PHYSICAL REVIEW D VOLUME 18, NUMBER 7 1 OCTOBER 1978

Right-handed currents and strong interactions at short distances

M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov
Institute of Nuclear Physics, Novosibirsk 90, Union of Soviet Socialist Republic
(Received 5 October 1977)

The effects of the introduction of right-handed currents are considered for weak interactions of ordinary hadrons. In particular, we derive the expressions for effective Hamiltonians of weak nonleptonic interactions with $\Delta S = 1$, $\Delta S = 2$ and of weak radiative decays. The coefficients occurring in the operator expansion of the effective Hamiltonians are found, taking into account strong interactions at short distances in the frame of quantum chromodynamics. Strong interactions diminish the effect of right-handed currents. Our conclusion is that right-handed currents are not important for weak interactions of ordinary hadrons.

I. INTRODUCTION

The spectrum of elementary particles seems to be richer than one might think. Almost certainly, there is a new heavy lepton.[1] It is difficult to imagine that this lepton is not accompanied by at least one new quark and, moreover, there is a recent experimental result indicating the production of new resonances decaying into a $\mu^+\mu^-$ pair, possibly a new ψ-like system.[2]

There is no doubt that heavy particles enter the Hamiltonian of weak interactions and, therefore, our knowledge of weak interactions is incomplete. Moreover, new particles can bring new kinds of interactions, and many authors have introduced right-handed currents.[3]

While it is difficult to judge now the validity of the various models for heavy particles it seems possible to estimate the effect of new currents and of heavy virtual states on the weak interactions of ordinary hadrons. In the present paper we will study from this point of view the role of (hypothetical) right-handed currents in coupling light and heavy quarks. Our interest in the subject was motivated mostly by the suggestions[3-5] that the introduction of right-handed currents resolves such long-standing problems of weak interactions as the origin of the $\Delta I = \frac{1}{2}$ rule in the nonleptonic decays of strange particles and possible violation of SU(3) flavor symmetry in the $\Sigma^+ \to p\gamma$ decay.

In fact, it is not a trivial matter to introduce new interactions in a consistent way. There are at least two reasons for this. First of all, the experimental data on the properties of strange particles are rich enough and, in some cases, put severe constraints on the structure of weak interactions. In particular, the $K_L - K_S$ mass difference is quite sensitive to the models of weak interactions. Secondly, an exchange of a heavy quark is a short-distance process, and we now have a reliable theory of strong interactions at short distances, i.e., quantum chromodynamics. Therefore, the effect of the introduction of new interactions with heavy quarks is not obscured by a lack of understanding of strong interactions.

We will exploit quantum chromodynamics (QCD) to calculate the effect of strong interactions at small distances on the Hamiltonian of weak nonleptonic interactions with right-handed currents. According to QCD the effective coupling constant of strong interactions is small at short distances,[6] and this makes the effect calculable. At large distances, i.e., at the distances of the order of radius of confinement, the interaction becomes indeed strong. For such distances one relies on some conventional phenomenological model of strong interactions, such as the quark model, to get an idea of the role of strong interactions.

The gluon exchanges at short distances can modify the weak Hamiltonian in a nontrivial way. In particular, for left-handed currents it was shown that strong interactions exhance the $\Delta I = \frac{1}{2}$ piece of the Hamiltonian and suppress the $\Delta I = \frac{3}{2}$ transitions.[7] Moreover, if one accounts for a new mass scale introduced by charmed quarks, then there arise new operators in the Hamiltonian.[8] Both observations can provide a key to the understanding of the $\Delta I = \frac{1}{2}$ rule.

The effects of strong interactions at short distances, in the case of right-handed currents, have been considered in a number of papers. In particular, in Refs. 3(c), 9, and 10 anomalous dimensions of transition operators relevant to $\Delta I = \frac{1}{2}$ have been found. The problem has not been investigated in full, however, mostly because of the technical difficulties. In particular, to account for the operator mixing, one has to evaluate the two-loop graphs in some cases.

The main purpose of the present paper is to derive the effective Hamiltonian of weak interactions for the $\Delta S = 1$, $\Delta S = 2$ transitions and for weak radiative decays within the models with right-handed currents. The results for $\Delta S = 1$ transitions were given by us in a letter[11] earlier and here we present some further details of derivations and discuss the correspondence with the results of other authors.

The method of evaluation of the effects of strong interactions at short distances is quite traditional in essence and makes use of the renormalization-group equations for summation of the logarithmic terms arising due to the gluon exchanges at short distances. We sum up explicitly the leading powers of $\ln m_W$, $\ln m_\sigma$, and $\ln m_c$, where m_W, m_σ, and m_c are the masses of the W boson, Higgs boson, and heavy (charmed) quark, respectively. All these masses are considered to be large in the mass scale of ordinary hadrons. We will present, however, a new formulation of the renormalization-group method which is useful for the purpose and can be helpful in other occasions as well.

Our main result is that strong interactions at short distances suppress the contribution of right-handed currents into the decays of ordinary hadrons. The result is by no means trivial since operators with rather high *positive* anomalous dimension are present in the expansion of the effective Hamiltonian. It turns out, however, that these operators enter with small numerical coefficients (such as $\frac{1}{19}$ for the transitions).

Numerically, the suppression factor varies from one kind of process to another. For weak radiative decays it ranges between $\frac{1}{3}$ and $\frac{1}{10}$ for reasonable choices of the parameters. For transitions with $\Delta S = 1$, the suppression factor is about 0.75. These results seem to indicate that right-handed currents do not play an essential role in weak interactions of ordinary hadrons.

For the $\Delta S = 2$ the situation is more complicated as far as comparison with experimental data is concerned. The point is that the result for the K_L-K_S mass difference depends rather heavily on the details of the models of weak interactions which are difficult to clarify at present. In particular, we will argue that the contribution of the box graph [see Fig. 1(a)] with W bosons and auxilary $\psi^\pm$ scalars in the intermediate state is not gauge invariant by itself. The gauge-dependent terms are canceled by gauge-dependent terms in correction to a single-Higgs-boson exchange [see Figs. 1(b) and 1(c)].

Thus, any estimate of the K_L-K_S mass difference cannot be performed consistently without making explicit assumptions on the structure of the Higgs sector of the model (this quite unusual situation was overlooked by the authors who made the estimates earlier[3b,3c]). Let us notice that the complexity arising in the Higgs sector of the models with right-handed currents was recognized earlier by Llewellyn Smith[12] for the currents coupling light quarks among themselves and in recent papers[13]

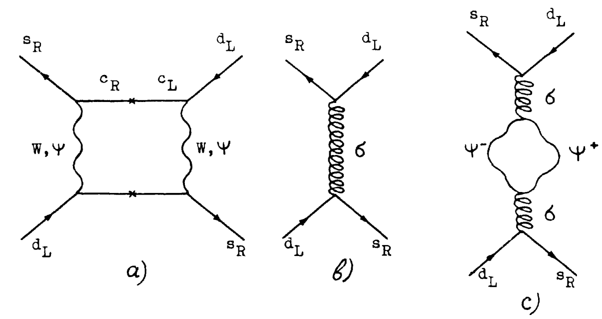

FIG. 1. Graphs describing the bare Hamiltonian of $\Delta S = 2$ transitions: (a) box graph with two-W exchange, (b) single Higgs-boson exchange, (c) radiative correction to graph (b). Cross denotes c-quark mass insertion, the wavy lines stand for W boson and unphysical Higgs scalars $\psi^\pm$, the curvy lines correspond to the physical Higgs boson.

concerning the currents coupling light and heavy quarks.

Our final expression for the K_L-K_S mass difference is a sum of several terms of opposite signs each of which is two orders of magnitude larger than the experimental number. Within the accuracy of the method used it is not possible to rule out that the cancellation between the various terms is practically complete so that the theoretical prediction for ΔM_{LS} is in no contradiction with experimental data. The reader who is reluctant to accept such a possibility would conclude that a coupling constant of right-handed currents must be small (of the order of $\frac{1}{10}$ of the Fermi coupling constant of weak interactions). This result strengthens the conclusion made above that right-handed currents do not play a major role in weak interactions of strange particles.

The procedure is as follows. In Sec. VIII the sent general remarks on the form of right-handed currents and discuss difficulties arising in the Higgs sector of the models. In Sec. III the notions of effective Hamiltonians and renormalization-group equations are introduced to sum up a power series of the logarithm of a large internal mass. The method is used in Secs. IV and V for construction of effective Hamiltonians of $\Delta S = 1$ transitions. Bare and effective Hamiltonians of $\Delta S = 2$ transitions are found in Secs. VI and VII. In Sec. VIII the effective Hamiltonian of weak radiative decays is presented. Section IX contains numerical estimates.

II. GENERAL REMARKS ON THE MODELS WITH RIGHT-HANDED CURRENTS

Before proceeding with a calculation of the short-distance effects, let us outline the models with right-handed currents. In particular, we would like to emphasize some difficulties existing in these models in the Higgs sector.

It is convenient to introduce left- and right-handed components of the quark fields

$$q_{L,R}=\tfrac{1}{2}(1\pm\gamma_5)q \tag{1}$$

and separate their contribution into the weak charged currents

$$j_\mu=j_\mu^L+j_\mu^R\,.$$

We will accept the following form of the currents j_μ^L and j_μ^R:

$$\begin{aligned} j_\mu^L&=\bar{s}_L\gamma_\mu(u_L\sin\theta_C+c_L\cos\theta_C)\\ &\quad+\bar{d}_L\gamma_\mu(u_L\cos\theta_C-c_L\sin\theta_C)\,,\\ j_\mu^R&=\sin\varphi\bar{s}_R\gamma_\mu c_R\,, \end{aligned} \tag{2}$$

where s, d, u are the fields of light quarks, c is a charmed quark, θ_C is the Cabibbo angle, and $\sin\varphi$ is a new parameter describing the coupling constant of right-handed currents.

Although Eq. (2) is not the most general one, it incorporates the models which are interesting from the practical point of view. In particular, we have not included the term $\bar{d}_R\gamma_\mu c_R$. Such a term violates the condition[14]

$$[V^i-A^i,H(\Delta S=1)]=0,\quad (i=1,2,3)\,, \tag{3}$$

where V^i-A^i are the generators of the $SU(2)_R$ group. The validity of Eq. (3) is confirmed by the success of the current-algebra predictions for $\Delta I=\frac{1}{2},\frac{3}{2}$ amplitudes in the decays $K\to 2\pi, 3\pi$. Moreover, the $\bar{d}_R\gamma_\mu c_R$ current would result in an unacceptiably large K_L-K_S mass difference.[3c] Therefore, the $\bar{d}_R\gamma_\mu c_R$ current, if it exists, cannot be important.

It is worth noting that $\sin\varphi\neq 1$ only if the number of quarks is larger than four. In the latter case there are some further terms in the currents but these terms do not manifest themselves in the processes under consideration and we have omitted them. Even if the number of quarks exceeds four, $\sin\varphi$ does not necessarily differ from 1. Thus, in the six-quark model proposed in Ref. 3b, $\sin\varphi=1$.

Currents (2) result in the following bare Hamiltonian of the $\Delta S=1$ transitions:

$$\begin{aligned} H^{(0)}(\Delta S=1)&=G\sqrt{2}\sin\theta_C\cos\theta_C(\bar{s}_L\gamma_\mu u_L\bar{u}_L\gamma_\mu d_L\\ &\quad-\bar{s}_L\gamma_\mu c_L\bar{c}_L\gamma_\mu d_L)\\ &\quad-G\sqrt{2}\sin\varphi\sin\theta_C\bar{s}_R\gamma_\mu c_R\bar{c}_L\gamma_\mu d_L\,. \end{aligned} \tag{4}$$

As for the $\Delta S=2$ transitions, they are induced both by two-W-boson and single-Higgs-boson exchanges (Fig. 1). Moreover, the Higgs-boson exchange is the leading one as far as counting in the weak-interaction coupling constant is concerned.

Existence of the coupling of the Higgs meson with a strangeness-changing source is specific for the models with right-handed currents. To substantiate the point, let us consider the structure of doublets with respect to the SU(2) group of weak interactions

$$L_1=\begin{pmatrix} u_L\\ d_L\cos\theta_C+s_L\sin\theta_C\end{pmatrix},$$

$$L_2=\begin{pmatrix} c_L\\ -d_L\sin\theta_C+s_L\cos\theta_C\end{pmatrix}, \tag{5}$$

$$R=\begin{pmatrix} c_R\\ s_R\end{pmatrix},$$

where we assumed that $\sin\varphi=1$ and omit the doublets constructed from heavy-quark fields alone.

The mass of the charmed quark arises because of the spontaneous symmetry breaking and the appearance of the vacuum expectation value of the Higgs field. The simplest multiplet of the Higgs mesons which is necessary for an introduction of mass is a triplet. Then the term responsible for quark masses is of the following form:

$$\begin{aligned} &-\frac{1}{2}m_c\left(\frac{1}{\langle\psi^3\rangle_0}\bar{R}\psi\tau L_2+\bar{R}L_2+\text{H.c.}\right)\\ &=-m_c\bar{c}c-g_W\frac{m_c}{\sqrt{2}m_W}[\psi^+\bar{c}_R(-d_L\sin\theta_C+s_L\cos\theta_C)\\ &\qquad+\psi^-\bar{s}_Rc_L+\text{H.c.}]\\ &\quad-g_W\frac{m_c}{2m_W}\sigma[\bar{c}_Rc_L-\bar{s}_R(-d_L\sin\theta_C+s_L\cos\theta_C)+\text{H.c.}]\,, \end{aligned} \tag{6}$$

where $\langle\psi^3\rangle_0=m_W/g_W$ is the vacuum expectation value of the neutral component of the Higgs field, and the σ field is defined as a deviation of ψ^3 from its vacuum expectation value, $\psi^3=\langle\psi^3\rangle_0+\sigma$.

It follows immediately from Eq. (6) that there exists a strangeness-changing neutral transition $s\to d\sigma$ with a well-defined coupling constant. Although Eq. (6) is model dependent, the conclusion on the existence of strangeness-changing coupling is more general. Moreover one can argue that such a coupling exists if the right-handed currents are responsible for the $\Delta I=\frac{1}{2}$ rule in weak nonleptonic interactions (see also Sec. VI).

It is worth emphasizing that the $\sigma\bar{s}d$ coupling is of the first order in g_W. Then the K_L-K_S mass difference arises, generally speaking, in the second order in g_W which corresponds to the first order in the Fermi coupling constant G. Therefore, the K_L-K_S mass difference is too large, in violent disagreement with experimental data.

There are three ways out of this difficulty: (i)

The mass of the Higgs meson is very large so that the coupling constant induced by the Higgs-meson exchange, g_W^2/m_σ^2, is of a magnitude comparable to the contributions of the fourth order in g_W. (ii) There are several Higgs fields and their coupling constants and masses satisfy such conditions that graphs of the type represented in Fig. 1(b) cancel each other.[12] (iii) The coupling constants (mixing angles) and masses of quarks are organized in such a way that $\sigma\bar{s}d$ coupling does not arise.[15]

If the possibility (iii) is chosen by nature, then the right-handed currents cannot be responsible for the $\Delta I = \frac{1}{2}$ rule or for the observed weak radiative decays (see Sec. VI). Possibilities (i) and (ii) mentioned above can save the idea on the dominating role of the right-handed currents but they do not look attractive by themselves. In case (i), increasing the mass of the Higgs boson results in the increase of the coupling constant of the self-interaction of Higgs scalars. Practically, we need an introduction of such a mass of the Higgs field that the perturbation theory breaks down. Case (ii), on the other hand, assumes quite a special organization of the Higgs-meson self-interaction for which there is not apparent reason.

While performing explicit calculations we will accept possibility (i), although all the results can be reformulated in a trivial way to case (ii).

Thus we see that consideration of the bare quark graphs provide arguments against the leading role of the right-handed currents. Further indications in the same direction come from a consideration of the effects of strong interactions at short distances which will be given in the subsequent sections.

III. LIMIT OF A LARGE INTERNAL MASS AND THE RENORMALIZATION GROUP

We will study the effective Hamiltonian of weak interactions of light quarks induced by exchanges of W bosons. The effective Hamiltonian arises after accounting for the strong-interaction effects at short distances in the bare Hamiltonian which, in the case of the $\Delta S = 1$ transitions, is given by a simple product of currents. Throughout this paper we assume the validity of quantum chromodynamics, i.e., we assume that the strong interactions are due to the exchanges of an octet of color gluons.

Thus there exist different scales associated with the mass of a W boson m_W and a characteristic hadron mass m. Moreover, there are heavy quarks in virtual states which bring a new mass scale m_c. We will assume that the masses satisfy the condition

$$m_W^2 \gg m_c^2 \gg m^2 \tag{7}$$

and will sum up the leading powers of the $\ln(m_w/m_c)$ and $\ln(m_c/m)$. For the mass of the W boson we accept the estimates common for the gauge theories of weak interactions, $m_w \sim 70-100$ GeV. The mass of the charmed quark is about 2 GeV. As for the typical hadronic mass, the estimates vary between

$$m^2 \sim m_\pi^2 \sim 0.02 \text{ GeV}^2 \text{ and } m^2 \sim m_\rho^2 \sim 0.5 \text{ GeV}^2 . \tag{8}$$

The former estimate corresponds to the proton electric radius while the latter one is traditional. By hadronic mass m it is convenient to understand the point where the effective coupling constant of strong interactions is of order unity. The choice of the low normalization point $m \sim m_\pi$ is supported by, say, analysis of the hadronic decays of ψ mesons which indicate that the effective coupling constant α_s is small, $\alpha_s(m_\psi) \sim 0.2$.[16] The coupling constant α_s can be determined, in principle, from the data on the e^+e^- cross section or from an analysis of the deep-inelastic scattering. At present, the experimental data, to our mind, are not accurate enough to provide for such a possibility.

Condition (7) corresponds to a choice of a low normalization point. Even if this is not true, we feel that it is useful to derive dependence on m in a self-consistent way.

We will adopt the method of summation of ln terms which is based on the introduction of an effective Hamiltonian. In the case of left-handed currents dependence on m_W was found first in Ref. 7, while summation of $\ln m_c$ terms was performed first in Refs. 17 and 8. The heavy-quark expansion was discussed in recent papers from a formal point of view.[18]

Consider first the case of a renormalized theory with one of the fields having mass M much larger than the others so that there exist two mass scales. Moreover, we are interested in the processes involving only light particles of low momenta, $p \ll M$. The presence of heavy particles can be described then by means of the effective Hamiltonian.

The effective Hamiltonian, unlike the fundamental one, does not contain the fields of heavy particles at all. Heavy particles enter only through loop graphs which induce new (quasi-) local vertices among the light particles. A well-known example of an effective Hamiltonian is a four-fermion weak interaction. It is commonly believed that this interaction is due to the W-boson exchange and is not a fundamental one. Nevertheless, it is adequate to describe weak interactions at low energies. The mass of a W boson determines the strength of the four-fermion interaction.

The coupling constants induced by virtual heavy particles tend to zero as mass M tends to infinity.

In the tree approximation, i.e., for graphs without loops, the corresponding vertices can be readily found and are proportional to M^{-n}. In the next orders in strong interactions there arise terms of order $M^{-n}(\ln M)^K$ and so on.

Introduction of effective Hamiltonians provides an easy way for summation of the ln terms. The procedure is as follows: Let us denote by g_i the constants giving the weight of various operators O_i in the operator expansion of the effective Hamiltonian. The operators are normalized in such a way that at some Euclidean point μ their matrix elements coincide with matrix elements of noninteracting fields. In other words, the generalized charges $g_i(\mu)$ are determined by the one-particle irreducible Green's functions at external momenta $p^2=-\mu^2$. Determination of $g_i(\mu)$ requires accounting for all the interactions at distances less than $1/\mu$. Therefore, the effective Hamiltonian can be treated as an ordinary one provided that all the integrations over the virtual momenta are cut off from above at $p=\mu$. The effects of strong interactions at distances larger than $1/\mu$ are accounted for in the matrix elements of operators $O_i(\mu)$.

The generalized charges g_i can be found as functions of μ by solving the renormalization-group equations which follow from the fact that physical amplitudes are independent of the choice of the normalization point μ. In other words, the change in the normalization point μ is equivalent to some change in the charges g_i. In this way we come to a set of differential equations of the form

$$\mu\frac{dg_i(\mu)}{d\mu}=\beta_i(g_1(\mu), g_2(\mu), \ldots), \tag{9}$$

where functions β_i depend on charges g_K but not on the point μ. Equation (9) assumes that all the charges are chosen to be dimensionless. This can always be achieved by introducing a proper power of μ into the definition of $g_i(\mu)$.

Using the perturbation theory in the effective Hamiltonian one can find the coefficients of expansion of functions β_i in powers of charges g_K. Indeed, the perturbation theory makes it possible to find the relations between the charges normalized at $p^2=-\mu^2$ and at $p^2=-(\mu+\Delta\mu)^2$. Expanding these relations in $\Delta\mu$, one can find explicit expressions for the functions β_i.

The charges induced by heavy-particle exchanges are of order M^{-n} and small. Usually all the consideration is confined to a certain order in $1/M$ and there is no difficulty to find the number of terms in the power expansion series in any charge g_K which must be kept explicit in the approximation considered.

There are some charges which are present in the fundamental Hamiltonian and do not depend on the large mass M. An example of this kind is the quark-gluon coupling constant. Use of perturbation theory in these charges is justified if smallness of the charges can be inferred in some way. In particular, in quantum chromodynamics the smallness of $\alpha_S(\mu)$ is a consequence of the asymptotic freedom of the theory provided that μ is large enough.

Thus far we have not used the renormalizability of the theory, and Eqs. (9) are valid in a nonrenormalizable theory as well, provided all possible independent operators are listed and included into the effective Hamiltonian. The renormalizability is invoked to define the initial conditions for the differential equations (9), i.e., the renormalizability guarantees that at $\mu\sim M$ the charges $g_i(\mu)$ can be expressed in terms of the coupling constant and masses entering the fundamental Hamiltonian. All the infinities encountered in the course of evaluation of $g_i(\mu\sim M)$ can be absorbed into renormalization of a finite number of terms in the Lagrangian and, moreover, there is no large ln term left.

Thus explicit expressions for functions $\beta_i(g)$ follow from the consideration of effective Hamiltonians, while initial conditions at $\mu\sim M$ depend on the initial theory.

In the literature the renormalization-group equations are mostly used in the Callan-Symanzyk form, i.e., in a form of linear partial differential equations. The generalized Gell-Mann-Low type equations (9) are more convenient and applicable in a nonlinear case such as many current amplitudes. Examples close to our approach can be found in Refs. 18 and 19.

If there is a sequence of masses such that $M_1 \gg M_2 \gg M_3 \gg \ldots$, it is convenient to introduce a sequence of effective Hamiltonians in such a way that the first one refers to the range of virtual momenta $M_1\gg\mu\gg M_2$, the second one can be viewed as a Hamiltonian for $M_2\gg\mu\gg M_3$, and so on. In the first region all the fields except for that of mass M_1 are considered as light, in the second region the field of mass M_2 is eliminated as well, and so on. Again, the procedure is especially helpful for the purpose of summation of the leading ln terms in each of the regions. In the following sections, we will give explicit examples of Eqs. (9).

Let us add some remarks on the choice of the set of independent operators O_i and corresponding charges g_i. First, there is no need to include operators which can be represented as a total derivative. The matrix elements of such operators vanish identically as far as all the particles are on mass shell and energy momentum conservation is respected. Since in the end only such matrix elements are of interest, we can disregard operators which are total derivatives. It is important that such operators are mixed by strong interactions only among themselves. Secondly, the set of independent operators can be somewhat reduced by using the equations of motion. There is no need to keep operators which are related to each other by equations of motion.

Moreover, operators O_i must be gauge invariant, and one can omit operators containing ghost fields. Let us present the arguments, disregarding for a moment the infrared problem which is inherent for QCD. Only gluons with transverse polarizations have physical meaning and all the operators can contain only the gluon field strength tensor $b^a_{\mu\nu}$ but not the potentials b^a_μ. The only delicate point here is that in practical calculations it is more convenient to use the covariant gauge conditions which imply introduction of the ghost fields. This seems to be a technical rather than a principle problem. Indeed, all the results of calculations can be reproduced in a ghost-free gauge and since the final answer is gauge invariant, any gauge can be used for intermediate calculations. Introduction of ghost fields for computational purposes does not imply the necessity of consideration of operators containing the ghost fields. If one still decides to do this, one must also take into account gauge-noninvariant operators. The combined effect on gauge-invariant operators would vanish.

The infrared cutoff introduced in QCD does not spoil gauge invariance, since we keep it only in ln terms $\ln^n m$ and consistently neglect powers of m.

Of course, if the set of operators includes some extra terms which can be eliminated following the line of reasoning outlined above, it does not mean that the final answer will be wrong. It is correct as far as all the calculations are made in a correct way. The only point is that one can economize the effort by eliminating unnecessary operators.

IV. EFFECTIVE HAMILTONIAN OF THE $\Delta S=1$ TRANSITIONS

As an application of the general method we will consider here a derivation of the effective Hamiltonian of the $\Delta S=1$ transitions in the models with right-handed currents of the form (2).

To this end let us enumerate first all the relevant generalized charges g_i. The strong interactions are described by the effective coupling constant $\alpha_s(\mu)$. The Hamiltonian depends also on the quark masses which we will renormalize at point μ and consider as some generalized charges. It is reasonable to neglect the masses of light quarks since they are not large compared to the inverse radius of confinement. Moreover, we will assume that all the heavy quarks have comparable masses and introduce explicitly only the mass of charmed quark $m_c(\mu)$. In fact, one must keep dependence of m_c on μ only for $m_W \gg \mu \gg m_c$. For the range of $m_c \gg \mu$ the mass operator does not contain ln terms and m_c can be considered as a constant.

The Hamiltonian of nonleptonic weak interactions can be represented as a sum over independent operators of dimension six

$$H(\Delta S=1)=\sum c_i O_i\,, \tag{10}$$

where local operators O_i are constructed from quark and gluon fields and the coefficients c_i are of order of the Fermi coupling constant of weak interactions G. Operators of higher dimension enter the expansion of the Hamiltonian with coefficients which are proportional to extra powers of m_W^{-2} and are negligible for this reason.

An explicit form of expansion (10) is different in the regions $m_W \gg \mu \gg m_c$ and $m_c \gg \mu \gg m$. In the former case one gets

$$H(\Delta S=1)=G\sqrt{2}\,\sin\psi\,\sin\theta[c_{B_1}B_1+c_{B_2}B_2+c_T T]\,, \tag{11}$$

where

$$B_1=\tfrac{4}{3}\bar{s}_R d_L \bar{c}_L c_R\,,\quad B_2=2\bar{s}_R t^a d_L \bar{c}_L t^a c_R\,,$$
$$T=\frac{ig(\mu)}{16\pi^2}m_c \bar{s}_R \sigma_{\mu\nu} t^a d_L b^a_{\mu\nu}\,. \tag{12}$$

Here $b^a_{\mu\nu}$ $(a=1\ldots 8)$ is the gluon field strength tensor, t^a are the Gell-Mann SU(3) matrices acting in the color space and normalized by condition $\mathrm{Tr}\{t^a t^b\}=2\delta^{ab}$, and g is a quark-gluon coupling constant, $g^2/4\pi=\alpha_S$.

The general form of the Hamiltonian (11) follows from the explicit constructing of all the possible operators. It is worth noting that we have not included into the list operators arising in the models with left-handed currents alone. For a discussion of these operators see Refs. 7 and 8.

Thus, the differential equations (9) in the region $m_W \gg \mu \gg m_c$ take the form

$$\mu\frac{d\alpha_s(\mu)}{d\mu}=-\frac{b}{2\pi}\alpha_s^2(\mu)\,,$$
$$\mu\frac{dm_c(\mu)}{d\mu}=-\delta\frac{\alpha_s(\mu)}{2\pi}m_c(\mu)\,, \tag{13}$$
$$\mu\frac{dc_i(\mu)}{d\mu}=-\frac{\alpha_s(\mu)}{2\pi}\gamma_{ij}c_j(\mu)\,,$$

where $b=11-2/3N$, N being the number of flavors, $\delta=-4$ is the anomalous dimension of the mass operator, and we kept only lowest-order terms in the expansion of functions β_i. The equations are by no means new. Their solutions look as follows (see, e.g., review in Ref. 19):

$$\alpha_s(\mu)=\frac{\alpha_s(\mu_0)}{1+b\alpha_s(\mu_0)/4\pi\ln\mu^2/\mu_0{}^2},$$
$$m_c(\mu)=m_c(\mu_0)\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{8/b}, \qquad (14)$$
$$c(\mu)=\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\gamma/b}c(\mu_0),$$

where matrix notations are used; γ is matrix γ_{ij} and c is the column c_i. Positiveness of the coefficient b is a manifestation of the asymptotic freedom.

As far as computational work is concerned, the central problem lies in an explicit evaluation of the γ_{ij} and we will present the results obtained below.

In the region $m_c \gg \mu \gg m$ the renormalization-group equations for the coefficients of the operator expansion look simpler. The reason is that in this region one must eliminate from consideration the operators containing the heavy-quark fields. Thus, we have in this region

$$\mu\frac{d\alpha_s(\mu)}{d\mu}=-\frac{b}{2\pi}\alpha_S{}^2(\mu),$$
$$\mu\frac{dc_T(\mu)}{d\mu}=-\left(\gamma_{\tilde{T}}+\frac{b}{2}\right)\frac{\alpha_S(\mu)}{2\pi}c_T(\mu), \qquad (15)$$

where $\gamma_{\tilde{T}}$ refers to the anomalous dimension of the operator $\tilde{T}=\bar{s}_R\sigma_{\mu\nu}t^a d_L b^a_{\mu\nu}$ and $\frac{1}{2}b$ arises due to including of $g(\mu)$ in the definition of T.

The only computation needed to find an explicit form of the renormalization-group equations (15) is that of the anomalous dimensions $\gamma_{\tilde{T}}$ of operator $\tilde{T}$. The latter is given by the one-loop graphs of Fig. 2 and can be readily found:[9-11]

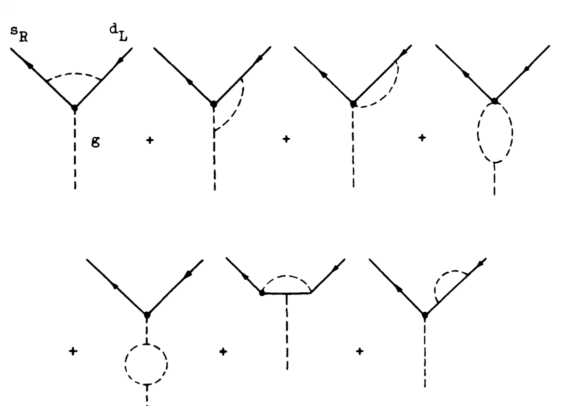

FIG. 2. Graphs relevant to computation of anomalous dimension of operator T (crossed graphs are not shown). Dashed line corresponds to gluon. Bold-faced point denotes operator T.

$$\gamma_{\tilde{T}}=-\tfrac{2}{3}+\tfrac{1}{2}b. \qquad (16)$$

Let us emphasize that, in the region of μ considered, the anomalous dimension of the operator T does not include anomalous dimension of mass $m_c(\mu)$. The reason was in fact already mentioned above: There are no ln terms in the mass operator at momenta $p< m_c$. For the same reason the value of b in Eq. (15) is fixed and equal to nine, independently of the number of flavors.

Before solving the renormalization-group equations explicitly let us summarize the whole procedure. In estimating the contribution of the right-handed currents into the low-energy decays one can consider only the operator T. All the other operators contain the charmed-quark fields and can be neglected. This corresponds to the smallness of heavy-quark admixtures in ordinary hadrons. Moreover, since heavy quarks appear only at short space and time intervals of the order $1/m_c$, the effect of heavy virtual quarks can be explicitly found within QCD since the effective coupling constant of strong interactions is small at short distances. In other words, heavy virtual states determine the value of coefficient c_T. The way of finding this, starting from the bare Hamiltonian, is rather lengthy: first we must solve renormalization-group equations (13) in the region $m_W \gg \mu \gg m_c$ using the bare Hamiltonian to fix the initial conditions, and then we must solve the differential equations (15) in the region $m_c \gg \mu \gg m$ using the results obtained in the region $\mu \gg m_c$ to fix the initial conditions.

V. SOLVING THE RENORMALIZATION-GROUP EQUATIONS FOR THE $\Delta S=1$ HAMILTONIAN

The bare Hamiltonian of the $\Delta S=1$ transition is given by the current product and fixes the values of the coefficients c_i in expansion (11) at $\mu\sim m_W$. Using the Fierz transformation one can convince oneself that Eq. (4) corresponds to

$$c_{B_1}(\mu\sim m_W)=1,\quad c_{B_2}(\mu\sim m_W)=1. \qquad (17)$$

To zero order in g, operator T does not appear. The lowest-order graph for operator T is presented in Fig. 3 and the corresponding value of c_T is equal to [remember that $g(\mu)$ is included in the definition of operator T]

$$c_T^{(1)}=1. \qquad (18)$$

Equations (17) and (18) represent the initial conditions for Eqs. (13). To get the matrix γ one must calculate the coefficients at ln terms arising in higher orders in α_S. In particular, diagonal matrix elements correspond to anomalous dimensions of operators B_1, B_2, and T. The anomalous dimensions of B_1 and B_2 are determined by the

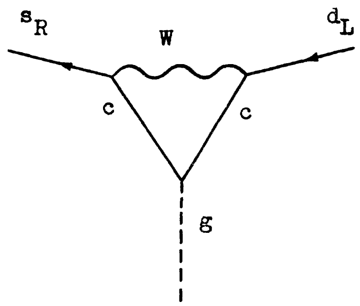

FIG. 3. Lowest-order graph giving rise to operator T.

graphs of Fig. 4 and equal to[3c]

$$\gamma_{11}=\gamma_{B_1}=8\ ,\ \gamma_{22}=\gamma_{B_2}=-1\ . \tag{19}$$

For the operator T one must sum up the dimensions of $\tilde{T}=\bar{s}_R\sigma_{\mu\nu}t^a d_L b^a_{\mu\nu}$, $g(\mu)$, and $m_c(\mu)$, since we included $g(\mu)$ and $m_c(\mu)$ in the definition of T.

$$\gamma_{33}=\gamma_T=\gamma_{\tilde{T}}-\tfrac{1}{2}b+\delta=(-\tfrac{2}{3}+\tfrac{1}{2}b)-\tfrac{1}{2}b-4=-\tfrac{14}{3}\ . \tag{20}$$

The meaning of nondiagonal matrix elements of γ is that they determine the mixing of operators. For example, γ_{31} corresponds to the arising of T from B_1 in the bare Hamiltonian. It is important that to find ln terms in the coefficient c_T at operator T one needs explicit calculations of two-loop graphs, which are represented in Fig. 5. After lengthy calculations of terms $g^3\ln\Lambda$ we get for mixing parameters

$$\gamma_{31}=\tfrac{2}{3}\ ,\ \gamma_{32}=\tfrac{5}{3}\ . \tag{21}$$

Thus, the matrix of anomalous dimensions has the form

$$\gamma=\begin{pmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ \frac{2}{3} & \frac{5}{3} & -\frac{14}{3} \end{pmatrix}. \tag{22}$$

Using the initial conditions [Eqs. (17) and (18)] and Eq. (14) we get

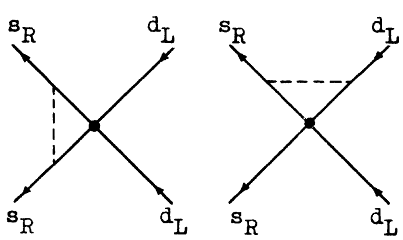

FIG. 4. Graphs contributing to the anomalous dimension of operators B_1, B_2.

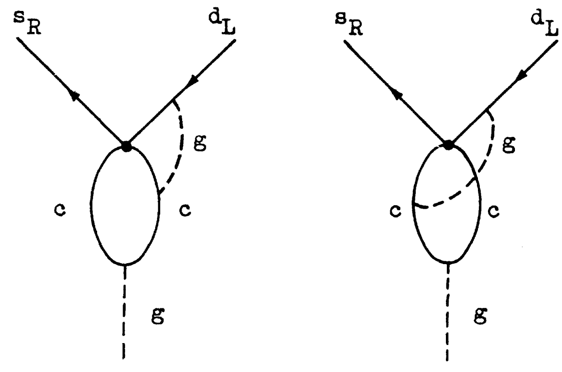

FIG. 5. An example of the graph relevant to computation of mixing of operators $B_{1,2}$ and T. Bold-faced point denotes the vertex induced by operators $B_{1,2}$.

$$c_{B_1}(\mu)=x^{8/b},\ c_{B_2}(\mu)=x^{-1/b}$$
$$c_T(\mu)=x^{-14/3b}\left[1+\tfrac{1}{19}(x^{38/3b}-1)+\tfrac{5}{11}(x^{11/3b}-1)\right]\ , \tag{23}$$

where $x=\alpha_S(\mu)/\alpha_S(m_W)$. To compute the matrix power we used the basis where matrix γ is diagonal.

In the region $\mu<m_c$, only operator T survives (see discussion in the previous section) and we obtain for $c_T(m)$ the following expression:

$$c_T(m)=x_2^{-2/27}x_1^{-14/3b}\left[1+\tfrac{1}{19}(x_1^{38/3b}-1)+\tfrac{5}{11}(x_1^{11/3b}-1)\right]\ , \tag{24}$$

where

$$x_1=\frac{\alpha_S(m_c)}{\alpha_S(m_W)}\ ,\ x_2=\frac{\alpha_S(m)}{\alpha_S(m_c)}\ . \tag{25}$$

Equation (24) represents our final result for the effective Hamiltonian of the $\Delta S=1$ transitions. To get an idea of the contributions of the right-handed current into observed decays, one must complement the derivation of the effective Hamiltonian by some model for the matrix element of operator T. In evaluating the matrix element, only the contribution of distances of order m^{-1} must be taken into account.

To conclude this section, let us compare the results presented above and first published in our letter[11] with the conclusions on the same subject by other authors. As was already mentioned, the first calculation of the anomalous dimension of operators $B_{1,2}$ entering the bare Hamiltonian in the models with right-handed currents was performed in Ref. 3c. It was emphasized by these authors that the anomalous dimension of operator B_1 is large and positive. It was conjectured on this ground that strong interactions enhance the contribution of right-handed currents. In our opinion, the matrix elements of operators $B_{1,2}$ are small since they contain the fields of the heavy quark. A consistent way to account for the operators $B_{1,2}$ is to calculate their mixing with operator T constructed from the fields of gluons and light quarks. In this sense the large anomalous dimension of operator B_1 is still relevant to the problem. Our result, however, is that this mixing is very small numerically [the corresponding coefficient is $\frac{1}{19}$, see Eq. (24)] so that enhancement of operator B_1 does not imply enhancement of the contribution of right-handed currents into the decay of ordinary hadrons.

The anomalous dimension of operator T was independently calculated by Ellis[9] and, later, by Wilczek and Zee.[10] We agree with the results of these authors on the value of $\gamma_{\tilde{T}}$ [see Eq. (16)]. However, it is not consistent, to our mind, to include the anomalous dimension of the mass of the charmed quark for momenta $p < m_c$ (for a discussion of this point see above) as the authors in Refs. 9 and 10 do. In the other words, the difference between m and m_c is disregarded in Refs. 9 and 10. Numerically, the effect is not too large, however. The second and very important point of difference is that in Refs. 9 and 10 the mixing of operator T with operators $B_{1,2}$ is not considered. As was repeatedly explained above, we consider this mixing as most important. It turns out to be much more complicated from the technical point of view than any other calculation. So the results of Refs. 9 and 10 do not allow checking the correctness of the most cumbersome part of the calculation made.

Let us also notice that Wilczek and Zee keep explicitly an operator (O_1 in their notation) which can be reduced to the other operators plus a total derivative by using the equations of motion. As was mentioned in Sec. III there is no necessity to keep such operators. The agreement of the results obtained in Ref. 10 for the anomalous dimension of operator $\tilde{T}$ with those presented here confirms the correctness of this general argument.

VI. BARE HAMILTONIAN OF $\Delta S = 2$ TRANSITIONS

In this and the following sections, we will derive the effective Hamiltonian of the $\Delta S = 2$ transitions which is responsible for the K_L-K_S mass difference. Here we will consider the bare Hamiltonian, by which we mean the Hamiltonian with strong interactions switched off. The emphasis is made on the Higgs-boson contribution.

There are three pieces in the Hamiltonian of $\Delta S = 2$ transitions associated with (i) a two-W-boson exchange together with exchanges where the W boson is replaced by unphysical Higgs scalars $\psi^{\pm}$ [see Fig. 1(a)], (ii) single-physical-Higgs-scalar σ exchange [see Fig. 1(b)], (iii) radiative corrections to the contribution (ii) [see Fig. 1(c)].

While the two-W-boson exchange is the most common one and was considered in a number of papers (see, e.g., Refs. 3b and c), the necessity of a calculation of the Higgs-boson exchange is specific for the models with right-handed currents. To substantiate the point let us give the result of a calculation of the amplitude associated with the graphs of Fig. 1(a) in the R_ξ gauge

$$M_{1a} = \frac{G^2 m_c^2}{2\pi^2} \sin^2\varphi \sin^2\theta_C \left\{ \left[4(\bar{s}_R d_L)^2 - (\bar{s}_R \sigma_{\mu\nu} d_L)^2\right] \times \left(\ln\frac{M_W^2}{m_c^2} - \frac{3}{2}\right) + (\bar{s}_R d_L)^2 \ln\xi \right\}. \tag{26}$$

We see that, while the $\ln m_W$ term is gauge independent, the rest of the answer depends on the choice of the form of the propagator of the W boson. Therefore, Eq. (26) cannot represent a complete answer for the bare Hamiltonian of the $\Delta S = 2$ transitions and we must add to it something else. In renormalizable theories this extra contribution comes from radiative corrections to the single-Higgs-boson exchange (see below). It is worth noting that, if there are left-handed currents alone, the W-boson contribution is gauge independent by itself.

Thus, any calculation of the $\Delta S = 2$ transitions requires some assumptions concerning the Higgs sector of the model and is not fixed by the structure of the group multiplets alone. We will consider in detail the case of a single Higgs field σ. To define the coupling constants of the Higgs mesons in a way as model independent as possible, we turn to the unitarity condition and consider first the W^+W^--boson production in $\bar{s}d$ collisions. The corresponding graph is presented in Fig. 6

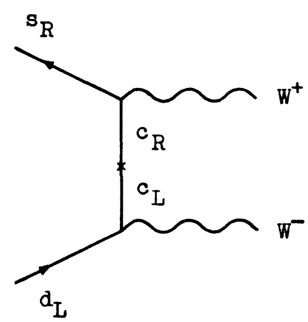

FIG. 6. The graph corresponding to the contribution of right-handed currents into the amplitude of $\bar{s}d \to W^+W^-$ transition. Cross denotes the mass insertion.

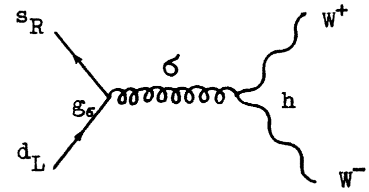

FIG. 7. Contribution of the physical Higgs boson σ to the amplitude for the $\bar{s}d \to W^+W^-$ transition.

and one readily finds for the matrix element

$$M_1(\bar{s}d \to W^+W^-) = -2\sqrt{2}\,\sin\varphi\,\sin\theta_C G\,m_c \bar{s}_R d_L \times\left(1+\frac{m_c^2}{p^2-m_c^2}\right) \underset{s\sim t\to\infty}{\longrightarrow} \text{const}\times\sqrt{s}\ , \qquad (27)$$

where m_c is the c-quark mass and p is the virtual c-quark momentum. We see that the high-energy behavior of the amplitude (27) cannot be reconciled with the unitarity condition.

Within renormalizable theories of weak interactions this growing contribution is canceled out by the graph with the Higgs-boson exchange (Fig. 7). The cancellation takes place at $s \gg m_c^2$ if the product of the coupling constants of the Higgs boson with s, d quarks and the W boson satisfies the condition

$$g_\sigma h = 2\sqrt{2}\ G\,m_c M_W^2 \sin\varphi\ , \qquad (28)$$

where the definition of the constants corresponds to the Lagrangian of the form:

$$\mathcal{L} = h\sigma W^+W^- - 2g_\sigma \sin\theta_C\,\sigma\,[\bar{s}_R d_L + \bar{d}_L s_R]\ , \qquad (29)$$

where σ is the physical Higgs scalar field.

Thus, we come to the same σsd vertex as that found in Sec. II. This conclusion can be avoided if there are several heavy quarks and their masses and mixing angles are organized in such a way that graphs of the type represented in Fig. 6 cancel among themselves (the general framework for constructing such models can be found in Ref. 15.) The crucial point is that the graph of Fig. 3, which is presumably responsible for nonleptonic weak interactions with $\Delta S=1$, is proportional just to the same product of masses and coupling constant as that of Fig. 6. Therefore the vanishing of graphs of the type in Fig. 6 implies the vanishing of the contribution of right-handed currents into decays of strange particles.

In the second order the $\sigma\bar{s}d$ coupling gives rise to the $\Delta S=2$ transitions:

$$H^\sigma(\Delta S=2) = -\frac{2g_\sigma^2}{m_\sigma^2}\sin^2\theta_C\,\bar{s}_R d_L \bar{s}_R d_L\ . \qquad (30)$$

The constant g_σ entering Eq. (30) can be found from Eq. (28) provided that the coupling constant h is known. The latter can be indeed found by imposing the unitarity condition on the amplitude of scattering of longitudinally polarized W bosons. The result is as follows:

$$h = m_W(16\sqrt{2}\,G\,m_W^2)^{1/2} \qquad \mathbf{(31)}$$

and, consequently,

$$g_\sigma = \left(\frac{Gm_c^2}{2\sqrt{2}}\right)^{1/2}\sin\varphi\ . \qquad (32)$$

Thus far we have considered the case of single-σ-boson exchange which is of the first order in the Fermi coupling constant G. Radiative corrections to it are of order Gm_σ^2 and can cancel the gauge-dependent terms in the graph associated with the double-W-boson exchange [see Eq. (26)]. An explicit answer for the radiative corrections [see Fig. 1(c)] looks as follows:

$$M_{1c} = \sin^2\varphi\,\sin^2\theta_C\,\frac{G^2 m_c^2}{2\pi^2}\ln\frac{m_c^2}{\xi m_W^2}(\bar{s}_R d_L)^2\ , \qquad (33)$$

and we see that the gauge-dependent terms are indeed canceled out in the sum of contributions (26) and (33). Equation (33) is valid in the limit of large mass of the σ boson, and we used the R_ξ gauge in which auxiliary Higgs fields are present. The $\sigma\psi^+\psi^-$ coupling is proportional to m_σ^2 and this factor cancels m_σ^{-2} arising from the σ-boson propagator. Thus, the graph of Fig. 1(c) with $\psi^\pm$ fields in the loop is the only one which gives a contribution to the σ exchange in the limit of large m_σ.

So, the graphs of Fig. 1 lead to the following expression:

$$H^{(0)}(\Delta S=2) = -\sin^2\varphi\sin^2\theta_C\,\frac{Gm_c^2}{m_\sigma^2}(\bar{s}_R d_L)^2 - \sin^2\varphi\,\sin^2\theta_C\,\frac{G^2 m_c^2}{4\pi^2} \times\left\{[4(\bar{s}_R d_2)^2 - (\bar{s}_R\sigma_{\mu\nu}d_L)^2]\left(\ln\frac{m_W^2}{m_c^2}-\tfrac{3}{2}\right) + (\bar{s}_R d_L)^2\ln\frac{m_\sigma^2}{m_W^2}\right\}. \qquad (34)$$

Equation (34) represents the final result for the bare Hamiltonian of the $\Delta S=2$ transitions in the model considered, and in the next section we will find its modifications due to the gluon exchanges at short distances.

Here we would like to notice that Equation (34) implies that the mass of the σ boson must be very large. An estimate of the matrix element of the

Hamiltonian (34) indicates that it can be reconciled with experimental data only if

$$m_\sigma > 3000 m_c \frac{m_K}{m_s + m_d} \sin\varphi . \tag{35}$$

The charmed quark is certainly not lighter than 1 GeV. As for the light-quark masses m_s and m_d, a rough estimate gives

$$m_s + m_d \approx m_K ,$$

while the most motivated one seems to be that given by Leutwyller[20] and Gell-Mann[21], i.e.,

$$m_s + m_d \approx 150 \text{ MeV} .$$

In any case the mass of the physical Higgs particle must be not less than several thousand GeV for $\sin\varphi \sim 1$.

An introduction of such huge mass leads to troubles with perturbation theory. Indeed, one can see that for such a choice of the Higgs-meson mass the radiative correction due to weak interactions in higher orders exceed the Born term [see Eq. (34)].

As was already mentioned, the difficulty can be avoided by introducing several Higgs mesons and organizing their coupling constants in a proper way (see Sec. II).

VII. EFFECTIVE HAMILTONIAN OF THE $\Delta S = 2$ TRANSITIONS

In this section we will give results for the gluon corrections to the bare Hamiltonian derived in the previous section. Solving renormalization-group equations (9) turns out to be equivalent to the following recipe (for the sake of definiteness we consider double-W-boson exchange since the corrections to the Higgs-meson exchange are much easier to find):

(1) The graphs of zero order in strong interactions considered in the previous section are represented as an integral over the virtual momentum p.

(2) Blocks with W-boson exchanges entering these graphs are replaced by the effective Hamiltonian of the $\Delta S = 1$ transitions

$$H(\Delta S = 1) = \sum c_i(p) O_i$$

(integration over p here corresponds to integration over μ in general expressions of Sec. III).

(3) Without performing integration over p, explicitly represent the integrand as a sum

$$\sum f_K(p) \tilde{A}_K ,$$

where operator structures A_K are diagonal with respect to the gluon corrections.

(4) Multiply $f_K(p)$ by factors $[\alpha_S(m)/\alpha_S(p)]^{\gamma_K/b}$ accounting for anomalous dimensions γ_K of operators $\tilde{A}_K$.

(5) Integrating the expression obtained over p gives the final result for the effective Hamiltonian.

The validity of the recipe is proved in the appendix using the general approach discussed in Sec. III. In the diagram language, step 2 corresponds to gluon exchanges with momenta larger than that of the virtual W boson in blocks with W-boson exchange while the fifth step corresponds to the integration over gluons with momenta less than p exchanged between the external lines.

Using this recipe one can get the following result for the effective Hamiltonian:

$$\begin{aligned} H(\Delta S=2) = &- \sin^2\varphi \sin^2\theta_C \left[\frac{G m_c^2}{\sqrt{2} m_\sigma^2} + \frac{G^2 m_c^2}{4\pi^2} \ln \frac{m_\sigma^2}{m_W^2} \right] x_2^{4.84/9} x_3^{-3.16/b} (\bar{s}_R d_L)^2 \\ &- \sin^2\varphi \sin^2\theta_C \frac{G^2 m_c^2}{\pi \alpha_s(m)} \Bigg\{ (\bar{s}_R d_L)^2 x_2^{13.84/9} \left[-2.67 x_1^{16/b} \frac{1 - x_1^{1-(19.16/b)}}{19.16 - b} + 4.77 x_1^{-2/b} \frac{x_1^{1-(1.16/b)} - 1}{b - 1.16} \right] \\ &+ \left[1.9 (\bar{s}_R d_L)^2 - (\bar{s}_R \sigma_{\mu\nu} d_L)^2 \right] x_2^{-5.51/9} x_1^{-2/b} \frac{1 - x_1^{1-(11.51/b)}}{11.51 - b} \Bigg\}, \end{aligned} \tag{36}$$

$$x_1 = \frac{\alpha_s(m_c)}{\alpha_s(m_W)}, \quad x_2 = \frac{\alpha_S(m)}{\alpha_S(m_c)}, \quad x_3 = \frac{\alpha_S(m_c)}{\alpha_S(m_\sigma)} .$$

In the appendix we present some details of derivation.

VIII. EFFECTIVE HAMILTONIAN OF WEAK RADIATIVE DECAYS

In the language of field theory, weak radiative decays are due to a combined effect of weak nonleptonic interactions and of electromagnetic transition. Therefore one can differentiate between the contributions of short and long distances, where by distance we understand here the difference between coordinates of the photon emission and of the weak transition.

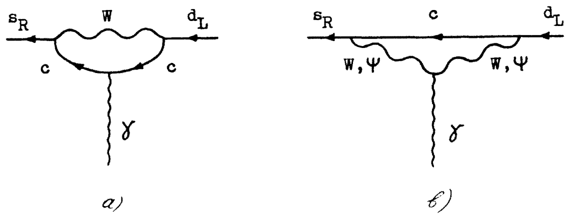

FIG. 8. Graphs describing $d\to s\gamma$ transition for bare quarks.

The contribution of long distances can be estimated in some way once the information on weak and electromagnetic transitions is granted (for attempts of such estimates see, e.g., Refs. 22). The contribution of short distances must be treated separately since it does not reduce to any kind of a product of known matrix elements. In this section we will use QCD to derive the expression for the effective Hamiltonian of weak radiative transitions associated with the contribution of short distances in the models with right-handed currents.

Let us list all the operators T_i which enter the effective Hamiltonian

$$H^{\gamma}(\Delta S=1)=\sum t_i T_i \,.$$

There are some conditions imposed on the operators. First of all, they must contain the electromagnetic field A_μ which, by virtue of gauge invariance, appears only through the field strength tensor $F_{\mu\nu}$. The operators can also contain the light-quark fields u, d, s and the gluon strength tensor $b^a_{\mu\nu}$ $(a=1,\ldots,8)$.

Moreover, we need count only operators of dimension $d\leq 6$. Operators of higher dimension contain factors m^2/m_c^2, m^2/m_W^2 and their contribution is small. The relevant operators of of dimension five look as follows:

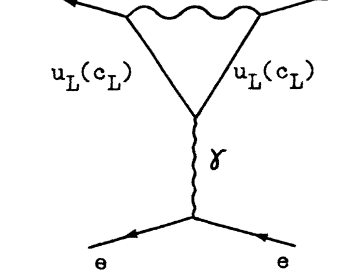

FIG. 9. Graph relevant to the computation of the coefficient $t_3^{(0)}$ [see Eq. (40)].

$$T_1=i\bar{s}_R\sigma_{\mu\nu}d_L F_{\mu\nu}\,,\qquad T_2=i\bar{s}_L\sigma_{\mu\nu}d_R F_{\mu\nu}\,. \tag{38}$$

As for dimension $d=6$, the two relevant operators

$$T_3=\bar{s}_L\gamma_\mu d_L\partial_\nu F_{\nu\mu} \quad\text{and}\quad T_4=\bar{s}_R\gamma_\mu d_R\partial_\nu F_{\nu\mu} \tag{39}$$

do not contribute to the decays with real photon emission. They are important, however, for a description of internal photon conversion into a lepton pair. Since the field d_R does not enter currents (2), the operator T_4 does not arise, so $t_4=0$.

From dimensional considerations it follows that the operators $T_{1,2}$ enter the expansion of the Hamiltonian with some mass factor. If the right-handed currents are present, this mass is m_c, generally speaking.

For bare quarks the radiative transitions are described by graphs in Figs. 8 and 9 and the corresponding coefficients can be readily found if the model (2) for weak currents is accepted. An explicit calculation gives

$$t_1^{(0)}=\sin\varphi\sin\theta_C\frac{eG\sqrt{2}}{24\pi^2}m_c\,,\quad t_2^{(0)}=t_4^{(0)}=0\,,$$
$$t_3^{(0)}=-\sin\varphi\sin\theta_C\frac{eG\sqrt{2}}{18\pi^2}\left(\ln\frac{m_c^2}{m^2}+\tfrac{5}{3}\right), \tag{40}$$

where for the sake of completeness we have included the contribution of left-handed currents as well (coefficient t_3). Expression for the coefficient t_3 is infrared divergent and we used cutoff at some mass m considering this mass is much larger than that of a nonstrange quark. The coefficient t_2 vanishes in the lowest order due to the cancellation of contributions of c and u quarks in the intermediate state. As we will show later this cancellation does not pursue if strong interactions are taken into account.

The general method for finding the gluon corrections to the bare graphs was outlined in Sec. III. As an example we will consider here in more detail the evaluation of the coefficient t_1. The renormalization-group equation for the coefficient t_1 looks as follows:

$$\mu\frac{dt_1(\mu)}{d\mu}=-\beta\frac{\alpha_s(\mu)}{2\pi}t_1(\mu)-\frac{\alpha_s(\mu)}{2\pi}\frac{em_c(\mu)}{16\pi^2}\sum\epsilon_i c_i(\mu)\,, \tag{41}$$

where β, ϵ_i are some numerical coefficients which can be found in perturbation theory.

In particular, the value of β is determined by the sum of the anomalous dimension of operator T_1 (see Fig. 10) and that of mass m_c. The coef-

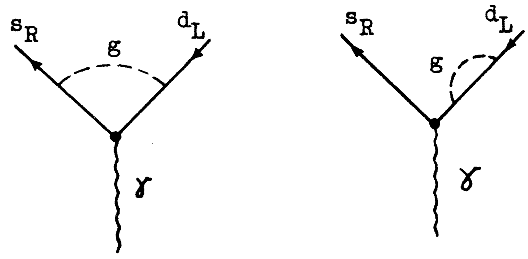

FIG. 10. Graphs contributing to the anomalous dimension of operator T_1.

ficients ϵ_i describe the mixing of operator T_1 and operators O_i entering the effective Hamiltonian of weak nonleptonic transitions with $\Delta S = 1$. The mixing is due to the two-loop graphs represented in Fig. 11. The necessity to consider the two-loop graphs can be inferred from the fact that to zero order in α_s there is no ln factor in the expression for t_1. In this respect the situation is similar to that considered in the case of the $\Delta S = 1$ decays (see Sec. V).

An explicit calculation of the two-loop graphs represented in Fig. 11 leads to the following values of the coefficients in Eq. (41):

$$\epsilon(B_1) = 0\,, \quad \epsilon(B_2) = -\tfrac{64}{9}\,, \tag{42}$$

where we have replaced the index i by the symbol of the operator which mixes with T_1.

Equation (41) holds for the virtual momenta $p > m_c$. In the region $p < m_c$ it is modified in the following way: The second term in the right-hand side of Eq. (41) vanishes since the corresponding Feynman integrals are cut off from below at $p = m_c$ and there is no logarithmic contribution associated with $p < m_c$. For the same reason there is no anomalous dimension of mass in this region and coefficient β coincides with the anomalous dimension

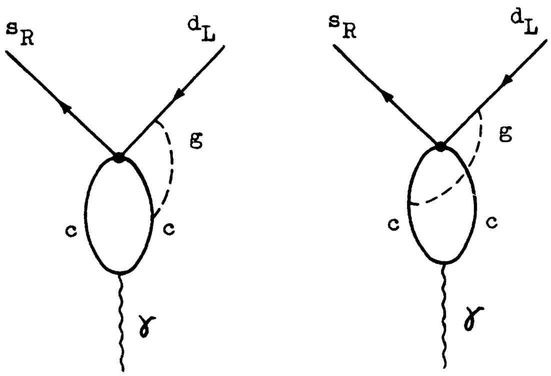

FIG. 11. Graphs giving rise to the mixing of operators T_1 and $B_{1,2}$. Bold-faced point denotes the vertex induced by operators $B_{1,2}$.

of operator T_1, i.e., $\beta = -\frac{4}{3}$. Integrating Eq. (41) gives

$$t_1(m) = \sin\varphi \sin\theta_C \frac{eG\sqrt{2}}{24\pi^2} m_c x_2^{-4/27} x_1^{-16/3b} \times \left[1 - \tfrac{32}{13}\left(x_1^{13/3b} - 1\right)\right], \tag{43}$$

where $x_{1,2}$ are defined in Eq. (25).

The coefficients $t_{2,3}$ which describe the effect of the left-handed currents can be found in a similar way. The only difference is that for $p < m_c$ there are ln terms present both in the effective mass of the d quark and in the mixing matrix. Moreover, one must consider mixing with operators O_i arising in the models with left-handed currents. The corresponding effective Hamiltonian of weak nonleptonic decays was found in Refs. 7 and 8. For the purposes of the present investigation the simplified analysis of Ref. 7 (which amounts to neglecting the difference between m and m_c) is sufficient. The corrections due to the difference in mass scales of ordinary and charmed hadrons in this case does not exceed several percent.

Omitting the results of intermediate calculations, let us give here the final result for $t_{2,3}$

$$\begin{aligned} t_2(m) &= -\sin\theta_C \cos\theta_C \frac{eG\sqrt{2}}{16\pi^2} m_d x_2^{-16/27} \\ &\quad\times\left[\tfrac{2}{7}\left(x_2^{28/27} - 1\right)x_1^{4/b} + \tfrac{4}{5}\left(x_2^{10/27} - 1\right)x_1^{-2/b}\right], \\ t_3(m) &= -\sin\theta_C \cos\theta_C \frac{eG\sqrt{2}}{18\pi^2}\, \frac{4\pi}{\alpha_s(m)} \\ &\quad\times x_2\left[-\tfrac{1}{5}\left(1 - x_2^{-5/9}\right) + \tfrac{2}{11}\left(1 - x_2^{-11/9}\right)x_1^{-2/b}\right]. \end{aligned} \tag{44}$$

Left-handed currents also contribute to the coefficient t_1. The corresponding expression can be found by substituting m_d in Eq. (44) for t_2 by m_s. It is comparable to the contribution of the right-handed currents if $\sin\varphi \sim \frac{1}{10}$.

Let us notice that the operator T_1 was first considered in Ref. 5. However, we disagree with the results of this paper both in zero order in α_s and with respect to gluon corrections. In Ref. 23 the coefficient t_1 was calculated as well as the anomalous dimension of operator T_1. At these points our results coincide with those of Ref. 23. The effect of mixing in coefficients t_1, t_2 are calculated here for the first time. The coefficient t_3 was derived in fact in Ref. 24 and is given here for the sake of completeness.

IX. NUMERICAL ESTIMATES

So far we have concentrated on the derivation of the effective Hamiltonians. In this section we will present some numerical estimates. Generally speaking, these estimates contain two parts: eval-

TABLE I. The effect of strong interactions for weak radiative decays. Coefficients of operator expansion t_i [see Eqs. (37)–(40)] are normalized to free-quark values in the case of t_1, t_3 (t_2 vanishes in the limit $\alpha_s \to 0$). The parameter m is the infrared cutoff, $\alpha_s(m)=1$, N is the number of flavors.

	$m=0.14$ GeV, $m_c=2$ GeV, $m_W=100$ GeV			$m=0.7$ GeV, $m_c=2$ GeV, $m_W=70$ GeV		
N	4	6	8	4	6	8
$t_1\left(\sin\varphi \sin\theta_C \frac{eG\sqrt{2}m_c}{24\pi^2}\right)^{-1}$	−0.07	−0.10	−0.13	−0.35	−0.41	−0.35
$t_2\left(\sin\theta_C \cos\theta_C \frac{eG\sqrt{2}m_d}{16\pi^2}\right)^{-1}$	−0.86	−0.87	−0.88	−0.58	−0.59	−0.61
$t_3\left(-\sin\theta_C \cos\theta_C \frac{eG\sqrt{2}}{18\pi^2}\ln\frac{m_c^2}{m^2}\right)^{-1}$	−0.40	−0.45	−0.50	−0.56	−0.66	−0.77

uation of the coefficients in this operator expansion of the effective Hamiltonians discussed and calculation of the corresponding matrix elements. In this paper we are interested in the effects of strong interactions at short distances and address ourselves mostly to the first part of the problem. As for the estimates of the matrix elements we will make only some random remarks here.

Let us start with weak radiative decays where the effect is the most prominent one. As was already mentioned, for numerical estimates we accept $m_W=70-100$ GeV, $m_c=2$ GeV. As for the typical hadronic mass we try two choices: $m\approx m_\rho=0.7$ GeV and $m\approx m_\pi=0.14$ GeV. The numerical results for coefficients $t_{1,2,3}$ are summarized in Table I where we indicated also the dependence on the number of flavors.

We see that the result practically does not depend on the number of heavy quarks. As for the dependence on choice of hadronic mass m, it is quite strong. In all the cases the strong interactions suppress considerably the contribution of the right-handed currents into radiative decays. The corresponding factor varies between $\frac{1}{3}$ and $\frac{1}{10}$.

This makes it implausible that the right-handed currents can dominate in, say, $\Sigma^+\to p\gamma$ decay. Most probably the radiative decays are determined by the distances of order of the confinement radius $1/m$. (For a detailed argument for the decays $K_L\to 2\gamma$ and $K^+\to\pi^+e^+e^-$, see Refs. 25 and 24, respectively.) If the violation of SU(3) symmetry in the $\Sigma^+\to p\gamma$ decay is confirmed experimentally, the key to the understanding of the phenomenon can be found most probably in conventional models of weak interactions. Let us notice in this connection that an example of strong violations of SU(3) symmetry in weak radiative decays was found in Ref. 26.

The effect of short distances on the weak nonleptonic decays with $\Delta S=1$ is described by the coefficient c_T introduced in Eq. (11). Table II summarizes numerical estimates for this coefficient. It turns out to be close to unity, or, rather, to 0.75, and is extremely stable against reasonable variations of the parameters of the theory.

Thus, the effect of strong interactions at short distances is very modest in this case. As was already mentioned, this conclusion is by no means trivial since at short distances operator T is mixed with operator B_1 which has a large positive anomalous dimension and is enhanced by strong interactions. Therefore, our main result is that this mixing is not important numerically.

It is quite a difficult problem to get a reliable estimate of the matrix element of operator T which describes the contribution of the right-handed currents into the decays of strange particles. Let us mention here a rough calculation which we feel is an overestimation of contribution of operator T rather than an underestimation of it.

Let us compare the width of the $K_S\to\pi^+\pi^-$ decay with the width of $\sigma\to\pi^+\pi^-$ decay of a (hypothetical) scalar σ meson. Strong interactions are described by the quark-gluon interaction $\frac{1}{2}g\bar{q}\gamma_\mu t^a q b^a_\mu$, while weak interactions are due to the Hamiltonian $\sin\varphi\sin\theta_c(G\sqrt{2}\,m_c/16\pi^2)g\bar{s}_R\sigma_{\mu\nu}t^a d_L b^a_{\mu\nu}$. Using the apparent similarity of the interactions we get for the ratio of the decay widths

TABLE II. The values of coefficient c_T which determines the contribution of right-handed currents into the effective Hamiltonian of $\Delta S=1$ transitions [see Eqs. (10)–(12) and (24)].

	$m=0.14$ GeV, $m_c=2$ GeV, $m_W=100$ GeV			$m=0.7$ GeV, $m_c=2$ GeV, $m_W=70$ GeV		
N	4	6	8	4	6	8
c_T	0.756	0.742	0.727	0.764	0.757	0.764

$$\frac{\Gamma^{(T)}_{\text{weak}}}{\Gamma_{\text{strong}}} \sim \left(\frac{Gm_c m\sqrt{2}}{8\pi^2} \sin\varphi \sin\theta_c\right)^2 ,$$

or (45)

$$\Gamma^{(T)}_{\text{weak}} \sim \tfrac{1}{8} \sin^2\varphi \Gamma_{\text{exp}} (K_S \to 2\pi) ,$$

if $m = m_\rho$, $\Gamma_{\text{strong}} = 0.3$ GeV. The value of $\sin\varphi$ is most probably much smaller than unity and we would like to conclude that operator T does not contribute significantly into the observed $K_S \to 2\pi$ decay.

Finally, let us turn to the discussion of the K_L-K_S mass difference. The situation here is somewhat reversed as compared to the previous case. The quark model seems to be quite reliable to use for an estimate of the matrix element. The effect of strong interactions, on the other hand, is quite unstable against the change of parameters since there is some cancellation among various contributions.

Consider first the contribution of a double W-boson exchange in the Feynman gauge. The coefficients in the operator expansion of the effective Hamiltonian of the $\Delta S = 2$ transitions are given in Table III for various choices of masses. We see that strong interactions suppress the contribution of the right-handed currents into the K_L-K_S mass difference. Still this contribution seems to be unacceptably large if $\sin\varphi \sim 1$. To substantiate the point let us give an estimate of the matrix element which assumes that the K meson consists of a quark-antiquark pair.

Then the matrix element of operator $(\bar{s}_R d_L)^2$ reduces to the product of matrix elements of the form $\langle K^0|\bar{s}_R d_L|0\rangle$. After accounting for all the possible ways of contracting the quark fields entering the operator $(\bar{s}_R d_L)^2$ and the wave function of the meson, we find

TABLE III. The effect of strong interactions for $\Delta S = 2$ transitions, induced by two-W-boson exchange. For a definition of the coefficients a_1, a_2 see Eqs. (A1) and (A5). The quantities $a_{1,2}^{(0)}$ are values of these coefficients in the limit of free quarks,

$$a_1^{(0)} = -\sin^2\varphi \sin^2\theta_C \frac{G^2 m_c^2}{\pi^2} \ln\frac{m_W^2}{m_c^2} ,$$

$$a_2^{(0)} = \sin^2\varphi \sin^2\theta_C \frac{G^2 m_c^2}{4\pi^2} \ln\frac{m_W^2}{m_c^2} .$$

	$m = 0.14$ GeV, $m_c = 2$ GeV, $m_W = 100$ GeV			$m = 0.7$ GeV, $m_c = 2$ GeV, $m_W = 70$ GeV		
N	4	6	8	4	6	8
$a_1/a_1^{(0)}$	−0.48	−0.64	−0.83	−1.06	−1.35	−1.75
$a_2/a_2^{(0)}$	0.19	0.18	0.18	0.20	0.19	0.18

$$\langle K^0|(\bar{s}_R d_L)^2|K^0\rangle = \tfrac{5}{12}\langle K^0|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_5 d|K^0\rangle . \quad (46)$$

To get an idea of the matrix element from the pseudoscalar density $\bar{s}\gamma_5 d$ we use the equation of motion

$$\partial_\mu(\bar{s}\gamma_\mu\gamma_5 d) = i(m_s + m_d)\bar{s}\gamma_5 d , \quad (47)$$

which is valid for interacting fields. Finally we get

$$\langle K^0|(\bar{s}_R d_L)^2|K^0\rangle = -\tfrac{5}{12} f_K{}^2 m_K{}^2 \left(\frac{m_K}{m_s + m_d}\right)^2 , \quad (48)$$

where f_K is the $K \to \mu\nu$ decay constant and we use for quark masses $m_s + m_d = 150$ MeV.[20,21]

Keeping this estimate in mind one readily finds that the contribution of the double-W-boson exchange into the K_L-K_S mass difference is comparable with experimental value only if

$$\sin\varphi \lesssim \tfrac{1}{10} . \quad (49)$$

The contribution of the Higgs-boson exchange to the K_L-K_S mass difference depends heavily on the structure of the Higgs multiplets and their interaction. This contribution is of opposite sign as compared to that considered above. In the examples which we have analyzed it cancels, say, half of the contribution of W bosons. Unfortunately, it turns out difficult to get a reliable calculation of the effect of cancellation. The point is that the result is sensitive, e.g., to the inclusion of the nonleading ln terms which we cannot calculate consistently but which can be estimated in part. In all the rather numerous estimates which we have made we never observed too strong a cancellation, but our feeling is that we cannot rule out such a possibility rigorously enough. Therefore, while estimate (49) still seems to be the best one, we cannot prove that $\sin\varphi$ is indeed thus small.

X. CONCLUSIONS

In this paper we have tried to pursue the idea that right-handed currents coupling light and heavy quarks give a dominant contribution to the weak decays of strange particles, as is proposed, e.g., in Refs. 3–5.

To this end we have considered the Higgs sector of the model, estimated some matrix elements, and found the effective Hamiltonian of weak nonleptonic interactions with $\Delta S = 1, 2$ and of weak radiative decays. The latter aim required major theoretical and computational effort and we hope that the results obtained are interesting by themselves, even outside the context of the concrete physical problem considered here.

In all the cases we have encountered with difficulties which indicate, to our mind, that right-handed currents do not play a major role in weak decays of strange particles.

There are possible ways of introducing right-handed currents which do not lead to strong effects in the observed decays of K-mesons and hyperons. One of the possibilities is provided by Bjorken, Lane, and Weinberg,[15] which we were not aware of in the course of the preparation of the present paper. The model amounts to a natural extension of the Glashow-Iliopoulos-Maiani mechanism to the case of right-handed currents. It might be worth emphasizing that our analysis does not result in any argument against such models. Right-handed currents can be present in nature. They seem to be unimportant for ordinary hadrons, however.

APPENDIX

In this appendix we will outline the derivation of Eq. (36) for $H(\Delta S=2)$,

$$H(\Delta S=2)=\sum a_i A_i \,, \tag{A1}$$

where A_i are local operators of dimension $d=6$ and the coefficients are of order $G^2 m_c^2$ for two-W-boson exchange. For operators with $d=8$ the coefficients are of order G^2, so that their contribution is suppressed by a factor $\sim m^2/m_c^2$.

The renormalization-group equations for $a_i(\mu)$ look like (for $\mu > m_c$)

$$\mu\frac{da_i(\mu)}{d\mu}=-\frac{\alpha_s(\mu)}{2\pi}\rho_{ij}a_j(\mu)-2\sigma_{ijk}c_j(\mu)c_K(\mu)m_c^2(\mu)\,, \tag{A2}$$

where ρ_{ij}, σ_{ijk} are numerical constants and the equations for effective charges $c_i(\mu)$ in $H(\Delta S=1)$ and $m_c(\mu)$ are given in the text [see Eqs. (13)] as well as their solutions [Eqs. (14)].

It is not difficult to write down the solution of Eqs. (A2), it is

$$a_i(\mu)=\left\{\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{\rho/b}\right\}_{ij}\left(a_j(\mu_0)+\int_{\mu^2}^{\mu_0^2}\frac{d\mu^2}{\mu^2}\left\{\left[\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right]^{-\rho/b}\right\}_{jl}\sigma_{lmn}c_m(\mu)c_n(\mu)m_c^2(\mu)\right)\,, \tag{A3}$$

where $\{\,\}_{ij}$ denotes the corresponding matrix element and $c_i(\mu)$ and $m_c(\mu)$ must be replaced by the explicit expressions (14) and (23).

It is convenient to use for computations the operator basis where matrices of anomalous dimensions are diagonal. For $\Delta S=1$ transitions just operators B_1, B_2 [see Eq. (12)] possess such a property once one neglects the contribution of operator T into a_i. This is legitimate since operator T contains an extra power of g. We remind the reader that the coefficients c_{B_1}, c_{B_2} have the form [see Eq. (23)]

$$c_{B_1}(\mu)=\left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)}\right]^{8/b}\,,\qquad c_{B_2}(\mu)=\left[\frac{\alpha_s(\mu)}{\alpha_s(m_W)}\right]^{-1/b}. \tag{A4}$$

Let us list now the operators A_i with $\Delta S=2$, which arise due to right-handed currents (2). There are two independent operators

$$A_1=(\bar{s}_R d_L)^2,\quad A_2=(\bar{s}_R\sigma_{\mu\nu}d_L)^2\,. \tag{A5}$$

From the calculations of the one-gluon correction (of the type in Fig. 5), it follows that the matrix of anomalous dimensions of A_1, A_2 have the form[3c]

$$\rho=\{\rho_{ij}\}=\begin{pmatrix} 5 & 20 \\ -\frac{1}{12} & -\frac{17}{3}\end{pmatrix}\,. \tag{A6}$$

The eigenvectors of the matrix ρ correspond to the following linear combinations of A_1 and A_2:

$$\tilde{A}_1=A_1-0.008A_2,\quad \{\rho_1=4.84\}\,,\qquad \tilde{A}_2=A_2-1.90A_1,\quad \{\rho_2=-5.51\}\,, \tag{A7}$$

where in the curly brackets the corresponding eigenvalues are displayed. We will neglect a small admixture of A_2 in the expression for $\tilde{A}_1$. In Eq. (A3), apart from initial conditions at $\mu=m_W$ which we find from the bare-quark calculations, we need the knowledge of constants σ_{lmn}. These are determined by ln terms in the graphs of Fig. 12, where bold-faced points denote vertices associated with operators B_1, B_2. One can find

$$\sigma_{\tilde{A}_1B_1B_1}=2.67\frac{G^2}{4\pi^2}\sin^2\varphi\sin^2\theta_C\,,\qquad \sigma_{\tilde{A}_1B_2B_2}=-4.77\frac{G^2}{4\pi^2}\sin^2\varphi\sin^2\theta_C\,,\qquad \sigma_{\tilde{A}_2B_2B_2}=\frac{G^2}{4\pi^2}\sin^2\varphi\sin^2\theta_C\,. \tag{A8}$$

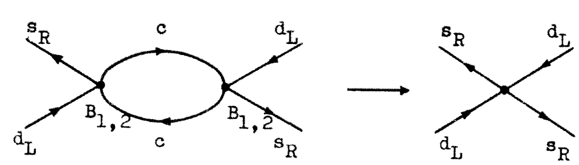

FIG. 12. Nonlinear mixing of operators with $\Delta S=1$ with operators in $H(\Delta S=2)$.

For the other combinations of indices, σ_{lmn} vanish.

Substituting the integration variable μ in Eq. (A3) by virtual momentum p, we arrive just at the recipe of calculation which was given in the text. After integration we come to the final result (36).

Note that in the region $m<\mu<m_c$, only the first term in the right-hand side of Eqs. (A2) and (A3) survives so that the effect of this region reduces to factors $x_2^{\rho i/9}$ in the coefficients of operators $\tilde{A}_1, \tilde{A}_2$.

The discussion above refers mainly to the two-W-boson exchange, since this case is in fact the most complicated one. As far as the physical-Higgs-scalar exchange is concerned, the calculation of strong-interaction effects is much simpler and reduces to powers of $\alpha_S(m)/\alpha_S(m_\sigma)$ determined by anomalous dimensions of operators $\tilde{A}_1, A_2$ and that of the effective mass $m_c(\mu)$.

A careful reader might notice that Eq. (36) in the limit $\alpha_S \to 0$ does not coincide exactly with the result of the bare-quark calculation, Eq. (34). The reason is that we omitted in Eq. (36) the terms which are smaller by $\ln m_W/m_c$ times than those kept. These terms are beyond the accuracy of our derivation.

[1]M. L. Perl et al., Phys. Rev. Lett. 35, 704 (1975); Phys. Lett. 63B, 466 (1976). For more recent reviews see, e.g., talks by A. Litke and U. Timm, at European Conference on Particle Physics, Budapest, 1977 (unpublished).

[2]W. R. Innes *et al.*, Phys. Rev. Lett. 39, 1240 (1977).

[3](a) A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. 35, 69 (1975); (b) H. Fritzsch, M. Gell-Mann, and P. Minkowski, Phys. Lett. 59B, 256 (1975); (c) F. Wilczek, A. Zee, R. Kingsley, and S. B. Treiman, Phys. Rev. D 12, 2768 (1975); R. Kingsley, F. Wilczek, and A. Zee, Phys. Lett. 61B, 259 (1976).

[4]H. Fritzsch and P. Minkowski, Phys. Lett. 61B, 275 (1976).

[5]M. A. Ahmed and G. G. Ross, Phys. Lett. 59B, 293 (1975).

[6]D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, Phys. Rev. Lett. 30, 1346 (1973).

[7]M. K. Gaillard and B. W. Lee, Phys. Rev. Lett. 33, 108 (1974); G. Altarelli and L. Maiani, Phys. Lett. 52B, 351 (1974).

[8]A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. Pis'ma Red. 22, 123 (1975) [JETP Lett. 22, 55 (1975)]; Zh. Eksp. Teor. Fiz. 72, 1275 (1977) [Sov. Phys.—JETP 45, 670 (1977)]; M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B120, 316 (1977).

[9]R. K. Ellis, Nucl. Phys. B108, 239 (1976).

[10]F. Wilczek and A. Zee, Phys. Rev. D 15, 2260 (1977).

[11]A. I. Vainshtein, V. I. Zakharov, and M. A. Shifman, Zh. Eksp. Teor. Fiz. Pis'ma Red. 23, 656 (1976) [JETP Lett. 23, 602 (1976)].

[12]C. H. Llewellyn Smith, CERN Report No. CERN-TH 1710, 1973 (unpublished); in *Proceedings of the Fifth Hawaii Topical Conference on Elementary Particle Physics, 1973,* edited by P. N. Dobson, Jr., V. Z. Peterson, and S. F. Tuan (Univ. of Hawaii Press, Honolulu, 1974). This reference was brought to our attention by E. P. Shabalin.

[13]S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977); M. S. Chanowitz, J. Ellis, and M. K. Gaillard, Nucl. Phys. B128, 506 (1977).

[14]E. Golowich and B. Holstein, Phys. Rev. Lett. 35, 831 (1975).

[15]J. D. Bjorken, K. Lane, and S. Weinberg, SLAC Report, 1977 (unpublished).

[16]T. Appelquist and H. Politzer, Phys. Rev. Lett. 34, 43 (1975).

[17]M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. 60B, 71 (1975).

[18]E. Witten, Nucl. Phys. B104, 445 (1976); B122, 109 (1977).

[19]H. Politzer, Phys. Rep. 14C, 130 (1974).

[20]H. Leutwyller, Phys. Lett. 48B, 45 (1974); Nucl. Phys. B76, 413 (1974).

[21]M. Gell-Mann, Oppenheimer Lectures, Reports of Inst. Adv. Study, Princeton, 1975 (unpublished).

[22]R. E. Berends, Phys. Rev. III, 1691 (1958); B. R. Holstein, Nuovo Cimento 6A, 561 (1971); G. Farrar, Phys. Rev. D 4, 212 (1971).

[23]N. Vasanti, Phys. Rev. D 13, 1889 (1976).

[24]M. A. Shifman, L. B. Okun', A. I. Vaĭnshtein, and V. I. Zakharov, Report No. ITEP-98, 1975 (unpublished); A. I. Vainshtein, V. I. Zakharov, L. B. Okun', and M. A. Shifman, Yad. Fiz. 24, 820 (1976) [Sov. J. Nucl. Phys. 24, 427 (1976)].

[25]M. K. Gaillard and B. W. Lee, Phys. Rev. D 10, 897 (1974).

[26]V. I. Zakharov and A. B. Kaĭdalov, Yad. Fiz. 5, 369 (1967) [Sov. J. Nucl. Phys. 5, 259 (1967)].

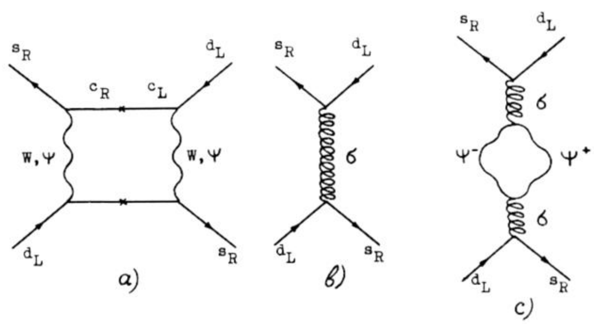

FIG. 1. Graphs describing the bare Hamiltonian of $\Delta S=2$ transitions: (a) box graph with two-W exchange, (b) single Higgs-boson exchange, (c) radiative correction to graph (b). Cross denotes c-quark mass insertion, the wavy lines stand for W boson and unphysical Higgs scalars $\psi^{\pm}$, the curvy lines correspond to the physical Higgs boson.

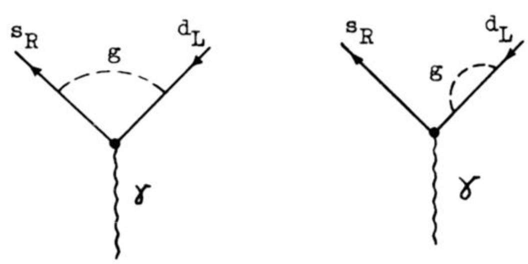

FIG. 10. Graphs contributing to the anomalous dimension of operator T_1.

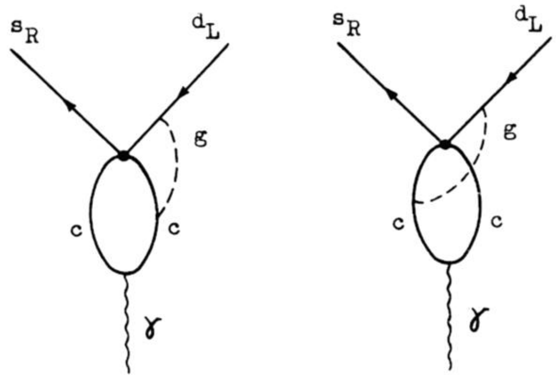

FIG. 11. Graphs giving rise to the mixing of operators T_1 and $B_{1,2}$. Bold-faced point denotes the vertex induced by operators $B_{1,2}$.

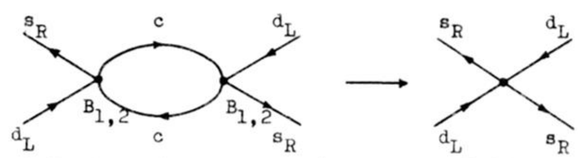

FIG. 12. Nonlinear mixing of operators with $\Delta S = 1$ with operators in $H(\Delta S = 2)$.

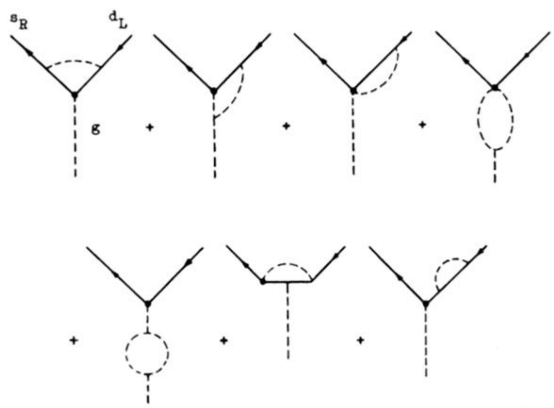

FIG. 2. Graphs relevant to computation of anomalous dimension of operator T (crossed graphs are not shown). Dashed line corresponds to gluon. Bold-faced point denotes operator T.

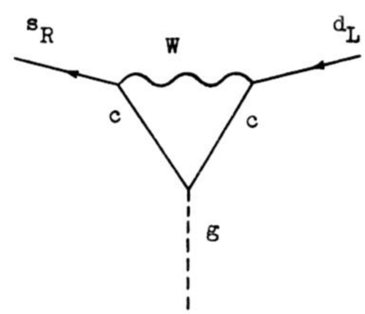

FIG. 3. Lowest-order graph giving rise to operator T.

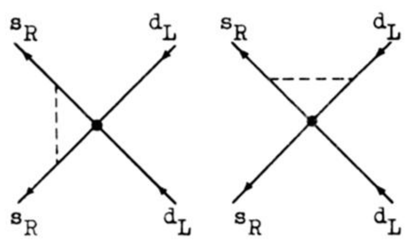

FIG. 4. Graphs contributing to the anomalous dimension of operators B_1, B_2.

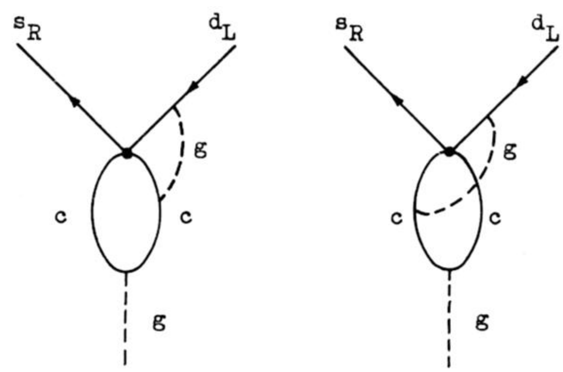

FIG. 5. An example of the graph relevant to computation of mixing of operators $B_{1,2}$ and T. Bold-faced point denotes the vertex induced by operators $B_{1,2}$.

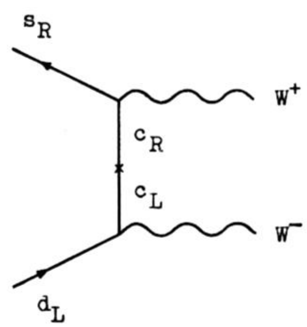

FIG. 6. The graph corresponding to the contribution of right-handed currents into the amplitude of $\bar{s}d \to W^+W^-$ transition. Cross denotes the mass insertion.

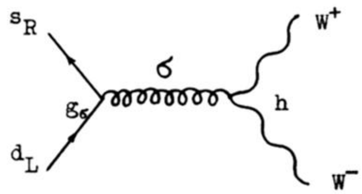

FIG. 7. Contribution of the physical Higgs boson σ to the amplitude for the $\bar{s}d \to W^+ W^-$ transition.

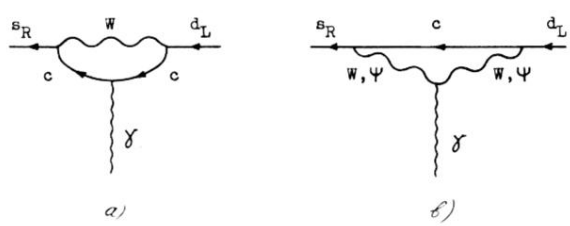

FIG. 8. Graphs describing $d \to s\gamma$ transition for bare quarks.

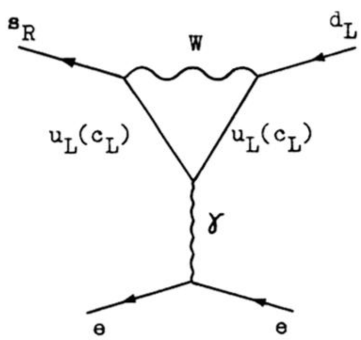

FIG. 9. Graph relevant to the computation of the coefficient $t_3^{(0)}$ [see Eq. (40)].