PHYSICAL REVIEW D VOLUME 18, NUMBER 7 1 OCTOBER 1978

Radiative decays of mesons in a relativistic quark model

L. Maharana and S. P. Misra
Institute of Physics, A/105 Saheed Nagar, Bhubaneswar-751007, India
(Received 18 August 1977)

Radiative decays of mesons, both $V \rightarrow P\gamma$ and $P \rightarrow V\gamma$, are calculated in a relativistic quark model of hadrons proposed earlier by one of the authors. The matrix elements for the above processes are calculated directly by considering appropriately Lorentz-boosted hadronic states. The electromagnetic current operator in terms of the appropriately Lorentz-boosted quark field operators are utilized. Only the wave functions of hadrons enter the picture. No vector dominance is assumed. Mixing angles of mesons are taken from quadratic mass formula in SU(3). Thus with *no* adjustable free parameters or concepts, there is reasonable agreement with experimental results.

I. INTRODUCTION

In a quark model, the radiative decay of mesons has been one of the simplest hadronic problems and therefore has attracted attention from the very beginning.[1] The form of the interaction is known; the complicacy involved is in describing the relativistic state of the outgoing meson and the appropriate consideration of the photon vertex. This is resolved by considering an interaction vertex,

$$\tfrac{1}{4} f \epsilon_{\mu\nu\lambda\sigma}(\partial_\mu V_\nu)(\partial_\lambda A_\sigma)P , \qquad (1.1)$$

and comparing the effect of (1.1) with that of a quark model at *nonrelativistic* energies, thus in a way determining f. Clearly this involves *ad hoc* approximations of interactions as well as kinematics for essentially a relativistic problem. The same problem has been tackled by Feynman *et al.*[2] in the relativistic harmonic-oscillator model. However, like the earlier approaches, the electromagnetic interaction is postulated not through a field operator, but a wave function. The Bethe-Salpeter wave functions used here also do not have a clear physical meaning,[3] and the timelike modes of the harmonic-oscillator wave functions are to be suppressed.[4] The same problem has also been considered using vector dominance, replacing (1.1) with

$$\tfrac{1}{4} g D_{abc}\epsilon_{\mu\nu\lambda\sigma}(\partial_\mu V_\nu^a)(\partial_\lambda V_\sigma^b)P^c , \qquad (1.2)$$

where D_{abc} includes effects of SU(3) breaking.[5] The results are either unsatisfactory or have too many parameters to adjust.[6]

Recently one of the authors has proposed a field-theoretic description of hadrons[7] from which one can construct a relativistic theory by Lorentz boosting.[8] This may be contrasted with the Lorentz boosting of wave functions,[9] or the replacing of two-component spinors by four-component spinors in an *ad hoc* manner. The model yields correct $\Gamma(\eta \rightarrow 2\gamma)/\Gamma(\pi^0 \rightarrow 2\gamma)$ without using partial conservation of axial-vector current (PCAC).[7] The nonrelativistic harmonic-oscillator wave function appears to be adequate for a description of low lying hadrons, including an almost quantitative generation of strong-interaction couplings from quark model parameters,[8] and is consistent with the Okubo-Zweig-Iizuka rule[10] with contributions similar to quark-pair-creation terms of earlier models.[11] In this model, the quark field operators describing hadrons in motion and the electromagnetic current in terms of quark operators already have preassigned meanings.[8] The quark parameters have also been determined. We chose a meson oscillator wave-function radius to be known from the charge radius of the pion, and to be universal. Also, we take the mixing of mesons to be known from the quadratic mass formula. Then, the matrix elements for the radiative decays have *no* adjustable parameter.

We calculate the widths for these decays and obtain a fair agreement with experimental results. We have not included any strong-interaction effects which could be responsible for the disagreement of $\Gamma(\omega \rightarrow \pi\gamma)$.

II. GENERAL THEORY

We note that in our model we first describe mesons at rest in terms of quark and antiquark creation operators.[7] The space dependence of these four-component field operators is given a specific but unconventional form, different from the free Dirac field operator. The constituent quarks are assumed to occupy fixed energy levels in hadrons in their rest frame, which naturally yield the time dependence of the quark field operators. Thus the space-time dependence of the quark field operators as constituents of hadrons becomes known in the rest frame of the hadron. The quark field operators corresponding to hadrons in motion are then determined by Lorentz boosting,[8] knowing that they are Dirac field operators whose transformation properties are known. Thus $Q^{L(p)}(x)$, the quark-annihilation field operator corresponding to a hadron of four-momentum p, is given as

$$Q^{L(p)}(x) = U(L(p))S(L(p))Q(L(p)^{-1}x)U^{-1}(L(p))$$
$$= (2\pi)^{-3/2}\left(\frac{p^0}{m}\right)^{1/2} \int \sum_r u_r^{L(p)}(\vec{k})Q_{Ir}(L(p)k)$$
$$\times \exp[-i(L(p)k)\cdot x]d^3k\,. \tag{2.1}$$

In the above equation,

$$u_r^{L(p)}(\vec{k}) = S(L(p))\begin{bmatrix} f_Q \\ g_Q\vec{\sigma}\cdot\vec{k}\end{bmatrix} u_{Ir}\,, \tag{2.2}$$

and

$$U(L(p))Q_{Ir}(k)U^{-1}(L(p)) = \left(\frac{p^0}{m}\right)^{1/2} Q_{Ir}(L(p)k)\,. \tag{2.2$'$}$$

We note that we have included spin rotation $S(L(p))$ in (2.2) and not in (2.2′). Also $k=(\vec{k},k^0)$ with $k^0=\omega=$ the quark energy level in the hadron of mass $m=(p^2)^{1/2}$, and $S(L(p))$ is the spinor-transformation matrix corresponding to the Lorentz transformation $L(p)$. We assume here the usual anticommutation rules:

$$[Q_{Ir}(Lk), Q_{Is}^\dagger(L'k')]_+ = \delta_{rs}\delta_3(Lk - L'k')\,. \tag{2.3}$$

For the hadron of momentum $\vec{p}$, $|h(\vec{p})\rangle$ is obtained by Lorentz boosting of $|h(\vec{0})\rangle$, and thus is described in terms of quark field operators $Q_{Ir}^\dagger(L(p)k)$ when the description of $|h(\vec{0})\rangle$ is known. The modifications in the above are trivial when we include the color and flavor degrees of freedom. We do so hence forth.

The electromagnetic-current field operator is taken as

$$J^\mu(x) = \sum_{i,Q} e_Q\bar{\psi}_Q^{ig}(x)\gamma^\mu\psi_Q^{ig}(x)\,. \tag{2.4}$$

In the above, Q stands for the flavor, i for the color index, and $\psi_Q(x) = Q(x) + \tilde{Q}(x)$, including both quark annihilation and antiquark creation operators. Further, the index g stands for the fact that the field operators in (2.4) can belong to any hadron in any frame of reference.

We now utilize this description to consider electromagnetic decays $M\to M'+\gamma$. The matrix element for this process in the lowest order is given by

$$\langle M'(\vec{p}\,')\gamma(\vec{k},\lambda)|S|M(\vec{p})\rangle = \delta_4(p_f - p_i)M_{fi}\,, \tag{2.5}$$

where by translational invariance,

$$M_{fi} = -i(2\pi)^4\langle M'(\vec{p}\,')|J^\mu(0)|M(\vec{p})\rangle$$
$$\times (2\pi)^{-3/2}\frac{1}{(2k^0)^{1/2}}\, e_\mu(\vec{k},\lambda)\,. \tag{2.6}$$

In the above, the notations are obvious. $J^\mu(0)$ has quark field operators $Q^g(0)$ or $\tilde{Q}^g(0)$, which will get contracted with quark field operators describing $|M(\vec{p})\rangle$ or $\langle M(\vec{p}\,')|$. In such a case, $Q^g(0)$ will be replaced by $Q^{L(p)}(0)$, with the application of (2.1) and (2.3).

Thus we note that M_{fi} has no arbitrary parameters which are yet to be determined. The relativistic nature of M' has already been taken into account in a specific manner in our earlier paper,[8] and the field operator also has a known form.[7] We take the nonrelativistic harmonic-oscillator wave functions for the mesons[7,8] and calculate M_{fi}. We now proceed to illustrate this for different cases. The final results have some similarity to predictions of nonrelativistic quark models, but as will be obvious from the details, the nature of the contributions is totally different. The way we have taken Lorentz boosting plays an essential role in the quantitative predictions.

III. APPLICATIONS

We shall give some details for the evaluation of $M_{fi}(\rho^-\to\pi^-\gamma)$ and quote the results for the other cases.

A. $\rho^-\to\pi^-\gamma$

Here with the colored-quark model we have,[8] *with the summation convention for repeated indices*,

$$|\rho_1^-(\vec{0})\rangle = \frac{1}{\sqrt{3}}\int \delta_3(k_1+k_2)d^3k_1d^3k_2u_\rho(\vec{k}_1)\mathfrak{N}_{I\,(1/2)}^i(k_1)^\dagger\tilde{\mathcal{P}}_{I\,(1/2)}^i(k_2)|\text{vac}\rangle\,, \tag{3.1}$$

and

$$|\pi^-(\vec{p})\rangle = \left(\frac{m_\pi}{p^0}\right)^{1/2} U(L(p))|\pi^-(\vec{0})\rangle$$
$$= \frac{1}{\sqrt{6}}\left(\frac{p^0}{m_\pi}\right)^{1/2}\int \delta_3(k_1+k_2)d^3k_1d^3k_2u_\pi(\vec{k}_1)u_{Ir}^\dagger v_{Is}\mathfrak{N}_{Ir}^i(L(p)k_1)^\dagger\tilde{\mathcal{P}}_{Is}^i(L(p)k_2)|\text{vac}\rangle\,. \tag{3.2}$$

Thus by (2.6) we get

$$M_{fi}(\rho^- \to \pi^- \gamma) = -i(2\pi)^4(2\pi)^{-3/2}\,\frac{e_\mu(\vec{k},\lambda)}{(2k^0)^{1/2}}\,\langle \pi^-(\vec{p})|J^\mu(0)|\rho_1^-(\vec{0})\rangle\,, \tag{3.3}$$

where we effectively have (2.4) replaced by

$$\begin{aligned} J^\mu_{\rm eff}(0) &= [e_{\mathfrak{N}}\bar{\mathfrak{N}}^{i\,L(p)}(0)\gamma^\mu \mathfrak{N}^i(0) + e_{\mathcal{P}}\bar{\mathcal{P}}^i(0)\gamma^\mu \mathcal{P}^{i\,L(p)}(0)] \\ &= (2\pi)^{-3}\int d^3k\, d^3k'\left(\frac{p^0}{m_\pi}\right)^{1/2}[e_{\mathfrak{N}}\bar{u}_r^{L(p)}(\vec{k}')\gamma^\mu u_s(\vec{k})\mathfrak{N}^i_{Ir}(L(p)k')^\dagger \mathfrak{N}^i_{Is}(k) \\ &\qquad + e_{\mathcal{P}}\bar{v}_s(\vec{k})\gamma^\mu v_r^{L(p)}(\vec{k}')\tilde{\mathcal{P}}^i_{Is}(k)^\dagger \tilde{\mathcal{P}}^i_{Ir}(L(p)k')]\,. \end{aligned} \tag{3.4}$$

In the above, $u_r^{L(p)}(\vec{k}')$ is given by (2.2), and

$$v_r^{L(p)}(\vec{k}') = S(L(p))\begin{bmatrix} g_Q\vec{\sigma}\cdot\vec{k}' \\ f_Q \end{bmatrix} v_{Ir}\,, \tag{3.5}$$

where, as before

$$v_{I(1/2)} = \begin{pmatrix} 0 \\ -i \end{pmatrix} \quad\text{and}\quad v_{I(-1/2)} = \begin{pmatrix} i \\ 0 \end{pmatrix}.$$

Using straightforward algebra, we then obtain, with repeated applications of (2.3) and some integrations,

$$\begin{aligned} M_{fi} = \frac{-i}{(2\pi)^{1/2}}\,\frac{1}{\sqrt{18}}\,3\left(\frac{m_\pi}{p^0}\right)\frac{e_\mu(\vec{k},\lambda)}{(2k^0)^{1/2}}\,i\Big[& e_{\mathfrak{N}}\int d^3k_1 u_\pi^*(\vec{k}_1')u_\rho(\vec{k}_1)\bar{u}^{L(p)}_{-1/2}(\vec{k}_1')\gamma^\mu u_{1/2}(\vec{k}_1) \\ & + e_{\mathcal{P}}\int d^3k_2 u_\pi^*(-\vec{k}_2')u_\rho(-\vec{k}_2)\bar{v}_{1/2}(\vec{k}_2)\gamma^\mu v^{L(p)}_{-1/2}(\vec{k}_2')\Big]\,. \end{aligned} \tag{3.6}$$

In (3.6),

$$\vec{k}_1' = \underline{L}(p)^{-1}\vec{k}_1 + \lambda_2(m_\pi/p^0)\vec{p}\,, \tag{3.7}$$

and

$$\vec{k}_2' = \underline{L}(p)^{-1}\vec{k}_2 + \lambda_1(m_\pi/p^0)\vec{p}\,, \tag{3.8}$$

where the matrix $\underline{L}(p)_{ij} = \{L_{ij}(p):\ i,j = 1,2,3\}$, and $\lambda_1 m_\pi = \omega_{\mathfrak{N}}$, $\lambda_2 m_\pi = \omega_{\mathcal{P}}$, i.e., they are fractional energies of the quark and antiquark in π^-. Also we take $e_\mu(\vec{k},\lambda) = (0, \vec{e}(\vec{k},\lambda))$. We then obtain, for the first term on the right-hand side of (3.6),

$$\begin{aligned} \bar{u}^{L(p)}(\vec{k}_1')\gamma^\mu e_\mu(\vec{k},\lambda)u(\vec{k}_1) &= -u^\dagger(\vec{k}_1')S[L(p)]^\dagger(\vec{\alpha}\cdot\vec{e})u(\vec{k}_1) \\ &= -[f_1f_1'b(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{e}) + ag_{\mathfrak{N}}f_1(\vec{\sigma}\cdot\vec{k}_1')(\vec{\sigma}\cdot\vec{e}) \\ &\qquad + f_1'ag_{\mathfrak{N}}(\vec{\sigma}\cdot\vec{e})(\vec{\sigma}\cdot\vec{k}_1) + bg_{\mathfrak{N}}^2(\vec{\sigma}\cdot\vec{k}_1')(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{e})(\vec{\sigma}\cdot\vec{k}_1)]\,. \end{aligned} \tag{3.9}$$

In (3.9), we have

$$f_1 = (1 - g_{\mathfrak{N}}^2\vec{k}_1^2)^{1/2} \simeq 1 - \tfrac{1}{2}g_{\mathfrak{N}}^2\vec{k}_1^2\,, \tag{3.10}$$

$$f_1' \simeq 1 - \tfrac{1}{2}g_{\mathfrak{N}}^2\vec{k}_1'^2\,, \tag{3.10$'$}$$

$$S(L(p)) = \begin{pmatrix} a & b\vec{\sigma}\cdot\vec{p} \\ b\vec{\sigma}\cdot\vec{p} & a \end{pmatrix}, \tag{3.11}$$

with $a = [(p^0 + m_\pi)/2m_\pi]^{1/2}$ and $b = [2m_\pi(p^0 + m_\pi)]^{1/2}$. We shall now substitute as follows:

$$\beta_1 = b\int d^3k_1 f_1 f_1' u_\pi^*(\vec{k}_1')u_\rho(\vec{k}_1)\,, \tag{3.12}$$

$$\beta_2(\vec{\sigma}\cdot\vec{p}) = ag_{\mathfrak{N}}\int d^3k_1 f_1(\vec{\sigma}\cdot\vec{k}_1')u_\pi^*(\vec{k}_1')u_\rho(\vec{k}_1)\,, \tag{3.13}$$

$$\beta_3(\vec{\sigma}\cdot\vec{p}) = -ag_{\mathfrak{N}}\int d^3k_1 f_1'(\vec{\sigma}\cdot\vec{k}_1)u_\pi^*(\vec{k}_1')u_\rho(\vec{k}_1)\,, \tag{3.14}$$

and

$$i\beta_4\vec{\sigma}\cdot(\vec{p}\times\vec{e}) = bg_{\mathfrak{N}}^2\int d^3k_1(\vec{\sigma}\cdot\vec{k}_1')(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{e})(\vec{\sigma}\cdot\vec{k}_1)u_\pi^*(\vec{k}_1')u_\rho(\vec{k}_1)\,. \tag{3.15}$$

Equations (3.13) and (3.14) are written from rotational invariance, and for (3.15) the fact that for decay at rest $\vec{e}\cdot\vec{p}=0$ has been utilized.

Parallel to (3.9), we obtain

$$\bar{v}(\vec{k}_2)\gamma^\mu e_\mu(\vec{k},\lambda)v^{L(p)}(\vec{k}_2') = -[bf_2f_2'(\vec{\sigma}\cdot\vec{e})(\vec{\sigma}\cdot\vec{p}) + af_2g_\varphi(\vec{\sigma}\cdot\vec{e})(\vec{\sigma}\cdot\vec{k}_2')$$
$$+af_2'g_\varphi(\vec{\sigma}\cdot\vec{k}_2)(\vec{\sigma}\cdot\vec{e}) + bg_\varphi^2(\vec{\sigma}\cdot\vec{k}_2)(\vec{\sigma}\cdot\vec{e})(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{k}_2')]\,, \tag{3.16}$$

where

$$f_2 \simeq 1-\tfrac{1}{2}g_\varphi^2\vec{k}_2^2,\quad f_2'\simeq 1-\tfrac{1}{2}g_\varphi^2\vec{k}_2'^2\,. \tag{3.17}$$

Also, similar to (3.12) through (3.15), we define the following:

$$\beta_1' = b\int d^3k_2 f_2 f_2' u_\pi^*(-\vec{k}_2')u_\rho(-\vec{k}_2)\,, \tag{3.18}$$

$$\beta_2'(\vec{\sigma}\cdot\vec{p}) = ag_\varphi\int d^3k_2 f_2(\vec{\sigma}\cdot\vec{k}_2')u_\pi^*(-\vec{k}_2')u_\rho(-\vec{k}_2)\,, \tag{3.19}$$

$$\beta_3'(\vec{\sigma}\cdot\vec{p}) = -ag_\varphi\int d^3k_2 f_2'(\vec{\sigma}\cdot\vec{k}_2)u_\pi^*(-\vec{k}_2')u_\rho(-\vec{k}_2)\,, \tag{3.20}$$

and

$$i\beta_4'\vec{\sigma}\cdot(\vec{e}\times\vec{p}) = bg_\varphi^2\int d^3k_2(\vec{\sigma}\cdot\vec{k}_2)(\vec{\sigma}\cdot\vec{e})(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{k}_2')u_\pi^*(-\vec{k}_2')u_\rho(-\vec{k}_2)\,. \tag{3.21}$$

Thus, with the above substitutions, we obtain, from (3.6),

$$M_{fi} = \frac{1}{(2\pi)^{1/2}}\frac{1}{\sqrt{2}}\frac{m_\pi}{p^0}\frac{1}{(2k^0)^{1/2}}\{-e_{\mathfrak{N}}\beta_{\mathfrak{N}}u^\dagger_{I-1/2}[i\vec{\sigma}\cdot(\vec{p}\times\vec{e})]u_{I1/2} - e_\varphi\beta_\varphi' v^\dagger_{I(1/2)}[i\sigma\cdot(\vec{e}\times\vec{p})]\,v_{I(-1/2)}\}$$
$$= \frac{-i}{(2\pi)^{1/2}}\frac{1}{\sqrt{2}}\frac{m_\pi}{p^0}\frac{1}{(2k^0)^{1/2}}(e_{\mathfrak{N}}\beta_{\mathfrak{N}} + e_\varphi\beta_\varphi')u^\dagger_{I(-1/2)}\vec{\sigma}\cdot(\vec{p}\times\vec{e})u_{I(1/2)}\,. \tag{3.22}$$

In the above equation we have substituted

$$\beta_{\mathfrak{N}} = \sum_{i=1}^4 \beta_i\,, \tag{3.23}$$

and

$$\beta_\varphi' = \sum_{i=1}^4 \beta_i'\,. \tag{3.24}$$

We now proceed to obtain $\beta_{\mathfrak{N}}$ and β_φ'. For this purpose, we take all the meson wave functions as

$$u_M(\vec{k}_1) = \left(\frac{R^2}{\pi}\right)^{3/4}\exp(-\tfrac{1}{2}R^2\vec{k}_1^2)\,. \tag{3.25}$$

We are to evaluate the integrals for β_i and β_i' in Eqs. (3.12)–(3.15) and (3.18)–(3.21). We make the substitution

$$\vec{k}_1 = \underline{D}\vec{\chi} - \alpha_1\vec{p}\,, \tag{3.26}$$

where we shall subsequently determine the matrix $\underline{D}$ and the constant α_1. We further choose that

$$\underline{D}_{ij} = \delta_{ij} + dp^ip^j\,. \tag{3.27}$$

Then, with (3.7) we get

$$\vec{k}_1' = \underline{D}'\vec{\chi} + \alpha_1'\vec{p}\,, \tag{3.28}$$

where

$$\underline{D}' = \underline{L}(p)^{-1}\underline{D}\,,$$

and

$$\alpha_1' = \frac{m_\pi}{p^0}(\lambda_2 - \alpha_1)\,. \tag{3.29}$$

We then obtain

$$\vec{k}_1'^2 + \vec{k}_1^2 = 2\vec{\chi}^2 + (\alpha_1^2 + \alpha_1'^2)\vec{p}^2\,, \tag{3.30}$$

provided we take, for (3.26) and (3.27),

$$D = \det\underline{D} = 1 + d\vec{p}^2 = \left(\frac{2p^{02}}{p^{02}+m_\pi^2}\right)^{1/2} \tag{3.31}$$

and

$$\alpha_1 = \frac{\lambda_2 m_\pi^2}{p^{02}+m_\pi^2}\,. \tag{3.32}$$

We also note that

$$D' = \det \underline{D}' = \frac{m_\pi}{p^0} D\,, \tag{3.33}$$

and

$$\alpha_1' = \frac{p^0}{m_\pi} \alpha_1\,. \tag{3.34}$$

Now, from (3.12) we obtain, with (3.10) and (3.10′) and the substitutions above,

$$\beta_1 = Ab\left\{1 - \tfrac{1}{2} g_{\mathfrak{N}}^2 \left[\frac{D^2 + D'^2 + 4}{2R^2} + (\alpha_1^2 + \alpha_1'^2)\vec{p}^2\right]\right\}, \tag{3.35}$$

where

$$A = D \exp[-\tfrac{1}{2}R^2(\alpha_1^2 + \alpha_1'^2)\vec{p}^2]\,. \tag{3.36}$$

Similarly, we obtain the following from (3.13), (3.14), and (3.15):

$$\beta_2 = Aag_{\mathfrak{N}}\left\{\alpha_1'\left[1 - \tfrac{1}{2} g_{\mathfrak{N}}^2\left(\frac{D^2+2}{2R^2} + \alpha_1^2\vec{p}^2\right)\right] + \alpha_1 g_{\mathfrak{N}}^2 \frac{DD'}{2R^2}\right\}, \tag{3.37}$$

$$\beta_3 = Aag_{\mathfrak{N}}\left\{\alpha_1\left[1 - \tfrac{1}{2} g_{\mathfrak{N}}^2\left(\frac{D'^2+2}{2R^2} + \alpha_1'^2\vec{p}^2\right)\right] + \alpha_1' g_{\mathfrak{N}}^2 \frac{DD'}{2R^2}\right\}, \tag{3.38}$$

and

$$\beta_4 = Abg_{\mathfrak{N}}^2\left(\alpha_1\alpha_1'\vec{p}^2 - \frac{DD'}{2R^2}\right). \tag{3.39}$$

The evaluation of integrals (3.18)–(3.21) proceeds in a similar manner, and we finally obtain, for $i = 1, 2, 3, 4$,

$$\beta_i' = \beta_i(\lambda_2 \to \lambda_1 \text{ and } g_{\mathfrak{N}} \to g_{\mathcal{P}})\,. \tag{3.40}$$

The effect of (3.40) is to be recognized in all the defining equations. In all the integrals above, we have included only up to $\vec{x}^2$ terms in the evaluation of the integrals.

Equation (3.22) now yields

$$\Gamma(\rho^- \to \pi^-\gamma) = \frac{m_\pi^2|\vec{k}|^3}{3\pi m_\rho p^0}(e_{\mathfrak{N}}\beta_{\mathfrak{N}} + e_{\mathcal{P}}\beta_{\mathcal{P}}')^2. \tag{3.41}$$

As in Ref. 8, we take $\lambda_1 = \lambda_2 = \frac{1}{2}$, and $g_{\mathcal{P}} = 1.62\ \text{GeV}^{-1}$, $g_{\mathfrak{N}} = 1.71\ \text{GeV}^{-1}$. We then finally obtain[12]

$$\Gamma(\rho^- \to \pi^-\gamma) = \frac{4\alpha}{27}\frac{m_\pi^2|\vec{k}|^3}{m_\rho p^0}(2\beta_{\mathcal{P}}' - \beta_{\mathfrak{N}})^2 = 36.6\ \text{keV}\,. \tag{3.42}$$

We have taken $R^2 = 22\ \text{GeV}^{-2}$ as was indicated in Ref. 8 for pion charge radius. We note that this result as well as all the subsequent results are not very sensitive to R^2; there is about a 10% variation in the numbers.

B. $\omega \to \pi^0\gamma$

For this process, Eq. (3.42) is replaced by

$$\Gamma(\omega \to \pi^0\gamma) = \frac{4\alpha}{27}\frac{m_\pi^2|\vec{k}|^3}{m_\omega p^0}(2\beta_{\mathcal{P}} + \beta_{\mathfrak{N}})^2 \simeq 311\ \text{keV}\,. \tag{3.43}$$

For the evaluation of $\beta_{\mathcal{P}}$ and $\beta_{\mathfrak{N}}$, Eqs. (3.35)–(3.39) with corresponding kinematics, have been utilized.

C. $K^{0*} \to K^0\gamma$ and $K^{+*} \to K^+\gamma$

For the calculation of these processes, we had seen in Ref. 8 that in (3.7) and (3.8) we have $\lambda_1 = 0.32$ and $\lambda_2 = 0.68$ as has been determined from $\phi \to 2K$. Tentatively taking these values, we get

$$\Gamma(K^{0*} \to K^0\gamma) = \frac{4\alpha}{27}\frac{m_K^2|\vec{k}|^3}{m_{K^*}p^0}(\beta_{\mathfrak{N}} + \beta_\lambda')^2 \simeq 115.5\ \text{keV}\,. \tag{3.44}$$

Proceeding in a similar manner, we have

$$\Gamma(K^{+*} \to K^+\gamma) = \frac{4\alpha}{27}\frac{m_K^2|\vec{k}|^3}{m_{K^*}p^0}(2\beta_{\mathcal{P}} - \beta_\lambda')^2 \simeq 49.9\ \text{keV}\,. \tag{3.45}$$

D. $\phi \to \eta\gamma$

We are to include here SU(3) mixing, and thus we take

$$|\phi_1(\vec{0})\rangle = \int \delta_3(k_1+k_2)d^3k_1\,d^3k_2\,u_\phi(\vec{k}_1)\left[\frac{\cos\theta_\phi}{\sqrt{3}}\lambda_I^i{}_{(1/2)}(k_1)^\dagger\tilde{\lambda}_I^i{}_{(1/2)}(k_2) + \frac{\sin\theta_\phi}{\sqrt{6}}q_I^i{}_{(1/2)}(k_1)^\dagger\tilde{q}_I^i{}_{(1/2)}(k_2)\right]|\text{vac}\rangle\,, \tag{3.46}$$

and

$$|\eta(\vec{p})\rangle = \left(\frac{p^0}{m_\eta}\right)^{1/2}\int \delta_3(k_1+k_2)\,d^3k_1\,d^3k_2\,u_\eta(\vec{k}_1) \times\left[-\frac{\cos\theta_\eta}{\sqrt{6}}\lambda_I^i(L(p)k_1)^\dagger\tilde{\lambda}_I^i(L(p)k_2) + \frac{\sin\theta_\eta}{2\sqrt{3}}q_I^i(L(p)k_1)^\dagger\tilde{q}_I^i(L(p)k_2)\right]|\text{vac}\rangle\,. \tag{3.47}$$

In (3.46) and (3.47), we have taken

$$q=\begin{pmatrix}\mathcal{P}\\ \mathfrak{N}\end{pmatrix}.$$

The calculation can be be performed as for $\rho^-\to\pi^-\gamma$. In the formula corresponding to (3.40), $\lambda_1=\lambda_2=\frac{1}{2}$ such that we finally get

$$\Gamma(\phi\to\eta\gamma)=\frac{4\alpha}{27}\frac{m_\eta{}^2|\vec{k}|^3}{m_\phi p^0}\left[-2\beta_\lambda\cos\theta_\phi\cos\theta_\eta+(2\beta_{\mathcal{P}}-\beta_{\mathfrak{N}})\sin\theta_\phi\sin\theta_\eta\right]^2 . \tag{3.48}$$

Further, the quadratic mass formula for mesons yields $\theta_\phi\simeq 5°$, $\theta_\eta\simeq -25°$, such that we obtain,

$$\Gamma(\phi\to\eta\gamma)\simeq 112\ \text{keV}. \tag{3.49}$$

E. $\phi\to\pi^0\gamma$

Here we obtain

$$\Gamma(\phi\to\pi^0\gamma)=\frac{4\alpha}{27}\frac{m_\pi{}^2|\vec{k}|^3}{m_\phi p^0}(2\beta_{\mathcal{P}}+\beta_{\mathfrak{N}})^2\sin^2\theta_\phi \simeq 2.7\ \text{keV}. \tag{3.50}$$

If we take the mixing angle as 6°, then we get

$$\Gamma\simeq 4.0\ \text{keV}. \tag{3.50'}$$

F. $\phi\to\eta'\gamma$

We define $|\eta'(\vec{p})\rangle$ as orthogonal to the state in (3.47), which yields

$$\Gamma(\phi\to\eta'\gamma)\simeq 0.19\ \text{keV}. \tag{3.51}$$

G. $\omega\to\eta\gamma$ and $\rho^0\to\eta\gamma$

With the mixing angles taken earlier, we obtain

$$\Gamma(\omega\to\eta\gamma)\simeq 0.85\ \text{keV} \tag{3.52}$$

and

$$\Gamma(\rho^0\to\eta\gamma)\simeq 16.9\ \text{keV}. \tag{3.53}$$

H. $\eta'\to\rho\gamma$ and $\eta'\to\omega\gamma$

We note that this has the form $P\to V\gamma$ as compared to $V\to P\gamma$. However, calculations remain almost unchanged, and we obtain with the same definition of β_Q as earlier, similar to (3.22), with only a change of sign;

$$M_{fi}(\eta'\to\rho\gamma)=\frac{i}{(2\pi)^{1/2}}\frac{1}{\sqrt{2}}\frac{m_\rho}{p^0}\frac{1}{(2k^0)^{1/2}}\times\cos\theta_\eta(e_{\mathcal{P}}\beta_{\mathcal{P}}-e_{\mathfrak{N}}\beta_{\mathfrak{N}})\times u^\dagger_{I(1/2)}\vec{\sigma}\cdot(\vec{p}\times\vec{e})u_{I(-1/2)}, \tag{3.54}$$

which yields

$$\Gamma(\eta'\to\rho\gamma)=\frac{4\alpha}{27}\frac{m_\rho{}^2|\vec{k}|^3}{m_{\eta'}p^0}\cos^2\theta_\eta(2\beta_{\mathcal{P}}+\beta_{\mathfrak{N}})^2 \simeq 45.5\ \text{keV}. \tag{3.55}$$

Similarly, we get

$$\Gamma(\eta'\to\omega\gamma)=\frac{4\alpha}{27}\frac{m_\omega{}^2|\vec{k}|^3}{m_{\eta'}p^0}[\cos\theta_\eta\cos\theta_\phi(2\beta_{\mathcal{P}}-\beta_{\mathfrak{N}})-2\sin\theta_\eta\sin\theta_\phi\beta_\lambda]^2 \simeq 4.8\ \text{keV}. \tag{3.56}$$

We list in Table I the results along with the experimental data.[12]

IV. DISCUSSION

The results of the calculations as tabulated are suprisingly good when we recall that in the process of calculations no constants have been adjusted, and the calculations have been carried out from "first principles" in the quark model in the lowest order without any vector-dominance or subsidiary hypothesis.[13] However, as in almost all such papers, $\Gamma(\omega\to\pi\gamma)$ as compared to $\Gamma(\rho\to\pi\gamma)$ remains a puzzle.[13,14] We note that we have *completely* ignored strong-interaction effects. We have a suspicion that this may be at the basis of the disagreement mentioned above.[15] Because of the nearness of ρ and ω masses, the correction for $\Gamma(\omega\to\pi\gamma)$ may be important here, and not be so important elsewhere. As noted, this puzzle of $\Gamma(\omega\to\pi\gamma)/\Gamma(\rho\to\pi\gamma)$ remains here, as in other models; e.g., in Ref. 6 nothing much better can be

TABLE I. Radiative decays of mesons. There is no adjustable parameter, and no strong-interaction effect has been included.

Decay Process	Calculated width (keV)	Experimental width (keV)
$\rho^-\to\pi^-\gamma$	36.6	35 ± 10
$\omega\to\pi^0\gamma$	311	880 ± 50
$K^{0*}\to K^0\gamma$	115.5	75 ± 35
$K^{+*}\to K^+\gamma$	49.9	<80
$\phi\to\eta\gamma$	112	82 ± 16
$\phi\to\pi^0\gamma$	2.7–4	5.7 ± 2
$\phi\to\eta'\gamma$	0.19	<40
$\omega\to\eta\gamma$	0.85	<50
$\rho^0\to\eta\gamma$	16.9	<160
$\eta'\to\rho\gamma$	45.5	<250
$\eta'\to\omega\gamma$	4.8	<50

achieved even while including symmetry breaking.

In the nonrelativistic quark models, kinematic corrections due to the relativistic nature of the problem were impossible to assess. This problem has been resolved with a specific theory of Lorentz boosting of the hadrons from rest as developed by one of the authors,[8] and this model seems to find further justification here. We note that a glance at Eqs. (3.35)–(3.39) clearly reveals the enormous quantitative differences of this model compared to any earlier quark model, although the final results are similar. It is nice to see that the structure of the hadrons along with the familiar form of electromagnetic interactions is mostly adequate to describe the above electromagnetic decays when Lorentz boosting is properly taken into account.

ACKNOWLEDGMENT

One of the authors (L.M.) would like to thank University Grants Commission for financial assistance.

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[14]Ref. 5 and 6. $\Gamma(\omega \rightarrow \pi\gamma)$ and $\Gamma(\rho \rightarrow \pi\gamma)$ are often taken as inputs, which lead to inconsistency elsewhere.

[15]See discussions at the end of Ref. 8.