Hyperon magnetic moments in SU(3)*

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(Received 18 July 1977)

The hyperon magnetic moments and the magnetic transition moment $\mu_{\Sigma^0\Lambda}$ have been calculated under various assumptions for the electromagnetic current operator within the spectrum-generating SU(3) approach. None of these assumptions, including the conventional ones using the Gell-Mann–Nishijima formula and SU(3) symmetry, lead to an acceptable fit to the experimental data.

Several experiments have provided the values for the magnetic moments of hyperons.[1] However, their theoretical understanding within the SU(3) scheme has so far caused difficulties.[2] It also appears that no reasonable assumption of SU(3) breaking can account for the discrepancies between the experimental and group-theoretical values calculated from the Gell-Mann–Nishijima formula

$$V_\mu^{\text{el}} = V_\mu^{\pi^0} + \frac{1}{\sqrt{3}} V_\mu^\eta \tag{1}$$

for the electromagnetic current, where $V_\mu^{\pi^0}$ and V_μ^η are the ($I=1$, $I_3=0$, $Y=0$) and ($I=0$, $I_3=0$, $Y=0$) components of an SU(3) octet operator, respectively.

Recently,[3] in connection with the calculation of the radiative decays of vector mesons $V\to P\gamma$ within SU(4), it was suggested that in addition to a symmetry-breaking factor one should also consider an SU(3) scalar term in the electromagnetic current operator. In particular it was argued that if the experimental value[4] $\Gamma(\rho\to\pi\gamma)=35\pm10$ keV is correct, then it is inevitable to generalize Eq. (1). The generalization which is, among others, capable of explaining the radiative decays of vector mesons is[3]

$$V_\mu^{\text{el}} = V_\mu^{\pi^0} + \frac{1}{\sqrt{3}} V_\mu^\eta + V_\mu^S, \tag{2}$$

where V_μ^S is an SU(3)-scalar operator.

In the present paper, Eq. (1) as well as Eq. (2) will be used for a fit of the experimental values of the hyperon magnetic momenta. The breaking of SU(3) will be taken into account in the form of the spectrum-generating SU(3)_E approach[5] in which coupling constants and form factors are not only given by the Clebsch-Gordan coefficients but are also functions of the masses. Within this spectrum-generating SU(3)_E approach we shall also use the same particular assumptions that were used for the calculations of $V\to P\gamma$. It will be shown that with Eq. (1), any symmetry-correction factor will give a rather poor fit of the experimental hyperon magnetic moments. Using Eq. (2) the agreement between experimental data and theoretical predictions seems to be better; however, taking all the experimental values seriously, the fit does not really improve.

The magnetic moment of the hyperon α is written as $(e/2m_p)\mu_\alpha$, where μ_α is the value of the magnetic moment in units of the nuclear magneton and m_p is the proton mass. Expressed in terms of the conventional form factors this is given by

$$\frac{e}{2m_p}\mu_\alpha = \frac{e}{2m_\alpha}(f_1^\alpha + 2m_\alpha f_2^\alpha), \tag{3}$$

where f_1^α and f_2^α are defined by

$$\langle p'\alpha|V_\mu^{\text{el}}|\alpha p\rangle = \bar{u}_\alpha(p')(f_1^\alpha\gamma_\mu + f_2^\alpha i\sigma_{\mu\nu}q^\nu)u_\alpha(p). \tag{4}$$

Usually f_1^α and f_2^α or $m_\alpha f_2^\alpha$ are written as

$$f_i^\alpha = \sum_\gamma C(\gamma, \alpha, \text{el}, \alpha) f_i^{(\gamma)}, \tag{5}$$

where $C(\gamma, \alpha, \text{el}, \alpha)$ are the antisymmetric (for $\gamma=F$) and the symmetric (for $\gamma=D$) SU(3) Clebsch-Gordon coefficients; and the $f_i^{(\gamma)}$ are supposed to be SU(3)-invariant form factors. However, if one takes SU(3) breaking into account by considering SU(3) as a spectrum-generating group, one obtains

$$f_1^\alpha = 2m_\alpha{}^{q-1} \sum_\gamma C(\gamma, \alpha, \text{el}, \alpha) F_1^{(\gamma)}, \tag{6}$$

$$f_2^\alpha = 2m_\alpha{}^{q-2} \sum_\gamma C(\gamma, \alpha, \text{el}, \alpha) F_2^{(\gamma)}, \tag{7}$$

where the $F_i^{(\gamma)}$ are the form factors invariant with respect to SU(3)_E, which is now considered not as a symmetry group but just as a particle-classifying group. Except for the suppression factor $m_\alpha{}^{q-1}$, Eqs. (6) and (7) are an immediate consequence of the Wigner-Eckart theorem for the irreducible tensor operator V_μ^{el}. A derivation of (6) and (7), i.e., essentially of the suppression factor $m_\alpha{}^{q-1}$, from the assumption that SU(3) is a spectrum-generating group, SU(3)_E, is given in the Appendix and makes use of results obtained in Refs. 6 and 7. The value of q depends upon the particular symmetry-breaking assumption. From the radiative decays of vector mesons q was determined empirically to be $q=1$, $\frac{1}{2}$, or $\frac{3}{2}$,[3(b)] of which $q=1$ was the theoretically preferred value.[5]

If one uses Eq. (1) for the electromagnetic interaction operator, then the Clebsch-Gordan coefficients in Eqs. (6) and (7) are $C(F, \alpha, \text{el}, \alpha)$, the antisymmetric octet-octet-octet coefficients which are proportional to the charges and $C(D, \alpha, \text{el}, \alpha)$, the symmetric ($D$-type) octet-octet-octet coefficients. If one uses Eq. (2) one has in addition to these two terms a term proportional to the octet-single-octet coefficient $C(S, \alpha, \text{el}, \alpha) \sim \delta_{\alpha\alpha}=1$ coming from the SU(3)-scalar operator V_μ^s. Thus one has four [in case of Eq. (1)] or six [in case of Eq. (2)] arbitrary parameters $F_1^{(F)}, F_1^{(D)}, F_1^{(S)}, F_2^{(F)}, F_2^{(D)}, F_2^{(S)}$ to fit from the charges and magnetic moments of the baryons.

In any case as f_1^α [or the factor of γ_α in Eq. (A3)] must be proportional to the charges, one obtains

$$f_1^\alpha = 2m_\alpha{}^{q-1}C(F, \alpha, \text{el}, \alpha)F_1^{(F)} , \tag{8}$$

i.e.,

$$F_1^{(D)}=0, \quad F_1^{(S)}=0 . \tag{9}$$

For f_2^α one obtains explicitly

$$f_2^\alpha = 2m_\alpha{}^{q-2}[C(F, \alpha, \text{el}, \alpha)F_2^{(F)} + C(D, \alpha, \text{el}, \alpha)F_2^{(D)}] , \tag{10}$$

for case (1), and

$$f_2^\alpha = 2m_\alpha{}^{q-2}[C(F, \alpha, \text{el}, \alpha)F_2^{(F)} + C(D, \alpha, \text{el}, \alpha)F_2^{(S)}+F_2^{(S)}], \tag{10$'$}$$

for case (2).

Inserting Eqs. (8), (9), and (10) into Eq. (3) one obtains

$$\mu_\alpha = \frac{m_p}{m_\alpha} 2m_\alpha{}^{q-1}[C(F, \alpha, \text{el}, \alpha)(F_1^{(F)}+2F_2^{(F)}) + C(D, \alpha, \text{el}, \alpha)2F_2^{(D)}], \tag{11}$$

$$\mu_\alpha = \frac{m_p}{m_\alpha} 2m_\alpha{}^{q-1}[C(F, \alpha, \text{el}, \alpha)(F_1^{(F)}+2F_2^{(F)}) + C(D, \alpha, \text{el}, \alpha)2F_2^{(D)}+2F_2^{(S)}]. \tag{11$'$}$$

For the comparison with the experimental data we define new parameters,

$$\begin{aligned} f &= 2m_p{}^{q-1}(F_1^{(F)}+2F_2^{(F)}) , \\ d &= 2m_p{}^{q-1}2F_2^{(D)} , \\ s &= 2m_p{}^{q-1}2F_2^{(S)} , \end{aligned} \tag{12}$$

in terms of which the magnetic moments in units of the nuclear magneton, μ_α, are expressed for case (1):

$$\mu_\alpha = \left(\frac{m_\alpha}{m_p}\right)^{q-2} [C(F, \alpha, \text{el}, \alpha)f + C(D, \alpha, \text{el}, \alpha)d] , \tag{13}$$

or for case (2):

$$\mu_\alpha = \left(\frac{m_\alpha}{m_p}\right)^{q-2} [C(F, \alpha, \text{el}, \alpha)f + C(D, \alpha, \text{el}, \alpha)d + s] . \tag{13$'$}$$

The factor m_p/m_α in Eq. (13), arises from the use of nuclear-magneton units; the factor $(m_\alpha/m_p)^{q-1}$ is the suppression factor describing the SU(3)-symmetry breaking; and the third factor in brackets is the effect that arises from the property of the SU(3) group.

For the Σ^0-Λ magnetic transition moment $\mu_{\Sigma^0\Lambda}$ (in nuclear-magneton units) the expression is slightly more complicated but obtained in a similar way from the formulas in the Appendix. It is given by

$$\mu_{\Sigma^0\Lambda} = \frac{m_p}{m_\Lambda + m_{\Sigma^0}} \frac{m_\Lambda{}^q + m_{\Sigma^0}{}^q}{m_p{}^q} \frac{m_p}{(m_\Lambda m_{\Sigma^0})^{1/2}} \times [C(F, \Sigma^0, \text{el}, \Lambda)f + C(D, \Sigma^0, \text{el}, \Lambda)d + s] . \tag{14}$$

As the suppression factor $(m_\alpha/m_p)^{q-1}$ with arbitrary q is very general for the magnetic moments ($\alpha'=\alpha$), but a consequence of the specific assumption (A6) in the Appendix for the transition moment ($\alpha' \neq \alpha$), we will consider arbitrary values of q for the magnetic moments. For the transition moment we shall then consider only the value $q=1$, which will turn out to be the best value for the hyperon magnetic moments. In this case Eq. (14) will be given by

$$\mu_{\Sigma^0\Lambda} = \frac{m_p}{(m_{\Sigma^0} m_\Lambda)^{1/2}} \left(-\frac{1}{\sqrt{15}} d + s\right) , \tag{15}$$

where we have already inserted the values of the Clebsch-Gordan coefficients.

After the value of q ($q=1$ is the theoretically preferred value and is also obtained empirically from the application of the spectrum-generating group approach to other processes[3,4,6]) has been chosen there are two [in case of Eq. (1)] or three [in case of Eq. (2)] arbitrary parameters, essentially reduced matrix elements. The Clebsch-Gordan coefficients are listed in Table I.

In case (1) the two parameters f and d will be determined from the value of neutron and proton magnetic moments and are obtained to be

$$f=\sqrt{3} \times 1.86, \quad d=-\sqrt{15} \times 0.98 .$$

The predictions of Eq. (13) for the magnetic moments are listed in columns 3–7 of Table II for various values of q. [The prediction for $\mu_{\Sigma^0\Lambda}$ in

TABLE I. SU(3) Clebsch-Gordan coefficients occurring in the matrix elements of the electromagnetic current operator.

$\alpha\,\alpha'$	$C(F,\alpha,el,\alpha')$	$C(D,\alpha,el,\alpha')$
p	$1/\sqrt{3}$	$-1/\sqrt{15}$
n	0	$2/\sqrt{15}$
Λ	0	$1/\sqrt{15}$
Σ^+	$1/\sqrt{3}$	$-1/\sqrt{15}$
Σ^-	$-1/\sqrt{3}$	$-1/\sqrt{15}$
Ξ^-	$-1/\sqrt{3}$	$-1/\sqrt{15}$
Σ^0	0	$-1/\sqrt{15}$
Ξ	0	$2/\sqrt{15}$
$\Sigma^0\Lambda$	0	$-1/\sqrt{5}$

the 6th column is not for $q=2$ but is the value of $C(F,\Sigma^0,\mathrm{el},\Lambda)f+C(D,\Sigma^0,\mathrm{el},\Lambda)d$, which is the conventional prediction without the SU(3)-breaking correction factor and which is identical with the prediction of Eq. (13′) with $q=2$ for the μ_α.] The experimental values, as far as they are known, are given in column 2 of Table II.[9] [The number for the $\Sigma^0\Lambda$ magnetic transition moment is the absolute value.) Comparison of the predictions and the experimental values shows that there is no value of q which leads to agreement between the predictions of Eq. (1) and the experimental data. The best predictions are obtained for $q=1$, but even for this value the predictions are off from the experimental values by 1 or 2 standard deviations for each value. Already the prediction for $\mu_{\Sigma^+}/\mu_{\Sigma^-}$, which is independent of any symmetry-breaking effect, differs from the experimental value by 2 standard deviations.

For case (2) we proceed in two ways: First we determine the three free parameters f, d, and s from μ_p, μ_n, and the ratio of the experimental values $\mu_{\Sigma^+}/\mu_{\Sigma^-}$, because these three quantities are independent of the value of q, which depends upon the particular symmetry-breaking assumption. We will see that the agreement between the experimental and theoretical values is again poor, even in the best case, which turns out to be again $q=1$. In order to investigate the cause of the disagreement, we will then in a second step fix f and d from the experimental values of μ_p and μ_n and determine s from the values of the hyperon magnetic moments and $|\mu_{\Sigma^0\Lambda}|$; and we will see that μ_Λ and $|\mu_{\Sigma^0\Lambda}|$ will require a positive value for s and μ_{Σ^-} and μ_{Ξ^-} will require a negative value for s.

From (13′) and the Clebsch-Gordan coefficients in Table I, one obtains with the experimental values of μ_p, μ_n, $\mu_{\Sigma^+}/\mu_{\Sigma^-}$:

$$\mu_p=2.79=\frac{1}{\sqrt{3}}f-\frac{1}{\sqrt{15}}d+s\,,$$

$$\mu_n=-1.91=\frac{2}{\sqrt{15}}d+s\,,$$

and

$$\mu_{\Sigma^+}/\mu_{\Sigma^-}=-1.91\pm 28\%=\frac{(1/\sqrt{3})f-(1/\sqrt{15})d+s}{-(1/\sqrt{3})f-(1/\sqrt{15})d+s}=\frac{2.79}{-(1/\sqrt{3})f-(1/\sqrt{15})d+s}\,.$$

From these three equations one determines

$$s=-0.23\pm 0.15\,,\qquad \frac{1}{\sqrt{15}}d=-0.84\pm 0.08\,,\qquad \frac{1}{\sqrt{3}}f=2.18\pm 0.17\,. \tag{16}$$

The values that one obtains for $C(F,\alpha,\mathrm{el},\alpha)f+C(D,\alpha,\mathrm{el},\alpha)d+s$ with these values of the param-

TABLE II. Comparison between the experimental values of the magnetic moments and the predictions from the Gell-Mann-Nishijima current operator for various cases of SU(3)-symmetry breaking.

		$\left(\frac{m_p}{m_\alpha}\right)^{2-q}[C(F,\alpha,\mathrm{el},\alpha)f+C(D,\alpha,\mathrm{el},\alpha)d$				
α	$\mu_\alpha^{\mathrm{exp}}$	$q=0$	$q=\frac{1}{2}$	$q=1$	$q=2$	$q=3$
p	2.79	input	input	input	input	input
n	-1.91	input	input	input	input	input
Λ	-0.67 ± 0.06	-0.68	-0.74	-0.81	-0.96	-1.14
Σ^+	2.83 ± 0.25	1.73	1.95	2.20	2.79	3.53
Σ^-	-1.48 ± 0.37	-0.53	-0.60	-0.68	-0.87	-1.12
Ξ^-	-1.85 ± 0.75	-0.44	-0.51	-0.62	-0.87	-1.23
$\Sigma^0\Lambda$	$1.82^{+0.25}_{-0.18}$			+1.38	1.66	

TABLE III. Comparison between the experimental values of the magnetic moments and the predictions from the generalized electromagnetic current operator for various cases of SU(3)-symmetry breaking.

	$\left(\frac{m_p}{m_\alpha}\right)^{2-q}[C(F,\alpha,\text{el},\alpha)f+C(D,\alpha,\text{el},\alpha)d+s]$				
α	$q=2$	$q=1$	$q=\frac{1}{2}$	$q=0$	μ_α^{exp}
Λ	-1.07 ± 0.17	-0.90 ± 0.14	-0.82 ± 0.13	-0.76 ± 0.12	-0.67 ± 0.06
Σ^+	2.79	2.21	1.95	1.73	2.83 ± 0.25
Σ^-	-1.59 ± 0.46	-1.24 ± 0.36	-1.10 ± 0.32	-0.97 ± 0.28	-1.48 ± 0.37
Ξ^-	-1.59 ± 0.46	-1.13 ± 0.33	-0.94 ± 0.27	-0.080 ± 0.23	-1.85 ± 0.75
Σ^0	$+0.84\pm0.07$	$+0.66\pm0.06$	$+0.59\pm0.05$	$+0.52\pm0.04$	...
Ξ^0	-1.67 ± 0.15	-1.19 ± 0.11	-1.00 ± 0.09	-0.85 ± 0.08	...
$\Sigma^0\Lambda$	1.22 ± 0.15	1.00 ± 0.12			$1.82^{+0.25}_{-0.18}$

eters s, d, and f are given in column 2 of Table III. They are for the magnetic moments the predictions in the case $q=2$ and for $\mu_{\Sigma^0\Lambda}$ the prediction without the SU(3)-breaking correction factor. Columns 3–5 of Table III contain the predictions of μ_α for other values of q. Column 6 gives again the experimental values of μ_α for comparison. Except for the value of μ_Λ, the case $q=2$ would give a good fit to the experimental data, but $q=1$ is again the case which leads to least disagreement with all experimental data; however, it also gives a rather poor fit.

We shall now show that the cause for this poor fit is the discrepancy between the value for the scalar term s required by μ_Λ and $\mu_{\Sigma^0\Lambda}$ on the one hand and the value for s required by μ_{Σ^-} and μ_{Ξ^-} on the other. Using the Clebsch-Gordan coefficients of Table I and expressing the hyperon magnetic moments and the magnetic transition moment in terms of μ_p, μ_n, and s one obtains:

$$\mu_\Lambda=\left(\frac{m_p}{m_\Lambda}\right)^{2-q}\left(\frac{\mu_n}{2}+\frac{s}{2}\right),$$

$$\mu_{\Sigma^+}=\left(\frac{m_p}{m_{\Sigma^+}}\right)^{2-q}\mu_p,$$

$$\mu_{\Sigma^-}=\left(\frac{m_p}{m_{\Sigma^-}}\right)^{2-q}(-\mu_n-\mu_p+3_s), \tag{17}$$

$$\mu_{\Xi^-}=\left(\frac{m_p}{m_{\Xi^-}}\right)^{2-q}(-\mu_n-\mu_p+3_s),$$

$$\mu_{\Sigma^0\Lambda}=\frac{m_p}{\sqrt{m_{\Sigma^+}m_\Lambda}}\left[-\frac{\sqrt{3}}{2}\mu_n+\left(\frac{\sqrt{3}}{2}+1\right)s\right]$$

for $q=1$ only

The experimental value of $\mu_{\Sigma^+}/\mu_{\Sigma^-}$ rules out all values of q except for $q=1$, $\frac{3}{2}$, and 2. We shall restrict ourselves to $q=1$ and $q=2$. The values of s obtained from the experimental data using (17) are given in Table IV. For example, for $q=1$, the values from μ_{Σ^-} and μ_{Ξ^-} combine to give $s=-0.37\pm0.15$ and the values from μ_Λ and $\mu_{\Sigma^0\Lambda}$ combine to give $s=+0.32\pm0.11$.

To summarize the results, it has been shown that the present experimental values of the hyperon magnetic moments cannot be explained by the Gell-Mann–Nishijima formula (1) and SU(3), even when SU(3) is considered as a spectrum-generating group and the symmetry breaking is taken into account. If an SU(3) scalar term is included in the electromagnetic current, as given by (2), the situation for the hyperon magnetic moments improves slightly, especially if one excludes some of the experimental data for μ_Λ. However, if all data are taken into account, including the magnetic transition moment $|\mu_{\Sigma^0\Lambda}|$, a discrepancy between the predictions for μ_Λ and $\mu_{\Sigma^0\Lambda}$ on the one hand and μ_{Σ^-} and μ_{Ξ^-} on the other becomes apparent. Equation (1), as well as (2), under the assumption of a spectrum-generating SU(3) with all possible SU(3)-breaking assumptions (including SU(3) as a symmetry group) do not lead to a reasonable fit of the experimental data. Besides an SU(3)-scalar term, a modification of (1) by some other new SU(3) irreducible tensor operators may be possible. However, already the present results

TABLE IV. Values of s for $q=1$ and $q=2$.

$q=2$	$q=1$	
-0.20 ± 0.12	-0.33 ± 0.16	from μ_{Σ^-}
-0.32 ± 0.25	-0.58 ± 0.35	from μ_{Ξ^-}
0.58 ± 0.12	$+0.32\pm0.14$	from μ_Λ
0.10 ± 0.14	$+0.32\pm0.16$	from $\lvert\mu_{\Sigma^0\Lambda}\rvert$ with the choice of sign $\mu_{\Sigma^0\Lambda}=1$

should be sufficient to show that the common practice of using $F_2^{(F)}$ and $F_2^{(D)}$ determined from the experimental values of μ_p and μ_n in the fits of the hyperon semileptonic decays (Cabibbo fits) is very questionable[8] as such a procedure does not even provide an explanation of the magnetic moments.

Discussions with A. Garcia, R. B. Teese, and J. Werle have been very valuable for the conclusion reached in this paper. Part of the work was done while at the International Centre for Theoretical Physics, Trieste and the author is grateful to the Centre and to Professor A. Salam for the hospitality extended to him.

APPENDIX

In this Appendix the formulas (6) and (7) will be derived with the help of results obtained in Refs. 6 and 7. Since the group-theory part of (6) and (7) is obvious, all that will be done here is to give a justification for the suppression factor.

If SU(3) is not a symmetry group but is considered to be a spectrum-generating group $SU(3)_E$ that commutes with the 4-velocity operator $\hat{P}_\mu = P_\mu/M$ (P_μ = hadron momentum operator, M = hadron mass operator), then the form factors $f_1^{(\gamma)}$ for the matrix element of an SU(3) tensor operator $\tilde{V}_\mu^\beta$ appearing in

$$\langle \alpha' p' | \tilde{V}_\mu^\beta | p\alpha\rangle = \bar{u}_{\alpha'}(p')(f_1^{\alpha'\beta\alpha}\gamma_\mu + f_2^{\alpha'\beta\alpha} i\sigma_{\mu\nu}q^\nu)u_\alpha(p)\,, \tag{A1}$$

with

$$f_i^{\alpha'\beta\alpha} = \sum_\alpha C(\gamma, \alpha, \beta, \alpha') f_i^{(\gamma)}$$

and α, α', β denoting the $SU(3)_E$ labels, are not $SU(3)_E$-invariant reduced matrix elements. Instead of (A1) one has to consider the following matrix elements of an $SU(3)_E$ tensor operator $\tilde{V}_\mu^\beta$:

$$\langle \alpha'\hat{p}' | \tilde{V}_\mu^\beta | \hat{p}\alpha\rangle = \bar{u}_{\alpha'}(\hat{p}')\Big(\sum_\gamma C(\gamma, \alpha, \beta, \alpha') F_1^{(\gamma)}\gamma_\mu + \sum_\gamma C(\gamma, \alpha, \beta, \alpha') F_2^{(\gamma)} i\sigma_{\mu\nu}\hat{q}^\nu\Big) u_\alpha(\hat{p})\,, \tag{A2}$$

where $|\hat{p}, \alpha\rangle$ are generalized eigenvectors of the velocity operator $\hat{P}_\mu$, and

$$\hat{q}_\mu = \hat{p}'_\mu - \hat{p}_\mu = (p'/m_{\alpha'} - p/m_\alpha)_\mu\,.$$

If $\tilde{V}_\mu^\beta$ is an $SU(3)_E$-octet operator, then (A2) is a consequence of the Wigner-Eckart theorem and the $F_i^{(\gamma)}(\hat{q}^2)$ are $SU(3)_E$-invariant form factors and functions of the $SU(3)_E$-invariant parameter $\hat{q}^2$.

The matrix element of an operator H_μ^β, which is a function of an octet operator $\tilde{V}_\mu^\beta$ and the mass operator M, has the form

$$\langle \alpha'\hat{p}' | H_\mu^\beta | \hat{p}\alpha\rangle = \bar{u}_{\alpha'}(\hat{p}')\phi^{\alpha'\alpha}\Big(\sum_\gamma C(\gamma, \alpha, \beta, \alpha') F_1^{(\gamma)}\gamma_\mu + \sum_\gamma C(\gamma, \alpha, \beta, \alpha') F_2^{(\gamma)} i\sigma_{\mu\nu}\hat{q}^\nu\Big) u_\alpha(p)\,, \tag{A3}$$

where $\phi^{\alpha'\alpha}$ is a function of the masses $m_{\alpha'} m_\alpha$. The connection between the conventional form factors $f_i^{\alpha'\beta\alpha}$ and the $SU(3)_E$-invariant form factors $F_i^{(\gamma)}$ and suppression factors $\phi^{\alpha'\alpha}$ is given by Eqs. (23) and (24) in Ref. 7:

$$f_1^{\alpha'\beta\alpha} = \frac{\phi^{\alpha'\alpha}}{(m_\alpha m_{\alpha'})^{3/2}} \sum_{\gamma=F,D} C(\gamma, \alpha, \beta, \alpha') \times\left[F_1^{(\gamma)} + \left(2 - \frac{(m_\alpha + m_{\alpha'})^2}{2m_\alpha m_{\alpha'}}\right)F_2^{(\gamma)}\right], \tag{A4}$$

$$f_2^{\alpha'\beta\alpha} = \frac{\phi^{\alpha'\alpha}}{2(m_\alpha m_{\alpha'})^{3/2}} \sum_{\gamma=F,D} C(\gamma, \alpha, \beta, \alpha')(m_\alpha + m_{\alpha'})F_2^{(\gamma)}\,.$$

The particular form of the suppression factor $\phi^{\alpha'\alpha}$ depends upon the particular assumption for H_μ^β. A very general assumption is that

$$\phi^{\alpha\alpha} = 2m_\alpha^{\;q+2} \quad q = 1, \tfrac{1}{2}, \tfrac{3}{2}, \ldots\,. \tag{A5}$$

For example, one may take for H_μ^β the electromagnetic interaction operator H_μ^{el} (corresponding to the electromagnetic current) and assume for it the same properties that were assumed in the calculation of the radiative decays[5]:

$$H_\mu^{el} = \{M, \{M^q, V_\mu^{el}\}\}\,, \tag{A6}$$

$$\tilde{V}_\mu^\beta = \{M^{-1}, V_\mu^\beta\} = \text{irreducible tensor operator}\,,$$

(β denotes the component of the SU(3)-tensor operator and may be π^0, $\pi^\pm$, K^0, K^+, $\bar{K}^0$, K^-, S, or el if the linear combinations of (1) and (2) are meant). Then

$$\langle \alpha'\hat{p}' | H_\mu^{el} | \hat{p}\alpha\rangle = (m_\alpha + m_{\alpha'})(m_\alpha^{\;q} + m_{\alpha'}^{\;q})\langle \alpha'\hat{p}' | V_\mu^{el} | \hat{p}\alpha\rangle = m_\alpha m_{\alpha'}(m_\alpha^{\;q} + m_{\alpha'}^{\;q})\langle \alpha'\hat{p}' | V_\mu^{el} | \hat{p}\alpha\rangle\,. \tag{A7}$$

Comparison of this with (A3) and (A2) then shows that

$$\phi^{\alpha'\alpha} = m_\alpha m_{\alpha'}(m_\alpha^{\;q} + m_{\alpha'}^{\;q})\,. \tag{A8}$$

For $\alpha = \alpha'$ this leads to (A5) for the suppression factor. However, for $\alpha = \alpha'$ the expression (A5) is also obtained under many more and very general assumptions of H_μ^{el} as a function of an $SU(3)_E$ tensor operator and the mass operator.

If one inserts (A5) into (A4) for the case $\alpha' = \alpha$, $\beta = \text{el}$, one obtains

$$f_1^\alpha = f_1^{\alpha,\text{el},\alpha} = 2m_\alpha{}^{q-1} \sum_{\gamma=F,D,S} C(\gamma, \alpha, \text{el}, \alpha) F_1^{(\gamma)}\ , \tag{6}$$

$$f_2^\alpha = f_2^{\alpha,\text{el},\alpha} = 2m_\alpha{}^{q-2} \sum_\gamma C(\gamma, \alpha, \text{el}, \alpha) F_2^{(\gamma)}\ . \tag{7}$$

A comparison of this with (5) shows that $q=1$ is also the case of SU(3) symmetry, or more precisely the case in which the conventional form factors f_1^α and f_2^α [in the normalization of (4)] are proportional to the Clebsch-Gordan coefficients. The case in which the magnetic moments μ_α in units of the nuclear magneton are proportional to the Clebsch-Gordan coefficients is $q=2$.

*Research supported in part by the Energy Research and Development Administration Contract No. E(40-1) 3992 and the US National Science Foundation Grant No. GF 42060.

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