PHYSICAL REVIEW D VOLUME 18, NUMBER 7 1 OCTOBER 1978

SU(3)-symmetry breaking and nonleptonic decays in the quark-density model

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(Received 26 October 1977)

The phenomenology of the p-wave nonleptonic decays in the quark-density model is reviewed. We find that a good fit can be obtained by considering contributions arising due to SU(3) breaking either in the hadronic mass factors or at the strong vertices. However, when considered simultaneously, the two effects tend to cancel each other. It seems impossible to avoid the Coleman-Glashow theorem, in any consistent manner, if the symmetry-breaking tadpoles are introduced explicitly in the Hamiltonian.

I. INTRODUCTION

The "tadpole-dominance" models,[1] e.g., the quark-density model[2] in which symmetry-breaking interactions are controlled by bilinear quark tadpoles, offer an elegant and perhaps the only satisfactory resolution of the problem of octet dominance in the nonleptonic decays. Interestingly, recent attempts at generating octet dominance for nonleptonic decays in field-theoretic models—especially in the field theory of quarks and gluons ("quantum chromodynamics")—have resulted in the introduction of new terms in the Hamiltonian which are explicitly of the tadpole form.[3] Particularly relevant here is the model of Fritzsch and Minkowski.[3] The model is not only of the tadpole form, but the tadpoles dominating the weak nonleptonic interaction also have the same transformation properties as the quark-density tadpoles. In spite of the persistent interest in the quark-density model, the phenomenology of the nonleptonic decays—especially of the p-wave nonleptonic decays—has not yet been presented unambiguously. The problem is that given the tadpole-type symmetry breaking one can perform a rotation in the SU(3) space so as to diagonalize all symmetry-breaking terms.[1] This makes the p-wave nonleptonic decays in the quark-density model vanish. Nonzero results for these decays have been obtained by various means, such as introducing *ad hoc* breaking of SU(3),[4] allowing the hadron masses to acquire non-SU(3)-invariant values,[5] or by letting the strong couplings (baryon-baryon-meson couplings) deviate from their SU(3) values.[6] In this paper we review the phenomenological situation for p-wave decays. We reach the conclusion that if the SU(3)-breaking effects are introduced systematically and consistently, the p-wave nonleptonic decays in the quark-density model remain zero. We show that this result, though surprising, is still an obvious consequence of the Coleman-Glashow theorem.

In Sec. II, the problem is presented in detail, and the "universality" hypothesis is discussed. We then discuss SU(3) symmetry-breaking effects in masses and coupling constants in Secs. III and IV, respectively. We conclude in Sec. V by analyzing the situation where the two symmetry-breaking effects are considered simultaneously.

II. THE QUARK-DENSITY MODEL

In the quark-density model the weak Hamiltonian H_W is given by

$$H_W = G_S P_7 + G_P S_6, \tag{1}$$

where S_6 and P_7 are, respectively, scalar and pseudoscalar tadpoles, bilinear in quarks. Using the Hamiltonian of Eq. (1), the algebra of currents, and the partial conservation of axial-vector current (PCAC) we can write the p-wave amplitudes as[2]

$$\begin{aligned}
B(\Lambda^0_-) &= \frac{G_P}{\sqrt{2} f_\pi}\left[g_{\Lambda N K} - \frac{2f_\pi}{\sqrt{3}\,\delta M}\,\frac{m_\Lambda + m_N}{2m_N}\left(\sqrt{3}\, g_{\pi NN} - \frac{2m_N}{m_\Lambda + m_\Sigma} g_{\Lambda\Sigma\pi}\right)\right],\\
B(\Xi^-_-) &= \frac{G_P}{\sqrt{2} f_\pi}\left[g_{\Xi\Lambda K} + \frac{2f_\pi}{\sqrt{3}\,\delta M}\,\frac{m_\Lambda + m_\Xi}{2m_\Lambda}\left(\sqrt{3}\, g_{\Xi\Xi\pi} + \frac{m_\Lambda}{m_\Xi} g_{\Lambda\Sigma\pi}\right)\right],\\
B(\Sigma^+_+) &= \frac{G_P}{\sqrt{2} f_\pi}\,\frac{2f_\pi}{\sqrt{3}\,\delta M}\,\frac{m_\Sigma + m_N}{2m_N}\left[-\sqrt{2}\, g_{NN\pi} + \frac{1}{\sqrt{2}}\,\frac{m_N}{m_\Sigma} g_{\Sigma\Sigma\pi} + \frac{2m_N}{m_\Lambda + m_\Sigma}\left(\frac{3}{2}\right)^{1/2} g_{\Lambda\Sigma\pi}\right],\\
B(\Sigma^-_-) &= \frac{G_P}{\sqrt{2} f_\pi}\left[\sqrt{2}\, g_{\Sigma N K} - \frac{2f_\pi}{\sqrt{3}\,\delta M}\,\frac{m_\Sigma + m_N}{2m_N}\left(-\left(\frac{3}{2}\right)^{1/2}\frac{2m_N}{m_\Lambda + m_\Sigma} g_{\Lambda\Sigma\pi} + \frac{m_N}{m_\Sigma}\,\frac{1}{\sqrt{2}} g_{\Sigma\Sigma\pi}\right)\right].
\end{aligned} \tag{2}$$

In Eq. (2), δM is the symmetry-breaking part in the mass operator

$$M = M_0 + \delta M S_8. \quad (3)$$

In deriving Eq. (2) it has been assumed that S_8 of Eq. (3) and S_6 of Eq. (1) belong to the same octet. This assumption, that the medium strong and weak symmetry-breaking tadpoles belong to the same octet, is the "universality" hypothesis. Universality is an essential feature of tadpole-type models. The original motivation for proposing the tadpole-dominance scheme[1] of symmetry-breaking interactions was to obtain a universal mechanism for the three interactions, viz, medium strong, electromagnetic, and weak. It must be realized that the Coleman-Glashow rotation can be carried out only if universality holds. Therefore it is tempting to try relaxing the universality hypothesis and thus to avoid the vanishing of the p-wave decays, as is attempted by Fritzsch and Minkowski.[3] However, the reasons for insisting on universality in the quark-density model are not purely aesthetic or historic. The universality is forced upon us by the phenomenology of s-wave decays. It is known that the d/f ratio for the S_7 tadpole required to fit the s-wave decays is $-\frac{1}{3}$, the same as the d/f ratio found for semistrong baryon mass splittings. If the quark-density model is to have any predictive power at all then the S_6 tadpole generating the s-wave decays must have the same d/f ratio as the S_7 tadpole. Hence, S_6 and S_8 must belong to the same octet.

Returning to Eq. (2)—which is derived under the assumptions of PCAC, current algebra, and universality—it can be seen that the p-wave decays vanish if the coupling constants and the hadron masses retain their SU(3) value, and if $-\frac{1}{2}\sqrt{3}\,\delta M = f\pi$. This latter relation is valid because it leads to δM having the value required for the Gell-Mann-Okubo mass splittings. Thus we see explicitly that the Coleman-Glashow theorem works—at least when SU(3) symmetry is exact for the hadron masses and the coupling constants.

III. SU(3) BREAKING IN HADRON MASSES

We have noticed in Sec. II that the p-wave decay amplitudes become zero only if the mass factors in Eq. (2) are constrained to their SU(3)-symmetric value 1. This amounts to neglecting the rapid variation of the baryon poles in the soft-pion limit, i.e., in the notation of Ref. 2, ignoring the term $R_c^{pc}(0)$. Kaushal and Khanna[5] have shown that if this term is not neglected—i.e., if one uses physical values for the mass factors in Eq. (2) instead of using SU(3) values (while leaving the couplings SU(3) invariant)—a good fit to the p-wave decays can be obtained. The fit obtained by them is shown in column 2 of Table I. The Coleman-Glashow theorem apparently fails. We shall comment on the reasons for this in the concluding section. However, a serious objection to the procedure of Ref. 5 is that the whole of the decay amplitudes are being generated by the $R_c^{pc}(0)$ term which is of the order $\Delta m_B/2m_B$ compared to the Born terms, and hence is expected to be small. Fitting the p-wave amplitudes using only this term is expected to require G_P of an order of magnitude higher than G_S. This will severely violate the principle of maximal parity-violation. Interestingly, actual calculation shows that this fear is unfounded. It seems that small symmetry-breaking effects of order $\Delta m_B/2m_B$ conspire constructively to give a large factor; and the G_P/G_S actually required to fit the decays is only about 3. Thus we obtain

$$H_W \sim G_S(P_7 + 3S_6). \quad (4)$$

This enhancement of the parity-conserving part of H_W with respect to the parity-violating part—though a little surprising—is, nevertheless, not unimaginable in view of the fact that the weak-in-

TABLE I. Amplitudes for the parity-conserving nonleptonic decays generated through SU(3) symmetry breaking in hadron masses and in strong coupling constants. The amplitudes are in units of $10^5 m^{-1/2}\text{sec}^{-1/2}$ (Ref. 2).

Process	Amplitude with SU(3)-broken mass factors	Amplitude with SU(3)-broken strong couplings	Experimental amplitudes (Ref. 2)
$B(\Lambda_-^0)$	14.28	11.58	10.644 ± 0.475
$B(\Xi_-^-)$	6.46	5.89	6.831 ± 0.574
$B(\Sigma_+^+)$	19.08	19.08	19.078 ± 0.347 [a]
$B(\Sigma_-^-)$	0.51	0	-0.549 ± 0.386
$G_P/\sqrt{2}\,f_\pi$	3.85×10^5	-4.44×10^5	···

[a] Input for fixing $G_P/\sqrt{2}\,f_\pi$.

teraction processes encompass a whole hierarchy of effective coupling strengths.

More corroborating evidence in favor of this type of fitting comes from the weak-radiative decays. In the conventional pole model the expressions for parity conserving and parity-violating decay plitudes for the process $\Sigma^+ \to p\gamma$ are, respectively,[2]

$$C = e\left(\frac{\mu_p}{2m_N} - \frac{\mu_{\Sigma^+}}{2m_\Sigma}\right)\frac{b}{m_\Sigma - m_N},$$
$$D = e\left(\frac{\mu_p}{2m_N} + \frac{\mu_{\Sigma^+}}{2m_\Sigma}\right)\frac{-b'}{m_\Sigma + m_N}, \tag{5}$$

where μ_p and μ_Σ are anomalous magnetic moments for the proton and the Σ particle. b and b' are weak-interaction couplings, and in the case of weak-interaction Hamiltonian of Eq. (4) one obtains

$$\frac{b}{b'} = \frac{G_P}{G_S} \simeq 3. \tag{6}$$

The asymmetry parameter for the process $\Sigma^+ \to p\gamma$ is given by

$$\alpha = \frac{2\,ReC^*D}{|C|^2 + |D|^2}. \tag{7}$$

Using the SU(3) result $\mu_{\Sigma^+}/\mu_p = 1$ in Eq. (5) and the ratio $b/b' \simeq 3$, we obtain $\alpha = -0.6$, which is close to the experimental value[7] $\alpha = -1.0 \pm 0.5$. Thus the use of physical mass factors in Eq. (2) instead of the SU(3)-symmetric values leads to a satisfactory solution of not only the problem of vanishing p-wave decays but also of the problem of weak-radiative decays.

IV. SU(3) BREAKING AT THE STRONG VERTICES

In Sec. III we have noticed that small symmetry-breaking effects in hadron masses of order $\Delta m_B / 2m_B$ are sufficient to generate reasonably large contributions to the p-wave nonleptonic decays. One may therefore expect similarly large contributions from possibly small SU(3) breaking at the strong (baryon-baryon-meson) vertices. The situation regarding the physical values of these couplings is not as clear as in the case of hadron masses. However, the available information on these couplings does not rule out the possibility of the couplings being SU(3) noninvariant. In any case, the presence of an S_8 tadpole term shall, in general, break the SU(3) both at the strong masses and at the strong vertices. There is no justification for assuming, *a priori*, the SU(3) invariance of the strong couplings. One should start by assuming the couplings to be SU(3) broken by the S_8 term; and the breaking parameters should be fixed from experimental information. This is what we intend to do in this section. Since we want to see the effect of SU(3) breaking at the vertices alone, on the p-wave nonleptonic decays, we shall for the purpose of this section assume SU(3) to be intact for the mass factors in Eq. (2). This type of analysis, to be sure, has already been carried out by Gavroglu.[6] However, that paper makes too many assumptions for the numerical results to be of any significance. In the following we evaluate the contributions of the SU(3)-breaking parts of the coupling constants to the p-wave decays, taking care to keep the number of assumptions at a minimum. As already stated in this section, we ignore the contribution from SU(3) breaking in masses computed in Sec. III.

Following Muraskin and Glashow,[8] we write the SU(3) noninvariant baryon-baryon-meson interaction term as

$$H_{\rm int} = \sqrt{2}\, dg\, {\rm Tr}(\bar{B}PB + BBP) + \sqrt{2}\, fg\, {\rm Tr}(\bar{B}PB - \bar{B}BP)$$
$$+ g_2\, {\rm Tr}\bar{B}\lambda_8 BP + g_5\, {\rm Tr}\bar{B}PB\lambda_8 + g_3\, {\rm Tr}\bar{B}[\lambda_8, P]_+ B$$
$$+ g_7({\rm Tr}\bar{B}P\, {\rm Tr}B\lambda_8 + {\rm Tr}\bar{B}\lambda_8\, {\rm Tr}BP) \tag{8}$$

(where $d + f = 1$). The ${\rm Tr}\bar{B}B\,{\rm Tr}P\lambda_8$ term is irrelevant for the coupling constants required in Eq. (2) and hence has been ignored in $H_{\rm int}$. Using Eq. (8), the seven couplings required in Eq. (2) can be written in terms of 6 parameters as follows:

$$g_{NN\pi} = g - \frac{2}{\sqrt{3}}(g_5 - g_3),$$
$$g_{\Sigma\Sigma\pi} = 2fg - \frac{2}{\sqrt{3}}(\tfrac{1}{2}g_2 - \tfrac{1}{2}g_5 - g_3),$$
$$g_{\Xi\Xi\pi} = (2f - 1)g + \frac{2}{\sqrt{3}}g_2,$$
$$g_{\Lambda\Sigma\pi} = \frac{2(1-f)}{\sqrt{3}}g + \tfrac{1}{3}(g_2 + g_5 + 2g_3 + 6g_7), \tag{9}$$
$$g_{\Xi\Lambda K} = \frac{(4f-1)g}{\sqrt{3}} + \tfrac{1}{3}(4g_2 + g_5 - g_3 + 6g_7),$$
$$g_{\Lambda NK} = -\frac{(1+2f)g}{\sqrt{3}} + \tfrac{1}{3}(g_2 + 4g_5 + 2g_3 + 6g_7),$$
$$g_{\Sigma NK} = (1 - 2f)g + \frac{1}{\sqrt{3}}g_2.$$

Six parameters are still too many to allow a determination from experimental information on the strong couplings. To place some conditions on these parameters, we use the expression of Eq. (9) in Eq. (2) (with $-\frac{1}{2}\sqrt{3}\,\delta M = f_\pi$, and mass factors equaling 1), and require that the nonleptonic decays so obtained must satisfy the Lee-Sugawara rule and the condition $B(\Sigma_-^-) = 0$. We remark that these conditions are reasonably well satisfied by the mass-breaking contributions of Sec. III. Using these two conditions, the number of parameters is reduced to 4, which we determine from the following experimental numbers:

$g_{\Lambda\Sigma\pi}{}^2/4\pi = 11.4$, Ref. 9

$g_{\Sigma\Sigma\pi}{}^2/4\pi = 12.5$, Ref. 9

$g_{NN\pi}{}^2/4\pi = 14.7$, Ref. 10

$g_{\Lambda NK}{}^2/4\pi = 16$, Ref. 11.

Thus we are able to calculate all the parameters required. Incidentally, for f we obtain the value 0.4 which is the same as the SU(6) value. Using these parameters in Eqs. (2) we get the p-wave decay amplitudes listed in Column 3. of Table I. Notice that the G_P required for the fit is of the same order of magnitude as in Sec. III, but has the opposite sign. We shall comment on the consequences of this in the next section. Here, we only remark that since the G_P/G_S ratio is now the negative of what it was in Section III, the asymmetry parameter α for the decay $\Sigma^+ \to p\gamma$ changes sign. Thus the fit of the p-wave decays through SU(3)-breaking effects in the coupling constants is bad in as much as it gives the wrong sign for the asymmetry parameter of the $\Sigma^+ \to p\gamma$ decay.

V. SU(3)-BROKEN MASSES AND VERTICES

We observe that the consideration of SU(3) symmetry breaking, either in the hadronic masses or at the strong vertices, leads to a good fit for the p-wave nonleptonic decays, though in the latter case the correct sign for the asymmetry parameter for the radiative decay $\Sigma^+ \to p\gamma$ is not obtained. However, the more important observation is that the two effects—viz. SU(3) splitting in masses and SU(3) splitting in the coupling constants—are of the same order of magnitude and are opposite in sign. This implies that in a consistent evaluation, using physical values for both the mass factors and the strong couplings, the two effects will cancel each other, and the p-wave decays will be zero. This is an expected consequence of the Coleman-Glashow theorem. When a rotation is carried out in the SU(3) space so as to diagonalize the symmetry breaking terms, SU(3) splitting is in general generated both at the vertices and in the mass operator.[12] A consistent treatment requires that both hadron masses and strong vertices be considered SU(3) noninvariant, and the breaking parameters be fixed from physically measured values of the masses and the couplings. If physical (i.e., symmetry-broken) values of coupling constants and hadron masses are used, the Coleman-Glashow theorem directs the p-wave decays to be-zero. In Secs. III and IV this zero result has been avoided by circumventing the theorem in considering either one of the two symmetry-breaking effects. In view of the calculated near equality of the two effects, the treatment of both Secs. III and IV is clearly erroneous. We remark that the two effects do not exactly cancel each other, only because in our treatment we have neglected the second-order symmetry breaking (i.e., effects involving cross terms in mass splitting and coupling pling-constant splitting) and because the information on the coupling constants is not very good. If we knew all the coupling constants and used these along with the physical values of the mass factors in Eq. (2) we would obtain the null result predicted by the Coleman-Glashow theorem.

In conclusion we want to point out that there is a way out of this impasse. Instead of putting the symmetry-breaking tadpoles explicitly in the Hamiltonian, one can equally well start with a completely symmetric Hamiltonian and generate these tadpoles through a spontaneous symmetry-breaking mechanism. In a chiral Lagrangian model of the mesons it has been shown by Bajaj and Kapoor[3] that nonleptonic-tadpole terms can indeed be generated through a spontaneous symmetry-breaking mechanism, and that in this case the consistency conditions are such that these tadpole terms cannot be transformed away. Unfortunately, the extension of the model to baryons is not straightforward.

ACKNOWLEDGMENT

We are grateful to Professor R. E. Marshak and Dr. A. K. Kapoor for stimulating discussions. Financial assistance from the C.S.I.R., New Delhi is gratefully acknowledged.

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