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SO(4) × U(1) model of weak and electromagnetic interactions

J. Leveille,* S. Rajpoot, and S. D. Rindani
Blackett Laboratory, Imperial College, London SW7 2BZ, England
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We study an SO(4) × U(1) extension of the standard model of weak and electromagnetic interactions, which accommodates the new leptons postulated as an explanation for trimuon events. Universality is shown to be natural in the model. A phenomenological analysis gives good agreement with present data.

I. INTRODUCTION

The observation of high-energy trimuon events in νN collisions reported recently[1] seems to indicate the existence of new heavy leptons M^- (~7 GeV) and L^0 (~3.5 GeV).[2] In the framework of gauge theories of weak and electromagnetic interactions, the new lepton M^- has to be accommodated in the same multiplet as the muon and its neutrino. In SU(2) × U(1) models, this would mean a mixing of either ν_μ or μ with new leptons. This leads to violation of μ-e universality,[3] unless universality is preserved by a correlated-mixing scheme.[4] Better experimental data on π_{l2} decays as well as on trimuon events will have to be awaited in order to decide if universality is violated to a degree sufficient to account for the trimuon events.[3]

On the other hand, if one demands natural universality, the gauge group would have to be extended beyond SU (2)× U(1), so that M^- is produced by a neutrino via a new charged gauge boson, distinct from the usual W. Lee and Weinberg[5] have recently proposed such a model based on the group SU(3) × U(1) which naturally insures universality. We propose another such model based on the group SO(4) × U(1).

The SO(4) × U(1) model has several features in common with the SU(3) × U(1) model of Lee and Weinberg, and we utilize a mechanism similar to theirs to preserve universality. However, although the two models have the same number of charged gauge bosons ($W^\pm, U^\pm$), there are two less neutral gauge bosons in the SO(4) × U(1) model—here the X_1 and X_2 of the SU(3) × U(1) theory,[5] which correspond to nondiagonal neutral currents, are absent. Thus, all neutral currents are diagonal.

The second major difference is the presence in this model of a right-handed $\bar{d}_R y_R$ current (where y is a quark of charge $-\frac{4}{3}$) which couples to the usual vector boson W. It is thus possible to accommodate the high-y anomaly reported in $\bar{\nu}N$ reactions.[6] In fact, the model constitutes an extension of an "exotic" SU(2) × U(1) model[7] with quarks of charge $-\frac{4}{3}$. This exotic model has not only been found to be in agreement with charged-current[8] and neutral-current[7] data, but has, in addition, desirable features such as naturally diagonal neutral currents and natural absence of right-handed $\bar{u}_R d_R$ and $\bar{u}_R s_R$ currents.

Besides the fact that the SO(4) × U(1) model has exotic $q=-\frac{4}{3}$ quarks, it also differs from the SU(3) × U(1) model in having doubly charged leptons.

The model accommodates the observed rate of trimuon events, and fits the established data on neutrino- (antineutrino-) induced neutral-current reactions. Parity violation in atoms is small, and the $\nu_\mu e$ elastic cross section is predicted to be equal to the $\bar{\nu}_\mu e$ cross section, which is not inconsistent with the present data.

We describe the model and the pattern of spontaneous symmetry breaking in Sec. II. In Sec. III, we discuss universality and the discrete symmetry RU responsible for it. In Secs. IV–VI we deal with the phenomenological implications of the model. The charged-current interactions are discussed in Sec. IV; results of neutral-current reactions are compared with experiment in Sec. V; and some higher-order effects are examined in Sec. VI. We conclude with Sec. VII.

II. THE MODEL

We shall make use of the SU(2) × SU(2) × U(1) notation ($T_1^{(1)}$, $T^{(2)}$, Y) for the representations of SO(4) × U(1), where $T^{(1)}$, $T^{(2)}$ denote the representations of two SU(2)'s, and Y is the weak hypercharge. The electric charge is given by $Q=T_3^{(1)}+T_3^{(2)}+Y$. The quarks belong to the representations $(\frac{1}{2},\frac{1}{2},-\frac{1}{3})$, $(0,0,\frac{2}{3})$, and $(0,0,-\frac{4}{3})$, and the leptons belong to $(\frac{1}{2},\frac{1}{2},-1)$, $(0,0,0)$, and $(0,0,-2)$. With the convention that the first SU(2) acts downwards and the second SU(2) acts from left to right, the representations of the fermions can be written as (possible mixings are not explicitly shown)

$$\begin{pmatrix} u & b \\ d & x \end{pmatrix}_L , \quad \begin{pmatrix} c & b' \\ s & x' \end{pmatrix}_L ;$$

$$\begin{pmatrix} t & d \\ b & y \end{pmatrix}_R, \quad \begin{pmatrix} t' & s \\ b' & y' \end{pmatrix}_R;$$

$$\begin{pmatrix} \nu_e & E^- \\ e & \eta \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu & M^- \\ \mu & \eta' \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau & T^- \\ \tau & \eta'' \end{pmatrix}_L;$$

$$\begin{pmatrix} E^0 & e \\ E^- & \xi \end{pmatrix}_R, \quad \begin{pmatrix} M^0 & \mu \\ M^- & \xi' \end{pmatrix}_R, \quad \begin{pmatrix} T^0 & \tau \\ T^- & \xi'' \end{pmatrix}_R.$$

The above is completed by appropriate SO(4) singlets. Here, b, b' are quarks with charge $-\frac{1}{3}$, t, t' are quarks with charge $+\frac{2}{3}$ and x, x', y, y' are quarks with charge $-\frac{4}{3}$. The E^-, M^-, T^- are new negatively charged leptons, the M^- being responsible for the trimuon events, and τ^- is the heavy lepton observed at SLAC.[9] Of the three heavy neutral leptons E^0, M^0, and T^0, a mixture of M^0 with the other two is identified with the decay product L^0 of the M^-. The ξ's and η's have charge -2.

We impose a symmetry R which prevents bare-mass couplings. Under R, the left-handed fermions change sign and the right-handed fermions are invariant. However, as we wish to give masses to the fermions via scalars, the latter (to be specified) are odd under R. The gauge bosons are invariant under R.

The scalar fields (their number is left unspecified) are chosen to belong to the complex representations $(1,1,0)$ and $(\frac{1}{2},\frac{1}{2},1)$, represented in matrix form by

$$\phi_i(1,1,0) \equiv \begin{pmatrix} \phi_i^{++} & \phi_i^{+0} & \phi_i^{+-} \\ \phi_i^{0+} & \phi_i^{00} & \phi_i^{0-} \\ \phi_i^{-+} & \phi_i^{-0} & \phi_i^{--} \end{pmatrix};$$

$$\Omega_i(\tfrac{1}{2},\tfrac{1}{2},1) \equiv \begin{pmatrix} \Omega_i^{++} & \Omega_i^{+0} \\ \Omega_i^{0+} & \Omega_i^{00} \end{pmatrix},$$

where the charges are the sums of the two superscripts. The ϕ_i couple to two $(\frac{1}{2},\frac{1}{2})$ fermion multiplets and the Ω_i to a $(\frac{1}{2},\frac{1}{2})$ multiplet and a singlet.

A general SO(4) × U(1)-invariant R-invariant potential constructed out of the ϕ_i and Ω_i possesses, for a certain range of the parameters, a minimum for nonzero values of ϕ_i^{+-}, ϕ_i^{-+}, and Ω_i^{00}, but $\phi_i^{00}=0$. This gives masses to the fermions, but prevents certain mixings which are undesirable from the point of view of universality. We discuss this in the next section.

After symmetry breaking, the diagonal vector-boson states are $W^\pm$, $U^\pm$, and

$$\begin{aligned} A &= \frac{W^0+U^0}{\sqrt{2}} \sin\theta + Y\cos\theta, \\ Z_1 &= \frac{W^0+U^0}{\sqrt{2}} \cos\theta - Y\sin\theta, \\ Z_2 &= \frac{W^0-U^0}{\sqrt{2}}, \end{aligned} \tag{1}$$

where W^i, U^i correspond to the generators of the two SU(2)'s, and Y corresponds to the generator of the U(1). Of the neutral states A, Z_1, and Z_2, A is the photon, and Z_1, Z_2 are massive. The mixing angle θ is defined by

$$\sin\theta = \frac{\sqrt{2}g'}{(g^2+2g'^2)^{1/2}}$$

where g, g' are the coupling constants for the SO(4) and U(1) groups. The masses of the vector bosons are given by

$$\begin{aligned} m_W^2 &= m_U^2 = g^2(2\mu+\nu), \\ m_{Z_1}^2 &= 4\mu g^2 \sec^2\theta, \\ m_{Z_2}^2 &= 4\nu g^2, \end{aligned} \tag{2}$$

where

$$\mu = \tfrac{1}{4}\sum |c_i|^2,$$
$$\nu = 2\sum (|a_i|^2 + |b_i|^2),$$

with

$$a_i = \langle \phi_i^{+-}\rangle, \quad b_i = \langle \phi_i^{-+}\rangle, \quad c_i = \langle \Omega_i^{00}\rangle.$$

III. THE SYMMETRY RU AND UNIVERSALITY

The choice of $\langle \phi_i^{00}\rangle = 0$ and nonzero vacuum expectation values for other neutral scalars preserves a discrete symmetry RU, where U is a diagonal 4×4 matrix, represented by $(-1, -i, i, -1)$ in the space of the four-dimensional eigenvectors of $T_3^{(1)} \otimes T_3^{(2)}$. In other words, U is the SU(2) × SU(2) transformation

$$\begin{pmatrix} e^{+i\pi/4} & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix} \otimes \begin{pmatrix} e^{i3\pi/4} & 0 \\ 0 & e^{-i3\pi/4} \end{pmatrix}.$$

Under RU, the fermions transform as

$$\begin{aligned} &(u, c, \nu_e, \nu_\mu, \nu_\tau) \to (u, c, \nu_e, \nu_\mu, \nu_\tau), \\ &(d, s, e, \mu, \tau) \to i(d, s, e, \mu, \tau), \\ &(b, b', E^-, M^-, T^-) \to -i(b, b', E^-, M^-, T^-), \\ &(x, x', \eta, \eta', \eta'') \to (x, x', \eta, \eta', \eta''), \\ &(t, t', E^0, M^0, T^0) \to -(t, t', E^0, M^0, T^0), \\ &(y, y', \xi, \xi', \xi'') \to -(y, y', \xi, \xi', \xi''). \end{aligned} \tag{3}$$

Since the vacuum expectation values chosen for the scalars are RU-symmetric, there can be no mixing of any of $u, c, d, s, x, x', \nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau, \eta, \eta', \eta''$ (set A) with any of $t, t', b, b', y, y', E^0, M^0, T^0, E^-, M^-, T^-, \xi, \xi', \xi''$ (set B). However, there can be mixing among states of the same charge within set A and set B.

The vector bosons transform under RU as

$$W^\pm \to \mp i W^\pm ,$$
$$U^\pm \to \pm i U^\pm , \tag{4}$$
$$W^0, U^0, Y \to W^0, U^0, Y .$$

Hence $W^\pm$ and $U^\pm$ cannot mix, but W^0, U^0, and Y can, as seen from (1).

The transformation properties (4) as well as the transformation properties of fermions in (3) imply that the $W^\pm$ induce transitions between ν_{e_L} and e_L, ν_{μ_L} and μ_L, etc., with equal strengths and also between u_L and d_{C_L} (where d_C is the usual Cabibbo-rotated d state) with the same strength. Thus we have natural universality in the theory.

The RU symmetry does not allow mixing between states belonging to singlet and $(\frac{1}{2}, \frac{1}{2})$ representations or between states belonging to the same representation, but having different values of $(T_3^{(1)}, T_3^{(2)})$. Hence exchange of neutral gauge bosons conserves (quark and lepton) flavors.

IV. CHARGED-CURRENT INTERACTIONS

The effective charged-current Lagrangian can be written as

$$\mathcal{L}_{\rm eff} = \frac{G_F}{\sqrt{2}} (j_1^\mu + J_1^\mu)(j_{1\mu} + J_{1\mu})^\dagger + \frac{G_F}{\sqrt{2}} (j_2^\mu + J_2^\mu)(j_{2\mu} + J_{2\mu})^\dagger , \tag{5}$$

where j_1^μ, J_1^μ, the currents coupling to W_μ, are given by

$$j_1^\mu = 2\bar{\nu}_e \gamma_L^\mu e + 2\bar{E}^- \gamma_L^\mu \eta + 2\bar{E}^0 \gamma_R^\mu E^- + 2\bar{e}\gamma_R^\mu \xi$$
$$+ \text{similar terms from other multiplets},$$
$$J_1^\mu = 2\bar{u}\gamma_L^\mu d + 2\bar{b}\gamma_L^\mu x + 2\bar{t}\gamma_R^\mu b + 2\bar{d}\gamma_R^\mu y$$
$$+ \text{similar terms from other multiplets},$$

and j_2^μ, J_2^μ, the currents coupling to U_μ, are given by

$$j_2^\mu = 2\bar{\nu}_e \gamma_L^\mu E^- + 2\bar{e}\gamma_i^\mu \eta + 2\bar{E}^0 \gamma_R^\mu e + 2\bar{E}^- \gamma_R^\mu \xi$$
$$+ \text{similar terms from other multiplets},$$
$$J_2^\mu = 2\bar{u}\gamma_L^\mu b + 2\bar{d}\gamma_L^\mu x + 2\bar{t}\gamma_R^\mu d + 2\bar{b}\gamma_R^\mu y$$
$$+ \text{similar terms from other multiplets},$$

where $\gamma_{L,R}^\mu = \frac{1}{2}\gamma^\mu(1 \mp \gamma_5)$. The Fermi constant $G_F/\sqrt{2} = g^2/8m_W{}^2$. In the above expressions possible allowed mixings should be understood.

With the help of the effective Lagrangian (5) it can be seen that if the double charged leptons are assumed to be heavier than the neutral and singly charged ones, the trimuon events in νN collisions arise in the same way as in the model of Lee and Weinberg.[5] Thus, a rate of $\mu^-\mu^-\mu^+$ production at Fermilab which is $(5-10) \times 10^{-4}$ times the rate of μ^- production can be accommodated.

The fact that u_R is a singlet implies the absence of right-handed $\bar{u}_R d_R$ or $\bar{u}_R s_R$ currents. However, a right-handed $\bar{d}_R y_R$ current coupling to the W exists. The high-y anomaly in antineutrino reactions[6] can therefore be accommodated by the model. The possibility of producing $M^-(M^+)$ in $\nu N \to M^- X (\bar{\nu} N \to M^+ X)$ (via U exchange), which subsequently decays into a single $\mu^-(\mu^+)$ has also to be considered in analyzing the $\nu N, \bar{\nu} N$ data. However, $M^\pm$ production is suppressed by phase space, as both the $M^\pm$ and the hadrons produced are heavy, and should give rise to small corrections.

Higher-order charged-current processes are considered later in Sec. VI.

V. NEUTRAL-CURRENT INTERACTIONS

The neutral currents which couple to Z_1 and Z_2 are given by

$$J_{Z_1}^\mu = \frac{g}{\sqrt{2}\cos\theta} [\bar{u}\gamma_L^\mu u - \tfrac{2}{3}\sin^2\theta\, \bar{u}\gamma^\mu u + \tfrac{1}{3}\sin^2\theta\, \bar{d}\gamma^\mu d + \bar{\nu}_e \gamma_L^\mu \nu_e + \bar{\nu}_\mu \gamma_L^\mu \nu_\mu + \sin^2\theta\, \bar{e}\gamma^\mu e + \sin^2\theta\, \bar{\mu}\gamma^\mu \mu + \ldots] , \tag{6a}$$

$$J_{Z_2}^\mu = \frac{g}{\sqrt{2}} [\bar{d}\gamma_\mu\gamma_5 d + \bar{e}\gamma_\mu\gamma_5 e + \bar{\mu}\gamma_\mu\gamma_5\mu + \ldots] , \tag{6b}$$

where only the terms involving the u, d quarks and ν_e, ν_μ, e, μ are shown.

As previously observed, both neutral currents are diagonal in all flavors. Another observation is that Z_2 does not couple to neutrinos, and hence does not contribute to neutrino reactions. Thus we can parametrize the neutrino neutral-current interactions by $x = \sin^2\theta$ and $\kappa^2 = \cos^2\theta\, m_{Z_1}{}^2/2m_W{}^2$ [$\kappa^2 < 1$ due to the pattern of symmetry breaking, as can be seen from (2)]. We present a comparison of theoretical predictions with experimental results for (i) parity violation in atoms, (ii) $\nu N, \bar{\nu} N$ inelastic neutral-current scattering, (iii) $\nu p, \bar{\nu} p$ elastic scattering, (iv) $\nu_\mu e, \bar{\nu}_\mu e$ elastic scattering, and (v) $\bar{\nu}_e e$ elastic scattering. We obtain a good fit to most results for x and κ^2 around 0.45 and $\frac{3}{4}$, respectively. The results are given for $x = 0.45$ and $\kappa^2 = 0.75$.

A. Parity violation in atoms

As can be seen from (6b), Z_2 exchange is parity conserving. Furthermore, from (6a), the electron

TABLE I. Comparison of results on νN, $\bar{\nu} N$ inelastic neutral-current scattering with data from Gargamelle (GGM) (Ref. 10), HPWF group (Ref. 11), and CITF group (Ref. 12).

		Experimental		
	Theoretical	GGM	HPWF	CITF
$R_{\nu N}$	0.25	0.25 ± 0.04	0.31 ± 0.06	0.24 ± 0.02
$R_{\bar{\nu} N}$	0.38	0.39 ± 0.06	0.39 ± 0.10	0.39 ± 0.06
R	0.51	0.59 ± 0.14	≤0.61 ± 0.25	

coupling to Z_1 is purely vector. Hence parity violation in atoms is proportional to nuclear spin and therefore small.

B. $\nu N, \bar{\nu} N$ inelastic scattering

The usual parton-model analysis of $\nu(\bar{\nu})N \to \nu(\bar{\nu})X$ below charm threshold, neglecting the sea-quark contribution, gives for the ratio of the neutral- to charged-current cross sections

$$R_{\nu N} \equiv \frac{\sigma^{\rm NC}}{\sigma^{\rm CC}}(\nu N) = \frac{1}{12\kappa^4}\left(3 - 4x + \frac{20}{9}x^2\right),$$

$$R_{\bar{\nu} N} \equiv \frac{\sigma^{\rm NC}}{\sigma^{\rm CC}}(\bar{\nu} N) = \frac{1}{4\kappa^4}\left(1 - \frac{4x}{3} + \frac{20}{9}x^2\right).$$

The ratio of $\bar{\nu}$ and ν neutral-current cross sections is

$$R \equiv \frac{\sigma^{\rm NC}(\bar{\nu}N)}{\sigma^{\rm NC}(\nu N)} = \frac{3 - 4x + 20/3x^2}{9 - 12x + 20/3x^2}.$$

The results for $x = 0.45$ and $\kappa^2 = 0.75$, together with the experimental values from Gargamelle (GGM),[10] the Harvard-Pennsylvania-Wisconsin-Fermilab (HPWF) group,[11] and the Caltech-Fermilab (CITF)[12] are shown in Table I.

C. $\nu p, \bar{\nu} p$ elastic scattering

With the usual assumptions of first-class currents, conserved vector currents, etc., we can write the νp and $\bar{\nu} p$ elastic and quasi-elastic cross sections in terms of the proton electric, magnetic, and axial form factors G_E, G_M, and G_A. We use the following dipole parametrization for these:

$$G_E(Q^2)/G_E(0) = G_M(Q^2)/G_M(0) = \left(1 + \frac{Q^2}{M_V{}^2}\right)^{-2},$$

$$G_A(Q^2)/G_A(0) = \left(1 + \frac{Q^2}{M_A{}^2}\right)^{-2},$$

$$(Q^2 = -t > 0).$$

$M_V{}^2$ is obtained as 0.71 GeV^2 from electron-proton scattering. $m_A{}^2$, however, is less precisely determined from quasi-elastic νN scattering[13] to be

TABLE II. Comparison of results on νp, $\bar{\nu} p$ elastic scattering with data from HPW (Ref. 14) and CIR (Ref. 15) groups.

	Theoretical	Experimental
$R_{\rm el}^{\nu}$	0.16	0.17 ± 0.05 (HPW)
$R_{\rm el}^{\bar{\nu}}$	0.28	0.23 ± 0.09 (CIR)
$\frac{\sigma(\nu p \to \nu p)}{\sigma(\bar{\nu} p \to \bar{\nu} p)}$	0.60	0.4 ± 0.2 (HPW)

between 0.71 GeV^2 and 1.32 GeV^2. We choose the value $m_A{}^2 = 0.71$ GeV^2 for our calculations.

In our model,

$$G_E(0) = \frac{1-x}{\kappa^2}, \quad G_M(0) = \frac{1}{\kappa^2}(1.84 - 2.79x),$$

$$G_A(0) = \frac{0.496}{\kappa^2}.$$

We compare in Table II the results for $R_{\rm el}^{\nu} \equiv \sigma(\nu p \to \nu p)/\sigma(\nu n \to \mu^- p)$ and $R_{\rm el}^{\bar{\nu}} \equiv \sigma(\bar{\nu} p \to \bar{\nu} p)/\sigma(\bar{\nu} p \to \mu^+ n)$ for the range $0.3 < Q^2 < 0.9$ GeV^2 using $m_A{}^2 = 0.71$ GeV^2, $x = 0.45$, and $\kappa^2 = 0.75$, and the BNL neutrino spectra, with the data of the Harvard-Pennsylvania-Wisconsin (HPW)[14] and the Columbia-Illinois-Rockefeller (CIR)[15] collaborations. We have also plotted the differential cross section $d\sigma/dQ^2$ as a function of Q^2 in Fig. 1.

D. $\nu_\mu e, \bar{\nu}_\mu e$ elastic scattering

Since the coupling of Z_1 to electron is purely vector, a prediction of the model is $\sigma(\nu_\mu e) = \sigma(\bar{\nu}_\mu e)$.

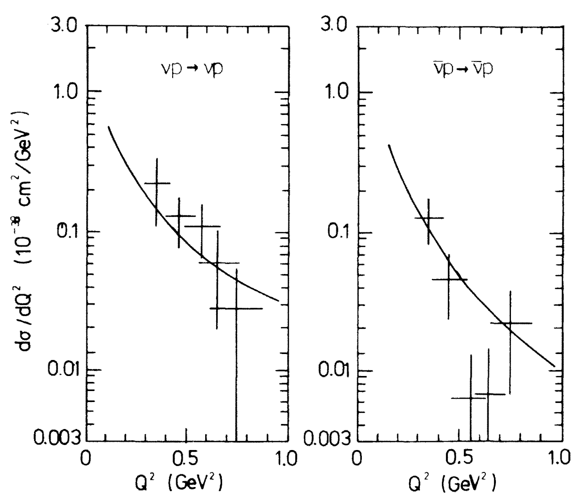

FIG. 1. A plot of $d\sigma/dQ^2$ vs Q^2 for νp and $\bar{\nu} p$ elastic scattering using $m_A{}^2 = 0.71$ (GeV)^2. The data points are from Ref. 14.

TABLE III. Comparison of the results for $\nu_\mu e$ and $\bar{\nu}_\mu e$ elastic scattering with data from Gargamelle (GGM) (Ref. 10) and the Aachen-Padova (AP) (Ref. 16) group. The values for $0.4 < E_e < 2.0$ GeV were those mentioned in a note added to the Aachen report.

	Theoretical	Experimental	E_e (GeV)
$\sigma_{\nu_\mu e}/E_\nu$ (10^{-42} cm^2/GeV)	2.01	2.4 ± 1.2 (AP)	0.2–2.0
		2.1 ± 1.2 (AP)	0.4–2.0
$\sigma_{\bar{\nu}_\mu e}/E_{\bar{\nu}}$ (10^{-42} cm^2/GeV)	2.01	$1.0^{+1.3}_{-0.5}$ (GGM)	0.3–2.0
		5.4 ± 1.7 (AP)	0.2–2.0
		3.0 ± 1.8 (AP)	0.8–2.0
		2.4 ± 1.3 (AP)	0.4–2.0

Though this is consistent with recent Aachen-Padova data,[16] the errors are too large to check this result accurately.

The neutrino and antineutrino cross sections (in the rest frame of the electron) are given by

$$\sigma_{\nu,\bar{\nu}} = \frac{G_F{}^2 m_e E_\nu}{8\pi\kappa^4}\left[4x^4\left(\frac{4}{3} - \frac{m_e}{2E_\nu}\right)\right],$$

where E_ν is the $\nu, \bar{\nu}$ energy, and m_e is the mass of the electron. We compare in Table III the values of $\sigma_{\nu,\bar{\nu}}/E_\nu$ obtained using $x = 0.45$ and $\kappa^2 = 0.75$ with the experimental values of Gargamelle[10] and the Aachen-Padova group (AP).[16]

E. $\bar{\nu}_e e$ elastic scattering

In this case the charged current and the neutral current both contribute to the cross sections. The differential cross section in the electron rest frame is given by

$$\frac{d\sigma}{dE_e} = \frac{G_F{}^2 m_e E_\nu}{8\pi\kappa^4}\left[4x^2 + (4\kappa^2 + 2x)^2\left(1 - \frac{E_e}{E_\nu}\right)^2 - 2x(4\kappa^2 + 2x)\frac{m_e E_e}{E_\nu{}^2}\right],$$

where E_e is the final energy of the electron.

To compare our prediction with the data of Reines *et al.*,[17] we fold the above expression into the antineutrino energy spectrum and calculate the cross section for the ranges $1.5 < E_e < 3$ MeV and $3 < E_e < 4.5$ MeV. The results for $\sigma_{\bar{\nu}_e e}/\sigma_{V-A}$ (σ_{V-A} is the charged-current cross section in the $V-A$ theory) are compared with experiment in Table IV.

It is seen that the agreement with experiment is good for νN inelastic scattering and νp elastic scattering. The data on $\nu_\mu e$, $\bar{\nu}_\mu e$, and $\bar{\nu}_e e$ elastic scattering is less certain, and although we obtain reasonable agreement with recent Aachen-Padova data for $\nu_\mu e$ and $\bar{\nu}_\mu e$ scattering, the agreement with the data of Reines *et al.* for $\bar{\nu}_e e$ scattering in the range $1.5 < E_e < 3$ MeV is poor. To obtain agreement for this range, we require a lower value of x, and this is also the case with the Weinberg-Salam model.[17] Clearly, we must await better data.

TABLE IV. Comparison of the results for $\bar{\nu}_e e$ elastic scattering with the data of Reines *et al.* (Ref. 17).

		Theoretical	Experimental
$\sigma_{\bar{\nu}_e e}/\sigma_{V-A}$	$1.5 < E_e < 3$ MeV	1.91	0.87 ± 0.25
$\sigma_{\bar{\nu}_e e}/\sigma_{V-A}$	$3 < E_e < 4.5$ MeV	2.18	1.70 ± 0.44

VI. SOME HIGHER-ORDER EFFECTS

A. K_L-K_S mass difference

The $W^\pm$ and $U^\pm$ vector bosons can induce transitions of the d and s quarks to u, c, x, x' (left-handed) and t, t', y, y' (right-handed) quarks. Hence, we get contributions to the process $d\bar{s} \to \bar{d}s$ from diagrams with pairs of $q = \frac{2}{3}$ and $q = -\frac{4}{3}$ quarks. Following the procedure of Gaillard and Lee,[18] we get the following ratio of K_L-K_S mass difference to the average mass of the K:

$$\frac{m_L - m_S}{m_K} \approx \frac{G_F}{\sqrt{2}}\frac{\alpha}{3\pi}\frac{f_K{}^2 \epsilon}{m_W{}^2 \sin^2\theta}$$

with

$$\epsilon = \sum_{a=u,t,x,y} \epsilon_{aa} + \sum_{(a,b)=(u,y)\,(t,x)} \epsilon_{ab}\,.$$

In the above expressions, f_K is the K decay constant, and if we assume that $m_a \approx m_a'$ (i.e., $m_{a'} - m_a \ll m_a$, $|m_b - m_a|$),

$$\epsilon_{aa} \approx \tfrac{4}{3}(m_{a'} - m_a)^2 \sin^2(\theta_C^L - \theta_a)\cos^2(\theta_C^L - \theta_a)\,, \quad (a = u, x)\,,$$

$$\approx \tfrac{4}{3}(m_{a'} - m_a)^2 \sin^2(\theta_C^R - \theta_a)\cos^2(\theta_C^R - \theta_a)\,, \quad (a = t, y)\,,$$

$$\epsilon_{ab}\ (a \neq b) \approx 0\,,$$

where θ_a is the mixing angle for the quarks a, a' and $\theta_C^{L,R}$ is the left- (right-) handed Cabibbo angle.

Thus, we must have

$$\sum_{a=u,t,x,y} (m_{a'} - m_a)^2 \sin^2(\theta_C^{L,R} - \theta_a)\cos^2(\theta_C^{L,R} - \theta_a) \approx 0.15\ (\text{GeV})^2$$

to explain the observed K_L-K_S mass difference. This restricts the t, x, y quark mass differences and/or mixing angles to be small.

B. Nonconservation of muon number

Muon number will not be conserved if there is mixing between E^0 and M^0, η and η', or ξ and ξ'. The processes $\mu \to e\gamma$ and $\mu \to 3e$ will get contributions from diagrams involving E^0, M^0 (if there is E^0-M^0 mixing) as in the Cheng and Li model,[19] and from diagrams involving the doubly charged leptons ξ, ξ' and η, η' (if there is $\xi - \xi'$, $\eta - \eta'$ mixing), as in the model of Wilczek and Zee.[20] One would expect the branching ratio for $\mu \to e\gamma$ to be $\sim 10^{-10} \times \sum_i (\Delta m_i^2)^2 \sin^2 2\omega_i$ where Δm_i^2 are the differences in the squares of masses (in GeV^2), and ω_i are the mixing angles of the heavy leptons.

C. Anomalous magnetic moment of muon

Contrary to the Georgi-Glashow model, the neutral heavy leptons do not contribute to Δg to order $G_F m_\mu m_F/\pi^2\sqrt{2}$ (with m_F the heavy mass), due to the purely right-handed nature of the couplings. For similar reasons the doubly charged leptons do not contribute to that order. To leading order the total weak contribution is

$$\Delta g = \frac{G_F m_\mu^2}{\pi^2\sqrt{2}}\left(\frac{x^2}{3\kappa^2} - \frac{5}{6}\frac{1}{1-\kappa^2} - \frac{8}{3}\right).$$

The first term comes from the Z_1 exchange, the second is the Z_2 contribution, and the last term is from doubly charged lepton contributions. It is interesting to note that the neutrino and neutral heavy-lepton contribution is canceled at this order by a contribution from the doubly charged lepton. For $x = 0.45$, $\kappa^2 = 0.75$ we find a total contribution of -5×10^{-8}, which is within experimental limits.

VII. CONCLUSIONS

We have presented a detailed analysis of a model which is in many respects a natural extension of the old SU(2) × U(1) theory. The fit with the data is very good. We have not attempted a detailed fit of the high-y anomaly due to the inadequate statistics of the present data. Since the shape of the y distribution is fairly sensitive to the masses and mixing angles of the new quarks and leptons in the model, such an attempt would be unwise at the moment. For similar reasons we have not computed precisely the rate for trimuon events.

An interesting variation of the model could be obtained by introducing a $(0, \frac{1}{2}, \frac{1}{2})$ Higgs multiplet which would give a higher mass to the $U^\pm$ states. Such a variation would greatly suppress the extra reactions involved in the model.

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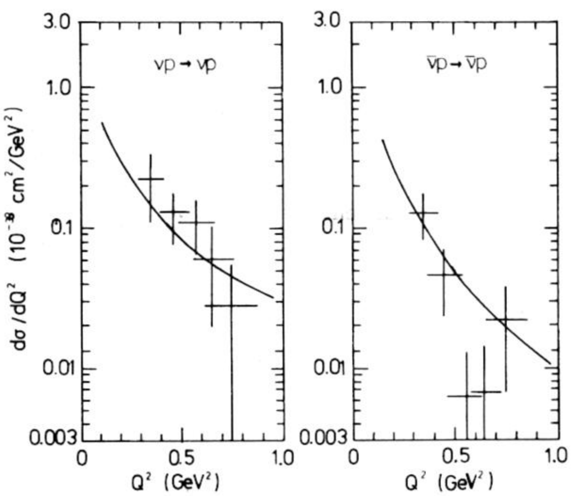

FIG. 1. A plot of $d\sigma/dQ^2$ vs Q^2 for νp and $\bar{\nu} p$ elastic scattering using $m_A{}^2 = 0.71$ $(\text{GeV})^2$. The data points are from Ref. 14.