Pattern of symmetry breaking with two Higgs doublets

Nilendra G. Deshpande and Ernest Ma
Institute of Theoretical Science, University of Oregon, Eugene, Oregon 97403
(Received 13 June 1977)

We analyze fully the pattern of symmetry breaking of an SU(2) × U(1) gauge model with two Higgs doublets. We find the phenomenon of spin-zero leptons to be a very general one, and obtain a solution previously obtained by one of us as a limiting case.

In the standard SU(2) × U(1) gauge model of the weak and electromagnetic interactions,[1] the minimum number of Higgs doublets required is one; but if there are more than one, then many interesting effects can arise, such as *CP* nonconservation,[2] muon-number nonconservation,[3] and observable charged and neutral Higgs bosons.[4] It is therefore of some importance to analyze *fully* the pattern of symmetry breaking with two or more Higgs doublets. In this paper, we limit our discussion to two Higgs doublets, and display all possible solutions in the most general case, as well as all admissible special cases. We find the phenomenon of spin-zero leptons[5] to be a very general one, and obtain the solution of Ref. 5 as a limiting case. We also find that massless neutral Goldstone bosons are possible under certain conditions, as well as charged "pseudo-Goldstone" particles. The experimental implications of these results will be briefly discussed.

Let

$$\begin{aligned} V = {} & \mu_1{}^2\Phi_1^\dagger\Phi_1 + \mu_2{}^2\Phi_2^\dagger\Phi_2 + \tfrac{1}{2}\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \tfrac{1}{2}\lambda_2(\Phi_2^\dagger\Phi_2)^2 \\ & + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ & + \tfrac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \tfrac{1}{2}\lambda_5^*(\Phi_2^\dagger\Phi_1)^2 \,. \end{aligned} \tag{1}$$

This is the most general SU(2) × U(1)-gauge-invariant, renormalizable Higgs potential for two doublets which is also invariant under the discrete transformations $\Phi_1 \to -\Phi_1$ and $\Phi_2 \to -\Phi_2$, where $\Phi_1 = (\phi_1^+, \phi_1^0)$ and $\Phi_2 = (\phi_2^+, \phi_2^0)$, respectively. The requirement that V should be bounded from below is expressed by the following conditions:

$$\begin{aligned} & \lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -(\lambda_1\lambda_2)^{1/2}, \\ & \lambda_3 + \lambda_4 \pm |\lambda_5| > -(\lambda_1\lambda_2)^{1/2} \,. \end{aligned} \tag{2}$$

Let $\langle\phi_1^0\rangle = v_1$, $\langle\phi_2^0\rangle = v_2$;[6] then $|v_1|$, $|v_2|$, and the relative phase between v_1 and v_2 are determined in the tree approximation by the conditions:

$$\begin{aligned} & |v_1|[\mu_1{}^2 + \lambda_1|v_1|^2 + (\lambda_3+\lambda_4)|v_2|^2 + \mathrm{Re}\,\lambda_5(v_1^*v_2)^2/|v_1|^2] = 0 \,, \\ & |v_2|[\mu_2{}^2 + \lambda_2|v_2|^2 + (\lambda_3+\lambda_4)|v_1|^2 + \mathrm{Re}\,\lambda_5(v_1^*v_2)^2/|v_2|^2] = 0 \,, \\ & \mathrm{Im}\,\lambda_5(v_1^*v_2)^2 = 0 \,. \end{aligned} \tag{3}$$

There are, of course, four possible solutions:

$$\begin{aligned} & \text{(A)}\ v_1 = 0, \quad v_2 = 0 \,; \\ & \text{(B)}\ v_1 = 0, \quad v_2 \neq 0 \,; \\ & \text{(C)}\ v_1 \neq 0, \quad v_2 = 0 \,; \\ & \text{(D)}\ v_1 \neq 0, \quad v_2 \neq 0 \,. \end{aligned} \tag{4}$$

They correspond to *mutually exclusive* sectors of the Higgs potential as given by the conditions below:

$$\begin{aligned} & \text{(A)}\ \mu_1{}^2 > 0, \quad \mu_2{}^2 > 0 \,; \\ & \text{(B)}\ \mu_2{}^2 < 0, \quad \lambda_2\mu_1{}^2 > \lambda_3\mu_2{}^2, \quad \lambda_2\mu_1{}^2 > (\lambda_3+\lambda_4 \pm |\lambda_5|)\mu_2{}^2 \,; \\ & \text{(C)}\ \mu_1{}^2 < 0, \quad \lambda_1\mu_2{}^2 > \lambda_3\mu_1{}^2, \quad \lambda_1\mu_2{}^2 > (\lambda_3+\lambda_4 \pm |\lambda_5|)\mu_1{}^2 \,; \\ & \text{(D)}\ \mu_1{}^2 < 0, \quad \mu_2{}^2 < 0, \quad \lambda_2\mu_1{}^2 < (\lambda_3+\lambda_4-|\lambda_5|)\,\mu_2{}^2 \,, \\ & \qquad \lambda_1\mu_2{}^2 < (\lambda_3+\lambda_4-|\lambda_5|)\mu_1{}^2, \quad \lambda_4 - |\lambda_5| < 0 \,, \\ & \qquad \lambda_5(v_1^*v_2)^2 = -|\lambda_5|\,|v_1|^2|v_2|^2 \,. \end{aligned} \tag{5}$$

These conditions are simply derived on the basis that $|v_1|^2$, $|v_2|^2$, and all Higgs-boson masses squared must be positive. For example, the Higgs-boson mass matrix for (C) is given by

$$\begin{aligned} V^{(2)} = {} & 2\lambda_1(\mathrm{Re}\,v_1^*\phi_1^0)^2 + (\mu_2{}^2 + \lambda_3|v_1|^2)\phi_2^-\phi_2^+ \\ & + [\mu_2{}^2 + (\lambda_3+\lambda_4)|v_1|^2]\,\bar\phi_2^0\phi_2^0 \\ & + \tfrac{1}{2}\lambda_5(v_1^*\phi_2^0)^2 + \tfrac{1}{2}\lambda_5^*(v_1\bar\phi_2^0)^2 \,, \end{aligned} \tag{6}$$

where $|v_1|^2 = -\mu_1{}^2/\lambda_1$. The five Higgs-boson mass eigenstates are $\sqrt{2}\,\mathrm{Re}(v_1^*\phi_1^0)/|v_1|$, $\phi_2^\pm$, ϕ_H^0, and ϕ_L^0, with masses squared of $2\lambda_1|v_1|^2$, $\mu_2{}^2 + \lambda_3|v_1|^2$, and $\mu_2{}^2 + (\lambda_3+\lambda_4 \pm |\lambda_5|)|v_1|^2$, respectively. The two neutral states ϕ_H^0 and ϕ_L^0 are linear combinations of ϕ_2^0 and $\bar\phi_2^0$ as follows:

$$\phi_H^0 = \sqrt{2}\,[\mathrm{Re}(v_1^*\phi_2^0)\cos\theta - \mathrm{Im}(v_1^*\phi_2^0)\sin\theta]/|v_1| \,,$$

where $\tan\theta = (|\lambda_5| - \mathrm{Re}\,\lambda_5)/\mathrm{Im}\,\lambda_5$, and ϕ_L^0 is the orthogonal combination. Note that $\mu_2{}^2$ is not necessarily positive or negative for this solution.

Case (A) is uninteresting because there is no symmetry breaking; and of the other three cases, because (B) is exactly like (C), we need only consider (C) and (D). In both of these cases, the original global SU(2) × U(1) symmetry of the Higgs potential V is broken down to U(1) which results in three would-be Goldstone bosons which become the longitudinal components of the three vector gauge bosons $W^{\pm}$ and Z. (With just one Higgs doublet, the original global symmetry is O(4) which is broken down to O(3), also resulting in three would-be Goldstone bosons.) However, there is one very important difference between (C) and (D): the discrete symmetry $\Phi_2 \to -\Phi_2$ remains unbroken in (C) but not in (D). This means that a new *multiplicative* quantum number is conserved in any SU(2) × U(1) weak-interaction gauge model which chooses the solution (C). The following scenario can then be envisioned: ϕ_2^+, ϕ_2^- are produced in e^+e^- annihilation, but because of this new conserved multiplicative quantum number, ϕ_2^+ and ϕ_2^- must either be stable or else decay into ϕ_2^0 and $\overline{\phi}_2^0$, which are linear combinations of the physical eigenstates ϕ_H^0 and ϕ_L^0, the latter being the lightest must then be stable. This situation is very similar to that of a sequential heavy lepton, and can therefore be invoked to explain the anomalous $e\mu$ events seen in e^+e^- annihilation,[7] as was recently proposed.[4]

If $\lambda_5=0$, the original global symmetry of the Higgs potential is enlarged to SU(2) × U(1) × U(1). Solution (C) now breaks the symmetry down to U(1) × U(1), preserving the symmetry $\phi_2 \to \phi_2 e^{i\alpha}$ This means that a conserved *additive* quantum number can now be defined for the doublet $\Phi_2 = (\phi_2^+, \phi_2^0)$, and its analogy to the lepton doublets (ν_e, e^-) and (ν_μ, μ^-) is even closer in this case. [For the solution (D), the residual symmetry is U(1) whether $\lambda_5 \neq 0$ or $\lambda_5 = 0$. Therefore, in the latter case, a massless neutral Goldstone boson

$$\sqrt{2}\,[|v_2|^2 \mathrm{Im}(v_1^* \phi_1^0) - |v_1|^2 \mathrm{Im}(v_2^* \phi_2^0)]/|v_1||v_2|(|v_1|^2+|v_2|^2)^{1/2}$$

will appear.]

Although ϕ_2^0 can be interpreted as a spin-zero neutrino, its mass is still arbitrary for the case $\lambda_5=0$. However, the remarkable fact is that we can enlarge the original global symmetry of the Higgs potential even further, so that ϕ_2^0 (and $\overline{\phi}_2^0$) can be obtained as strictly massless Goldstone particles. Let $\mu_1{}^2 = \mu_2{}^2$ and $\lambda_1=\lambda_2=\lambda_3+\lambda_4$ in addition to $\lambda_5=0$, then the Higgs potential V is invariant under the global symmetry SU(2) × SU(2) × U(1), where the second SU(2) comes from the invariance of V under the transformation of (Φ_1, Φ_2) as a doublet. [It should be noted that these conditions on the Higgs potential parameters are only sensible because they correspond to a larger symmetry. For example, if $\lambda_1 \neq \lambda_2$, then the condition $\lambda_1\lambda_2 = (\lambda_3+\lambda_4)^2$ cannot be maintained in the field theory beyond the tree approximation, although the condition $\lambda_5=0$ alone is perfectly legitimate since it introduces an extra U(1) symmetry.] For this special case of SU(2) × SU(2) × U(1) symmetry, (C) and (D) are *equivalent* solutions, and the breakdown is into U(1) × U(1), resulting in three would-be Goldstone bosons belonging to the first SU(2) group which is directly related to the local gauge group, plus two *true* Goldstone bosons belonging to the second SU(2) group which has nothing to do with the local gauge group. It can be seen from (6) that the two physical massless particles are indeed ϕ_2^0 and $\overline{\phi}_2^0$, and we obtain exactly the model of Ref. 5.

Another special situation is when $|\lambda_4| = |\lambda_5|$, in which case the global symmetry becomes O(4). (This can be easily verified with a little algebra.) For the solution (C), O(4) is broken down to O(3), so that either ϕ_H^0 or ϕ_L^0 becomes degenerate in mass with $\phi_2^{\pm}$ and together they form an O(3) triplet; but this latter symmetry is not a true symmetry of the complete theory, because the condition $|\lambda_4| = |\lambda_5|$ is impossible to maintain in the presence of vector-gauge-boson interactions.[5] For the solution (D), O(4) is broken down to O(2), resulting in five would-be Goldstone bosons: the three usual ones associated with $W^{\pm}$ and Z, plus two others which combine to make up one charged "pseudo-Goldstone" boson,[8] which is expected to have a small mass due to radiative effects.

If $\lambda_4=\lambda_5=0$, then the symmetry is even larger, namely O(4) × O(4) which breaks down to O(3) × O(4) and O(3) × O(3) for (C) and (D), respectively. In the latter case, there will be three "pseudo-Goldstone" bosons. If $\mu_1{}^2=\mu_2{}^2$ and $\lambda_1=\lambda_2=\lambda_3$ in addition to $\lambda_4=\lambda_5=0$, then the symmetry is the largest of all, i.e., O(8) which breaks down to O(7) for (C) or (D), resulting in four "pseudo-Goldstone" bosons, and one neutral Higgs particle.

In conclusion, for an SU(2) × U(1) gauge model with two Higgs doublets, it is not necessary that both develop vacuum expectation values to have spontaneous symmetry breakdown. It is equally likely for the Higgs potential V to choose the solution (B) or (C) instead of (D). Therefore, the phenomenon of spin-zero leptons is a very general one; it depends only on the existence of a discrete symmetry and either of the conditions (B) or (C) stated in (5). Under more special circumstances, when the original global symmetry of V is extended to SU(2) × SU(2) × U(1), it is even possible to have ϕ_2^0 and $\overline{\phi}_2^0$ as truly massless Goldstone bosons, so that the identification of (ϕ_2^+, ϕ_2^0) as a spin-zero lepton doublet is even more compelling. Experimentally, $\phi_2^{\pm}$ can be directly produced in e^+e^- annihilation and be responsible for the anomalous $e\mu$ events.[7] The tests for such a hypothesis are discussed in Ref. 4. They can also appear as decay products of high-mass particles in exactly the same way as the ordinary spin-one-half letpon doublets (ν_e, e^-) and (ν_μ, μ^-). For example, the process $D^0 \to \pi^- \phi_2^+ \overline{\phi}_2^0$ is possible, if it were kinematically allowed. Although it remains to be seen whether there are indeed such particles, their discovery would be an exciting confirmation of the whole framework of spontaneously broken gauge models via the Higgs mechanism.

This research was supported in part by U.S. Energy Research and Development Administration under Contract No. AT(45-1)-2230.

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