should be finite. From (10), we derive as $\kappa\to 0$, at the boundary

$$(E^a_n)_{\mathrm{in}} = (H^a_t)_{\mathrm{in}} = 0, \tag{11}$$

which is the boundary condition postulated for the MIT bag.[2] Thus as $\kappa\to 0$, at fixed m_V, *the infinite-volume vacuum acts as a perfect "dia-electric" substance*, just as the usual superconductor which is a perfect diamagnet.

[The limit $\kappa\neq 0$ but $m_V\to\infty$ corresponds to the one discussed by Creutz and Soh.[3] In this case, in the outside region near the boundary $\vec{\mathrm{E}}^a$ and $\vec{\mathrm{H}}^a$ are $O(e^{-m_V d})$, where d is the normal distance from the boundary. Thus, we find

$$(V^a_\mu)_{\mathrm{out}} = O(m_V^{-1}e^{-m_V d}). \tag{12}$$

As $m_V\to\infty$, $(V^a_\mu)_{\mathrm{out}}\to 0$, and therefore the gauge field is confined within the inside region. Since V^a_μ is continuous across the boundary, we have in the same limit $(V^a_\mu)_{\mathrm{in}}=0$ at the boundary, which implies, in contrast to (11), the boundary condition

$$(E_t^a)_{in} = (H_n^a)_{in} = 0. \tag{13}$$

Thus, the infinite-volume vacuum behaves like a perfect "conductor." In this limit, even if the total color of $(j_\mu^a)_{ext}$ is nonzero, the action integral remains finite. This follows because the space integral of $m_V^2 (V_\mu^a)_{out}{}^2 \to 0$ as $m_V \to \infty$, on account of (12). Consequently, quarks can exist in free form, though their mass may be heavy.]

III. SOLITON MODEL

The scalar field σ in the soliton model is now regarded as a phenomenological field.[9] As mentioned in Sec. I, any scalar color singlet function ϕ of the gauge field that characterizes the long-range order of the infinite-volume vacuum can be used as the interpolating field for σ. In the soliton description, the value of σ inside the hadron is very different from that outside: inside σ is approximately 0, but outside σ must $\to \sigma_{vac}$ as one moves away from the hadron. The change in the value of σ from σ_{vac} can be viewed as a phenomenological description of the breakdown of the long-range order in QCD. Thus, the region occupied by the hadron plays a role similar to that of the domain structure in, say, an infinite ferromagnet. In the ferromagnetic case, the change in the long-range order can be described by a classical magnetization function, even though the underlying mechanism for ferromagnetism is quantum mechanical. Here, one expects that a similar classical description may serve at least as the zeroth approximation, except that because of relativistic invariance, the corresponding function must be a Lorentz scalar, and hence the scalar σ field.

In order to incorporate the short-distance effects of QCD in the soliton model, the vacuum in the interior of the hadron is assumed to be that of QCD for a *finite* volume. Thus, inside the hadron, one may apply the usual perturbation series of QCD. Some typical diagrams for mesons and baryons are given in Fig. 1. With the inclusion of the vector flux lines, the hadron radius can be determined by examining either the matter density of the quarks or the energy density of the vector field. Let R and L be, respectively, the hadron radii determined by these two methods. For definiteness, we shall assume the permanent quark confinement mechanism discussed in Sec. II. The radius L can then be defined by using the boundary condition (11):

$$E_n^a = H_t^a = 0, \text{ at } r = L. \tag{14a}$$

To define R, we require the quark wave function ψ to satisfy

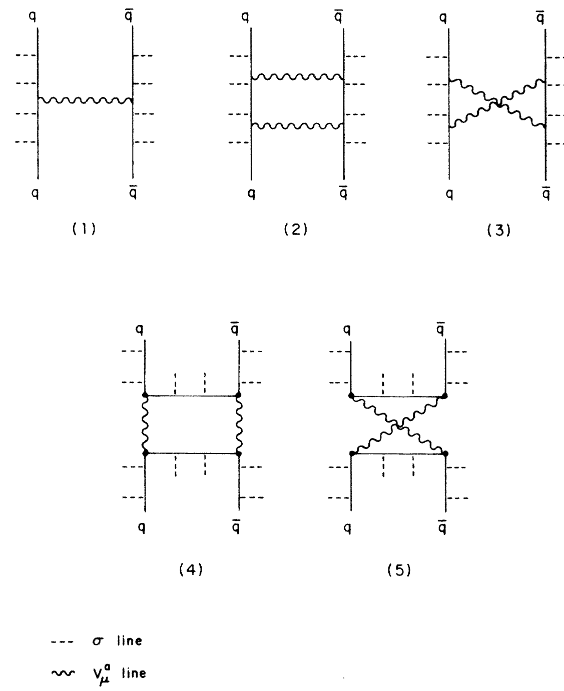

FIG. 1. Examples of QCD correction diagrams to meson energy.

$$\psi^\dagger \beta \psi = 0, \text{ at } r = R. \tag{14b}$$

For simplicity, we assume a spherical shape for the hadron in both (14a) and (14b). Since quarks are the source of the color-gauge fields, we expect

$$L \geq R. \tag{15}$$

The ratio L/R should in principle be determined by minimizing the total energy of the system. The actual calculation of the energy dependence of these "collective coordinates" is a difficult one. For a phenomenological description, we may just as well regard L and R as independent parameters.

In the following, we shall examine the perturbation series of the hadron in terms of QCD in a finite volume ($r \leq L$):

$$M = M_0 + \alpha M_1 + \alpha^2 M_2 + \cdots, \tag{16}$$

where α is the "fine-structure constant" of QCD. As we shall see, in the energy range of interest, the value of α is not small, $\sim\frac{1}{2}$. Consequently, one cannot expect a perturbative calculation of the hadron mass to be too accurate. Furthermore, because of the large number of degrees of freedom $\mathfrak{N}$ due to internal symmetry (color and flavor), the actual expansion parameters in (16) consist of both α and $\mathfrak{N}\alpha$. In the following, we shall try to include QCD corrections up to

$O(\alpha^2)$. Fortunately, as we shall see, to that order except for η and η' mesons there is no especially large $\mathfrak{N}\alpha$ term in the mass calculation of other hadrons.

IV. ZEROTH ORDER

In the zeroth-order calculation, we neglect the exchange of vector mesons. The Lagrangian in a soliton model consists of only the scalar σ field and the quark field ψ (which beside being a color triplet also has F flavors):

$$\mathcal{L} = -\frac{1}{2}\left(\frac{\partial\sigma}{\partial x_\mu}\right)^2 - U(\sigma) - \psi^\dagger\gamma_4\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + f\sigma + m\right)\psi, \qquad (17)$$

where m is the mass matrix for quarks inside the hadron. In this note, we shall concern ourselves mainly with light-quark hadrons; hence,

$$m \cong 0.$$

The potential function $U(\sigma)$ is assumed to have a local minimum at $\sigma = 0$, and an absolute minimum at $\sigma = \sigma_{\text{vac}}$, with

$$U(0) - U(\sigma_{\text{vac}}) \equiv p. \qquad (18)$$

Since σ is only a phenomenological field, describing the long-range collective effects of QCD, its short-wavelength components do not exist in reality. We should, therefore, ignore the σ-loop diagrams. The remaining σ diagrams are all tree diagrams, which correspond to the quasiclassical approximation used in Ref. 1. Thus, ψ and σ can be reduced to c-number functions which satisfy (for $m = 0$)

$$(-i\vec{\alpha}\cdot\vec{\nabla} + f\beta\sigma)\psi = \epsilon\psi$$

and (19)

$$-\nabla^2\sigma + U'(\sigma) = -fN\psi^\dagger\beta\psi\ ,$$

where $U'(\sigma) = dU/d\sigma$, $\vec{\alpha}$ and β are the usual Dirac matrices, ψ satisfies $\int\psi^\dagger\psi d^3r = 1$, and N is the total number of quarks and antiquarks,

$$N = 2, \text{ for mesons,}$$

and (20)

$$N = 3, \text{ for baryons.}$$

As shown in Ref. 1, under very general assumptions, (19) can be further reduced, through scaling, to a simple system of two coupled first-order differential equations:

$$\frac{du}{d\rho} = (-1 + u^2 - v^2)v$$

and (21)

$$\frac{dv}{d\rho} + \frac{2v}{\rho} = (1 + u^2 - v^2)u,$$

where $\rho = \epsilon r$,

$$\psi \propto \begin{pmatrix} u \\ i(\vec{\sigma}\cdot\vec{r}/r)v \end{pmatrix} S\ ,$$

$$S = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{or} \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad (22)$$

$\vec{\sigma}$ is the usual Pauli spin matrix, and the variables ρ, u, and v are all dimensionless. Although (21) does not contain any explicit parameters, its solutions form a one-parameter family. Here we give only a brief summary of the results derived in Ref. 1. As $\rho \to 0$, one has $v \to 0$ and $u \to u(0)$. For every $u(0)$ between 0 and a critical value $u_c = 1.7419$, there is a solution of (21). The solution can be obtained by direct integration from $\rho = 0$ to $\rho = \xi_0$. At $\rho = \xi_0$, one has $u(\xi_0) = v(\xi_0)$ and therefore $\psi^\dagger\beta\psi = 0$. The radius of the hadron is given by

$$R = \xi_0/\epsilon. \qquad (23)$$

A convenient parameter to label these solutions can be either $u(0)$ or the integral

$$n \equiv \int (u^2 + v^2) d^3\rho \qquad (24)$$

over the region $\rho \leq \xi_0$. As the initial value $u(0) \to 0$, one has $n \to 0$, but as $u(0) \to u_c = 1.7419$, $n \to \infty$.

The physical description of the soliton solution then resembles that of a gas bubble (i.e., the hadron) inside a medium (i.e., the vacuum). The hadron mass is determined by three parameters:

$$p, s, \text{ and } n \qquad (25)$$

where p is defined by (18) which presents the pressure of the medium on the bubble, s is the surface tension which arises because σ changes from 0 to σ_{vac} across the soliton surface, and n determines the gas pressure inside the bubble which is due to both the kinetic energy of quarks and the excitation energy of σ. In either of the limits $n \to 0$ or $n \to \infty$ the hadron mass has the form, in the notation of (16),

$$M_0 = \frac{N\xi_0}{R} + \tfrac{1}{3}4\pi R^3 p + 4\pi R^2 s, \qquad (26)$$

where

$$\xi_0 = 2.0428, \text{ when } n \to 0$$

and (27)

$$\xi_0 = 1, \text{ when } n \to \infty.$$

The double limit $n \to 0$ and $s \to 0$ gives the MIT bag,[2,3] and the double limit $n \to \infty$ and $p \to 0$ gives the SLAC bag.[4]

V. FIRST ORDER

To evaluate the first-order correction in α, we include only the diagram (1) in Fig. 1 for mesons, and the corresponding one in Fig. 2 for baryons.[10] The propagator D of V_μ^a depends on the boundary condition (14a). For example, for $L \gg R$, where L and R are, respectively, the hadron radii determined by (14a) and (14b), one has

$$D(k) \to \frac{-i}{k^2}, \tag{28}$$

which is the free propagator.

For low-lying hadron states, the quarks are all in the $s_{1/2}$ state. In the approximation of zero quark mass ($m=0$), their first-order mass correction M_1 can always be written in the form

$$M_1 = -N\alpha(I_{el} - \mu I_{mag})/R, \tag{29}$$

where I_{el} and I_{mag} are functions of the ratio R/L, but otherwise independent of the spin-parity of the hadron, and μ varies with hadrons, e.g.,

$$\begin{aligned} \mu = -3&,\ \text{for } \pi \\ -1&,\ \text{for } N \\ +1&,\ \text{for } \rho \text{ and } \Delta. \end{aligned} \tag{30}$$

To see the N dependence, we denote the eight Gell-Mann 3×3 SU_3 matrices of the ith quark by λ_i^a. For a color-singlet state we have

$$\frac{1}{2}\sum_{i\neq j}\sum_{a=1}^{8}\langle\lambda_i^a\lambda_j^a\rangle = -\frac{1}{2}\sum_{i=1}^{N}\sum_{a=1}^{8}\langle(\lambda_i^a)^2\rangle = -\tfrac{8}{3}N. \tag{31}$$

The spin dependence can be derived by a similar simple calculation of the average of $\sum_{i\neq j}\vec{\sigma}_i\cdot\vec{\sigma}_j$, which gives (30). That M_1 should be proportional to R^{-1} follows from pure dimensional considerations, provided $m=0$, which is a good approximation only for hadrons made of nonstrange quarks.

In this section, we shall concentrate on π, ρ, N, and Δ. (Except for η and η', because of the success of the Gell-Mann–Okubo mass formula,[11] the mass splitting between other SU_3 partners of π, ρ, N, and Δ can be fitted with a nonvanishing strange quark mass m_s. The details will be given in a separate publication. The η-η' anomaly will be discussed in Sec. VIII.)

For simplicity, we shall set the surface tension $s=0$. Then, in either the MIT bag limit ($n\to0$) or the SLAC-type bag limit ($n\to\infty$), the nadron masses for π, ρ, N, and Δ are given by

$$M = \frac{N\xi}{R} + \tfrac{1}{3}4\pi R^3 p + O(\alpha^2), \tag{32}$$

where

$$\xi = \xi_0 - \alpha(I_{el} - \mu I_{mag}) \tag{33}$$

and ξ_0 is given by (27). The evaluation of I_{el} and I_{mag} depends on the ratio R/L and the quark wave functions. For $L\gg R$, and using the solution (22) that is valid for the MIT bag limit ($n\to0$), we find

$$I_{el} = 3.409 \text{ and } I_{mag} = 0.363. \tag{34}$$

Our calculation may be compared to one already made for the MIT bag, in which different assumptions are made. In the MIT bag calculation,[12] for zero-mass quarks there is no "electric" energy, and instead, there is a "zero-point" energy.[13] For clarity of comparison, our $\alpha=\alpha_c$ used in the MIT bag calculation.[12]

In either the limit $n\to0$ or $n\to\infty$, the constant ξ_0 in (33) is known. Also, at any given ratio R/L, I_{el}, and I_{mag} can be calculated. The hadron mass M, given by (32), then depends on two parameters p and α. Hence, to first order in α, one can derive two *mass formulas* among the four masses M_π, M_ρ, M_N, and M_Δ of π, ρ, N, and Δ,

$$\left(\frac{3}{2}\right)^{3/4}\frac{M_\rho}{M_\Delta} = 1 \tag{35a}$$

and

$$\frac{3(M_\rho^{4/3} - M_\pi^{4/3})}{4(M_\Delta^{4/3} - M_N^{4/3})} = 1. \tag{35b}$$

The experimental values of these two ratios are 0.847 for (35a) and 1.187 for (35b). Considering

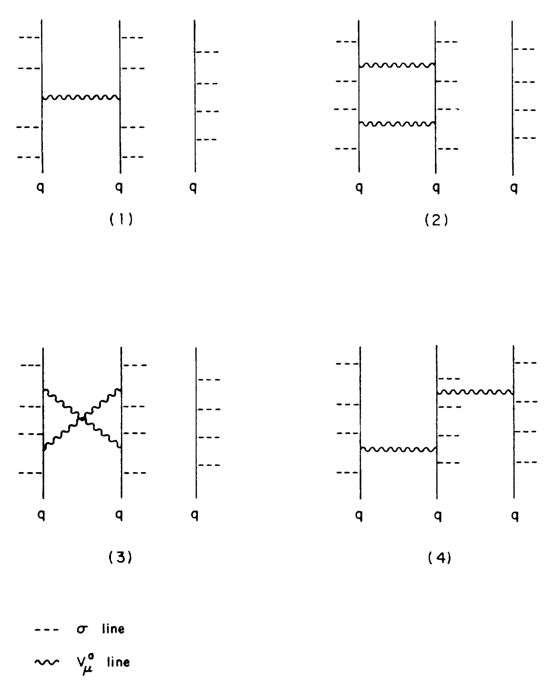

FIG. 2. Examples of QCD correction diagrams to baryon energy.

that correction terms of the order of $\alpha^2 \approx 20\%$ have not yet been included, the agreement can be considered to be a good one. We note that because of the specific form (32) for the hadron mass, these two mass formulas (35a) and (35b) are valid independently of the specific numerical values of ξ_0, I_{el}, and I_{mag}.

VI. SECOND ORDER

To second order in α, there are many diagrams. Besides the ones given in Figs. 1 and 2, there are also other diagrams owing to the renormalization of the V_μ^a propagator and that of the V_μ^a-quark vertex. As in the preceding section, we also concentrate first on the masses of π, ρ, N, and Δ. As we shall see, by inclusion of $O(\alpha^2)$ corrections, (32) becomes

$$M = N\frac{\xi}{R} + \tfrac{1}{2}N(N-1)\frac{\eta}{R} + \tfrac{1}{3}4\pi p R^3, \tag{36}$$

where, just as in (33)

$$\xi = \xi_0 - \alpha(I_{el} - \mu I_{mag}) + O(\alpha^2), \tag{37}$$

$$\eta = \eta_{el} - \mu\eta_{mag} = O(\alpha^2), \tag{38}$$

and ξ_0 and μ are, respectively, given by (27) and (30).

To show this, let us examine first the N dependence. Because we are interested only in two specific N values: $N=2$ for mesons and $N=3$ for baryons, any N-dependent function can be expressed in the form (36). Actually, in diagrams (2) and (3) of either Fig. 1 and Fig. 2, the two vector mesons in the t channel can be in either an SU_3 octet or an SU_3 singlet; in the former, the resulting amplitude is linear in N, and in the latter it is proportional to $\frac{1}{2}N(N-1)$. Diagrams (4) and (5) of Fig. 1 do not contribute, since as we shall see in Sec. VIII, they are nonzero only for η and η'. Diagram (4) of Fig. 2 exists only for the baryons; therefore, for $N=2$ and 3, it is proportional to $N-2=\frac{1}{3}[N(N-1)-N]$.

Next, we discuss the spin dependence of (36)–(38). In diagrams (2) and (3) of either Fig. 1 or Fig. 2, the interaction is between only a single quark pair; hence, the spin dependence must be of the form

$$\frac{1}{2}\sum_{i\neq j}\vec{\sigma}_i\cdot\vec{\sigma}_j, \tag{39}$$

where $\vec{\sigma}_i$ is the Pauli spin matrix of the ith quark (where the "spin" refers to the total angular momentum of its $s_{1/2}$ wave function). In diagram (4) of Fig. 2, in principle there is yet another spin-dependent invariant: $\epsilon_{ijk}\vec{\sigma}_i\cdot(\vec{\sigma}_j\times\vec{\sigma}_k)$. However, because of time-reversal invariance, its diagonal matrix element must be zero. Thus, the spin-dependent part of these mass-correction diagrams must be proportional to (39), and that leads to the μ factor in (37) and (38).

The evaluation of these diagrams is complicated, because in the intermediate states both the quarks and the σ field are excited. Since the σ field itself represents a collective mode of multiple vector-meson states, there is always the question whether some of the second (and also higher) order effects may have already been included in the σ-field degree of freedom. Short of making the actual calculation, we may ask the following: If we require an exact fit of the masses of π, ρ, N, and Δ, is it true that all we need are the relatively small $O(\alpha^2)$ corrections? This is certainly a very weak test; however, in view of the large difference in these masses, from $\pi(140)$ to $\Delta(1232)$, it may not be a trivial criterion. For definiteness, we assume $\xi_0 = 2.0428$ (i.e., $n\to 0$), $L \gg R$ and therefore (34) holds. In addition, we neglect the $O(\alpha^2)$ correction in ξ. Thus, there are four parameters in (36): p, α, η_{el}, and η_{mag}. An *exact* fit of $\pi(140)$, $\rho(770)$, $N(940)$, and $\Delta(1232)$ gives

$$\alpha = 0.498,$$

$$\eta_{el}/I_{el} = 0.116 \sim \tfrac{1}{2}\alpha^2, \tag{40}$$

and

$$\eta_{mag}/I_{mag} = 0.136 \sim \tfrac{1}{2}\alpha^2,$$

which shows at least a certain degree of self-consistency of our expansion.

VII. NEAR-ZERO PION MASS

From (32) and (33) [or (36)–(38)], one sees that there exists a critical value of α, called α_c. For $\alpha<\alpha_c$, $M(R)$ is always >0. For $\alpha>\alpha_c$, $M(R)$ becomes negative at small R, and it $\to -\infty$ as $R\to 0$. Among the low-lying hadrons, the pion is the lowest energy state. [Both the "electric" and "magnetic" energies are attractive in the pion state; in addition, according to (30), $\mu=-3$.] Therefore, by chossing α close to α_c, there is no difficulty in obtaining a small pion mass. By using (33), (34), and $\xi_0 = 2.0428$, one sees that the critical value α_c is $\sim\frac{1}{2}$, which is very near the α value given by (40).

It may be of interest to speculate why the actual α should be near α_c. In our model, we must either limit ourselves to $M(R)\geq 0$, or instead of M, consider the hadron energy E at a given momentum $\vec{p}$, since $E=(\vec{p}^{\,2}+M^2)^{1/2}$ which is always positive, even if M may be negative. For convenience, let us take the former view. Furthermore, we may incorporate the "asymptotic-freedom" property[14] of QCD by regarding $\alpha=\alpha(R)$ with $\alpha(0)=0$. Now if we vary R, the minimum of the energy E of the lowest hadron state, which is the pion, occurs when $M=0$ and therefore $\alpha(R)=\alpha_c$. The actual nonzero, but small, pion mass would then be attributed to either the quark mass m inside the hadron being nonzero, or some quantum fluctuation effects not included in this simple picture.

VIII. ω-ϕ MIXING AND η-η' ANOMALY

It is well known that the usual SU_3 ω-ϕ mixing can be readily understood in any quark model, while the η-η' system is an anomaly.[12,15] Among the 1^- mesons, ρ^0 is $2^{-1/2}(\bar{u}u-\bar{d}d)$, ω^0 is $2^{-1/2}(\bar{u}u+\bar{d}d)$, and ϕ^0 is $\bar{s}s$. Let m_s, m_u, and m_d be, respectively, the masses of s, u, and d quarks (inside the hadrons). Since $m_u=m_d$ if we neglect electromagnetic corrections, but m_s is much larger, one has the approximate degeneracy between $\omega(783)$ and $\rho(770)$, but $\phi(1020)$ is of a much higher mass. The usual Gell-Mann–Okubo ω-ϕ mixing angle ~35° is simply the rotation[16] which transforms the SU_3 flavor singlet

$$3^{-1/2}(\bar{u}u+\bar{d}d+\bar{s}s) \tag{41a}$$

and the SU_3 flavor octet

$$6^{-1/2}(\bar{u}u+\bar{d}d-2\bar{s}s) \tag{41b}$$

to the bases

$$2^{-1/2}(\bar{u}u+\bar{d}d) \text{ and } \bar{s}s. \tag{42}$$

On the other hand, a similar identification of the 0^- mesons would lead to the ridiculous result that either $\eta'(958)$ or $\eta(549)$ should be approximately degenerate with the pion. It has been pointed out[12,15] that the difference between the vector and the pseudoscalar mesons lies in the annihilation diagrams, given by (4) and (5) of Fig. 1. These diagrams exist only for the 0^- mesons, but are absent for the 1^- mesons. However, the problem remains, since these annihilation diagrams are $O(\alpha^2)$, and it is strange that their amplitude should be so large, resulting especially in the rather heavy mass of η'.

We would like to resolve this anomaly in amplitude by observing that there is a large factor due to internal symmetry, which is present in the annihilation diagrams, but not in other $O(\alpha^2)$ diagrams. To see this, let us first decompose the mass matrix of the η-η' system into the sum of two terms

$$\mathfrak{M}_a+\mathfrak{M}_s, \tag{43}$$

where $\mathfrak{M}_a$ is due to the annihilation diagrams [i.e., (4) and (5) in Fig. 1] and $\mathfrak{M}_s$ is due to the difference in masses between the s and the u,d quarks. $\mathfrak{M}_a$ is diagonal if the base vectors are (41a) and (41b), while $\mathfrak{M}_s$ is diagonal if the base vectors are given by (42). The physical η and η' mesons are the eigenstates of the sum (43).

To simplify the discussions, we may, in the evaluation of $\mathfrak{M}_a$, neglect the mass difference between the s and the u,d quarks. Since the gauge field is flavorless, the $q\bar{q}$ state in these annihilation diagrams must be a flavor singlet; in addition, it is also a color singlet and a spin singlet. Thus, in terms of the SU_3-flavor and SU_3-color multiplets, the only $\bar{q}q$ state that has a nonzero annihilation amplitude is

$$|\bar{q}q\rangle=(18)^{-1/2}\sum_{c=1}^{3}[\bar{u}_\uparrow(c)u_\downarrow(c)+\bar{d}_\uparrow(c)d_\downarrow(c)+\bar{s}_\uparrow(c)s_\downarrow(c) \\ -\bar{u}_\downarrow(c)u_\uparrow(c)-\bar{d}_\downarrow(c)d_\uparrow(c)-\bar{s}_\downarrow(c)s_\uparrow(c)], \tag{44}$$

where c is the color index, $u_\uparrow(c)$ then denotes a u quark of color c in an $s_{1/2}$ state with its z-component angular momentum $=+\frac{1}{2}$, while $u_\downarrow(c)$ denotes that with its z-component angular momentum $=-\frac{1}{2}$, etc. The matrix element of annihilation of this $\bar{q}q$ state into two virtual gauge quanta (of color indices a and b, and space-time indices μ and ν) can be written as

$$\langle\text{vac}|\psi^\dagger\lambda^a\lambda^b M_{\mu\nu}\psi|\bar{q}q\rangle, \tag{45}$$

where $M_{\mu\nu}$ is colorless as well as flavorless. The product $\lambda^a\lambda^b$ consists of a color octet $(if^{abc}+d^{abc})\lambda^c$ and a color singlet $\frac{2}{3}\delta^{ab}$; only the latter contributes since $|\bar{q}q\rangle$ is a color singlet. Thus, (45) is equal to

$$\tfrac{2}{3}\delta^{ab}\langle\text{vac}|\psi^\dagger M_{\mu\nu}\psi|\bar{q}q\rangle. \tag{46}$$

According to (44), the $\bar{q}q$ state is a coherent mixture of 18 states. Now, the operator $\psi^\dagger M_{\mu\nu}\psi$ is a color singlet, a flavor singlet, and even under charge conjugation. Hence, its matrix elements satisfy

$$\begin{aligned}\langle\text{vac}|\psi^\dagger M_{\mu\nu}\psi|\bar{u}_\uparrow(c)u_\downarrow(c)\rangle \\ &=-\langle\text{vac}|\psi^\dagger M_{\mu\nu}\psi|\bar{u}_\downarrow(c)u_\uparrow(c)\rangle \\ &=\langle\text{vac}|\psi^\dagger M_{\mu\nu}\psi|\bar{d}_\uparrow(c)d_\downarrow(c)\rangle=\cdots,\end{aligned} \tag{47}$$

where c can be any given color. Thus, each of the 18 component states in $\bar{q}q$ gives an identical contribution to the annihilation amplitude (45); i.e., (45) can be written as

$$\tfrac{2}{3}(18)^{1/2}\delta^{ab}\langle\text{vac}|\psi^\dagger M_{\mu\nu}\psi|\bar{u}_\uparrow(c)u_\downarrow(c)\rangle. \tag{48}$$

The same coherence applies to the creation part of diagrams (4) and (5) of Fig. 1. Summing over the color indices of the gauge fields gives another factor $\sum_{a,b}\delta^{ab}\delta^{ab}=8$. Putting these factors together, we see that the diagonal matrix element of the mass matrix $\mathfrak{M}_a$ for the $\bar{q}q$ state (44) carries an unusually large coefficient

$$(\tfrac{2}{3})^2\times 18\times 8=64, \tag{49}$$

while all other matrix elements of $\mathfrak{M}_a$ are zero. This large coefficient (49) accounts for the abnormally large amplitude of the second-order annihilation diagrams, and thereby removes the puzzling part of the η-η' anomaly. The details of the diagonalization of (43) will be given in a separate publication.

IX. REMARKS

The above discussion connecting QCD with the soliton (or bag) model of hadrons is purely a phenomenological one. The aim of most efforts in the current literature is to derive quark confinement from QCD directly.[5] In this connection, we may recall the relation between QED and the phenomenon of superconductivity. A similarly direct attack would impel one to proceed by starting from QED, using its short-range Coulomb force to establish the existence of crystals, then obtaining the electron-phonon interactions, extracting from them the Bardeen-Cooper-Schrieffer (BCS) correlation energy, and finally deriving the BCS theory.[17] As yet, no one has succeeded through pure theoretical deduction even in the first step: proving the existence of crystal from QED.

At present, it is far from clear that QCD is the correct theory, and there is still the open question whether QCD has an infinite-volume limit. This uncertainty makes a phenomenological approach perhaps more effective and certainly in closer contact with observations. In any case, the soliton (or bag) model at least serves as an alternative way to understand the spectroscopy and the dynamics of hadrons.

ACKNOWLEDGMENT

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[5]See, e.g., the review article on quantum chromodynamics by W. Marciano and H. Pagels, Phys. Rep. 36C, 139 (1978) for various references on QCD.

[6]For example, in the case of a ferromagnet of volume L^3, let $M(H,L)$ be the magnetization per unit volume under an external magnetic field H. One has $\lim_{L\to\infty}\lim_{H\to 0} M(H,L)=0$ always, while $\lim_{H\to 0}\lim_{L\to\infty} M(H,L)$ is not.

[7]This still leaves open whether the vacuum for an infinite volume is invariant under a global color-gauge transformation or not. Presumably, in a Gell-Mann–Zweig quark model [M. Gell-Mann, Phys. Lett. 8, 214 (1964)], one would like to have the global color symmetry intact, while perhaps not in the model of M. Y. Han and Y. Nambu [Phys. Rev. 139, B1006 (1965)], since the color symmetry is violated by the electromagnetic interaction anyway.

[8]For QED, the corresponding $\mathcal{L}_{\text{eff}}=\frac{1}{2}(\vec{E}^2-\vec{H}^2)+(360\pi^2)^{-1}(e/m_e)^4[(\vec{E}^2-\vec{H}^2)^2+7(\vec{E}\cdot\vec{H})^2]+\cdots$, where $\vec{E}$ and $\vec{H}$ are the usual electric and magnetic fields. [See H. Euler, Ann. Phys. (Leipz.) 26, 398 (1936).]

[9]In the soliton model, there may be more than one phenomenological scalar field, in which case we may regard σ as a column matrix with these different scalar fields as its components. The quark mass is assumed to be a linear functional of σ. In QCD the quark field is coupled to V_μ^a and therefore, in principle, to any scalar color-singlet function $\phi(V_{\mu\nu}^a)$. A change in $\langle\text{vac}|: \phi(V_{\mu\nu}^a): |\text{vac}\rangle$ implies then a change in the quark mass, which in the soliton model corresponds to a change in σ. If we adopt the quark confinement mechanism discussed in Sec. II, then both κ and m_V in Eq. (4) should also be represented by phenomenological scalar fields, which can be incorporated into σ as its additional components. A complete discussion will be given in a separate publication.

[10]In addition, there is also the quark self-energy diagram which will lead to an $O(\alpha)$ correction to the value of ξ_0 given in (27). It can be readily shown that the inclusion of this correction term does not alter the general expression given below for the hadron mass. [For example, Eqs. (32) for the first order, (36) for the second order and the two mass formulas (35a) and (35b) all remain unchanged.]

[11]M. Gell-Mann, California Institute of Technology Synchrotron Laboratory Report No. CTSL-20, reprinted in M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (Benjamin, New York, 1964); S. Okubo, Prog. Theor. Phys. 27, 949 (1962).

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[13]We have difficulty in following either the argument for eliminating the "electric" energy, or that of the particular form of a negative zero-point energy used in Ref. 12. If we accept their argument, we may say that the hadron considered in Ref. 12 corresponds to $L=R$, while in our calculation Eq. (34) holds for $L \gg R$. The MIT zero-point energy, given by Eq. (2.9) of Ref. 12, should then be replaced by $-Z_0/L$ which, in the case of $L \gg R$, can be neglected. Likewise, for $L \gg R$, the

vector propagator is given by (28), which is the free propagator; hence, our calculations of I_{el} and I_{mag} also differ from that of Ref. 12. From our point of view, the energy cost of making the "bubble," at any fixed ratio R/L, can best be described by the phenomenological expression $\frac{1}{3}4\pi R^3 p + 4\pi R^2 s$. There should be no large zero-point energy associated with the scalar field. The σ field is a phenomenological field, representing the long-range order of the vacuum. Its loop diagrams, and therefore the corresponding zero-point energy, can be neglected.

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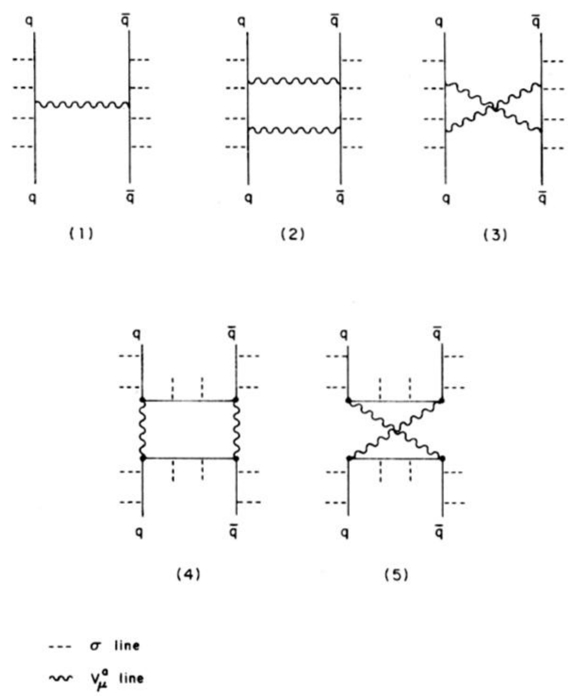

FIG. 1. Examples of QCD correction diagrams to meson energy.

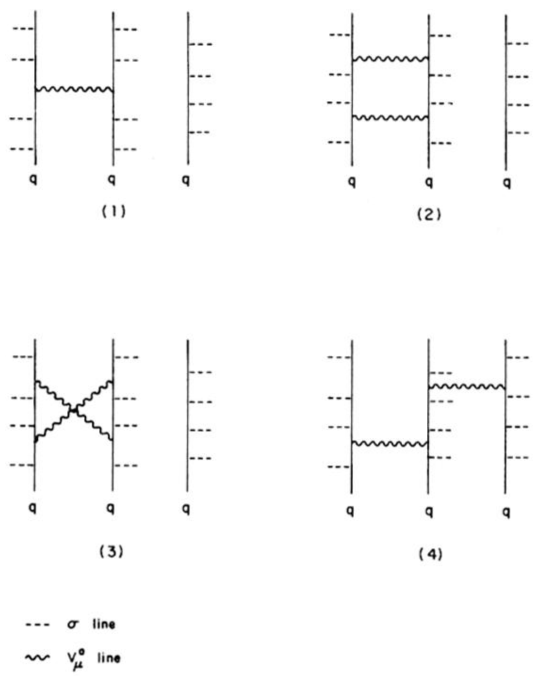

FIG. 2. Examples of QCD correction diagrams to baryon energy.