PHYSICAL REVIEW D VOLUME 18, NUMBER 7 1 OCTOBER 1978

Quark-cluster model of dibaryon resonances

D. B. Lichtenberg, E. Predazzi,* D. H. Weingarten, and J. G. Wills
Physics Department, Indiana University, Bloomington, Indiana 47401
(Received 25 May 1978)

We suggest that a recently discovered dibaryon resonance is composed of quark clusters such as a quadriquark and a diquark or a pentaquark and a quark.

I. INTRODUCTION

Some years ago, the existence of a Regge trajectory of dibaryon resonances was proposed,[1] based on evidence from a variety of experiments. Subsequently an experiment of Kamerud *et al.*[2] showed further evidence for such a trajectory, and very recently Auer *et al.*[3] obtained strong additional evidence for a diproton resonance with a mass of about 2260 MeV. Partial-wave analyses[3,4] suggest the resonance at 2260 MeV has spin 3 and negative parity.

In this note we take the existence of dibaryon resonances as established and consider a model for the structure of the leading Regge trajectory of dibaryons. We argue that the leading trajectory consists of a quadriquark (an unexcited system of four quarks) and a diquark (an unexcited system of two quarks) orbiting around their common center of mass. Almost degenerate are a trajectory corresponding to a pentaquark-quark configuration and one corresponding to a linear arrangement of three diquarks.

The idea we will present is a generalization to the six-quark system of a model proposed more than a decade ago[5,6] in which baryons were considered bound states of a quark and a diquark. Shortly after the quark-diquark model was suggested, it was pointed out[7] that the predicted spectrum of baryons includes fewer low-lying states than does the spectrum of a three-quark model with harmonic-oscillator forces. The present experimental evidence favors this simpler spectrum.[8] In Ref. 7 it was also proposed, in effect, that exotic mesons might be realized by replacing the quark and antiquark of a normal meson with a diquark and antidiquark, respectively.[9] Our present suggestion is that the leading dibaryon trajectories also correspond to configurations with quark clusters.

We begin by giving an argument for the quark-cluster picture using a simple classical string model suggested by quantum chromodynamics (QCD). As a corollary, this argument also leads to the quark-diquark configuration for the leading baryon trajectory and to a diquark-antidiquark configuration for the leading exotic meson trajectory. Thus, we treat normal and exotic mesons, baryons, and dibaryon resonances in a unified way.[10] Next we briefly consider consequences of the string model for the decays of dibaryons composed of quark clusters. Finally, we estimate the masses of such resonances using notions from a potential model.[11]

II. CLASSICAL STRING MODEL

A classical string model for spinless quarks can be constructed as follows. Strings are assumed to carry color $\underline{3}$ indices at one end and $\underline{3^*}$ indices at the other, while quarks carry $\underline{3}$ indices and antiquarks $\underline{3^*}$. Allowed configurations are obtained by joining quarks and strings into color singlets by contracting a $\underline{3}$ index with a $\underline{3^*}$, or contracting a triple of $\underline{3}$ indices or a triple of $\underline{3^*}$ indices with the alternating tensor ϵ_{ijk}. The action for each configuration consists of the sum of the Nambu-Goto action[12] for each string and the usual classical relativistic action for spinless particles for each quark. This action is the spinless-quark version of an action which has been derived[13,14] as an approximation to a class of terms in Wilson's strong-coupling expansion to lattice gauge theory; it has also been suggested[15] as an adaptation of the dual string model.

If closed string loops are ruled out, for the moment, then the most general way of connecting six quarks to form a dibaryon resonance is shown in Fig. 1(a). Any or all of these strings can be taken to have zero length or nearly so, yielding, for example, the configurations shown in Figs. 1(b)–1(h).

Now each Regge trajectory in this model consists of a continuous sequence of rotational excitations about the center of mass of some particular configuration. Calculations in Ref. 13 show that the strings of leading trajectories remain straight without vibrational excitations. For a given angular momentum, string vibrations increase the total energy and therefore decrease the trajectory's intercept. For strings which remain straight the equations of motion forbid configurations such as Fig. 1(b) with one or more quarks attached by a string of nearly zero length to a pair of colinear strings at a location other than the center of mass. The equations of motion also eliminate configurations such as Fig. 1(c) with colinear strings joined at a point and the number of strings in the one direction differing both from zero and from the number of strings in the opposite direction.

Three classes of configurations remain. Class I: A single linear string passes through the center of mass ending in unexcited clusters of quarks. The possibilities are shown in Figs. 1(d), 1(e), and 1(f). Class II: A single linear string passes through the center of mass ending in one or more rotationally excited clusters of quarks. A possibility is shown in Fig. 1(g). Class III: More than one string goes through the center of mass. A possibility is shown in Fig. 1(h).

Class I. For large values of angular momentum, the angular momentum and energy of any configuration is carried predominantly by the string. For a string with endpoints moving at nearly the speed of light the c.m.-system energy squared s and angular momentum $J(s)$ obey

$$J(s)/s \rightarrow K, \tag{1}$$

where the slope of the trajectory K is a constant whose magnitude depends on the strength of the interaction. Thus, asymptotically for large J, the model leads to linear trajectories with slope independent of the quarks at the string's ends. The asymptotic slope is the same as that of normal mesons, which consist of a quark and an antiquark joined by a single string. For small values of angular momentum, $J(s)$ and s can be related by the moment of inertia $I(s)$

$$[J(s)]^2 = 2I(s)(s^{1/2} - 6q), \tag{2}$$

where q is the mass of a quark. It follows from (2) that the configuration with maximum $I(s)$ gives the leading trajectory. It has been verified by explicit calculation of all class I trajectories that the diquark-quadriquark configuration of Fig. 1(d) has the highest value of $I(s)$. This trajectory remains leading for large s, since the asymptotic slope of all class I trajectories is the same. However, the quark-pentaquark trajectory corresponding to Fig. 1(e) and the trajectory corresponding to Fig. 1(f) lie only slightly lower.

Class II. Asymptotically the trajectory of Fig. 1(g) is again linear with slope K. If the main string is completely deexcited, the trajectory of Fig. 1(f) is obtained. This lies below the trajectory of 1(d). Moreover, the larger the mass at the string ends, the flatter the trajectory for small values of s. Thus, as the main string in Fig. 1(g) picks up angular momentum, the trajectory initially rises more slowly than that of 1(d). Similar arguments can be made for all other class II trajectories.

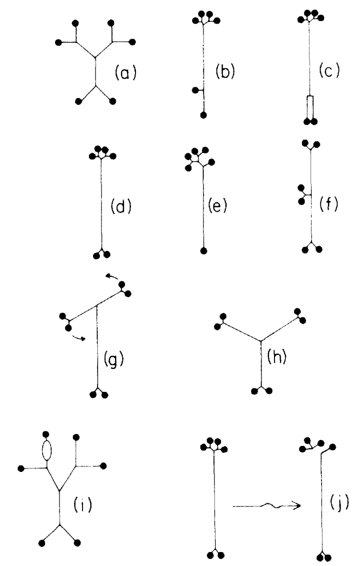

FIG. 1. Various configurations of six quarks in the string model.

Class III. When configuration 1(h) is highly excited, if each arm carries energy E, it carries angular momentum $\frac{1}{2}J(4E^2)$ with the same function $J(s)$ appearing in Eq. (1). If n equal arms radiate from the center of mass, the total angular momentum J_T and total energy squared s are related by[16]

$$\begin{aligned} J_T/s &= \tfrac{1}{2}nJ(4E^2)/(nE)^2 \\ &= (2/n)J(4E^2)/(4E^2) \\ &= 2K/n\,. \end{aligned}$$

If n is greater than two the trajectory is asymptotically flatter than any of those in class I or II. Explicit calculation also shows that the trajectory of 1(d) leads class III trajectories for small values of s.

We now relax our assumption that no closed loops of strings occur. A typical configuration with one loop is shown in Fig. 1(i). If the loop has zero length, Fig. 1(i) becomes identical to Fig. 1(a). If the loop has finite length, arguments similar to those we have already given show the resulting trajectories are all below the quadriquark-diquark trajectory. A similar conclusion holds for other loop configurations we have examined, although a completely general argument covering all possibilities has not been found. Nonetheless, our results strongly suggest the leading dibaryon trajectory is actually given by the diquark-quadriquark configuration, with the quark-pentaquark and linear three-diquark configurations lying slightly below.

If our arguments are applied to the three-quark system, one finds that the leading baryon trajectory does indeed have a quark-diquark configuration. For an exotic two-quark and two-antiquark system we find the leading trajectory consists of a diquark and an antidiquark. The slopes of these trajectories are predicted to be the same asymptotically as the slope of meson trajectories; for small s they are predicted to be somewhat less than the meson slope.

We next briefly consider decays of a dibaryon composed of quark clusters. For the diquark-quadriquark configuration, if some of the strings within the quadriquark reconnect, a decay to a ground-state baryon and an excited baryon can occur as shown in Fig. 1(j). For the quark-pentaquark configuration, recombination of strings in the pentaquark also yields a ground-state baryon and an excited baryon. On the other hand, recombination of the strings at the diquark at the center of mass of the three-diquark system of Fig. 1(f) yields a pair of excited baryons. Since each of these decays requires only a slight change in the structure of the state, we expect them to occur easily; thus the leading dibaryon resonances should be fairly broad. In fact the resonance at 2260 MeV has a width of about 200 MeV. Moreover we would expect decay by string recombination to yield a significant fraction of the total width of dibaryons. Thus diquark-quadriquark and quark-pentaquark systems should show an enhancement of asymmetric decays to an excited baryon and a ground-state baryon and a corresponding suppression of symmetric decays while the three-diquark system should have symmetric decays enhanced and asymmetric decays suppressed.

III. POTENTIAL MODEL

Now the classical string model without any quantum-mechanical corrections is more likely to be quantitatively accurate for large values of s. To estimate the masses of the lowest dibaryons we use instead a potential model suggested by QCD. We begin by considering the interaction between two quarks, or a quark and an antiquark, arising from the exchange of a single colored gluon. This interaction is attractive only for two quarks in a color $\underline{3}^*$ configuration and for a quark and an antiquark only in a color $\underline{1}$ configuration.[17] The diquark of the string model is therefore replaced, in the present discussion, by unexcited pairs of quarks bound in a $\underline{3}^*$ configuration. This is precisely the combination for which the string model suggests binding will occur.

We now observe that, except for spin-dependent forces, the interaction between two colored triplets or a triplet and an antitriplet should depend only on their color configurations. To this approximation we can take the interaction between a quark and a diquark to be the same as between a quark and an antiquark, and the interaction between two diquarks to be the same as between two antiquarks. Thus the quadriquark of the string model is replaced here by a pair of diquarks bound in a $\underline{3}$ configuration and a pentaquark is a bound state of quadriquark and quark in a $\underline{3}^*$ configuration.

Next we assume that only color singlets occur as free particles. In the string model this was insured by the rules for joining strings and quarks and by the choice of action. Here the corresponding mechanism of confinement is a potential rising linearly[18] with r added to the one-gluon potential acting between each pair of particles (quarks, diquarks, etc.) which in our earlier picture were joined by strings. (Other authors[19,20] have considered potentials which increase as another power of r or as $\ln r$.)

The mass of a bound state of two particles (quarks, diquarks, etc.) now becomes the sum of the masses of the constituents plus an interaction energy which depends only on the color multiplicity and on the masses of the constituents. For simplicity we consider only quarks of two flavors, u and d, and we neglect their mass difference. If the symbol for a particle denotes its mass, capital letters denote color singlets, and small letters denote $\underline{3}$ or $\underline{3}^*$ states, the mass of a meson M is given by

$$M = 2q + \delta_1(qq)\,, \tag{3}$$

where $\delta_1(qq)$ is the interaction energy of $q\bar{q}$ in an SU(3)-singlet state. Similarly the mass of a diquark d is $d = 2q + \delta_3(qq)$, where $\delta_3(qq)$ is the qq interaction energy in a $\underline{3}^*$ state. The mass of a baryon B, considered as a bound state of q and d, becomes

$$B = q + d + \delta_1(qd)\,, \tag{4}$$

and an exotic meson mass E, given by a $d\bar{d}$ bound state, is

$$E = 2d + \delta_1(dd)\,. \tag{5}$$

In the model a quadriquark f is a dd bound state in a $\underline{3}$ configuration and a pentaquark is a qf bound state in a $\underline{3}^*$ configuration. These have masses $f = 2d + \delta_3(\overline{dd})$ and $p = f + q + \delta_3(fq)$. For a dibaryon state D, which is a $\underline{1}$ configuration of fd or pq, we obtain

$$D_{fd} = f + d + \delta_1(fd), \quad D_{pq} = p + q + \delta_1(pq)\,. \tag{6}$$

(An estimate of the mass of the ddd configuration will be given below.)

Equations (3)–(6) contain too many parameters to be useful unless the behavior of the functions $\delta_1(m_1, m_2)$ and $\delta_3(m_1, m_2)$ is specified. In a potential model in which the potential varies as r^ϵ, simple scaling arguments show[20] that δ_1 and δ_3 behave like $\mu^{-\epsilon/(2+\epsilon)}$ where μ is the reduced mass of the constituent particles. Thus, for a linear potential, the δ's vary like $\mu^{-1/3}$. We have examined the behavior of δ_1 and δ_3 numerically in a variety of potential models which give qualitative fits to the observed meson spectra. We have found that these quantities are small compared to the quark masses and generally decrease as μ increases. Furthermore, if δ_1 and δ_3 start out positive they may even go negative. With a suitable choice of parameters, we can obtain a value for the lowest negative-parity dibaryon resonance close to the value of the observed resonance at 2260 MeV. However, for simplicity we restrict ourselves here to the case in which δ_1 and δ_3 are constants. Based on our numerical work we believe that this assumption will lead to upper limits on the masses of exotic mesons and dibaryons.

With δ_1 and δ_3 constants, Eqs. (3)–(6) become

$$M = 2q + \delta_1, \quad B = 3q + \delta_3 + \delta_1, \tag{7}$$

$$E = 4q + 2\delta_3 + \delta_1, \quad D = 6q + 4\delta_3 + \delta_1. \tag{8}$$

Note that in the approximation that δ_1 and δ_3 are constants, D_{fd} and D_{pq} are degenerate in mass. This degeneracy will be only slightly broken if δ_1 and δ_3 have a weak mass dependence like $\mu^{-1/3}$. If all of the strings in a qf, pq, or ddd configuration are nearly unexcited, the three possibilities become practically identical. Thus D in Eq. (8) should also be nearly the mass of the lowest ddd dibaryon. In any case the widths of these resonances are likely to be much greater than the splitting between the lightest states on the qf, pq, and ddd trajectories.

We assume that u and d quarks have masses of 335 MeV as would be the case if quarks have Dirac magnetic moments. Using the experimental values of meson and baryon masses, we can apply Eq. (7) to determine δ_1 and δ_3. Then, placing these values in Eq. (8), we can estimate the masses of exotic mesons and baryons.

As in our treatment with strings, we neglect spin dependence of the interactions. We do this partly for simplicity and partly because even for the qq interaction the usual spin-dependent terms do not correctly give the fine-structure splitting of charmonium.[21] We must therefore consider the values of M and B to be spin-averaged quantities. From the lowest nonstrange mesons $\rho\omega\pi\eta$ we obtain $M = 670$ MeV. Actually the η and η' mesons contain some admixture of strange quarks. This should have the effect of increasing the η' mass and decreasing the η mass so that M should perhaps be a bit greater than 670 MeV. Use of either the quark model or quark-diquark model with a spin-spin interaction leads to a spin-averaged baryon mass which is the mean of N and Δ weighted by their spin multiplicities. We obtain $B = 1085$ MeV.

With these values we obtain from Eq. (7) $\delta_1 = 0$, $\delta_3 = 80$ MeV. Eq. (8) then gives the following spin-averaged ground state values of D and E:

$$E = 1500 \text{ MeV}, \quad D = 2330 \text{ MeV}.$$

As we already mentioned, the string model suggests both of these trajectories have nearly the same slope as meson trajectories. Taking the meson slope as 1 GeV^{-2} and assuming degeneracy of even- and odd-parity trajectories, we obtain for the recurrences of $E(1500)$ and $D(2330)$:

$$\begin{gathered} E_1^-(1800),\ E_2^+(2060),\ E_3^-(2290), \ldots, \\ D_1^-(2540),\ D_2^+(2730),\ D_3^-(2900), \ldots, \end{gathered} \tag{9}$$

where signs denote parity and subscripts give orbital angular momentum.

The odd-parity dibaryon of lowest mass is the $D_1^-(2540)$. Its mass is considerably higher than that of the observed dibaryon resonance at 2260 MeV. However, the masses of Eq. (9) should be regarded as upper limits of the spin-averaged values, and therefore cannot be directly compared to the masses of observed dibaryon resonances. In any case, the data needed to determine the actual values of spin-averaged dibaryon and exotic meson masses are not currently available.

ACKNOWLEDGMENT

One of us (E.P.) would like to thank the members of the Physics Department at Indiana University for their kind hospitality during his visit. This work was supported in part by the U. S. Department of Energy and in part by the National Science Foundation.

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*Permanent address: Istituto di Fisica Teorica dell' Universita and Istituto Nazionale di Fisica Nucleare—Torino, Italy.

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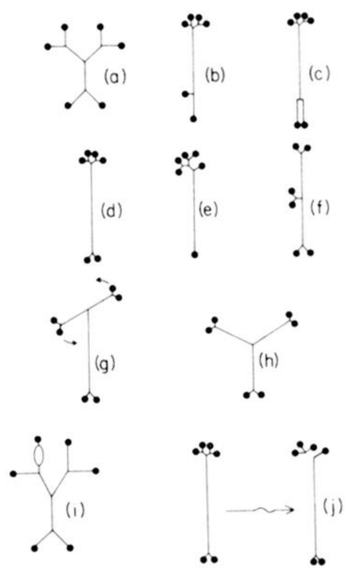

FIG. 1. Various configurations of six quarks in the string model.