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$M1$ transitions in the MIT bag model

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We investigate, in the MIT bag model, the $M1$ transitions of low-lying hadrons. We perform the following calculations: (1) We recompute 32 hadron masses with a choice of bag parameters designed to give the correct values for the proton magnetic moment, μ_p, and several masses, M_ρ, M_ω, M_Δ, M_Ω, and M_D; (2) we compute η, η', η_c mixing in an untrustworthy approximation; and (3) we compute the widths for 38 $M1$ transitions.

I. INTRODUCTION

In a series of papers[1-7] the properties of the simple, plausible, well-determined, confined-quark, MIT bag model have been explored. With only a few adjustable parameters the model has a variety of successful qualitative and quantitative predictions. The purpose of the present paper is to apply the model to $M1$ transitions ($\Delta \to N+\gamma$, $\rho\to\pi+\gamma$, etc.).

Within the model, the most extensively studied systems are static bags, those with s-wave quarks, without radial excitation, and without excitation of the bag surface. Most of the work to date has been within the framework of the spherical cavity approximation in which the bag shape is *not* properly treated as a dynamical variable and the quark wave functions are parametrized by a (static-) bag radius. The present work is also within this approximation, and, to a great extent, its chief result is to illuminate the limitations of the approximation.

The usual bag parametrization gives a proton magnetic moment too small by about 30%. In Sec. II we introduce a new parameter into the bag energy, adjust it so that the proton magnetic moment is predicted correctly, and then recalculate the masses, radii, magnetic moments, charge radii, and axial-vector coupling constants of static bags. In Sec. III we calculate baryon and boson, static-bag, $M1$ transitions. Calculations involving isoscalar, pseudoscalar mesons suffer from additional uncertainty arising from mixing induced by transitions to pure gluon states. We segregate these calculations into an appendix. In Sec. IV we conclude briefly. We include in our calculations the full panoply of static bags within SU(4) symmetry.[8]

Static-bag $M1$ transitions for a few key cases have been studied by Hays and Ulehla[9] and by Katz and Tatur.[10] These authors used the usual bag parametrization with the bad prediction for the proton magnetic moment and thus could not be expected to get good results for transitions such as $\Delta\to N+\gamma$. In addition, in Ref. 10, a simplifying assumption was made in evaluating the integrals. Related work on non-$M1$ transitions to nonstatic-bag states has been performed by Hey, Holstein, and Sidhu.[11] Many other authors have discussed $M1$ transitions within other models.[12]

II. RECALCULATION OF BAG PARTICLE PROPERTIES

A. The bag model

The calculation of quark wave functions, bag masses, and bag radii has been reviewed in several places.[3,7] Its essential features are as follows (1) The free Dirac equation is solved for each quark subject to a linear boundary condition that ensures the vanishing at the surface of the bag of all (vector) currents carrying quantum numbers. (2) Color-gluon fields interacting with quark fields are included; their electric and magnetic energies are calculated; (3) The total energy is found from adding to the energies found in (1) and (2) a volume bag energy and a zero-point energy. (4) A quadratic boundary condition is imposed that the sum of the quark and gluon pressures balances the external pressure B locally on the bag's surface. For the $l=0$ states with no radial or surface excitations (static bags) this last step reduces to performing the above steps for varying bag radius R and then minimizing the total energy as a function of R. This calculation was performed in Ref. 3 for noncharmed quarks and in Ref. 4 for charmed quarks. The free parameters were fitted to the N, Δ, ω, Ω, and ψ masses. Good results were obtained for ratios of magnetic moments to the proton magnetic moment. The proton magnetic moment itself, however, came out too small by about $\frac{1}{3}$. In calculating $M1$ transitions it seems desirable to add an extra term phenomenologically, preferably one that has some theoretical justification, so as to ensure a good value of other magnetic quantities including μ_p. Kuti[13] has pointed out that a confinement scheme based on a surface tension rather than a volume pressure provides an alternative basis for a phenomenology. Including both a volume energy $\frac{4}{3}\pi BR^3$ and a surface energy $4\pi SR$ and refitting all bag masses and radii while adjusting S to produce the correct value of μ_p is not, however, possible. The bag model makes a strong correlation between boson and fermion masses and radii. Within the parametrization of Ref. 3, the only differences in the form of the boson and fermion masses are an extra quark kinetic energy and different coefficients for the magnetic gluon energy. Both energies vary as $1/R$ for bags of nonstrange quarks; μ_p, on the other hand, varies as R. Thus it is not possible to add any extra term to the energy that has the same form for mesons and baryons and fit μ_p, M_p, M_ω, and M_Δ.

The simplest term that one can add to the energy that does differ between baryons and mesons is $C\,|N_q - N_{\bar{q}}|$ where N_q and $N_{\bar{q}}$ are the numbers of quarks and antiquarks while C is a constant. Such a term is a crude approximation to effects arising from differences between mesons and baryons in valence-quark, higher-order, gluon-exchange diagrams.[14]

With this addition the expression for the energy is

$$M(R) = E_V + E_0 + E_Q + E_M + E_E + \Delta E\,, \tag{1}$$

where the six terms are

(i) the bag volume energy

$$E_V = \tfrac{4}{3}\pi BR^3\,; \tag{2}$$

(ii) the finite part of the zero-point energy

$$E_0 = -\frac{Z_0}{R}\,; \tag{3}$$

(iii) the sum of single-particle quark energies

$$E_Q = \sum_i \omega_i\,; \tag{4}$$

(iv) the color-magnetic energy

$$E_M = -4\pi\alpha_c \sum_a \sum_{i>j} \int_{\text{bag}} d^3x\, \vec{B}_i^a \cdot \vec{B}_j^a\,, \tag{5}$$

where α_c is the square of the rationalized quark-gluon coupling constant, a is the color index, and i and j are quark labels;

(v) the color-electric energy

$$E_E = 2\pi\alpha_c \sum_a \sum_{i,j} \int_{\text{bag}} d^3x\, \vec{E}_i^a \cdot \vec{E}_j^a\,; \tag{6}$$

(vi) the "bag-type" energy, not present in previous papers,

$$\Delta E = C\,|N_q - N_{\bar{q}}|\,. \tag{7}$$

E_M and E_E are evaluated in detail in Ref. 3. The quark energy ω is given by

$$\omega(m,R) = \frac{1}{R}(x^2 + \lambda^2)^{1/2}\,, \tag{8}$$

where $\lambda = mR$ and x obeys the eigenvalue equation

$$\tan x = x/[1 - \lambda - (x^2 + \lambda^2)^{1/2}]\,. \tag{9}$$

Magnetic moments are given by

$$\mu = \frac{R}{6}\,\frac{4\alpha + 2\lambda - 3}{2\alpha(\alpha - 1) + \lambda}, \tag{10}$$

where $\alpha = R\omega$. We recall that when the quark mass vanishes, the solution of (9) is $x(mR) = x(0) = 2.0428$. We now determine the parameters B, Z_0, α_c, m_l (the light-quark mass), m_c (the charmed-quark mass), and C by fitting (1) to the masses M_ω, M_p, M_Δ, M_Ω, and M_D and (10) to the value of the proton magnetic moment, μ_p. In Table I we compare the present values of the parameters with the results of Refs. 3 and 4. In Tables II and III we compare the values of the masses and radii for baryons and mesons. In general, the fit to experimental particle masses[15]

TABLE I. Values of the bag parameters. Energies are in GeV and distances are in GeV^{-1}.

	B	Z_0	α_c	m_1	m_s	m_c	C
Refs. 3 and 4	4.54×10^{-4}	1.84	0.55	0	0.279	1.551	···
This paper							
Solution A	1.21×10^{-4}	0.567	0.826	0	0.266	1.506	0.139
Solution B	6.65×10^{-5}	0.497	1.16	0.079	0.317	1.534	0.139

TABLE II. Baryon masses (GeV) and radii (GeV^{-1}) for the three sets of parameters of Table I.

					This paper			
			Ref. 3 or 4		Solution A		Solution B	
Particle	Quarks	M_{exp}	M	R	M_A	R_A	M_B	R_B
p	lll	0.938	0.938	5.00	0.937	7.34	0.938	8.27
Λ	lls	1.116	1.105	4.95	1.109	7.24	1.110	8.10
Σ	lls	1.189	1.144	4.95	1.165	7.25	1.170	8.19
Ξ	lss	1.321	1.289	4.91	1.304	7.13	1.308	7.97
N^* (Δ)	lll	1.236	1.233	5.48	1.236	8.17	1.236	9.61
Σ^*	lls	1.385	1.382	5.43	1.386	8.07	1.385	9.45
Ξ^*	lss	1.533	1.529	5.39	1.528	7.98	1.530	9.27
Ω^-	sss	1.672	1.672	5.35	1.666	7.88	1,672	9.10
C_1 (Σ_c)	llc	2.430	2.357	4.78	2.495	7.65	2.428	8.41
C_0 (Λ_c)	$c(ll)_{anti}$	2.260	2.214	4.63	2.253	6.84	2.257	7.68
S	$c(sl)_{sym}$		2.507	4.75	2.564	7.14	2.576	8.22
A	$c(sl)_{anti}$		2.396	4.58	2.450	6.75	2.459	7.54
T	ccs		2.653	4.71	2.705	7.04	2.720	8.03
X_U	ccl		3.538	4.27	3.636	6.43	3.654	7.33
X_s	ccs		3.690	4.25	3.781	6.36	3.803	7.15
C_1^*	cll		2.461	5.12	2.495	7.65	2.499	8.97
S^*	$c(sl)_{sym}$		2.603	5.02	2.633	7.54	2.640	8.76
T^*	css		2.742	5.02	2.766	7.43	2.778	8.55
X_U^*	ccl		3.661	4.69	3.727	7.01	3.743	8.15
X_s^*	ccs		3.795	4.64	3.854	6.88	3.875	7.90
Ω_c	ccc		4.827	4.21				

of the new parameterization is as good as that of Refs. 3 and 4. It is significantly worse in only one case, that of the K-meson mass which is now too high by about 50 MeV. The π meson is similarly too high as it was in Ref. 3; following their discussions of the effects of raising the light-quark mass from zero, one infers that raising it in this case would give the K mass correctly and move the π mass toward 140 MeV. We therefore give a second solution of the parameters with $m_l \neq 0$ adjusted so as to give the proper value for the kaon mass. The parameters of this solution (B) are also given in Table I and the corresponding mass and radius values are given in Tables II and III. One sees that the pion mass moves, in going from solution A to solution B, by about 40 MeV toward its experimental value. Also, the proton radius increases in B to compensate for the decrease in the light quark magnetic moment.

In Table IV we give values, deduced from the quark wave function according to the prescription of Table I of Ref. 3, for static magnetic moments and electric charge radii for some particles, comparing with experiment where possible.[15,16] The

TABLE III. Boson masses (GeV) and radii (GeV^{-1}) for the three sets of parameters of Table I.

					This paper			
			Ref. 3 or 4		Solution A		Solution B	
Particle	Quarks	M_{exp}	M	R	M_A	R_A	M_B	R_B
ρ	$l\bar{l}$	0.77	0.783	4.71	0.785	7.30	0.783	8.62
K^*	$l\bar{s}$	0.892	0.928	4.65	0.928	7.18	0.926	8.42
ω	$l\bar{l}$	0.783	0.783	4.71	0.785	7.30	0.783	8.62
ϕ	$l\bar{l}$	0.019	1.068	4.61	1.063	7.09	1.063	8.21
K	$l\bar{s}$	0.495	0.497	3.26	0.545	5.15	0.495	4.01
π	$l\bar{l}$	0.139	0.280	3.34	0.298	5.28	0.243	4.33
D	$c\bar{l}$	1.865	1.726	2.80	1.864	5.31	1.870	5.71
F	$c\bar{s}$		1.885	2.84	2.015	5.27	2.022	5.55
D^*	$c\bar{l}$	2.007	1.969	4.18	2.015	6.59	2.020	7.72
F^*	$c\bar{s}$		2.099	4.12	2.139	6.44	2.149	7.44
ψ	$c\bar{c}$	3.095	3.095	3.53	3.194	5.60	3.216	6.39

TABLE IV. Magnetic moments and charge radii for cases of interest for the three sets of bag parameters of Table I.

Particle	$(\mu/\mu_p)_{\rm exp}$	Ref. 3 μ/μ_p	Solution A μ/μ_p	Solution B μ/μ_p	$(R_c)_{\rm exp}$	Ref. 3 R_c	Solution A R_c	Solution B R_c
p	1.0	1.0	1.0	1.0	0.88 ± 0.03	0.73	1.06	1.14
n	−0.685	−0.67	−0.67	−0.67	-0.12 ± 0.01	0	0	0
Σ^+	0.938 ± 0.15	0.97	0.95	0.96			1.08	1.16
Σ^0		0.31	0.30	0.30			0.27	0.27
Σ^-	-0.530 ± 0.13	−0.36	−0.36	−0.36			1.01	1.10
Ξ^0		−0.56	−0.52	−0.51			0.37	0.37
Ξ^-	-0.663 ± 0.27	−0.23	−0.19	−0.19			0.96	1.04
Λ	-0.240 ± 0.021	−0.255	−0.23	−0.23			0.27	0.27
π^+	···	···	···	···	0.56 ± 0.04	0.49	0.76	0.60
π^0	···	···	···	···			0	0
K^+	···	···	···	···			0.72	0.55
K^0	···	···	···	···			0.17	0.11

axial-vector coupling constant can also be deduced from Table I of Ref. 3. Table V gives values for the correction factor to the SU(6) values. For example, for $n\to pe\bar{\nu}$, the ratio g_A/g_V as given by the parameters of Ref. 3 would be 0.653 times the SU(6) value, $\frac{5}{3}$, namely 1.09. Our case B gives a better value, 0.717, so we find $g_A/g_V=1.20$ which is closer to the experimental value of 1.25. Thus one sees that the new $C\neq0$ parametrization gives at least as good a fit to these static quantities as the $C=0$ parameterization of Refs. 3 and 4, while having the important advantage of giving quark wave functions that give the proper μ_p. For some quantities, such as g_A, the fit is much better. We will use these solutions in Sec. III in calculating $M1$ transition rates but two important reservations as to their creditability are necessary: (1) Both solutions give a large quark gluon coupling constant, $\alpha_c\simeq1$, while the expressions of (5) and (6) are explicitly first order in α_c. Inclusion of higher-order contributions could make important modifications in predicted bag wave functions so that the present calculations of $M1$ transition rates, while suggestive, cannot be considered definitive bag tests.[17] (2) For $c\bar{c}$ and $3c$ states there is little quark kinetic energy and inclusion of higher-order terms in α_c is considered to be still more important. This is because, in the absence of outward quark pressure to balance the inward bag pressure, one expects gluon pressure to provide the chief balancing force. A treatment using, instead of the approximation of Ref. 3, a Born-Oppenheimer approach is in progress.[18] $M1$ transitions involving $c\bar{c}$ and $3c$ systems in the present paper must therefore be considered a merely suggestive basis for comparison with the results expected from the Born-Oppenheimer approach.

We note that both solution A ($m_l=0$) and solution B ($m_l\neq0$) are surprisingly accurate for the masses of Σ_c and A_c when compared to the tentative experimental values given in Ref. 14. The "old" bag results of Ref. 4 are not so close. Because Ref. 4 fixed m_c from ψ while we fix m_c from D, this result tends to give credence to both the general bag picture and to assertion (2) above that the techniques of Refs. 3 and 4 and the present paper are less reliable for the $c\bar{c}$ and $3c$ states.

III. THE $M1$ TRANSITION RATES

We now calculate the $M1$ transition rates. Following Ref. 6, electromagnetic transition rates are given by the imaginary part of the magnetic contribution to the mass difference,

$$\frac{\Gamma}{2}=\text{Im}\left\{\frac{e^2}{(2\pi)^2}\sum_{\eta,\eta'}\int_{\text{bag}}d^3x\,d^3y\,\frac{\sin k|\vec{x}-\vec{y}|}{|\vec{x}-\vec{y}|}\langle i,\eta|J_\mu(\vec{x},0)|f,\eta\rangle\langle f,\eta'|J^\mu(y,0)|i,\eta'\rangle\right\}. \tag{11}$$

In (11) the sum on (η,η') is a sum over the emitting quark; both $\eta=\eta'$ and $\eta\neq\eta'$ are to be kept. J_μ is given by

$$J_\mu(x)=\sum_\alpha Q_\alpha\bar{q}_\alpha(x)\gamma_\mu q_\alpha(x)\,, \tag{12}$$

where Q_α is the charge of the αth quark. Expanding (12) yields

$$\vec{J}(\vec{x},0)=-\frac{1}{4\pi}\sum_{\alpha}\sum_{m_1,m_2}N_{\alpha_1}N_{\alpha_2}b^{+}_{\alpha_2}(m_2)Q_{\alpha}b_{\alpha_1}(m_1)U^{+}_{m_2}(\hat{x}\times\vec{\sigma})U_{m_1}$$
$$\times\left[j_0\left(\frac{x_{\alpha_1}x}{R_1}\right)j_1\left(\frac{x_{\alpha_2}x}{R_2}\right)\left(\frac{\omega_{\alpha_1}+m}{\omega_{\alpha_1}}\right)^{1/2}\left(\frac{\omega_{\alpha_2}-m}{\omega_{\alpha_2}}\right)^{1/2}+j_0\left(\frac{x_{\alpha_2}x}{R_2}\right)j_1\left(\frac{x_{\alpha_1}x}{R_1}\right)\left(\frac{\omega_{\alpha_1}-m}{\omega_{\alpha_1}}\right)^{1/2}\left(\frac{\omega_{\alpha_2}+m}{\omega_{\alpha_2}}\right)^{1/2}\right]. \tag{13}$$

Here N_α is the quark normalization

$$N_\alpha^{-2}=R^3[j_0(x_\alpha)]^2\,\frac{2\omega_\alpha(\omega_\alpha-1/R)+m_\alpha/R}{\omega_\alpha(\omega_\alpha-m_\alpha)}, \tag{14}$$

and we have used the expression for the quark wave function

$$q(r,t)=\frac{N}{\sqrt{4\pi}}\begin{pmatrix}\left(\frac{\omega+m}{\omega}\right)^{1/2}ij_0\left(\frac{xr}{R}\right)U\\ -\left(\frac{\omega-m}{\omega}\right)^{1/2}j_1\left(\frac{xr}{R}\right)\vec{\sigma}\cdot\hat{r}U\end{pmatrix}. \tag{15}$$

The expression in brackets in (13) takes into account the fact that the general wave functions in the initial (i) and final (f) bags are different. In (11) we use the expression

$$\frac{e^{ik|x-y|}}{|x-y|}=4\pi ik\sum_{l=0}^{\infty}j_l(kx)h_l^{(1)}(ky)\sum_{m=-l}^{l}Y_{lm}(\Omega_x)Y_{lm}(\Omega_y). \tag{16}$$

We then obtain for the full $M1$ width

$$\Gamma=\frac{e^2}{4\pi}k^3(\tfrac{16}{3})\sum_{\substack{\alpha,\beta\\ \alpha\geq\beta}}\mu_\alpha\mu_\beta C^{PQ}_{\alpha\beta}, \tag{17}$$

where the transition moment μ_α is given by

$$\mu_\alpha=\frac{1}{2k}N_{\alpha_1}N_{\alpha_2}\int_0^R dx\,x^2j_1(kx)$$
$$\times\left[j_0\left(\frac{x_{\alpha_1}x}{R_1}\right)j_1\left(\frac{x_{\alpha_2}x}{R_2}\right)\left(\frac{\omega_{\alpha_1}+m_\alpha}{\omega_{\alpha_1}}\right)^{1/2}\left(\frac{\omega_{\alpha_2}-m_\alpha}{\omega_{\alpha_2}}\right)^{1/2}+j_0\left(\frac{x_{\alpha_2}x}{R_2}\right)\left(j_1\,\frac{x_{\alpha_1}x}{R_1}\right)\left(\frac{\omega_{\alpha_2}+m_\alpha}{\omega_{\alpha_2}}\right)^{1/2}\left(\frac{\omega_{\alpha_1}-m_\alpha}{\omega_{\alpha_1}}\right)\right]^{1/2}. \tag{18}$$

In the limit of $k\to 0$ and $R_1=R_2$, this reproduces the expression for the static magnetic moment of Ref. 3. The magnetic transition coefficients $C^{PQ}_{\alpha\beta}$ are, as in Ref. 6, given by

$$C^{PQ}_{\alpha\beta}=\sum_{m_1,m_2}\sum_{k_1,k_2}\langle P|b^{\dagger}_{\alpha}(m_2)Q_\alpha b_\alpha(m_1)|Q\rangle\langle Q|b^{\dagger}_{\beta}(k_1)Q_\beta b_\beta(k_2)|P\rangle U^{\dagger}_{m_2}\sigma^iU_{m_1}U^{\dagger}_{k_1}\sigma_iU_{k_2}. \tag{19}$$

The $C^{PQ}_{\alpha\beta}$ are listed in Tables VI and VII for the cases of interest to the present work. In Tables VIII and IX we give the values of the transition moments, from (18), for fermions and bosons for the two cases considered in Sec. II.

TABLE V. Correction factors for the SU(6) values of g_A/g_V.

	Ref. 3	Solution A	Solution B
$\Delta S=0$	0.653	0.653	0.717
$\Delta S=1$	0.707	0.723	0.776

In evaluating the integrals in (11) or (18) it is necessary to choose the value of the upper limit of the integral and its relation to the radii R_1 and R_2 in the integrands. We have chosen the following ansatz: Given R_1 and R_2, for a particular transition, from the results of Sec. II, we take $R=\frac{1}{2}(R_1+R_2)$; with this value of R we recalculate the eigenvalue $x(R)$ in (9) and replace R_1 and R_2 by R everywhere in (18). Then $\alpha_1=\alpha_2=\alpha$. The motivation for this ansatz is the realization that the static-cavity model of Ref. 3 is an approximation to the "real" physical situation in which the bag shape is a dynamical variable with its own wave

TABLE VI. The magnetic transition coefficients $C_{\alpha\beta}^{PQ}$ as defined in (19) for fermions.

Transition	q_1/q_1	q_1q_3	q_1q_3	q_2q_2	q_2q_3	q_3q_3
$N^* \to \Sigma^+\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{4}{27}$
$\Sigma^{*+} \to \Sigma^+\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{4}{27}$
$\Sigma^{*0} \to \Sigma^0\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$-\frac{4}{27}$	$\frac{4}{27}$	$-\frac{4}{27}$	$\frac{1}{27}$
$\Sigma^{*-} \to \Sigma^-\gamma$	$\frac{4}{27}$	$-\frac{4}{27}$	$-\frac{4}{27}$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{27}$
$\Sigma^{*0} \to \Lambda\gamma$	0	0	0	$\frac{12}{27}$	$\frac{12}{27}$	$\frac{3}{27}$
$\Xi^{*0} \to \Xi^0\gamma$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{8}{27}$	$\frac{1}{27}$	$\frac{8}{27}$	$\frac{16}{27}$
$\Xi^{*-} \to \Xi^-\gamma$	$\frac{1}{27}$	$\frac{2}{27}$	$-\frac{4}{27}$	$\frac{1}{27}$	$-\frac{4}{27}$	$\frac{4}{27}$
$\Sigma^0 \to \Lambda\gamma$	0	0	0	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{1}{27}$
$\Sigma_c^{*++} \to \Sigma_c^{++}\gamma$	$\frac{16}{27}$	$-\frac{16}{27}$	$-\frac{16}{27}$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{4}{27}$
$\Sigma_c^{*+} \to \Sigma_c^+\gamma$	$\frac{16}{27}$	$\frac{8}{27}$	$-\frac{32}{27}$	$\frac{1}{27}$	$-\frac{8}{27}$	$\frac{16}{27}$
$\Sigma_c^{*0} \to \Sigma_c^0\gamma$	$\frac{16}{27}$	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{27}$
$\Sigma_c^{*+} \to \Lambda_c\gamma$	0	0	0	$\frac{3}{27}$	$\frac{12}{27}$	$\frac{12}{27}$
$\Sigma c \to \Lambda c\gamma$	0	0	0	$\frac{4}{27}$	$\frac{4}{27}$	$\frac{1}{27}$
$S^{*+} \to S^+\gamma$	$\frac{16}{27}$	$\frac{8}{27}$	$-\frac{32}{27}$	$\frac{1}{27}$	$-\frac{8}{27}$	$\frac{16}{27}$
$S^{*+} \to A^+\gamma$	0	0	0	$\frac{3}{27}$	$\frac{12}{27}$	$\frac{12}{27}$
$X_U^{*++} \to X_U^{++}\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$-\frac{16}{27}$	$\frac{4}{27}$	$-\frac{16}{27}$	$\frac{16}{27}$
$X_U^{*+} \to X_U^+\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{4}{27}$
$X_s^{*+} \to X_s^+\gamma$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{4}{27}$	$\frac{8}{27}$	$\frac{4}{27}$
$T_0^* \to T_0^*\gamma$	$\frac{16}{27}$	$\frac{8}{27}$	$\frac{8}{27}$	$\frac{1}{27}$	$\frac{2}{27}$	$\frac{1}{27}$

TABLE VII. The magnetic transition coefficients $C_{\alpha\beta}^{PQ}$ of (19) for bosons. The parameters α_i, β_i, and γ_i are given in Table XII.

Transition	q_1q_1	q_1q_2	q_2q_2
$\rho \to \pi\gamma$	$\frac{12}{27}$	$-\frac{12}{27}$	$\frac{3}{27}$
$\rho \to \eta\gamma$	$\frac{12}{27}\alpha_1{}^2$	$\frac{12}{27}\alpha_1{}^2$	$\frac{3}{27}\alpha_1{}^2$
$\omega \to \pi\gamma$	$\frac{12}{27}$	$\frac{12}{27}$	$\frac{3}{27}$
$\omega \to \eta\gamma$	$\frac{12}{27}\alpha_1{}^2$	$-\frac{12}{27}\alpha_1{}^2$	$\frac{3}{27}\alpha_1{}^2$
$K^{*+} \to K^+\gamma$	$\frac{12}{27}$	$-\frac{12}{27}$	$\frac{3}{27}$
$K^{*0} \to K^0\gamma$	$\frac{3}{27}$	$\frac{6}{27}$	$\frac{3}{27}$
$\phi \to \eta\gamma$	$\frac{3}{27}\beta_1{}^2$	$\frac{6}{27}\beta_1{}^2$	$\frac{3}{27}\beta_1{}^2$
$\phi \to \eta'\gamma$	$\frac{3}{27}\beta_2{}^2$	$\frac{6}{27}\beta_2{}^2$	$\frac{3}{27}\beta_2{}^2$
$\eta' \to \rho\gamma$	$\frac{12}{27}\alpha_2{}^2$	$\frac{12}{27}\alpha_2{}^2$	$\frac{3}{27}\alpha_2{}^2$
$\eta' \to \omega\gamma$	$\frac{12}{27}\alpha_2{}^2$	$-\frac{12}{27}\alpha_2{}^2$	$\frac{3}{27}\alpha_2{}^2$
$\eta_c \to \phi\gamma$	$\frac{3}{27}\beta_3{}^2$	$\frac{6}{27}\beta_3{}^2$	$\frac{3}{27}\beta_3{}^2$
$\eta_c \to \omega\gamma$	$\frac{12}{27}\alpha_3{}^2$	$-\frac{12}{27}\alpha_3{}^2$	$\frac{3}{27}\alpha_3{}^2$
$\eta_c \to \rho\gamma$	$\frac{12}{27}\alpha_3{}^2$	$\frac{12}{27}\alpha_3{}^2$	$\frac{3}{27}\alpha_3{}^2$
$\psi_c \to \eta\gamma$	$\frac{12}{27}\gamma_1{}^2$	$\frac{24}{27}\gamma_1{}^2$	$\frac{12}{27}\gamma_1{}^2$
$\psi_c \to \eta'\gamma$	$\frac{12}{27}\gamma_2{}^2$	$\frac{24}{27}\gamma_2{}^2$	$\frac{12}{27}\gamma_2{}^2$
$\psi_c \to \eta_c\gamma$	$\frac{12}{27}\gamma_3{}^2$	$\frac{24}{27}\gamma_3{}^2$	$\frac{12}{27}\gamma_3{}^2$
$D^{*+} \to D^+\gamma$	$\frac{12}{27}$	$-\frac{12}{27}$	$\frac{3}{27}$
$D^{*0} \to D^0\gamma$	$\frac{12}{27}$	$\frac{24}{27}$	$\frac{12}{27}$
$F^{*+} \to F^+\gamma$	$\frac{12}{27}$	$-\frac{12}{27}$	$\frac{3}{27}$

function. It should be recognized that our procedure gives a slightly larger transition moment than choosing $R = \min(R_1, R_2)$.

An important point to notice is that the r dependence of the current operator in the transition moment is

$$\frac{1}{k} j_1(kr) = O_{\text{tr}}$$

in comparison to the simple static dependence

$$O_s = r\,.$$

TABLE VIII. Baryon quark transition moments, μ_α, from (18). They are calculated by using the bag masses of Table II, not the experimental masses.

	Solution A			Solution B		
	μ_{q_1}	μ_{q_2}	μ_{q_3}	μ_{q_1}	μ_{q_2}	μ_{q_3}
$N^* \to N\gamma$	0.5913	0.5913	0.5913	0.5586	0.5586	0.5586
$\Sigma^* \to \Sigma\gamma$	0.4573	0.6600	0.6600	0.4427	0.6476	0.6476
$\Sigma^* \to \Lambda\gamma$	0.4248	0.6076	0.6076	0.4000	0.5781	0.5781
$\Xi^* \to \Xi\gamma$	0.4527	0.4527	0.6498	0.4352	0.4352	0.6311
$\Sigma \to \Lambda\gamma$	0.5056	0.7252	0.7252	0.5009	0.7235	0.7235
$\Sigma_c^* \to \Sigma_c\gamma$	0.1563	0.7374	0.7374	0.1549	0.7586	0.7586
$\Sigma_c^* \to \Lambda_c\gamma$	0.1373	0.6098	0.6098	0.1305	0.5912	0.5912
$S^* \to S\gamma$	0.1566	0.5070	0.7304	0.1552	0.5102	0.7474
$S^* \to A\gamma$	0.1459	0.4602	0.6509	0.1471	0.4705	0.6755
$X_U^* \to X_U\gamma$	0.1543	0.1543	0.6641	0.1522	0.1522	0.6771
$X_s^* \to X_s\gamma$	0.1553	0.1553	0.4750	0.1541	0.1541	0.4782
$T_0^* \to T_0\gamma$	0.1568	0.5043	0.5043	0.1556	0.5056	0.5056

TABLE IX. Boson quark transition moments, μ_α, from (18). They are calculated using the bag masses from Table II.

	Solution A		Solution B	
	μ_{q_1}	μ_{q_2}	μ_{q_1}	μ_{q_2}
$\rho \to \pi\gamma$	0.4720	0.4720	0.4240	0.4240
$\omega \to \pi\gamma$	0.4730	0.4730	0.4243	0.4243
$K^* \to K\gamma$	0.3707	0.4938	0.3369	0.4381
$D^* \to D\gamma$	0.1494	0.5731	0.1475	0.5821
$F^* \to F\gamma$	0.1511	0.4299	0.1492	0.4298

This difference tends to cut down the size of the transition moment for $kr > 1$. For transitions in this region it means that experimental static and transition moments provide independent information; it is not true that changing the bag parametrization to ensure a large R and a correct μ_p, as was done in Sec. II, will give a large moment for the corresponding transition, $\Delta \to p + \gamma$. For a given transition, once $kR \gtrsim 1$, increasing R becomes progressively less effective in increasing the transition moment, μ_{tr}. Roughly, μ_{tr} measures the product of one large and one small component of the two different wave functions at $r = 1/k$, while the static moment μ_s measures the product of large and small components of the same wave function at $r = R$.

In Tables X and XI, we give the results of the two (A for $m_l = 0$, B for $m_l \neq 0$) bag parametrizations for $M1$ radiative widths.

The comparison between the predicted and experimental values for the measured cases (ΔN, $\rho\pi$, $\omega\pi$, and K^*K) is not spectacular. The 9:1 ratio between $\omega\pi$ and $\rho\pi$ is a result of ω-ρ degeneracy in any quark model and no amount of adjusting the model will ever change it. The ΔN value is too small, despite our parametrization that gives a good value for μ_p, for the reasons mentioned above. In this regard it is interesting to notice that all the cases correspond to $kR > 1$.

IV. DISCUSSION

Consider first our reparametrization of the static-bag model. This gives, as discussed, reasonably good results for magnetic moments and masses (except for K and, of course, π), satis-

TABLE X. Baryon $M1$ transition rates in keV for the two cases of Table I (solution A is m_l equal to zero and solution B is m_l equal to 79 MeV). The rates in parentheses use experimental masses for each particle with the radius of Table II.

Transition	Experiment	Solution A	Solution B
$N^* \to N\gamma$	700 ± 70	338.51 (338.51)	291.50 (291.50)
$\Sigma^{*+} \to \Sigma^+\gamma$		153.27 (113.06)	135.86 (108.10)
$\Sigma^{*0} \to \Sigma^0\gamma$	<1800	30.08 (22.13)	26.43 (20.99)
$\Sigma^{*-} \to \Sigma^-\gamma$		1.99 (1.50)	1.89 (1.53)
$\Sigma^{*0} \to \Lambda\gamma$	<2200	222.79 (211.23)	197.69 (190.91)
$\Xi^{*0} \to \Xi^0\gamma$		158.39 (146.33)	145.03 (137.15)
$\Xi^{*-} \to \Xi^-\gamma$	<360	2.00 (1.86)	1.93 (1.84)
$\Sigma^0 \to \Lambda\gamma$		1.11 (2.67)	1.36 (2.66)
$\Sigma_c^{*++} \to \Sigma_c^{++}\gamma$		3.27	2.88
$\Sigma_c^{*+} \to \Sigma_c^+\gamma$		1.52	1.36
$\Sigma_c^{*0} \to \Sigma_c^0\gamma$		2.67	2.26
$\Sigma_c^{*+} \to \Lambda_c\gamma$		176.73	166.17
$\Sigma_c^+ \to \Lambda_c\gamma$		22.91	24.57
$S^{*+} \to S^+\gamma$		1.46	1.26
$S^{*+} \to A^+\gamma$		74.01	40.24
$X_U^{*++} \to X_U^{++}\gamma$		4.35	5.75
$X_U^{*+} \to X_U^+\gamma$		3.96	5.03
$X_s^{*+} \to X_s^+\gamma$		1.35	1.29
$T_0^* \to T_0\gamma$		0.85	0.73

TABLE XI. Boson $M1$ transition rates in keV for the two cases of Table I. The values in parentheses use the experimental particle masses.

Transition	Experiment	Solution A	Solution B
$\phi \to \pi\gamma$	5.7 ± 2.1	0	0
$\rho \to \pi\gamma$	35 ±10	36.54 (43.45)	34.43 (37.47)
$\omega \to \pi\gamma$	880 ±60	326.85 (398.72)	310.30 (344.04)
$K^{*+} \to K^+\gamma$	<80	7.45 (7.71)	8.69 (7.48)
$K^{*0} \to K^0\gamma$	75 ±35	90.80 (93.72)	93.94 (81.90)
$D^{*+} \to D^+\gamma$		1.00 (0.82)	1.07 (0.89)
$D^{*0} \to D^0\gamma$		27.73 (22.57)	27.75 (23.05)
$F^{*+} \to F^+\gamma$		0.12	0.14

factory results for axial-vector coupling constants, and not unreasonable results for charge radii. It cannot, however, be claimed that there is any evidence for the form of our reparametrization—the addition of $C|N_q - N_{\bar{q}}|$ to the energy. In the first place it might be that the solution of the problem posed by the low value of μ_p lies in an anomalous quark magnetic moment rather than in a larger proton radius, although this seems somewhat outside the spirit of a quark model. But even if modification of the static-bag wave functions is called for, our addition to the Hamiltonian is not unique; other changes, such as making the constant Z_0 in the zero-point energy different for baryons than for bosons, are possible. Our value for C does, however, indicate the size of the necessary change.

The universal slope α' of Regge trajectories is another semimeasurable quantity. Johnson and Thorn[2] have shown that for massless light quarks, α' is proportional to $(B\alpha_c)^{1/2}$ and that the old bag parameters give a good numerical value when compared to slopes derived from particle spectroscopy. Our reparametrization gives a larger α_c but a smaller B so its corrections are appropriately directed. Numerically, however, α' falls for model A (B) to 63% (55%) of its "old bag" value; we consider this comparison uncomfortable but not necessarily compelling.

Consider now the $M1$ transitions obtained with models A and B. One of our most important results is probably the realization that increasing the bag radius for Δ and p in order to fit μ_p does not give the $M1$ transition rate correctly. The rate for $\Delta \to N\gamma$ is suppressed by two other factors: (1) The maximum value of the matrix element of the current in Eq. (13) falls with increasing $\Delta R = R_\Delta - R_N$; this tends to suppress transition moments relative to static moments. (2) The ratio $j_1(kR)/kR$ is approximately $1 - k^2R^2/10$; for Δ-N this effect suppresses the transition moment relative to the static moment by about 40% for our parametrization and 20% in the "old bag." The result of these two effects is that the rate for $\Delta \to N\gamma$ in the "old bag" of Ref. 3, 200 keV, is not easily brought into agreement with experiment. If the rate is scaled upward by $[(\mu_p)_{\text{exp}}/(\mu_p)_{\text{Bag}}]^2 \simeq 2$, in a crude attempt to take into account an anomalous quark magnetic moment, it is still 14 standard deviations short of experiment. It should be noted that introduction of anomalous quark moments in the bag model raises calculational difficulties in evaluating the contribution of hadron center-of-mass motion to momentum-dependent terms; the q in $\bar{\psi}_j\, \sigma_{\mu\nu} q^\nu \psi_i$ is $p_j - p_i$, where $p_i = -i\,(\partial_R + \partial_r)$.

Our value for $K^* \to K\gamma$ is well within experimental error with either the predicted or experimental masses, but since our K mass is too heavy and the error is large this is, at best, a modest success. The ratio of $\eta' \to \rho\gamma$ to $\eta' \to \omega\gamma$ has recently been measured accurately.[19] Our solutions for the mixing, as explained in the Appendix, make the η essentially pure $s\bar{s}$ so that we can only predict zero for both these decays. Finally the radiative decays into pions are not, because of the large momentum transfer involved, expected to be reliable in the static-bag approximation.

In general the available data are much too limited to make a meaningful assessment of bag predictions or to serve as a guide for modifications of the model. It is highly desirable for this model and others[12] that further careful $M1$ measurements be undertaken.

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APPENDIX

We treat here $M1$ transitions involving the isoscalar pseudoscalar mesons (η, η', η_c). It is generally thought that within both the bag model and other confinement schemes, these particle states are dynamically mixed through the SU(4)-singlet, two colored gluon, intermediate state. Each is a linear combination of $L\bar{L}$, $s\bar{s}$, and $c\bar{c}$, where $L\bar{L}$ is the isosinglet combination of $u\bar{u}$ and $d\bar{d}$ quarks. Our approach, following that of Ref. 3, is to make a three-parameter expression for the mixing and to determine the three parameters by requiring the eigenvalues of the mass matrix, when minimized with respect to the radius R, to be the experimental masses. This approach suffers from two difficulties: (1) The $c\bar{c}$ system, as discussed in Sec. II, should not be treated to lowest order in α_c. (2) One result of the static-cavity approximation in which the radius R is treated as a fixed parameter rather than a quantum-mechanical variable is that the wave functions we find will not be orthogonal if they have different radii. The orthogonality that they must enjoy by virtue of the self-adjointness of the bag Hamiltonian with the two bag boundary conditions requires the R wave functions to check numerically. In the present approximation bag states having different R's are considered orthogonal by virtue of the unknown R wave function.

We write the mass matrix as

$$\mathfrak{M} - MI = \begin{pmatrix} E_0 - M + 2\beta & \sqrt{2}\,\beta & \sqrt{2}\,\beta \\ \sqrt{2}\,\beta & E_s - M + \beta & \beta \\ \sqrt{2}\,\beta & \beta & E_c - M + \beta \end{pmatrix}. \tag{A1}$$

Here E_0, E_s, and E_c are the $L\bar{L}$, $s\bar{s}$, and $c\bar{c}$ system energies as determined by (1) in Sec. II. M is the eigenvalue to be determined. β is the singlet-state two-gluon mixing. The two-gluon state is assumed, by its singlet nature, to couple equally to $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, and $c\bar{c}$ states; hence the couplings to $L\bar{L}$, $s\bar{s}$, and $c\bar{c}$ are in the ratio $\sqrt{2}$, 1, 1. We parametrize β as follows:

$$\beta = a + bM + c/R\ . \tag{A2}$$

We must solve for the six quantities a, b, c, R_η, $R_{\eta'}$, and R_{η_c}. We define $f(R, \beta(R,M), M)$ by

$$f(R, \beta(R,M), M) = \det(\mathfrak{M} - MI)\ . \tag{A3}$$

TABLE XII. (a) Values of the η, η', and η_c radii and of the mixing parameters in the parametrization of Eq. (A2). (b) Values of the η, η', and η_c direction cosines along $|L\bar{L}\rangle$, $|s\bar{s}\rangle$, and $|c\bar{c}\rangle$ for the $m_l = 0$ solution.

(a)

R_η	$R_{\eta'}$	R_{η_c}	a	b	c
5.00	2.32	3.62	0.696	−0.237	−1.09

(b)

i	α_i	β_i	γ_i
η	−0.765	0.642	0.053
η'	0.002	1.0	0.0
η_c	0.117	0.101	−0.988

We have three mass equations,

$$f(R_i, \beta(R_i, M_i) M_i) = 0, \quad i = \eta, \eta', \eta_c \tag{A4}$$

and three radius equations,

$$0 = \frac{dM}{dR} = \left(\frac{\partial f}{\partial M} + \frac{\partial f}{\partial \beta}\frac{\partial \beta}{\partial M}\right)^{-1} \left(\frac{\partial f}{\partial R} + \frac{\partial f}{\partial \beta}\frac{\partial \beta}{\partial R}\right). \tag{A5}$$

We have solved (A4) and (A5) numerically for the $m_l = 0$ bag-parameter cases of Table I. The results for R_i, a, b, and c, and the $L\bar{L}$, $s\bar{s}$, and $c\bar{c}$ mixing are shown in Table XII. The mixing

TABLE XIII. $M1$ transition rates in keV involving isoscalar, pseudoscalar mesons for the $m_l = 0$ solutions of Tables III and XII.

Transition	Experiment	Bag (A)
$\phi \to \eta\gamma$	64 ± 10	43.72
$\phi \to \eta'\gamma$		2.39
$\psi_c \to \eta_c\gamma$	<3.5	21.00
$\psi_c \to \eta\gamma$	0.55 ± 0.01	0
$\psi_c \to \eta'\gamma$	0.152 ± 0.117	0
$\eta' \to \rho\gamma$	<300	0
$\eta' \to \omega\gamma$	<50	0
$\rho \to \eta\gamma$	50 ± 13 76 ± 15	58.33
$\omega \to \eta\gamma$	$3^{+2.5}_{-1.8}$ 29 ± 7	6.36
$\eta_c \to \phi\gamma$		0
$\eta_c \to \omega\gamma$		0.06
$\eta_c \to \rho\gamma$		0.54

parameters are defined as

$$\alpha_i \mid L\bar{L}\rangle + \beta_i \mid s\bar{s}\rangle + \gamma_i \mid c\bar{c}\rangle . \qquad \text{(A6)}$$

No solution to (A4) and (A5) was found using the $m_l \neq 0$ case from Table I. Using these results we may calculate the $M1$ widths involving the η's by proceeding as in Sec. III and then multiplying by α_i^2, β_i^2, or γ_i^2 as appropriate for the quark structure of the meson. It is important to remember to multiply the pseudoscalar into vector plus photon rates by 2 to take into account the sum over final helicity states. The results of these calculations are given in Table XIII.

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