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Semiclassical approach to the quark-string model and the hadron spectrum

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A semiclassical approach to systems including both bosons and fermions is formulated, and a useful path-integration formula is given for fixed fermion and antifermion number. The quark-string model derived from a continuum limit of the lattice gauge theory is studied based on this method. The massless condition of the string with quarks at the ends is resolved within the context of the semiclassical approximation. The analysis shows a universality of the spring constant of the string and a good prediction for the η_c mass in the new-particle spectroscopy. Quantitative predictions due to quantum corrections (daughter trajectories) are not in good agreement within our approximation.

I. INTRODUCTION

In this and a forthcoming paper we study the hadron spectra, old and new mesons as well as baryons, from a unified point of view. The theory on which this work is based is that of the quark-string model[1] which has been derived from a continuum limit of the lattice gauge theory.[2] The model is a well defined model and furnishes the hadron with three basic properties: (i) quark confinement, (ii) asymptotic scaling, and (iii) relativistic invariance.

In a previous paper,[1] hereafter referred to as I, starting off with the strong-coupling approach of Wilson's lattice gauge theory, we have shown that a Lagrangian governing the motion of the quark string can be derived in a certain continuum limit of the lattice distance, and we have also shown that classical solutions to the quark string exhibit a good qualitative nature. In the second paper,[3] referred to as II, the quark string has been shown to exhibit a simple scaling behavior such that the photoproduction amplitude tends to that of the free-quark theory in the high-momentum-transfer limit. In the third paper,[4] referred to as III, the detailed properties of the classical solutions and the quantum corrections due to the string oscillation, disregarding correlations with the quark spin, have been studied. When applied to the hadron spectra, we have found two interesting features: (i) The large mass difference between η_c and ψ/J in the new-particle spectroscopy can be interpreted; an abnormal classical solution predicts a pseudoscalar particle at $m_{\eta_c} \simeq 2.8$ GeV if the ψ-particle mass is taken to be 3.1 GeV, (ii) The quantum corrections show good agreement with experimental spectra.

Encouraged by these results, we would like to study further the quantum corrections. In this paper we wish to focus discussion on some basic theoretical problems which have not been fully studied or stated in previous papers. First, the detailed mechanism of the appearance of abnormal solutions will be studied in Sec. II. Second, a full discussion of our semiclassical method will be given. To our knowledge, in the framework of path integrals the semiclassical approach for a system involving the fermion field has not been well formulated in a practical fashion. In Sec. III and Appendix A we give proof of a useful formula of path integrals for the boson-fermion system. Third, in obtaining the quantum corrections the massless condition of the string will be resolved. It will be shown that the longitudinal-oscillation modes of the string can be eliminated even if massive quarks are attached to the string ends (Appendix B). Both the type-1 oscillation, the vibration perpendicular to the rotation plane of the classical solution, and the type-2 oscillation, that parallel to the rotation plane, will be investigated. Fourth, the correlation between the quantum oscillation of the string and the quark spin, which has been previously neglected, will now be taken into account (Secs. III and IV). As a result of this term the numerical fit with experimental data will become slightly worse than before[4] in the low-mass region because the spin-string correlation turns out to be too large (Sec. IV).

The semiclassical method is considered to be a good approximation when the classical energy is large compared to quantum corrections. A problem which is not settled in our paper is that the center-of-mass coordinate of the classical solution is not included as a dynamical variable so that the recoil effect on the classical solution due to quantum oscillations is not taken into account. This may be one of the reasons that the numerical results of quantum corrections in the low-mass region are not in good agreement. It is considered, however, that the interpolation of Regge trajectories from the high-mass region to the low-mass region provides us with gross information on the physics of the quark string, and that the work is a good first approximation to the hadron spectroscopy.

II. THE MODEL AND CLASSICAL SOLUTIONS

In our theory the meson is represented by a piece of string with a quark and an antiquark attached to the ends. Let $X^\mu(\sigma,\tau)$ $(0<\sigma<\pi)$ be a point on the world sheet swept out by the string. Let either the quark or antiquark be attached at $\sigma=\sigma_1\equiv 0$, i.e., $X_1^\mu(\tau)\equiv X^\mu(\sigma_1,\tau)$ and the other at $\sigma=\sigma_2\equiv\pi$, i.e., $X_2^\mu(\tau)\equiv X^\mu(\sigma_2,\tau)$. The Lagrangian which governs the motion of the quark string is then defined[1,5] by

$$L=L_{st}+L_{q_1}+L_{q_2}\,, \tag{2.1}$$

where

$$L_{st}=\int_0^\pi \mathcal{L}_{st}d\sigma=-\gamma\int_0^\pi[(X_\tau\cdot X_\sigma)^2-X_\tau^2X_\sigma^2]^{1/2}d\sigma, \tag{2.2}$$

$$L_{q_i}=\frac{i}{2}[X_{i\tau}^\mu/(X_{i\tau}^2)^{1/2}]\bar\psi_i\gamma_\mu\overleftrightarrow{\partial}_\tau\psi_i-m_i(X_{i\tau}^2)^{1/2}\bar\psi_i\psi_i\,, \tag{2.3}$$

with $X_\tau^\mu=\partial X^\mu(\sigma,\tau)/\partial\tau$ and $X_\sigma^\mu=\partial X^\mu(\sigma,\tau)/\partial\sigma$ and $i=1$ and 2. The quark field $\psi_i(\tau)$ $(i=1,2)$ is assumed to be an anticommuting field. Since the Lagrangian L has the reparametrization invariance $\tau\to\tau'=f(\tau)$, with f being an arbitrary function, no ghost appears in the system. Taking advantage of this invariance we adopt the timelike gauge $X^0(\sigma,\tau)=\tau$ throughout this work.

In this section we first find a set of classical solutions to the Euler equations and quantize them by the Bohr-Sommerfield method to provide the meson spectra, which are associated with leading Regge trajectories. The quantum corrections to the classical trajectories will give us the daughter trajectories, which will be discussed in the next section.

Classical solutions

The Euler equations for this system are given by

$$\frac{\partial}{\partial\tau}\left[\frac{(X_\tau\cdot X_\sigma)X_\sigma^\mu-X_\sigma^2X_\tau^\mu}{(X_\tau\cdot X_\sigma)^2-X_\sigma^2X_\tau^2}\right]+\frac{\partial}{\partial\sigma}\left[\frac{(X_\tau\cdot X_\sigma)X_\tau^\mu-X_\tau^2X_\sigma^\mu}{(X_\tau\cdot X_\sigma)^2-X_\sigma^2X_\tau^2}\right]=0,\ \text{for } 0<\sigma<\pi \tag{2.4}$$

$$-\frac{d}{d\tau}p_i^\mu=(-1)^i\left[\frac{(X_\tau\cdot X_\sigma)X_\tau^\mu-X_\tau^2X_\sigma^\mu}{\{(X_\tau\cdot X_\sigma)^2-X_\tau^2X_\sigma^2\}^{1/2}}\right]_{\sigma=\sigma_i}\quad(i=1,2)\,, \tag{2.5}$$

$$i\not e_i\psi_{i\tau}+[\tfrac{1}{2}i\not e_{i\tau}-m_i(X_{i\tau}^2)^{1/2}]\psi_i=0\quad(i=1,2)\,, \tag{2.6}$$

where p_i^μ denotes the canonical momentum conjugate to X_i^μ and is given by

$$p_i^\mu=-\tfrac{1}{2}i(X_{i\tau}^2)^{-1/2}(g^{\mu\nu}-e_i^\mu e_i^\nu)(\bar\psi_i\gamma_\nu\psi_{i\tau}-\bar\psi_{i\tau}\gamma_\nu\psi_i)-m_ie_i^\mu\bar\psi_i\psi_i \tag{2.7}$$

and $e_i^\mu=X_{i\tau}^\mu/(X_{i\tau}^2)^{1/2}$ and $\not e_i=\gamma_\mu e_i^\mu$. In the above equations, $X^0(\sigma,\tau)=\tau$ is to be implied.

A useful set of solutions to (2.4) is that of rigid rotators.[6,7] Take the center of mass of the rotator at the origin. Assuming the solution to be

$$\vec X(\sigma,\tau)=\vec X_{cl}(\sigma,\tau)=(\rho(\sigma)\cos\omega\tau,\rho(\sigma)\sin\omega\tau,0), \tag{2.8}$$

one finds that (2.4) is satisfied for an arbitrary $\rho(\sigma)$. However, the solution which corresponds to the leading trajectory is known to be represented by

$$\rho(\sigma)=\frac{1}{\omega}\sin c(\sigma-\sigma_0) \tag{2.9}$$

with arbitrary constants c and σ_0. Equation (2.6) is homogeneous with respect to ψ_i and can be solved for ψ_i as a function of yet unknown coordinates $\vec X_i(\tau)$ and τ. Substituting

$$\psi_i(\tau)=\exp[-i\tau(\Omega_i+\tfrac{1}{2}\omega\sigma^3)]u_i \tag{2.10}$$

and $\vec X_i(\tau)=\vec X_{cl}(\sigma_i,\tau)$ into (2.6), one can determine Ω_i and u_i as follows:

$$u_i=u_{i,A(\epsilon_i,S_i)}=\left[\frac{1+\epsilon_i(1-\omega^2\rho_i^2)^{1/2}}{2(1-\omega^2\rho_i^2)^{1/2}}\right]^{1/2}\begin{bmatrix}\chi_{S_i}\\ \dfrac{2iS_i\omega\rho_i}{1+\epsilon_i(1-\omega^2\rho_i^2)^{1/2}}\chi_{-S_i}\end{bmatrix} \tag{2.11}$$

and

$$\Omega_i=\Omega_{i,A(\epsilon_i,S_i)}=\frac{\epsilon_i}{(1-\omega^2\rho_i^2)^{1/2}}[m_i(1-\omega^2\rho_i^2)-S_i\omega], \tag{2.12}$$

where A labels the state of the quark, $S_i(=\pm\frac{1}{2})$ denotes the third component of the spin quantum number of the ith quark [the definition of spin is given later by (2.33)], and $\epsilon_i(=\pm1)$ signifies the positive or negative frequency of the solution. The flavor index, if needed, is implicitly included in A. χ_{S_i}

is the Pauli spinor with two components and $\rho_i \equiv\rho(\sigma_i)$.

The unknown parameters ρ_i $(i=1,2)$, or equivalently c and σ_0 in (2.9), are determined by (2.5). The substitution of (2.8)–(2.12) into (2.5) provides us with the boundary condition of the string,

$$\frac{m_i\omega^2\rho_i}{(1-\omega^2\rho_i^2)^{1/2}}+\frac{S_i\omega^3\rho_i}{(1-\omega^2\rho_i^2)^{3/2}}=(-1)^i\frac{\gamma\omega}{c}\rho_\sigma(\sigma=\sigma_i) \tag{2.13}$$

or, by using (2.9),

$$\omega(m_i\cos^2\alpha_i+S_i\omega)=(-1)^i\gamma\frac{\cos^4\alpha_i}{\sin\alpha_i}\operatorname{sign}(\cos\alpha_i), \tag{2.14}$$

where $\alpha\equiv c(\sigma-\sigma_0)$ and $\alpha_i=c(\sigma_i-\sigma_0)$. It is this nonlinear boundary condition that characterizes the quark-string model.

On determining the boundary value α_i one notes that the right-hand side of (2.14) is periodic with the period π and odd under $\alpha_i\to-\alpha_i$. If the solution $\alpha(m,S,\omega)$ to

$$[m\cos^2\alpha(m,S,\omega)+S\omega]\omega=\gamma\frac{\cos^4\alpha(m,S,\omega)}{\sin\alpha(m,S,\omega)}\times\operatorname{sign}(\cos\alpha(m,S,\omega)) \tag{2.15}$$

is found for given m, S, and ω in the principal sector $0\leqslant\alpha\leqslant\pi$, then α_i are given by $\alpha_1=-\alpha(m_1,S_1,\omega)-l_1\pi$ and $\alpha_2=\alpha(m_2,S_2,\omega)+l_2\pi$ with l_1, $l_2=0,1,2,\ldots$. No difference appears in (2.15) for the quark or antiquark. Although the numerical values for α_i have to be obtained by calculator the gross behavior is able to be understood by inspecting Fig. 1, in which the right- and left-hand sides of (2.15) are separately shown. For a given classical frequency ω and the quark mass m there exists a single solution α_+ if $S=+\frac{1}{2}$,

$$\alpha(m,+\tfrac{1}{2},\omega)=\alpha_+,\quad(0\leqslant\alpha_+\leqslant\pi/2). \tag{2.16}$$

On the other hand, if $S=-\frac{1}{2}$ there are three possible solutions; one is

$$\alpha(m,-\tfrac{1}{2},\omega)=\alpha_-^{(1)},\quad(\pi/2\leqslant\alpha_-^{(1)}\leqslant\pi), \tag{2.17}$$

which exists for all positive values of m and ω, and the others are

$$\alpha(m,-\tfrac{1}{2},\omega)=\begin{cases}\alpha_-^{(2)}\\ \alpha_-^{(3)}\end{cases}(0\leqslant\alpha_-^{(2)}\leqslant\alpha_-^{(3)}<\pi/2), \tag{2.18}$$

which exist only if $m^2\geqslant 2\gamma$. (As will be shown later, all Regge slopes asymptotically tend to $\alpha'_\infty=1/2\pi\gamma$ in our theory. This condition, therefore, is equivalent to $m^2\alpha'_\infty\geqslant 1/\pi$, which was given in

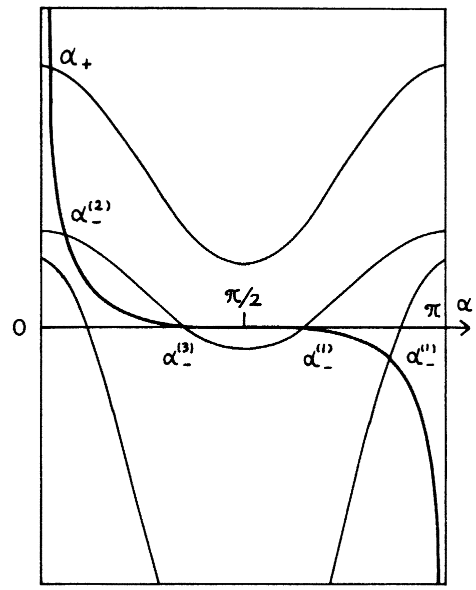

FIG. 1. The right-hand side of (2.15) is shown by the bold line, whose cross points with thin lines [the left-hand side of (2.15)] give the solutions $\alpha\pm^{(j)}$. The top and the second thin lines are, respectively, for $S=+\frac{1}{2}$ and $S=-\frac{1}{2}$. The bottom line shows a case with large ω for $S=-\frac{1}{2}$. The unit of vertical coordinate is arbitrary.

III.) Solutions α_+ and $\alpha_-^{(2)}$ have been called normal solutions, while $\alpha_-^{(1)}$ and $\alpha_-^{(3)}$ have been called abnormal solutions when $m^2\geqslant 2\gamma$ in III. It will be convenient for later discussion to know some asymptotic behaviors of the solutions.

(a) As $\omega\to\infty$ $(\omega^2\gg\gamma,\ \omega\gg m)$, the α's behave as

$$\alpha_+\sim 2\gamma/\omega^2\text{ and }\alpha_-^{(1)}\sim\pi-2\gamma/\omega^2. \tag{2.19}$$

Solutions $\alpha_-^{(2)}$ and $\alpha_-^{(3)}$ do not exist for $\omega\to\infty$.

(b) As $\omega\to 0$ $(\omega^2\ll\gamma,\ \omega\ll m)$, the α's behave as

$$\alpha_+\sim\frac{\pi}{2}-\left\{\frac{1}{2\gamma}[(m^2+2\gamma)^{1/2}+m)]\right\}^{1/2}\sqrt{\omega}, \tag{2.20}$$

$$\alpha_-^{(1)}\sim\frac{\pi}{2}+\left\{\frac{1}{2\gamma}[(m^2+2\gamma)^{1/2}-m]\right\}^{1/2}\sqrt{\omega}, \tag{2.21}$$

$$\alpha_-^{(2)}\sim\frac{\pi}{2}-\left\{\frac{1}{2\gamma}[m+(m^2-2\gamma)^{1/2}]\right\}^{1/2}\sqrt{\omega}, \tag{2.22}$$

$$\alpha_-^{(3)}\sim\frac{\pi}{2}-\left\{\frac{1}{2\gamma}[m-(m^2-2\gamma)^{1/2}]\right\}^{1/2}\sqrt{\omega}. \tag{2.23}$$

The last two, $\alpha_-^{(2)}$ and $\alpha_-^{(3)}$, are meaningful only when $m^2\geqslant 2\gamma$.

At this point we discuss why the $\alpha_-^{(1)}$ state of the quark is bound at $\pi/2\leqslant\sigma\leqslant\pi$. As one can easily confirm by substituting solutions into (2.7), the direction of the spatial momentum of the quark $\vec{p}$ is antiparallel to the velocity $\vec{X}_\tau$ due to a strong correlation between the spin and the acceleration $\vec{X}_{\tau\tau}$. (This is reminiscent of classical electrodynamics where $\vec{p}$ is different from velocity due to $\vec{p}=\vec{x}_\tau - e\vec{A}$.) The centrifugal force, in this case, works *inward*, and this balances with the outward centrifugal force of this string. Although this is an unfamiliar case, we do not find any reason to exclude the solution $\alpha_-^{(1)}$.

Now that all necessary solutions to the boundary condition are obtained, one is able to construct meson states by combining the solutions, one for the left end ($i=1$) and one for the right end ($i=2$) of the string. Let us denote the state by

$$M(\alpha(m_1,S_1,\omega),\alpha(m_2,S_2,\omega)),$$

which means $\alpha_1=-\alpha(m_1,S_1,\omega)$ and $\alpha_2=\alpha(m_2,S_2,\omega)$. [The integer l_i introduced below (2.15) is taken to be zero because $l_i\neq 0$ provides a nonleading trajectory.]

(i) $M(\alpha_+,\alpha_+)$. The quark spins form a triplet and correspond to the state whose total angular momentum J equals the orbital angular momentum L plus one in the nonrelativistic limit. The ρ meson, the ψ meson, etc. are associated with this.

(ii) $M(\alpha_+,\alpha_-^{(k)})$ with $k=1,2$, or 3. The quark spins form a singlet or a triplet, hence the states are degenerate for each k. In the nonrelativistic limit, the states correspond to those with $J=L$. The pion, the A_1 meson, etc. are associated with this solution.

(iii) $M(\alpha_-^{(j)},\alpha_-^{(k)})$ with j, $k=1$, 2, or 3. If $\alpha_-^{(j)}\neq\alpha_-^{(k)}$, the states are degenerate with respect to the charge conjugation. The quark spins form a triplet which is antiparallel to L. The ϵ particle, etc. are associated with this class. We note that in this case there is a special solution of $M(\alpha_-^{(1)},\alpha_-^{(1)})$ with no fold $[c(\sigma_2-\sigma_1)<\pi]$. It can be shown that this state always has negative angular momentum J_3 although the orbital angular momentum of the string part has positive component. We think this solution unstable and disregard this in the following.

Leading Regge trajectories

In order to get meson spectra, we calculate the classical energy (mass) E_{cl}, and the angular momentum (spin) J_{cl} of each solution obtained above. From E_{cl} and J_{cl}, which are functions of ω, we eliminate ω to obtain a relationship between E_{cl} and J_{cl} and impose the Bohr-Sommerfeld condition that J_{cl} should be an integer.

General expressions for the energy-momentum P^μ and the angular momentum $M^{\mu\nu}$ are derived from the Lagrangian (2.1) by the Noether theorem,

$$P^\mu = P^\mu_{\text{st}} + \sum_{i=1}^{2} p_i^\mu, \qquad (2.24)$$

$$M^{\mu\nu} = M^{\mu\nu}_{\text{st}} + \sum_{i=1}^{2} M^{\mu\nu}_{q,i}, \qquad (2.25)$$

where

$$P^\mu_{\text{st}} = -\gamma\int_0^\pi \frac{(X_\tau\cdot X_\sigma)X_\sigma^\mu - X_\sigma^2 X_\tau^\mu}{[(X_\tau\cdot X_\sigma)^2 - X_\sigma^2\cdot X_\tau^2]^{1/2}}\,d\sigma, \qquad (2.26)$$

$$M^{\mu\nu}_{\text{st}} = \gamma\int_0^\pi \frac{(X^\mu X_\sigma^\nu - X^\nu X_\sigma^\mu)(X_\tau\cdot X_\sigma) - (X^\mu X_\tau^\nu - X^\nu X_\tau^\mu)X_\sigma^2}{[(X_\tau\cdot X_\sigma)^2 - X_\sigma^2 X_\tau^2]^{1/2}}\,d\sigma, \qquad (2.27)$$

$$M^{\mu\nu}_{q,i} = -(X_i^\mu p_i^\nu - X_i^\nu p_i^\mu) + \Sigma_i^{\mu\nu}, \qquad (2.28)$$

$$\Sigma_i^{\mu\nu} = \tfrac{1}{4}\bar\psi_i\{\not e_i, \sigma^{\mu\nu}\}\psi_i. \qquad (2.29)$$

The quark momentum p_i^μ ($i=1,2$) is given by (2.7). The last quantity (2.29) is the spin angular momentum[5] of the ith quark. It will be instructive to give some of the explicit forms of the above quantities. The energy and the angular momentum for the quark are simplified by the use of (2.6) and are given by

$$E_{q,i} = p_i^0 = \frac{m_i}{(1-\vec{X}_{i\tau}^2)^{1/2}}\bar\psi_i\psi_i - \frac{\vec\Sigma_i\cdot(\vec{X}_{i\tau\tau}\times\vec{X}_{i\tau})}{1-\vec{X}_{i\tau}^2}, \qquad (2.30)$$

$$\vec{J}_{q,i} = \vec{M}_{q,i} = \vec{L}_{q,i} + \vec\Sigma_i, \qquad (2.31)$$

$$\vec{L}_{q,i} = (\vec{X}_i\times\vec{p}_i) = \frac{m_i\bar\psi_i\psi_i}{(1-\vec{X}_{i\tau}^2)^{1/2}}(\vec{X}_i\times\vec{X}_{i\tau}) - \frac{(\vec{X}_i\cdot\vec{X}_{i\tau\tau})\vec\Sigma_i - (\vec{X}_i\cdot\vec\Sigma_i)\vec{X}_{i\tau\tau}}{1-\vec{X}_{i\tau}^2}, \qquad (2.32)$$

$$\vec\Sigma_i = \frac{1}{4}\,\frac{1}{(1-\vec{X}_{i\tau}^2)^{1/2}}\,\bar\psi_i\{\gamma^0 - \vec\gamma\cdot\vec{X}_{i\tau}), \vec\sigma\}\psi_i. \qquad (2.33)$$

Note that the third component of the spin (2.33) has the eigenvalue $S_i(1-\vec{X}_{i\tau}^2)^{-1/2}$ with $S_i=\pm\frac{1}{2}$, whose absolute value is always larger than $\frac{1}{2}$ due to the

Lorentz factor $(1-\bar{X}_{i\tau}^2)^{-1/2}$. This is because the spin (2.33) is not defined in the rest system of the quark. Also note that the second term in (2.30) is essentially an L-S coupling energy because $\bar{X}_{\tau\tau} = -\omega^2\bar{X}$ for the classical solution (2.8), and this makes a negative contribution to $E_{q,i}$ if the spin is antiparallel to the orbital angular momentum.

Substituting the explicit form of classical solutions obtained above, one can write the classical values in terms of the boundary quantities α_i,

$$E_{st} = \gamma\int_0^\pi \left(\frac{\rho_\sigma^2}{1-\omega^2\rho^2}\right)^{1/2} d\sigma = \frac{\gamma}{\omega}(\alpha_2-\alpha_1), \tag{2.34}$$

$$J_{st}^{(3)} = \gamma\int_0^\pi \left(\frac{\rho_\sigma^2}{1-\omega^2\rho^2}\right)^{1/2} \omega\rho^2 d\sigma = \frac{\gamma}{2\omega^2}[(\alpha_2-\tfrac{1}{2}\sin 2\alpha_2)-(\alpha_1-\tfrac{1}{2}\sin 2\alpha_1)], \tag{2.35}$$

$$E_{q,i} = \frac{m_i}{(1-\omega^2\rho_i^2)^{1/2}} + \frac{S_i\omega^3\rho_i^2}{(1-\omega^2\rho_i^2)^{3/2}} = \frac{1}{|\cos\alpha_i|}(m_i + S_i\omega\tan^2\alpha_i), \tag{2.36}$$

$$J_{q,i}^{(3)} = \frac{m_i\omega\rho_i^2}{(1-\omega^2\rho_i^2)^{1/2}} + \frac{S_i\omega^2\rho_i^2}{(1-\omega^2\rho_i^2)^{3/2}} + \frac{S_i}{(1-\omega^2\rho_i^2)^{1/2}} = |\cos\alpha_i|^{-1}\left[\frac{m_i}{\omega}\sin^2\alpha_i + S_i(\tan^2\alpha_i+1)\right]. \tag{2.37}$$

Other spatial components of $\vec{J}_{st}$ and $\vec{J}_{qi}$ are vanishing.

The Regge trajectory is now obtained by eliminating ω from

$$E_{cl} = E_{st}(\omega) + \sum_{i=1}^{2} E_{q,i}(\omega) \tag{2.38}$$

and

$$J_{cl}^{(3)} = J_{st}^{(3)}(\omega) + \sum_{i=1}^{2} J_{q,i}^{(3)}(\omega). \tag{2.39}$$

It is important to keep in mind that the low- (high-) mass behavior is determined by the large (small) value of ω as is the case in usual string models. The asymptotic behaviors (2.19)–(2.23) enable us to confirm the following behaviors.

(a) As $\omega\to 0$, i.e., in the high-angular-momentum region, $E_{cl}\sim\pi\gamma/\omega$ and $J_{cl}\sim\pi\gamma/2\omega^2$, hence $J_{cl}\sim(1/2\pi\gamma)E_{cl}$. All trajectories therefore become parallel with the universal slope $\alpha'_\infty = 1/2\pi\gamma$.

(b) For the case of the low-angular-momentum limit ($\omega\to\infty$), the trajectories behave as follows.

(1) The trajectory $M(\alpha_+,\alpha_+)$, which is associated with ρ,ψ families, etc., behaves as

$$J(\alpha_+,\alpha_+) \sim 1 + \tfrac{3}{8}\gamma^{-2/3}(E-m_1-m_2)^{4/3}. \tag{2.40}$$

The ground-state mass with spin 1 equals the sum of constituent-quark masses (quark-mass additivity). This property will be used for determining the quark mass parameters in Sec. IV.

(2) The trajectory $M(\alpha_+,\alpha_-^{(1)})$ for $m_i^2\alpha'_\infty < 1/\pi$, which is associated with π, A_1 families, etc., behaves as

$$J(\alpha_+,\alpha_-^{(1)}) \sim \frac{1}{2\pi\gamma}(E-m_1-m_2)^2. \tag{2.41}$$

The ground-state mass with spin 0 equals m_1 plus m_2.

(3) The trajectory $M(\alpha_-^{(1)},\alpha_-^{(1)})$ for $m_i^2\alpha'_\infty < 1/\pi$, which is associated with the ϵ family, behaves as

$$J(\alpha_-^{(1)},\alpha_-^{(1)}) \sim -1 + \frac{1}{4\pi\gamma}(E-m_1-m_2)^2. \tag{2.42}$$

(4) The other trajectories for $m_i^2\alpha'_\infty > 1/\pi$ show complicated behavior in the low-angular-momentum region (Fig. 2). Since the charmed-quark mass is supposed to be above the critical mass $(1/\pi\alpha'_\infty)^{1/2} \simeq 0.56$ GeV the new-particle family meets the case. As will be discussed in Sec. IV the trajectory $M(\alpha_+,\alpha_-^{(3)})$ is associated with the η_c family.

III. QUANTUM CORRECTIONS

In this section we investigate quantum corrections to the classical solutions studied in the preceding section. The method we use is an extended version of the stability angle method which has been developed in nonlinear field theories by Dashen, Hasslacher, and Neveu,[8] and later applied to a string model by Kikkawa, Sato, and Uehara.[7] In the system including the fermion field the concept of the semiclassical approximation is not well known. Although the principal idea of our approach is equivalent to the one exploited in another

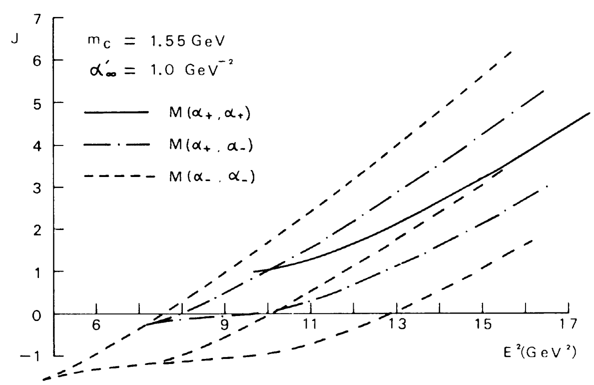

FIG. 2. Classical trajectories. Some almost degenerate trajectories along $M(\alpha_+, \alpha_-)$ and $M(\alpha_-, \alpha_-)$ are not shown.

paper by Dashen, Hasslacher, and Neveu[9] (DHN) in a study of the Gross-Neveu model, the method must be improved in a point. Since the string field $X^\mu(\sigma,\tau)$, which corresponds to the collective field σ in DHN, obeys a complicated nonlinear equation, it is not easy to find classical solutions other than those discussed in Sec. II. The problem is, therefore, solved by the following two steps.

In our approach we perform the fermion path integration as was done by DHN and obtain an effective action for the string that contains no fermion variables. In the first step of our approximation the classical solutions to the effective action are looked for by imposing the stationary condition on the action. This step is shown to be equivalent to the argument in Sec. II and corresponds to DHN. In the second step we make an expansion of the effective action around the classical solution and keep the terms up to the quadratic ones with respect to variations. This approximated effective action will be solved with the help of some auxiliary variables (see theorem below).

The spectra are obtained by inspecting poles in the propagator

$$D(E)\equiv\mathrm{Tr}\,\frac{1}{E-\hat{H}}=-i\int_0^\infty dT\,e^{-iET}\,\mathrm{Tr}[\exp(-i\hat{H}T)], \tag{3.1}$$

or, if those with a definite third component of $\vec{J}$ are of interest, in the propagator

$$D_{j_3}(E)\equiv\left\langle j_3\left|\frac{1}{E-\hat{H}}\right|j_3\right\rangle =-i\int_0^{2\pi}\frac{d(\Delta\theta)}{2\pi}\int_0^\infty dT\exp[i(ET-j_3\,\Delta\theta)]\times\mathrm{Tr}[\exp(-i\hat{H}T+i\hat{J}_3\Delta\theta)], \tag{3.2}$$

where $\hat{H}$ denotes the operator Hamiltonian of the system. By use of the path integral the trace part in (3.2) can be represented by

$$Q(T,\Delta\theta)\equiv\mathrm{Tr}[\exp(-i\hat{H}T+i\hat{J}_3\Delta\theta)] =\int\mathfrak{D}\vec{X}\prod_{i=1}^{2}\mathfrak{D}\bar{\psi}_i\mathfrak{D}\psi_i\prod_{i=1}^{2}\delta(\psi_i(T)+\exp(-\tfrac{1}{2}i\,\Delta\theta\sigma^3)\psi_i(0))\prod_\sigma\delta(\vec{X}(\sigma,T)-R(\Delta\theta)\vec{X}(\sigma,0)) \times\exp\left[i\int_0^T d\tau\left(L_{st}+\sum_{i=1}^{2}L_{q,i}\right)-iE_0T\right], \tag{3.3}$$

where

$$R(\Delta\theta)=\begin{pmatrix}\cos(\Delta\theta), & -\sin(\Delta\theta), & 0\\ \sin(\Delta\theta), & \cos(\Delta\theta), & 0\\ 0, & 0, & 1\end{pmatrix},$$

E_0 is the zero-point energy, and L_{st} and $L_{q,i}$ are defined by (2.2) and (2.3). $\mathfrak{D}\vec{X}\Pi\mathfrak{D}\bar{\psi}\mathfrak{D}\psi$ implies the functional integration with a certain measure including a gauge condition for $\vec{X}$. In the following we proceed with a discussion of $\Delta\theta=0$.

Before going into the semiclassical approximation we perform the fermion integration and get

$$Q(T,\Delta\theta=0)=\sum_{\{n_{i,A}\}}\int\mathfrak{D}\vec{X}\prod_\sigma\delta(\vec{X}(\sigma,T)-\vec{X}(\sigma,0))\exp\left\{i\int_0^T d\tau\,L_{st}-i\sum_{i,A}n_{i,A}\epsilon_{i,A}\zeta_{i,A}[\vec{X}]-i\left(E_0T-\sum_{\epsilon_{i,A}>0}\zeta_{i,A}[\vec{X}]\right)\right\}. \tag{3.4}$$

Here $\zeta_{i,A}$ is the "stability angle"[8] defined by the phase of $\psi_{i,A}$ such that

$$\psi_{i,A}(T)=\exp(-i\zeta_{i,A})\psi_{i,A}(0), \tag{3.5}$$

where $\psi_{i,A}(\tau)$ obeys the equation of motion

$$\left[i\not{e}_i\frac{d}{d\tau}+\frac{i}{2}\not{e}_{i\tau}-m_i(X_{\pi}{}^2)^{1/2}\right]\psi_{i,A}(\tau)=0 \tag{3.6}$$

with $\not{e}_i$ defined below (2.7). $\epsilon_{i,A}$ is the sign function introduced in (2.12). The integer $n_{i,A}$ ($i=1$ or 2) is the occupation number (0 and 1) of the state $A(\epsilon_i,S_i,f_i)$ with f_i being a flavor index of the ith quark. The last term in the exponent ($E_0T-\sum_{\epsilon_{i,A}>0}\zeta_{iA}$) is the contribution from the Dirac vacuum. The term would have been eliminated if one had begun with the normal-ordered Hamiltonian with respect to the fermion field,[10] hence we disregard this term hereinafter. If a quark with spin S_1 and an antiquark with spin S_2 are attached at $\sigma=\sigma_1$ and $\sigma=\sigma_2$, or vice versa, respectively, what we should consider is

$$Q_{\{S_1,S_2\}}(T,0)=\int \mathfrak{D}\vec{X}\prod_\sigma \delta(\vec{X}(\sigma,T)-\vec{X}(\sigma,0))\,\exp\left(i\int_0^T d\tau L_{st}-i\sum_{i=1}^2\epsilon_{i,A}\zeta_{i,A}[\vec{X}]\right). \tag{3.7}$$

The exponent of (3.7) is the effective action I_{eff} of our string. Note that I_{eff} does not involve the fermion field and is still exact except for the fermion and antifermion numbers being fixed.

As the first step of the semiclassical approximation we look for the stationary value of I_{eff}. In Appendix A the stationary condition

$$\frac{\delta I_{eff}}{\delta\vec{X}}=\int\frac{\delta L_{st}}{\delta\vec{X}}d\tau-\sum_{i=1}^2\frac{\delta\zeta_{i,A}}{\delta\vec{X}}\epsilon_{i,A}=0 \tag{3.8}$$

is shown to be equivalent to find classical solutions of the Euler equations (2.4)–(2.6) provided that the $\bar{\psi}$'s are auxiliary variables. From a particular set of solutions (the rigid rotators in Sec. II) the leading Regge trajectories were already obtained.

In the second step of our approximation we take account of quantum fluctuations around the classical solutions $\vec{X}_{cl}(\sigma,\tau)$. Substituting

$$\vec{X}=\vec{X}_{cl}+\vec{Y} \tag{3.9}$$

for $\vec{X}$ in I_{eff} we make the Taylor expansion around $\vec{X}_{cl}$ and keep the terms up to the second order in $\vec{Y}$ disregarding higher-order terms. Note that this expansion is not equal to the $\hbar$ expansion.

The information that the quark is a fermion is included in that $n_{i,A}$ in (3.4) takes only 0 and 1. Let us assume that the fermion and antifermion numbers are fixed as in (3.7). In Appendix A, we introduce a commuting coordinate η ($\bar{\eta}$) associated with each fermion (antifermion) and give a proof of the following theorem.

Theorem. The path integral (3.7) with the effective action I_{eff} is equivalent, up to the second order in $\vec{Y}$, to that with the new local action involving $\vec{Y}$, and extra commuting coordinates η and $\bar{\eta}$:

$$Q_{\{A_1,A_2\}}(T)=\exp(iS_{cl})K/K_0. \tag{3.10a}$$

where

$$K\equiv\int\mathfrak{D}\vec{Y}\prod_{i=1}^2(\mathfrak{D}\bar{\eta}_i\mathfrak{D}\eta_i)\prod_\sigma\delta(\vec{Y}(\sigma,T)-\vec{Y}(\sigma,0))\times\prod_{i=1}^2\left[\delta(\eta_i(T)-\exp(-i\zeta_{i,A}^{(0)})\eta_i(0))\delta\left(\int_0^T\bar{\psi}_{i,A}^{(0)}\phi_i^{(0)}\eta_i d\tau\right)\right]\exp(iS_Q[\vec{Y},\eta])\,, \tag{3.10b}$$

$$K_0\equiv\int\prod_{i=1}^2(\mathfrak{D}\bar{\eta}_i\mathfrak{D}\eta_i)\prod_{i=1}^2\left[\left[\delta(\eta_i(T)-\exp(-i\zeta_{i,A}^{(0)})\eta_i(0))\delta\left(\int_0^T\bar{\psi}_{i,A}^{(0)}\phi_i^{(0)}\eta_i d\tau\right)\right]\exp(iS_Q[\vec{Y}=0,\eta]). \tag{3.10c}$$

Here $S_{cl}=I_{cl}-\sum_{i=1}^2\epsilon_{i,A}\zeta_{i,A}^{(0)}$ and I_{cl} is the classical action of the string part, and $\zeta_{i,A}^{(0)}=(\Omega_{i,A}+\omega S_i)T$ with Ω defined in (2.10). S_Q is, provided $Y^0=0$, given by

$$S_Q=\int_0^T\left(L_{st}^Q+\sum_{i=1}^2\epsilon_i L_{q,i}^Q\right)d\tau\,, \tag{3.11}$$

$$L_{st}^Q=\frac{1}{2}\int_0^\pi d\sigma\left[\left(\frac{\partial^2\mathcal{L}_{st}}{\partial X_\tau^\mu\partial X_\tau^\nu}\right)Y_\tau^\mu Y_\tau^\nu+2\left(\frac{\partial^2\mathcal{L}_{st}}{\partial X_\tau^\mu\partial X_\sigma^\nu}\right)Y_\tau^\mu Y_\sigma^\nu+\left(\frac{\partial^2\mathcal{L}_{st}}{\partial X_\sigma^\mu\partial X_\sigma^\nu}\right)Y_\sigma^\mu Y_\sigma^\tau\right], \tag{3.12}$$

$$L_{q,i}^Q=\left[\left(\frac{\partial^2L_{q,i}}{\partial\psi_{i\tau}\partial\bar{\psi}_i}\right)\bar{\eta}_i\eta_{i\tau}+\left(\frac{\partial^2L_{q,i}}{\partial\psi_i\partial\bar{\psi}_{i\tau}}\right)\bar{\eta}_{i\tau}\eta_i+\left(\frac{\partial^2L_{q,i}}{\partial\psi_i\partial\bar{\psi}_i}\right)\bar{\eta}_i\eta_i\right]$$
$$+\left[\left(\frac{\partial^2L_{q,i}}{\partial X_\tau^\mu\partial\psi_{i\tau}}\right)\eta_{i\tau}+\left(\frac{\partial^2L_{q,i}}{\partial X_\tau^\mu\partial\bar{\psi}_{i\tau}}\right)\bar{\eta}_{i\tau}+\left(\frac{\partial^2L_{q,i}}{\partial X_\tau^\mu\partial\psi_i}\right)\eta_i+\left(\frac{\partial^2L_{q,i}}{\partial X_\tau^\mu\partial\bar{\psi}_i}\right)\bar{\eta}_i\right]Y_\tau^\mu+\left[\left(\frac{\partial^2L_{q,i}}{\partial X_\tau^\mu\partial X_\tau^\nu}\right)Y_\tau^\mu Y_\tau^\nu\right]. \tag{3.13}$$

In (3.13), $(\cdots)$ means that X^μ and ψ_i in the bracket are to be replaced by the classical solutions X_{cl}^μ and $\psi_i^{(0)}$. Although (3.12) and (3.13) are formally equal to the correction parts of the original Lagrangian (2.2) and (2.3) when they are expanded by substitution $X^\mu\to X_{cl}^\mu+Y^\mu$ and $\psi\to\psi_{cl}+\eta$, the point of this theorem is that η is a commuting variable rather than an anticommuting one. We remark that, while the quark Lagrangian L_{qi}, (2.3), vanishes if the classical solution is substituted, the role of the classical action of the quark is played by the stability angle for the classical solution $\xi^{(0)}$.

This theorem is stated for the particular configuration of the fermion and antifermion as specified by (3.7). If the fermion configuration is different, other auxiliary coordinates must be introduced. For example, if a quark and an antiquark are attached both at $\sigma=\sigma_1$ and $\sigma=\sigma_2$, one must use four auxiliary coordinates: $\eta_1(\epsilon=+1)$ $[\eta_2(\epsilon=-1)]$ for the quark (antiquark) at $\sigma=\sigma_1$, and $\eta_3(\epsilon=+1)$ $[\eta_4(\epsilon=-1)]$ for the quark (antiquark) at $\sigma=\sigma_2$. The action S_Q in this case should consist of $\sum_{j=1}^{4}\epsilon_j L_{q,j}^{Q} + L_{st}^{Q}$, where $L_{q,j}^{Q}$ is given by (3.13) for $\eta_j (j=1,2,3,$ or 4).

The fact that both Y and η are commuting coordinates and the Lagrangian in (3.11) is quadratic with respect to these variables enable us to evaluate the path integral (3.10) by completing the square. As shown in Appendix B the resulting formula is given by

$$Q_{\{s_1,s_2\}}(T)=\sum_{\{N_a^{(p)}\}}\sum_{\substack{l=1\\(\omega T=2\pi l)}}^{\infty}\left(\frac{i}{2\pi}\right)^{1/2}\left|\frac{\partial^2 S_{cl}}{\partial T^2}\right|^{1/2}\exp\left[iS_{cl}(T)-i\sum_{p,a}N_a^{(p)}\omega\nu_a^{(p)}(\omega)T\right], \quad (3.14)$$

where p signifies the type of transversed oscillations and takes 2 and 3 (Ref. 11) (no longitudinal ($p=1$) oscillation appears), $\nu_a^{(p)}(\omega)$ the eigenfrequency with the node number a $(=0,1,2,\ldots)$, and $N_a^{(p)}(=0,1,2,\ldots)$ the occupation number of the quantum. As is well known the factor $|\partial^2 S_{cl}/\partial T^2|^{1/2}$ appears due to the zero-frequency mode and recovers the time-translation invariance which has been violated in the classical solution. Recovering the $\Delta\theta(\neq 0)$ effect is simple and only the difference appears in the relation between T and ω.

The propagator can now be obtained by substituting (3.14) for $\Delta\theta(\neq 0)$ into (3.1) or (3.2) and performing T integration,

$$-i\int_0^\infty dT\exp(iET)Q_{\{s_1,s_2\}}(T,\Delta\theta)\approx\sum_{\{N_a^{(p)}\}}(-i)\sum_{l=0}^{\infty}\int_0^\infty dt[(i/2\pi)(l+\Delta\theta/2\pi)]^{1/2}t\left|\frac{\partial^2 S_{cl}}{\partial t^2}\right|^{1/2}$$
$$\times\exp\left\{i(l+\Delta\theta/2\pi)\left[S_{cl}(t)-\sum_{p,a}N_a^{(p)}\omega\nu_a^{(p)}(\omega)t+Et\right]\right\}. \quad (3.15)$$

In going to the right-hand side we have change the integration variable from T to the classical period $t=2\pi/\omega$, which is related to the former through $\omega T=2\pi 1+\Delta\theta$ with 1 being the revolution number of the classical rotator.[7] The t integration in (3.15) is evaluated by the stationary-phase method. Keeping in mind that $\nu_a^{(p)}$ is the higher-order term, we must find the period t as a function of E by

$$\frac{d}{dt}S_{cl}(t)-2\pi\sum_{a,p}N_a^{(p)}\frac{d}{dt}\nu_a^{(p)}(\omega=2\pi/t)=-E. \quad (3.16)$$

Assuming that $t(E)$ can be expanded in accordance with our expansion [(A9) in Appendix A] as

$$t=t(E)=t_0(E)+t_1(E)+\cdots,$$

we substitute this into (3.16) and equate terms of the same order to obtain

$$-\frac{d}{dt_0}S_{cl}(t_0)=E, \quad (3.17)$$

$$t_1=\frac{\sum_{a,p}2\pi N_a^{(p)}\frac{d}{dt_0}\nu_a^{(p)}(\omega=2\pi/t_0)}{\frac{d^2}{dt_0^{\,2}}S_{cl}(t_0)}. \quad (3.18)$$

Since the left-hand side of (3.17) equals the classical energy (2.38), the relation between $\omega=2\pi/t_0$ and E is the same as in the classical case. The stationary value of the large square bracket in (3.15), therefore, turns out to be

$$W_{\{N_a^{(p)}\}}(E)=W_0(E)-2\pi\sum_{a,p}N_a^{(p)}\nu^{(p)}[\omega=2\pi/t_0(E)] \quad (3.19)$$

with

$$W_0(E)=S_{cl}(t_0(E))-Et_0(E). \quad (3.20)$$

It should be noted that (3.20) is nothing other than the classical angular momentum $2\pi J_{cl}$ for the energy E. The Regge trajectories are obtained from (3.17) and (3.19) by eliminating the running parameter $\omega=2\pi/t_0$.

The summation over 1 in (3.15) can be easily performed and the final expression for the propagator (3.1) is given by

$$D(E)=\sum_{\{i,A\}}\sum_{\{N_a^{(p)}\}}\frac{-it_0(E)\exp[iW_{\{N_a^{(p)}\}}(E)]}{1-\exp[iW_{\{N_a^{(p)}\}}(E)]} \quad (3.21)$$

and for (3.2) by

$$D_{j_3}(E)=\sum_{\{i,A\}}\sum_{\{N_a^{(p)}\}}\frac{2\pi t_0(E)}{W_{\{N_a^{(p)}\}}(E)-2\pi j_3}\times\frac{1-\exp[iW_{\{N_a^{(p)}\}}(E)-2\pi i j_3]}{1-\exp[iW_{\{N_a^{(p)}\}}(E)]}\,. \tag{3.22}$$

The particle spectra are given by the zeros of the denominator of (3.22), i.e.,

$$W_{\{N_a^{(p)}\}}(E)=2\pi j_3. \tag{3.23}$$

This is the Regge condition that particles appear when the trajectory crosses integers. As a particular case where $N_a^{(p)}=0$, (3.23) reduces to the Bohr-Sommerfeld trajectory obtained in the preceding section.

It may be worthwhile to remark that, in obtaining (3.19), zero-point-energy contributions have been disregarded. This contribution due to the string oscillation modifies (3.19) to

$$W_{\{N_a^{(p)}\}}=W_0-\sum_{a,p}2\pi N_a^{(p)}\nu_a^{(p)}+2\pi\alpha_0\,, \tag{3.24}$$

where α_0 is a constant (actually an infinity). The value of α_0 could be determined from the Lorentz-invariance condition as was done in the dual string model.[12] This problem is beyond the scope of the semiclassical method.

IV. MESON SPECTRA

In this section we study the meson spectra based on the theoretical analysis of the quark-string model shown in previous sections. As was pointed out in papers I–III our method may not be a good approximation in the low-mass and low-angular-momentum region. If the (lattice) gauge theory is the basic theory and the string model is an approximation to it, there are some reasons that the simple Lagrangian (2.1) may miss some ingredients of the gauge theory. Even if we take our string Lagrangian as a basic theory of the hadron, the semiclassical method we adopted may not be expected to work well in the low-mass region because (i) the semiclassical method is unreliable and (ii) the recoil effect of the classical solution due to the quantum oscillation may not be disregarded unless the classical mass is large compared to the quantum oscillation.

Nevertheless, encouraged by the very good results in the preliminary analysis in III, we interpolate our solutions from the high-angular-momentum region into the low-angular-momentum region. The result is considered to be a good first approximation to the hadron spectroscopy.

We point out that all the previous results concerning leading trajectories are unchanged. As far as the quantum correction is concerned, the previous result on the type-1 oscillation is modified because correlations between the quark spin and the quantum string oscillation [the first and the second term in (3.13)] are now taken into account. The main effect of the modification is that there appear to be some number of daughter trajectories with relatively small mass corrections in the low-mass region. The analysis of the type-2 oscillations is a new contribution of this paper.

We note that only the parameters contained in our theory are quark masses (m_u, m_d, m_s, m_c, etc.) and the universal spring constant γ of the string, which determines the scale of energy. The spring constant γ is related to the universal asymptotic Regge slope by $\alpha'_\infty=(2\pi\gamma)^{-1}$. We emphasize that the old and new particles, as well as baryons, have common spring constants. In this paper we take $\alpha'_\infty=(2\pi\gamma)^{-1}=1.0\ \text{GeV}^{-2}$ unless otherwise stated.

Leading trajectories

To the quark masses $m_u=m_d=0.385$ GeV, i.e., $m_\rho=m_u+m_d=0.770$ GeV, the ρ-meson trajectory becomes almost straight with $\alpha'_\rho\simeq\alpha'_\infty$. Since the quark masses are below the critical mass $(\pi\alpha'_\infty)^{-1/2}$, the π meson appears with the same mass as ρ. Our method is not powerful enough to obtain the $\rho-\pi$ mass difference in the low-mass region.

The classical trajectories for the $c\bar{c}$ states are shown in Fig. 2. The charmed-quark mass m_c is assumed to be 1.55 GeV, which makes $m_{\psi/J}=3.10$ GeV. A remarkable point in the $c\bar{c}$ states is that, because $m_c^2>(\pi\alpha'_\infty)^{-1}$, the solutions $M(\alpha_+,\alpha_-^{(1)})$ and $M(\alpha_+,\alpha^{(3)})$ become stabler than $M(\alpha_+,\alpha_-^{(2)})$. The trajectories $M(\alpha_+,\alpha_-^{(1)})$ and $M(\alpha_+,\alpha_-^{(3)})$ predict pseudoscalar particles at 2.80 GeV (η'_c) and 2.81 GeV (η_c), respectively. These values should be compared with the observed[13] η_c^{expt} (2.83). Although the two pseudoscalar particles around 2.8 GeV are expected, no experimental information is available yet. The 2^+ particle on the exchange partner of ψ/J indicates the mass 3.56 GeV, which should be compared with $\chi(3.561)$. Our string model predicts a set of four trajectories $M(\alpha_-^{(1)},\alpha_-^{(1)})$, $M(\alpha_-^{(3)},\alpha_-^{(1)})$, $M(\alpha_-^{(1)},\alpha_-^{(3)})$, and $M(\alpha_-^{(3)},\alpha_-^{(3)})$, all of which pass zero around 2.75 GeV, which we call ϵ_c. If some of them have nonvanishing residue in the propagator, $J^{PC}=0^{++}$ should be observed at this energy.

At this point we wish to comment on the wave function of these $c\bar{c}$ particles. In the ordinary charmonium model the $M1$ transition in $\psi\to\eta_c\gamma$ gives too large a prediction compared to observation, since the wave function of η_c is the same as

TABLE I. Angular momentum distribution of typical states.

State	l_q	S_q	l_{st}	J_3
$\psi(1^-)$	0	1.0	0	1.0
$\chi(2^+)$	0.93	1.04	0.03	2.0
$\eta_c(0^-)$	0.45	−0.80	0.35	0.0
$\eta_c'(0^-)$	−0.13	−0.86	0.99	0.0
$\epsilon_c(0^+)$	1.23	−3.31	2.08	0.0

that of ψ/J except for the spin wave function and the decay is allowed.[14] In contrast to this, both η_c and η_c' in our model have quite different structure from ψ/J. In Table I, we show the classical angular momentum (J_3) distribution of some typical states. Most of the angular momentum of normal states, $\psi(1^-)$ and $\chi(2^+)$, is carried by the quark spin (S_q) and the quark orbital angular momentum (l_q), while almost none is carried by the string ($l_{st} \simeq 0$). On the other hand, η_c and η_c' as well as ϵ_c have a non-negligible contribution from l_{st} and a large negative value of l_q+S_q which cancels the former to give vanishing total angular momentum. This unfamiliar situation occurs due to the relativistic effect of the quark motion. Note that the eigenvalues of our spin operator (2.33) are $\pm\frac{1}{2}(1-X_{\pi}{}^2)^{-1/2}$ so that S_3 can be smaller than $-\frac{1}{2}$. As will be inferred from this fact, the wave-function structure of ψ/J and those of these particles are quite different from each other, hence both

TABLE II. Particle preditions from classical (leading) trajectories. The $c\bar{c}$ states can be read from Fig. 2.

Theoretical predictions J^{PC}	Mass in GeV	Experimental candidates
$c\bar{c}$ states		
1^{--}	3.10 (input)	$\psi/J(3.095)$
1^{--}	3.86	$\psi'(3.772)$
2^{++}	3.56	$\chi(3.561)$
0^{-+}	2.81	$\eta_c(2.83)$
0^{-+}	2.80	?
0^{++}	2.75	?
0^{++}	3.17	?
0^{++}	3.59	$\chi(3.45)$
1^{++}	3.58	$\chi(3.51)$
$c\bar{u}$ or $c\bar{d}$ states		
1^-	2.006 (input)	$D^*(2.006)$
0^-	1.76	$D(1.870)$
$c\bar{s}$ states		
1^-	2.06 (input)	?
0^-	1.81	?
$b\bar{b}$ states		
1^-	9.50 (input)	$\Upsilon(9.50)$
0^-	6.71	?

the $M1$ transition $\psi \to \gamma\eta_c$ (η_c') and the $E1$ transition $\psi \to \gamma\epsilon_c$ are supposed to be small. This is in contrast to the ordinary charmonium model.

Other trajectories $M(\alpha_+, \alpha_-^{(2)})$, $M(\alpha_-^{(2)}, \alpha_-^{(3)})$, etc. are also shown in Fig. 2.

A similar situation occurs for the charmed particles (D, F, etc.), and the Υ family. Some predictions for these families are listed in Table II.

The excellent agreement of $c\bar{c}$-state spectra provides strong support for the idea that the old and new mesons are composed of quarks and string with a common spring constant. The small slope for the new-particle trajectories in the low-angular-momentum region is due to the large quark mass.[15]

Finally, we wish to comment on the string $\alpha_-^{(1)}$ solution. If the solution $\alpha_-^{(1)}$ is unstable against quantum corrections, some of the trajectories discussed above must be discarded. In particular, one of the η_c's is unstable and the degeneracy is removed. However, the pion and the A_1 trajectories, both belonging to $M(\alpha_+, \alpha_-^{(1)})$, must be discarded, too. As is seen below, although we have calculated real quantum frequencies, we have not fully analyzed complex ones.

Daughter trajectories

The daughter trajectories are derived as quantum corrections to the leading trajectories. According to (3.17) and (3.19) the trajectory $J(E)$ is obtained by eliminating ω from

$$J = J_{\rm cl}(\omega) - \sum_{a,p} N_a^{(p)} \nu_a^{(p)}(\omega)\,,$$

$$E = -\frac{d}{dt_0} S_{\rm cl}(t_0) = E_{\rm cl}(\omega). \tag{4.1}$$

As is shown in Appendix B, some ν's are exactly known, $\nu_0^{(2)} = \nu_0^{(3)} = 0$ and $\nu_1^{(2)} = \nu_1^{(3)} = 1$. Other ν's are numerically calculated as a function of ω.

The zero-frequency modes, $\nu_0^{(2)}$ and $\nu_0^{(3)}$, need not be considered.[8] Since we are identifying particles in terms of J_3 rather than total angular momentum, all quantum-corrected states should not be counted as independent states. In fact, the mode $\nu_1^{(3)}$ decreases j_3 by 1, $\Delta j_3 = -1$, but keeps the energy (mass) unchanged, $\Delta m = 0$. The states with $N_1^{(3)} \neq 0$ are, therefore, considered to be those which are generated by rotating the state with $N_1^{(3)} = 0$ around an axis perpendicular to the z axis, hence they should not be countered as independent states. All other states are considered to be independent. It should be noted, however, that some of the classical trajectories, say, $M(\alpha_-^{(2)}, \alpha_-^{(2)})$, can be the daughter trajectories of the other, say $M(\alpha_+, \alpha_+)$.

Some of daughter trajectories are shown in Fig.

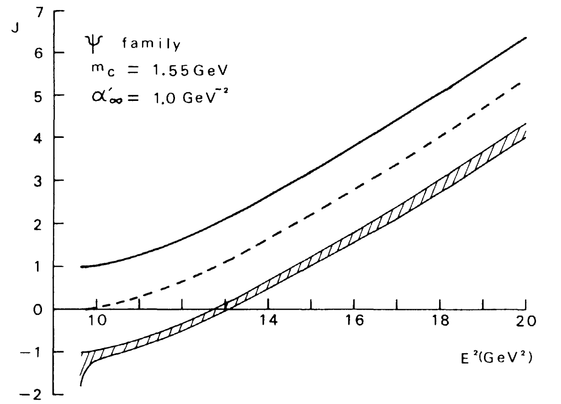

FIG. 3. Quantum corrections to the ψ/J trajectory. The broken line shows a trajectory with opposite parity to the leading one. Within the shaded area there are six trajectories, which separate themselves as J grows.

3 for the $c\bar{c}$ states. Contrary to expectation six trajectories with the same quantum number as the ψ/J trajectory gather around mass $\simeq 3.8$ GeV at $J=1$, and three trajectories around mass $\simeq 4.0$ GeV. As J grows, their mutual distance grows (Table III) to get unit intervals as the dual string model. The trajectories do not all necessarily imply the existence of $J^{PC}=1^{--}$ particles. To judge whether the trajectory has nonvanishing residue of the propagator pole, some more refined method than the stationary approximation in the calculation of (3.15) will be needed.

It is important, however, to point out the reason the trajectories gather in the low-mass region. This is the effect of the spin resonance with the orbital rotation. As is seen in (B18), the second term in the bracket has a pole at the energy $E = \omega\nu = \omega/\cos\alpha_i$, where $(\cos\alpha_i)^{-1} = (1-\omega^2\rho_i^2)^{-1/2}$ is the Lorentz factor. As was done in paper III, if the correlation between spin and quantum oscillation of the string [the first and the second terms in (3.13)] is disregarded, the singularity in ν in (B18) does not appear. The trajectories, then cross $J=1$ with about unit intervals.

V. CONCLUDING REMARKS

We have developed a systematic method of semiclassical approximation to the system including both bosons and fermions. The quark-string model has been studied by the method and the Regge trajectories for the meson have been obtained. Although the classical solutions, when quantized by the Bohr-Sommerfeld condition, provide excellent spectra for the new particles, the quantum correction does not give a good quantitative result due to the strong spin resonance.[16]

Taking account of the failure in getting the $\rho-\pi$ mass splitting, we feel that a new (nonsemiclassical) approximation is needed in the low-mass region. The semiclassical approach can never generate a large spin-spin interaction term in the quark-string model because the interaction between quarks is mediated by the classical string which produces the L-S force, while it seems that the spin-spin force plays an important role at least in the low-mass region.

In a forthcoming paper we will report on an analysis of the baryon trajectory.

APPENDIX A

In this appendix we present the semiclassical method for the system involving both boson and fermion fields. Although the discussion is given as for the quark-string model, the method can be easily translated into field theory.

Let us begin with reminding readers of some basic formulas of the DHN method.[9]

On performing integrations in (3.3) (we assume $\Delta\theta=0$ for simplicity) one is required to evaluate the eigenvalue of the differential operator

$$D[X] = i\not{e}\frac{d}{d\tau} + \frac{i}{2}\dot{\not{e}}_\tau - m(X_\tau^2)^{1/2} \tag{A1}$$

under the boundary condition $\psi(\tau) = -\psi(0)$. The index $i(=1,2)$ which discriminates the end points of the string as well as the flavor index is suppressed unless necessary. Suppose one has found a complete set of solutions ψ's, labeled by the

TABLE III. Quantum corrections $\nu_a^{(p)}$ for the $M(\alpha_+,\alpha_+)$ trajectory. Exactly known corrections $\nu_1^{(2)}(\omega) = \nu_1^{(3)}(\omega) = 1$ are not shown in the table. Although quantum corrections for other trajectories are available, they are not listed here.

$\nu_a^{(P)}$ \ ω	1.0	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10	~0
$\nu_3^{(2)}$	2.01	1.97	1.93	1.90	1.87	1.85	1.83	1.81	1.81	1.84	2.00
$\nu_4^{(2)}$	3.84	4.23	4.69	5.27	6.03	7.08	8.63	5.93	4.39	3.25	3.00
$\nu_2^{(3)}$	1.00	1.01	1.01	1.01	1.01	1.02	1.03	1.05	1.10	1.27	2.00
$\nu_4^{(3)}$	1.15	1.14	1.13	1.13	1.10	1.10	1.09	1.10	1.13	1.29	3.00

index A, which satsify

$$D[X]\psi_A(\tau)=0 \tag{A2}$$

and

$$\psi_A(T)=\exp(-i\zeta_A)\psi_A(0)\,, \tag{A3}$$

where the periodicity $X^\mu(T)=X^\mu(0)$ is assumed. Then a set of functions $\xi_{A,n}$ $(n=0,\pm1,\pm2\ldots)$,

$$\xi_{A,n}(\tau)=\exp\{i[(2n+1)\pi+\zeta_A]\tau/T\}\psi_A(\tau), \tag{A4}$$

obeys

$$\begin{aligned} &D[X]\xi_{A,n}=E_n(A)\not{e}\,\xi_{A,n}\,,\\ &\xi_{A,n}(T)=-\xi_{A,n}(0)\,,\\ &\int_0^T d\tau\,\bar{\xi}_{A,n}\not{e}\,\xi_{A',n'}=T\delta_{AA'}\delta_{nn'} \end{aligned} \tag{A5}$$

The eigenvalues of $D[X]$ are, therefore, given by

$$E_n(A)=-\frac{1}{T}[(2n+1)\pi+\zeta_A]\,. \tag{A6}$$

Then

$$\begin{aligned} \det|D[X]| &\propto \prod_{n,A}\left[1+\frac{\zeta_A}{(2n+1)\pi}\right]\\ &=\prod_A \cos(\zeta_A/2)\\ &=\sum_{\{n_A\}}\exp\left(-i\sum_A n_A\epsilon_A\zeta_A+i\sum_{\epsilon_A>0}\zeta_A\right), \end{aligned} \tag{A7}$$

where $n_A=0$ or 1 (note that $\det|\not{e}|=1$). The formula (A7) proves (3.4) in the text. In our string model a quark and an antiquark are excited at $\sigma=\sigma_1$, and $\sigma=\sigma_2$, respectively. Hence, the effective action becomes

$$I_{\rm eff}=\int_0^T L_{\rm st}d\tau-\sum_{i=1}^{2}\epsilon_{i,A}\zeta_{i,A}[\vec{X}]. \tag{A8}$$

We evaluate this effective action $I_{\rm eff}$ up to the second order with respect to Y where $X^\mu=X^\mu_{\rm cl}+Y^\mu$, $X^0=\tau$, and $Y^0=0$. The classical solution $X^\mu_{\rm cl}$ will be defined later. Supposing that all quantities are functionals of Y^μ, we make Taylor expansions as follows:

$$\begin{aligned} &D=D^{(0)}+D^{(1)}[Y]+D^{(2)}[Y]+\cdots\,,\\ &e_\mu=e_\mu^{(0)}+e_\mu^{(1)}[Y]+e_\mu^{(2)}[Y]+\cdots\,,\\ &E_n(A)=E_n^{(0)}(A)+E_n^{(1)}(A,[Y])+E_n^{(2)}(A,[Y])+\cdots\,,\\ &\xi_{A,n}=\xi^{(0)}_{A,n}+\xi^{(1)}_{A,n}[Y]+\xi^{(2)}_{A,n}[Y]+\cdots\,. \end{aligned} \tag{A9}$$

Substituting (A9) into (A5) we are going to evaluate $E_n^{(r)}(A)$ by following the same line of the perturbation calculation. The value $\zeta_A^{(r)}$ is determined through (A6).

In the zeroth-order approximation (A5) provides

$$\begin{aligned} &D^{(0)}\xi^{(0)}_{A,n}=E_n^{(0)}(A)\not{e}^{(0)}\xi^{(0)}_{A,n}\,,\\ &\xi^{(0)}_{A,n}(T)=-\,\xi^{(0)}_{A,n}(0) \end{aligned} \tag{A10}$$

with the normalization condition

$$\int_0^T\bar{\xi}^{(0)}_{A,n}\not{e}^{(0)}\xi^{(0)}_{A',n'}d\tau=T\delta_{AA'}\delta_{nn'} \tag{A11}$$

and the completeness condition

$$\frac{1}{T}\sum_{n,A}\xi^{(0)}_{A,n}(\tau)\bar{\xi}^{(0)}_{A,n}(\tau')\not{e}^{(0)}=\delta(\tau-\tau'). \tag{A12}$$

If one defines $\psi_A^{(0)}$ by

$$\xi^{(0)}_{A,n}(\tau)=\exp[-iE_n^{(0)}(A)\tau]\psi_A^{(0)}(\tau), \tag{A13}$$

$\psi_A^{(0)}$ satisfies

$$D^{(0)}\psi_A^{(0)}=0,\quad \psi_A^{(0)}(T)\exp(-i\zeta_A^{(0)})\psi_A^{(0)}(0), \tag{A14}$$

where $\zeta_A^{(0)}\equiv-[(2n+1)\pi+E_n^{(0)}(A)T]$, and is n independent. [From (A14) and (2.10), $\zeta_A^{(0)}=(\Omega_A+\omega S_A)T$.]

In the first-order approximation we obtain

$$[D^{(0)}-\not{e}^{(0)}E_n^{(0)}(A)]\xi^{(1)}_{A,n}+[D^{(1)}-\not{e}^{(1)}E_n^{(0)}(A)]\xi^{(0)}_{A,n}=E_n^{(1)}(A)\not{e}^{(0)}\xi^{(0)}_{A,n}\,. \tag{A15}$$

Using the new function

$$\begin{aligned} &\psi_A^{(1)}(\tau)\equiv\exp[iE_n^{(0)}(A)\tau]\xi^{(1)}_{A,n},\\ &\psi_A^{(1)}(T)=\exp(-i\zeta_A^{(0)})\psi_A^{(1)}(0)\,, \end{aligned} \tag{A16}$$

and $\psi_A^{(0)}$ defined by (A13), one translates (A15) into

$$D^{(0)}\psi_A^{(1)}+D^{(1)}\psi_A^{(0)}=E_n^{(1)}(A)\not{e}^{(0)}\psi_A^{(0)} \tag{A17}$$

and

$$E_n^{(1)}(A)=-\frac{\zeta_A^{(1)}}{T}=\frac{1}{T}\int_0^T\bar{\psi}_A^{(0)}D^{(1)}\psi_A^{(0)}d\tau \tag{A18}$$

or

$$-\zeta_A^{(1)}=\int_0^T\bar{\psi}_A^{(0)}D^{(1)}\psi_A^{(0)}d\tau\,, \tag{A18$'$}$$

where $\int_0^T\bar{\psi}_A^{(0)}\not{e}^{(0)}\psi_A^{(0)}d\tau=T$ has been used. The relation (A18) implies that $E_n^{(1)}(A)$ is n-independent. The relation (A17) can be solved for $\psi_A^{(1)}$ by the well known method

$$\psi_A^{(1)}=a\psi_A^{(0)}-\frac{1-P_A}{D^{(0)}}D^{(1)}\psi_A^{(0)}\,, \tag{A19}$$

where a is a constant and P_A is the projection operator into the state $\psi_A^{(0)}$. The constant a can be shown to be zero within our approximation by the normalization condition (A5).

At this point, we are able to inspect the stationary condition of the effective action I_{eff} (A8). The variation of I_{eff} with respect to $X^{\mu}_{(\sigma,\tau)}$ for $\sigma \neq \sigma_i$ $(i=1,2)$ simply provides us with the string equation (2.4) because $\zeta_{i,A}$ contains only the boundary value $X^{\mu}_i(\tau) = X^{\mu}(\sigma_i,\tau)$. The variation with respect to X^{μ}_i occurs in both L_{st} and $\zeta_{i,A}$, and that of the latter equals $\zeta^{(1)}_{i,A}$. Therefore, the stationary condition for X^{μ}_i is

$$\int_0^T d\tau' \left[\bar{\psi}^{(0)}_{i,A}(\tau') \frac{\delta D^{(1)}_i(\tau')}{\delta Y^{\mu}_i(\tau)} \psi^{(0)}_{i,A}(\tau') + \frac{\delta L_{\text{st}}(\tau')}{\delta X^{\mu}_i(\tau)} \right] = 0, \tag{A20}$$

where $\psi^{(0)}_{i,A}$ obeys (A14). Equation (A20) precisely coincides with (2.5). As a consequence, it has been shown that the stationary condition of I_{eff} with the help of (A14) is equivalent to obtaining the classical solution to (2.4)–(2.6) in Sec II. In general, any solution obeying (2.4)–(2.6) is called a set of classical solutions and is denoted by $(X^{\mu}_{\text{cl}}, \psi^{(0)}_A)$.

Let us return to the perturbation calculation. In the second-order approximation we obtain

$$TE^{(2)}_n(A) = \int_0^T \xi^{(0)}_{A,n}\{D^{(1)} - [\rlap{/}e^{(1)}E^{(0)}_n(A) + \rlap{/}e^{(0)}E^{(1)}_n(A)]\}\,\xi^{(1)}_{A,n}d\tau + \int_0^T \xi^{(0)}_{A,n}\{D^{(2)} - [\rlap{/}e^{(2)}E^{(0)}_n(A) + \rlap{/}e^{(1)}E^{(1)}_n(A)]\}\,\xi^{(0)}_{A,n}d\tau\,, \tag{A21}$$

or, in terms of $\psi^{(0)}_A$ and $\psi^{(1)}_A$,

$$\begin{aligned} TE^{(2)}_n(A) = -\zeta^{(2)}_A = \int_0^T \bar{\psi}^{(0)}_A D^{(1)}\psi^{(1)}_A\, d\tau - E^{(1)}_n(A) \\ \times \int_0^T \bar{\psi}^{(0)}_A \rlap{/}e^{(0)}\psi^{(1)}_A d\tau \\ + \int_0^T \bar{\psi}^{(0)}_A [D^{(2)} - \rlap{/}e^{(1)}E^{(1)}_n(A)]\psi^{(0)}_A d\tau. \end{aligned} \tag{A22}$$

If one takes the rigid-rotator solution (2.10)–(2.12) as the classical solution, one can easily show that $\bar{\psi}^{(0)}_A \rlap{/}e^{(1)}\psi^{(0)}_A = 0$. The result is, therefore,

$$-\zeta^{(2)}_A = -\int_0^T \bar{\psi}^{(0)}_A D^{(1)} \frac{1-P_A}{D^{(0)}} D^{(1)}\psi^{(0)}_A d\tau + \int_0^T \bar{\psi}^{(0)}_A D^{(2)}\psi^{(0)}_A d\tau \tag{A23}$$

Summing up all ζ's we obtain the final formula

$$I_{\text{eff}} \approx \left(I^{\text{st}}_{\text{cl}} - \sum_{i=1}^{2} \epsilon_i \zeta^{(0)}_{i,A} \right) + \left(\int_0^{\tau} d\tau L^{Q}_{\text{st}} - \sum_{i=1}^{2} \epsilon_i \zeta^{(2)}_{i,A} \right), \tag{A24}$$

where $I^{\text{st}}_{\text{cl}}$ denotes the classical action of the string part. The first-order correction terms have been eliminated because the classical solution satisfies the stationary condition. We emphasize that, although the quark part action (2.3) is vanishing if the classical solution (2.10) is substituted for $\psi(\tau)$, the quark term

$$\sum_{i=1}^{2} \epsilon_i \zeta^{(0)}_{i,A} = \sum_{i=1}^{2} \epsilon_i (\Omega_{i,A} + \omega S_i) T\,,$$

where $\Omega_{i,A}$ is given by (2.12), has appeared as the zeroth contribution from the stability angle. We call $S_{\text{cl}} \equiv (I^{\text{st}}_{\text{cl}} - \sum_i \epsilon_i \zeta^{(0)}_{i,A})$ the "classical action" of the quark string.

We are now in a position of proving the theorem (3.10) in the text. The classical-action part in (3.10a) can be immediately obtained from the first term in (A24). In terms of our simplified notation the quantum-correction part (3.10b) for $\Delta\theta = 0$ can be written as

$$K = \int \mathfrak{D}\vec{Y} \prod_i (\mathfrak{D}\bar{\eta}_i \mathfrak{D}\eta_i) \prod_i \delta\left(\int_0^T d\tau \bar{\psi}^{(0)}_{i,A} \rlap{/}e^{(0)}_i \eta_i \right) \times \prod_\sigma \delta(\vec{Y}(\sigma,T) - \vec{Y}(\sigma,0)) \prod_i \delta(\eta_i(T) - \exp(-i\zeta^{(0)}_{i,A})\eta_i(0)) \exp\left(i\int_0^T L^Q d\tau \right), \tag{A25}$$

where

$$L^Q = L^Q_{\text{st}} + \sum_{i=1}^{2} \epsilon_i L^Q_{q,i}$$

with

$$L^Q_{q,i} = \bar{\eta} D^{(0)}_i \eta_i + \bar{\psi}^{(0)}_{i,A} D^{(1)}_i \eta_i + \bar{\eta}_i D^{(1)}_i \psi^{(0)}_{i,A} + \bar{\psi}^{(0)}_{i,A} D^{(2)}_i \psi^{(0)}_{i,A}\,. \tag{A26}$$

The string part Lagrangian L^Q_{st} is given by (3.12). The boundary condition for η in (A25) is adopted in order for η to satisfy the same boundary condition as the corresponding field $\psi^{(0)} + \psi^{(1)}$ [see (A14) and (A16)].

Let us introduce the new coordinate η_0 defined by

$$\eta(\tau)=\eta_0(\tau)-\frac{1-P_A}{D^{(0)}}D^{(1)}\psi_A^{(0)}\,,\qquad \eta_0(T)=\exp(-i\zeta_A^{(0)})\eta_0(0)\,, \tag{A27}$$

where η_0 should be orthogonal to $\psi_A^{(0)}$. Then we can rewrite (A26) as

$$L_q^Q=\bar\eta_0 D^{(0)}\eta_0-\bar\psi_A^{(0)}D^{(1)}\frac{1-P_A}{D^{(0)}}D^{(1)}\psi_A^{(0)}+\bar\psi_A^{(0)}D^{(2)}\psi_A^{(0)}\,. \tag{A28}$$

The path integral (A25) now reads

$$K=K_0\times\int\mathfrak{D}\vec{Y}\prod_\sigma\delta(\vec{Y}(\sigma,T)-\vec{Y}(\sigma,0))\exp\left\{i\int_0^T d\tau\left[L_{st}^Q+\sum_{i=1}^2\epsilon_i\left(\bar\psi_{i,A}^{(0)}D_i^{(2)}\psi_{i,A}^{(0)}-\bar\psi_{i,A}^{(0)}D_i^{(1)}\frac{1-P_{i,A}}{D_i^{(0)}}D_i^{(1)}\psi_{i,A}^{(0)}\right)\right]\right\}, \tag{A29}$$

where

$$K_0=\int\prod_i(\mathfrak{D}\bar\eta_i\mathfrak{D}\eta_i)\prod_i\delta\left(\int_0^T\bar\psi_{i,A}^{(0)}\not{e}_i^{(0)}\eta_i d\tau\right)\prod_i\delta(\eta_i(T)-\exp(-i\zeta_{i,A}^{(0)})\eta_i(0))\exp\left(i\int_0^T d\tau\sum_{i=1}^2\epsilon_i\bar\eta_i D_i^{(0)}\eta_i\right). \tag{A30}$$

The formula (A29) divided by K_0 provides us with the effective action

$$I_{eff}=S_{cl}+\int_0^T d\tau\left[L_{st}^Q+\sum_{i=1}^2\epsilon_i\left(\bar\psi_{i,A}^{(0)}D_i^{(2)}\psi_{i,A}^{(0)}-\bar\psi_{i,A}^{(0)}D_i^{(1)}\frac{1-P_{i,A}}{D_i^{(0)}}D_i^{(1)}\psi_{i,A}^{(0)}\right)\right], \tag{A31}$$

which precisely equals the action (A24) with (A23) obtained by the stability angle method. This proves the theorem.

For later purposes we remark that (A30) can be easily calculated by the stability-angle method, and

$$K_0=\sum_{\{N_{i,B}\}}\prod_{i,B\neq i,A}\exp[-i(\zeta_{i,B}^{(0)}-\zeta_{i,A}^{(0)})N_{i,B}]\,, \tag{A32}$$

where B runs over the complete set of solutions to (A14) except for the initially given classical solution ψ_A. N_B runs over all integers $(0,1,2,\ldots)$.

APPENDIX B

This appendix is devoted to the evaluation of the path integral (3.10),

$$Q_{\{A_1,A_2\}}(T)=\exp(iS_{cl})K/K_0, \tag{B1}$$

with

$$K=\int\mathfrak{D}\vec{Y}\prod_i(\mathfrak{D}\bar\eta_i\mathfrak{D}\eta_i)\prod_\sigma\delta(\vec{Y}(\sigma,T)-\vec{Y}(\sigma,0))\prod_i\delta(\eta_i(T)-\exp(-i\zeta_{i,A}^{(0)})\eta_i(0))\prod_i\delta\left(\int_0^T\bar\psi_{i,A}^{(0)}\not{e}_i^{(0)}\eta_i d\tau\right)\exp(iS_Q[\vec{Y},\eta]), \tag{B2}$$

$$K_0=\int\prod_i(\mathfrak{D}\eta_i\mathfrak{D}\eta_i)\prod_i\delta(\eta_i(T)-\exp(-i\zeta_{i,A}^{(0)})\eta_i(0))\prod_i\delta\left(\int_0^T\bar\psi_{i,A}^{(0)}\not{e}_i^{(0)}\eta_i d\tau\right)\exp(iS_Q[\vec{Y}=0,\eta]), \tag{B3}$$

where S_Q is given by (3.11) [$\Delta\theta$ is taken to be zero for simplicity.] Since the rigid rotator solution for X_{cl} is chosen, it is most convenient to adopt the body fixed frame to the rotator by making the transformation

$$\vec{Y}(\sigma,\tau)=R(\theta=\omega\tau)\vec{Z}(\sigma,\tau),\qquad \eta_i(\tau)=\exp[-i\tau(\Omega_{i,A}+\tfrac12\omega\sigma^3)]\chi_i(\tau)\,,\qquad \psi_{i,A}^{(0)}(\tau)=\exp[-i\tau(\Omega_{i,A}+\tfrac12\omega\sigma^3)]u_{i,A}. \tag{B4}$$

In this frame of reference, all explicit τ dependence in S_Q disappears [see (B10) and (B11)], and the integrals (B2) and (B3) can be best evaluated by completing the square. Since both $\vec{Y}$ and η can be treated as commuting coordinates as shown in Appendix A, we can do this by finding a complete set of eigenfunctions to the Euler equations that follow from S_Q. (This method is equivalent to that of the stability angle presented in Appendix A.)

To the action S_Q for a given set of classical solutions $(X_{\rm cl}, \psi^{(0)}_{i,A})$, let the Euler equations in the body fixed frame be

$$\Delta Z = \bar{u}_A O \chi + \bar{\chi} O u_A , \tag{B5}$$

$$D^{(0)} \chi = - D^{(1)}[Z] u_A , \tag{B6}$$

where Δ, $D^{(0)}$, and $D^{(1)}[Z]$ are certain differential operators with no explicit τ dependence, and O is a 4×4 matrix. The formal solution to (B6) is given by

$$\chi = c \chi_\perp^{(0)} - \frac{1 - P_A}{D^{(0)}} D^{(1)}[Z] u_A \tag{B7}$$

with

$$D^{(0)} \chi_\perp^{(0)} = 0, \tag{B8}$$

where $\chi_\perp^{(0)}$ is implied to be orthogonal to u_A. The eigenfrequencies will be obtained by substituting (B7) into (B5).

Before entering into calculations, we point out that the eigenfunctions can be classified into two groups. One group is that with a nonzero inhomogeneous term ($c \neq 0$) in (B7), and the other group is that with a zero inhomogeneous term ($c = 0$).

The inhomogeneous group ($c \neq 0$) is not needed for our purpose. Since $\chi_\perp^{(0)}$ satisfies (B8) the eigenfrequency can be easily evaluated. Let it be

$$(\zeta_B^{(0)} - \zeta_A^{(0)})/T = (\Omega_B + \omega S_B) - (\Omega_A + \omega S_A) .$$

The second term comes from the transformation factor in (B4). Substituting (B7) into (B5) one easily finds that Z should also have the same eigenfrequency if $c \neq 0$. The contribution of this class of frequencies to (B2) is altogether

$$\sum_{\{N_B\}} \prod_{B \neq A} \exp[-i(\zeta_B^{(0)} - \zeta_A^{(0)}) N_B] \tag{B9}$$

As has been shown in (A32), this is equal to K_0, and can be factored out by the normalization in (B1). In the following, therefore, we consider the homogeneous class solutions only.

Substituting explicit classical solutions (2.8) and (2.11) with the boundary condition (2.14) into (3.12) and (3.13) and using (B4) we find that

$$\begin{aligned} L^Q_{\rm st} &= \int_{\alpha_1}^{\alpha_2} d\alpha \left[\frac{\gamma}{2(1-\omega^2\rho^2)\omega} \{ (Z_{2\tau} + \omega Z_1)^2 - 2\omega^3 \rho\rho_\alpha [(Z_{1\tau} - \omega Z_2) Z_{2\alpha} - (Z_{2\tau} + \omega Z_1) Z_{1\alpha}] - \omega^2 Z_{2\alpha}{}^2 \} + \frac{\gamma}{2\omega} (Z_{3\tau}{}^2 - \omega^2 Z_{3\alpha}{}^2) \right] \\ &= \int_{\alpha_1}^{\alpha_2} d\alpha \left[\frac{\gamma}{2(1-\omega^2\rho^2)\omega} (Z_{2\tau}{}^2 - \omega^2 Z_{2\alpha}{}^2 - \omega^2 Z_2{}^2) + \frac{\gamma}{2\omega} (Z_{3\tau}{}^2 - \omega^2 Z_{3\alpha}{}^2) \right] + \frac{\omega\gamma}{2} \left[\frac{\omega^2 \rho\rho_\alpha}{(1-\omega^2\rho^2)} \left(Z_2{}^2 + Z_1{}^2 + \frac{2}{\omega} Z_1 Z_{2\tau} \right) \right]_{\alpha_1}^{\alpha_2} \end{aligned} \tag{B10}$$

for $\alpha_1 < \alpha < \alpha_2$, and

$$\begin{aligned} L^Q_q = {} & \frac{i}{2(1-\omega^2\rho^2)^{1/2}} \left(\bar{\chi} \not{e}^{(0)} \chi_\tau - \bar{\chi}_\tau \not{e}^{(0)} \chi - i \bar{\chi} \left\{ \Omega_A + \frac{\omega}{2} \sigma_3, \not{e}^{(0)} \right\} \chi \right) - m (1-\omega^2\rho^2)^{1/2} \bar{\chi}\chi \\ & + \frac{i}{2(1-\omega^2\rho^2)^{1/2}} \left(\bar{u}_A B \chi_\tau - \bar{\chi}_\tau B u_A - i \bar{u}_A \left\{ \Omega_A + \frac{\omega}{2} \sigma_3, B \right\} \chi - i \bar{\chi} \left\{ \Omega_A + \frac{\omega}{2} \sigma_3, B \right\} u_A \right) \\ & + m \frac{3\rho\omega}{(1-\omega^2\rho^2)^{1/2}} (Z_{2\tau} + \omega Z_1)(\bar{u}_A \chi + \bar{\chi} u_A) \\ & - \frac{1}{2(1-\omega^2\rho^2)^{1/2}} \Big\{ 2\Omega_A (\bar{u}_A \vec{W}\vec{\gamma} \chi + \bar{\chi} \vec{W}\vec{\gamma} u_A) - \omega Z_{3\tau} [(\bar{u}_A \gamma^3 \sigma^3 \chi) + (\bar{\chi} \gamma^3 \sigma^3 u_A)] \\ & \qquad + \frac{i\omega\rho}{(1-\omega^2\rho^2)^{1/2}} (Z_{2\tau} + \omega Z_1)(\bar{u}_A \chi_\tau - \bar{\chi}_\tau u_A) - i (\bar{u}_A \vec{W}\vec{\gamma} \chi_\tau - \chi_\tau \vec{W}\vec{\gamma} u_A) \Big\} \\ & + \frac{m}{(1-\omega^2\rho^2)^{1/2}} (Z_{1\tau} - \omega Z_2)^2 + \frac{m}{(1-\omega^2\rho^2)^{1/2}} Z_{3\tau}{}^2 + \frac{1}{(1-\omega^2\rho^2)^{3/2}} \left(m + \frac{\omega^3 \rho^2 S}{1-\omega^2\rho^2} \right) (Z_{2\tau} + \omega Z_1)^2 \end{aligned} \tag{B11}$$

for $\alpha = \alpha_i$ ($i = 1, 2$). Here

$$B = \not{W} + \frac{\omega\rho W^2}{1-\omega^2\rho^2} \not{e}^{(0)} , \tag{B12}$$

$$W^\mu = (0, Z_{1\tau} - \omega Z_2, Z_{2\tau} + \omega Z_1, Z_{3\tau}) .$$

Z_1 denotes the longitudinal mode, Z_2 the transverse mode parallel to the rotation plane of the classical solution, and Z_3 the transverse mode perpendicular to the rotation plane. Though all quantities (ρ, Z, χ, and u_A) in (B11) should have an extra index $i (= 1, 2)$ which indicates the end points of the string, that is suppressed to avoid notational complexity.

The diagonalization of $L^Q = L^Q_{\rm st} + \sum_{i=1}^2 L^Q_{q,i}$ is straightforward. Assuming that the eigenfrequency is $\nu\omega$, i.e.,

$$Z_j = \exp(-i\nu\omega\tau)G_j + \exp(i\nu\omega\tau)G_j^* \quad (j=1,2,3)\,, \tag{B13}$$

$$\chi = \exp(-i\nu\omega\tau)\chi^{(+)} + \exp(i\nu\omega\tau)\chi^{(-)}\,, \tag{B14}$$

we solve (B7) and (B5) provided $c=0$.

It is important to point out that the Lagrangian for the string part (B10) does not provide the equation of motion for the longitudinal mode Z_1 due to the massless property of the string. In fact one can eliminate Z_1 out of L_{st}^Q by adding a certain boundary term as (B10). The boundary term, combined with the quark term (B11), then provides us with equations of motion for $Z_{1i}(\tau) = Z_1(\sigma_i, \tau)$. Because of this reason, $Z_1(\sigma,\tau)$ for $\sigma_1 < \sigma < \sigma_2$ is not considered as a dynamical variable, and could have been eliminated if an adequate condition had been imposed from the outset. One can also confirm that even if one begins with the model with massive string[7] instead of (2.2), the same result will follow in the massless limit.

On solving (B5) we eliminate the boundary quantities χ_i by the use of equations of motion. The eigenvalue equations eventually turn out to be the following.

The type-2 mode.[11] For $\alpha_1 < \alpha < \alpha_2$,

$$\frac{d^2}{d\alpha^2}G_2 + 2\tan\alpha\frac{d}{d\alpha}G_2 + (\nu^2-1)G_2 = 0 \tag{B15}$$

with the boundary condition

$$G_{2\alpha}/G_2\big|_{\alpha=\alpha_i} = (-1)^i\frac{\omega}{\gamma|\cos\alpha_i|^3}(1-\nu^2) \times\left[(m_i\cos^2\alpha_i + \omega S_i)(1-\nu^2\cos^2\alpha_i - \frac{(m_i\cos^2\alpha_i + 2\omega S_i)\cos^4\alpha_i(1-\nu^2)\nu^2}{2(m_i\cos^2\alpha_i+2\omega S_i) - (m_i\cos^2\alpha_i + 3\omega S_i)\cos^2\alpha_i(1-\nu^2)}\right]. \tag{B16}$$

The type-1 mode. For $\alpha_1 < \alpha < \alpha_2$,

$$\frac{d^2}{d\alpha^2}G_3 + \nu^2 G_3 = 0 \tag{B17}$$

with the boundary condition

$$G_{3\alpha}/G_3\big|_{\alpha=\alpha_i} = (-1)^i\frac{\omega\nu^2}{\gamma|\cos\alpha_i|} \times\left(m_i + \omega\tan^2\alpha_i\frac{S_i}{1-\nu^2\cos^2\alpha_i}\right). \tag{B18}$$

Here α_i denotes the boundary value of the classical solution determined in (2.14) and classified by (2.17) and (2.18), and S_i denotes the spin of the quark at the ith end.

Exact solutions for (B17) and (B15) are known, i.e., for (B17),

$$G_3 \sim \begin{Bmatrix}\cos(\nu\alpha)\\ \sin(\nu\alpha)\end{Bmatrix} \tag{B19}$$

and for (B15),

$$G_2 \sim \begin{Bmatrix}\sin\alpha\cos(\nu\alpha) - \nu\cos\alpha\sin(\nu\alpha)\\ \nu\cos\alpha\cos(\nu\alpha) + \sin\alpha\sin(\nu\alpha)\end{Bmatrix} \text{ for } \nu\neq 0,1\,, \tag{B20}$$

$$G_2 \sim \begin{Bmatrix}\sin\alpha\\ \alpha\sin\alpha + \cos\alpha\end{Bmatrix} \text{ for } \nu = 0\,, \tag{B21}$$

$$G_2 \sim \begin{Bmatrix}1\\ 2\alpha + \sin(2\alpha)\end{Bmatrix} \text{ for } \nu = 1\,. \tag{B22}$$

Both (B17) and (B15) have exact eigenvalues 0 and 1. The zero solution for G_3, $\nu_a^{(p)} = \nu_0^{(3)} = 0$, and the unit solution for G_2, $\nu_a^{(p)} = \nu_1^{(2)} = 1$, are immediately understood by inspecting the right-hand sides of (B18) and (B16). The zero solution $\nu_0^{(2)} = 0$ for G_2 can be confirmed by substituting $\sin\alpha$ for G_2 in (B16) and 0 for ν,

$$\cot\alpha_i = (-1)^i\frac{\omega}{\gamma|\cos\alpha_i|^3}(m_i\cos^2\alpha_i + \omega S_i)\,. \tag{B23}$$

This is equal to the boundary condition (2.14) for α_i and is satisfied. Similar situation occurs for $\nu_1^{(3)}$.

In general, eigenfrequencies $\nu_a^{(p)}$, where $p=2$ and 3, and $a=0,1,2,\ldots$, must be calculated numerically from the boundary condition (B18) and (B16). We have obtained most of the necessary frequencies for phenomenological arguments Sec. IV.

Let us return to the path integral (B1). Now that the eigenfrequencies $\nu_a^{(p)}$ are obtained, (B1) can be written as

$$Q_{\{A_1,A_2\}}(T) = \sum_{\substack{l=1\\(\omega T=2\pi l)}}^{\infty} \exp(iS_{cl}) \int \mathfrak{D}a_a^{(p)} \left(\frac{i}{2\pi}\right)^{1/2} \left|\frac{\partial^2 S_{cl}}{\partial T^2}\right|^{1/2} \exp\left\{ i\int_0^T \sum_{p,a} \tfrac{1}{2}[(a_{a\tau}^{(p)})^2 - \omega^2 \nu_a^{(p)2}(a_a^{(p)})^2]d\tau + \text{const}\right\}$$

$$= \sum_{\substack{l=1\\(\omega T=2\pi l)}}^{\infty} \left(\frac{i}{2\pi}\right)^{1/2} \left|\frac{\partial^2 S_{cl}}{\partial T^2}\right|^{1/2} \sum_{\{N_a^{(p)}\}} \exp\left(-i\omega \sum_{a,p} \nu_a^{(p)} N_a^{(p)} T\right), \qquad \text{(B24)}$$

where $N_a^{(p)}$ denotes the occupation number of the mode of $\nu_a^{(p)}$, and 1 the revolution number of the classical rotator within the period T. The first two factors in (B24) appear from the zero-frequency modes. This is the formula (3.14).

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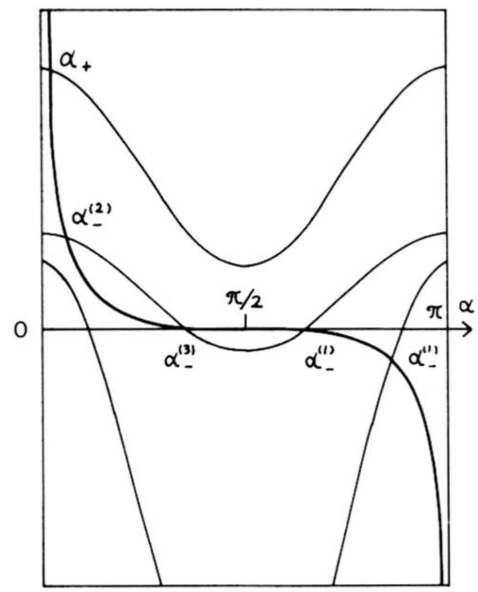

FIG. 1. The right-hand side of (2.15) is shown by the bold line, whose cross points with thin lines [the left-hand side of (2.15)] give the solutions $\alpha\pm^{(j)}$. The top and the second thin lines are, respectively, for $S=+\frac{1}{2}$ and $S=-\frac{1}{2}$. The bottom line shows a case with large ω for $S=-\frac{1}{2}$. The unit of vertical coordinate is arbitrary.

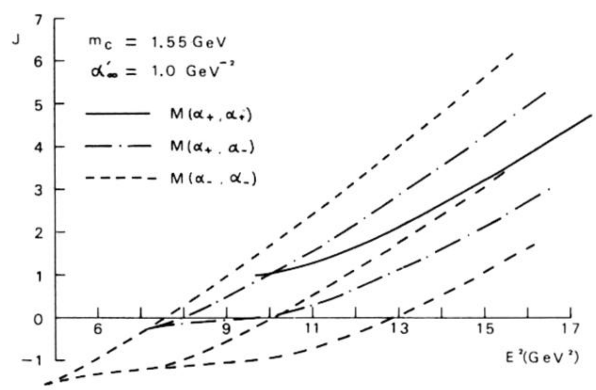

FIG. 2. Classical trajectories. Some almost degenerate trajectories along $M(\alpha_+, \alpha_-)$ and $M(\alpha_-, \alpha_-)$ are not shown.

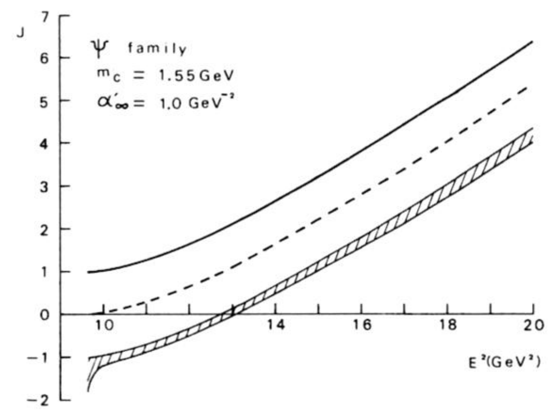

FIG. 3. Quantum corrections to the ψ/J trajectory. The broken line shows a trajectory with opposite parity to the leading one. Within the shaded area there are six trajectories, which separate themselves as J grows.