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Sum rules for weak $NN\pi$ amplitudes and theoretical descriptions of nonleptonic hyperon decays

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The sum rule based on current algebra predicts the $p \to n + \pi^+$ amplitude to be $A(p\,^+_+) = -\alpha 3.02 \times 10^{-7}$ ($\hbar = c = 1$). The weak-interaction-model–dependent parameter α can have the same value $\alpha = \tan\theta_C/\sqrt{2}$ for the Weinberg-Salam as for the Cabibbo model if the observed $|\Delta \vec{I}| = \frac{1}{2}$, $|\Delta S| = 1$ enhancement is explained by using PCAC (partially conserved axial-vector current). When the Melosh transformation is used with the soft-pion and infinite-momentum techniques, a very small amplitude $A(p\,^+_+) = -\alpha 3.02 \times 10^{-9}$ is found. A simple description of SU(3)-symmetry breaking, reproducing the observed nonleptonic decay amplitudes again, predicts $A(p\,^+_+) = -\alpha 3.04 \times 10^{-8}$. Useful insights may be gained from $\Delta I = 1$ parity-violating experiments, such as γ asymmetry in the $n + p \to d + \gamma$ reaction.

I. INTRODUCTION

Parity-violating effects in nuclear physics offer the possibility of looking at the long-standing problems in particle and/or nuclear physics from an entirely new angle and in an entirely different light, requiring more details than were necessary to produce some of the abundant fits of nonleptonic hyperon decay amplitudes.[1,2] We refer the reader to the literature connecting weak potentials and measured nuclear processes.[3,4] In this paper we want, however, to stress the importance of one measurement of parity-violating asymmetry in the angular distribution of photons emitted in the $n + p \to d + \gamma$ process.[5,6] As explained in the Appendix, this experiment, when perfected, will probably yield information on the strength of the pion-exchange contribution to weak potentials, namely, on the weak parity-violating (PV) nucleon-nucleon-pion ($NN\pi$) amplitude.

Using unified gauge theories with asymptotically free strong gluon interactions, it is possible to evaluate effective weak Hamiltonians (H_W^{eff}) (Refs. 7–9) which can be used as an input in further calculations of the $p \to n + \pi^+ [A(p\,^+_+)]$ amplitude. The whole chain of reasoning can be described graphically, as shown in Fig. 1.

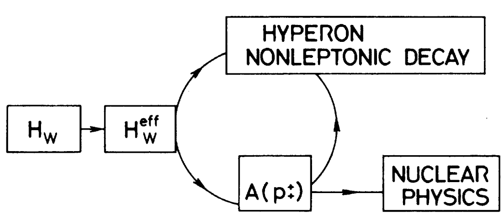

FIG. 1. Graphical chain for evaluating the $A(p\,^+_+)$ amplitude.

In this paper we want to concentrate on the loop in the diagram shown in Fig. 1, making some comments on the first and the last links. We study several approaches[1,2,10] that bear some definite formal and/or dynamical resemblance.

II. $SU(6)_W$ SYMMETRY AND THE WEAK NUCLEON-NUCLEON-MESON COUPLING

As weak Hamiltonians in unified gauge theories are built from current quarks and are taken between quarks constituting hadrons, it is of interest to look for physical consequences of the Melosh transformation in the case of $NN\pi$ weak coupling.

In Ref. 2, modified Lee–Sugawara relations were obtained by the soft-pion and infinite-momentum techniques with the Melosh transformation based on the $SU(6)_W$ algebra of currents. This led to a good fit for nonleptonic hyperon decays. This treatment can easily be extended to the sum rules containing the $A(p\,^+_+)$ amplitude, provided that the proton-neutron mass difference is used as a measure of isospin-symmetry breaking.

Using the appropriate effective bilinear terms for the current $\mathfrak{F}_\alpha^{05}$ and for the matrix elements of the weak Hamiltonian,[2] and performing the $SU(6)_W$ reduction, we find that

$$A(p_+^+) = \alpha(m_f - m_i)[2X^{35} - 2X^{405} + \tfrac{16}{405}(\tfrac{7}{15})^{1/2}(X^{56,0}_{405,(8_A,3)} + 2X^{56,\pm1}_{405,(8_A,3)}) - \tfrac{5}{162}(X^{70,0}_{405,(8_A,3)} + 2X^{70,\pm1}_{405,(8_A,3)})] . \quad (2.1)$$

Here m_i denotes the mass of the initial particle, i.e., a proton, while m_f refers to the final particle, i.e., a neutron.[11] All other notation closely follows Ref. 2. The first two terms come from both the current-commutator term and the sum over intermediate baryon states.

Since everything is expressed using variables common to Ref. 2, the new sum rule for the $A(p_+^+)$ amplitude can be read immediately as follows:

$$A(p_+^+) = -\alpha(m_i - m_f)\frac{2}{\sqrt{3}}\left[\frac{A(\Xi_-^-)}{m_\Xi - m_\Lambda} + 2\frac{A(\Lambda_-^0)}{m_\Lambda - m_N}\right]. \quad (2.2)$$

The amplitude has been derived under the assumption that weak Hamiltonian transforms as an SU(3) octet:

$$H_W^{(8)} = H_W^{(8)}(\Delta S = \pm 1) + \alpha H_W^{(8)}(\mathrm{PV}) . \quad (2.3)$$

Thus the weak-interaction-model-dependent parameter α measures the relative strengths of the strangeness-conserving (PV) and strangeness-changing $(\Delta S = \pm 1)$ octet parts of H_W. For the Cabibbo model, $\alpha = \tan\theta_C/\sqrt{2}$.[12] The new summation rule represents a considerable modification of the previous one, which was[12,13]

$$A(p_+^+) = -\alpha\frac{2}{\sqrt{3}}[A(\Xi_-^-) + 2A(\Lambda_-^0)] . \quad (2.4)$$

Numerically, Eq. (2.2) leads to a considerable reduction of the previous prediction,[12,14,15]

$$A(p_+^+) = -\alpha 3.02\times10^{-7} , \quad (2.5a)$$

giving

$$A(p_+^+) = -\alpha 3.02\times10^{-9} . \quad (2.5b)$$

Thus Eq. (2.5b) is in quantitative agreement with the previous conclusions[16,17] that simultaneous fits of s- and p-wave hyperon nonleptonic decay amplitudes imply smaller values for $A(p_+^+)$. An important ingredient in the Machacek-Tomozawa fit of nonleptonic weak decays was the particular form of the matrix element of the weak Hamiltonian between two baryon states. In the next section we discuss a theoretical scheme[10] leading to an effective weak Hamiltonian which is proportional to the pion four-momentum. For such a Hamiltonian, the infinite-momentum limit for baryons and $\mathrm{SU}(6)_W$ symmetry leads to the sum rule (2.2). Otherwise, as indicated in Sec. IV, the sum rule (2.2) would be valid for the Born terms only.

We feel that the result (2.5b), which is the smallest one deduced so far, probably represents the lower limit for the extrapolation procedure from the physical nonleptonic hyperon decay amplitudes to the $A(p_+^+)$ amplitude.

III. UNIFIED GAUGE MODEL WITH PCAC

Deductions in Sec. II were based on the assumption that the effective weak Hamiltonian is a one-body operator (i.e., bilinear in spinors),[2] which seems to be at variance with the usual approach to unified gauge theories.[18,19,9] However, this is not the case if the pion is contracted by the PCAC (partially conserved axial-vector current) condition before performing the spatial-coordinate integration over the intermediate-vector-boson propagator.[10] If the effective charmed-quark mass is much larger than the other quark masses (i.e., $\bar{m}_{\mathcal{P}'}{}^2 \gg \bar{m}_{\mathcal{P}}{}^2$), then the effective $\Delta S = 1$ Lagrangian is proportional to

$$q_\mu \bar{\mathfrak{N}}(0)\gamma^\mu(1-\gamma_5)\lambda(0) , \quad (3.1)$$

where $\mathfrak{N}$ and λ refer to the respective quarks, and q is the pion four-momentum.

Such a mechanism is contrived to explain the $|\Delta\vec{I}| = \frac{1}{2}$ selection rule for nonleptonic decays. This can be tested independently by studying parity-violating processes in nuclei. Contrary to the standard approach, which predicts an enhancement[8,14,20,21] of the $A(p_+^+)$ amplitude in the Weinberg-Salam model[22] in comparison with the Cabibbo model, there is no enhancement in the approach of Ref. 10. The usual enhancement comes from neutral currents. Since these currents contain a charmed quark only in pairs $\bar{\mathcal{P}}'\mathcal{P}'$, they do not contribute to an effective operator of the type (3.1). The only contribution can come from a bilinear combination of the terms in the charged current,

$$\sim\cos\theta_C(\bar{\lambda}\mathcal{P}') - \sin\theta_C(\bar{\mathfrak{N}}\mathcal{P}') , \quad (3.2)$$

where θ_C is the Cabibbo angle. The dominant terms in the effective Lagrangian with $\Delta S = 0, 1$, $\Delta I = 1, \frac{1}{2}$ are

$$L_{\mathrm{eff}} \cong (-)\sin\theta_C\cos\theta_C\, q_\mu\bar{\mathfrak{N}}(0)\gamma^\mu(1-\gamma_5)\lambda(0) + \sin^2\theta_C\, q_\mu\bar{\mathfrak{N}}(0)\gamma^\mu(1-\gamma_5)\mathcal{P}(0) , \quad (3.3)$$

thus giving the effective strength of the isovector contribution to H_W^{PV}, the same as in the Cabibbo model.

IV. SU(3) SYMMETRY BREAKING AND THE WEAK $NN\pi$ COUPLING

The modified Lee-Sugawara relation of Ref. 2 was previously deduced[1] by the inclusion of a version of the SU(3)-symmetry-breaking mechanism in the approach based on the soft-pion formalism and chiral invariance.[23,24] As the model of Ref. 1 fits both s- and p-wave decay amplitudes, it seems instructive to search for its predictions about the $A(p_+^+)$ amplitude.

Inclusion of SU(3)-symmetry breaking resulted in the appearance of PV Born terms that satisfy the relation analogous to Eq. (2.2). Using the fits of experimental amplitudes presented in Ref. 1, we can readily determine the contribution from the Born term and the contribution from the current-commutator term separately. Using fit (3) from Table I of Ref. 1 as an example, we find that

$$\begin{aligned} &A(\Lambda_-^0)_{CC} = 1.91\times10^{-7}, \quad A(\Xi_-^-)_{CC} = -4.09\times10^{-7},\\ &A(\Lambda_-^0)_B = 1.09\times10^{-7}, \quad A(\Xi_-^-)_B = -0.77\times10^{-7},\\ &A(p_+^+)_{CC} = 0.6\times10^{-8}, \quad A(p_+^+) = 0.575\times10^{-8},\\ &A(p_+^+)_B = -0.025\times10^{-8}, \end{aligned} \tag{4.1}$$

in the system of units $\hbar = c = 1$. The result (4.1), which is about six times smaller than the naive sum-rule prediction (2.5a), is of a similar nature as earlier estimates employing vector or decuplet pole terms instead of nucleon pole terms.[16,17] The D/F ratio is nearly unity, i.e., $D/F = -1.08$, instead of the naive sum-rule fit ratio (2.4) $D/F = -0.57$. The deduction is, nevertheless, essentially different from the $SU(6)_W$ prediction based on (2.5b), thus illustrating to what extent the loop in Fig. 1 depends on precise dynamical assumptions.

This result also supports the remark[8] that SU(3)-symmetry-breaking effects cast serious doubts on the validity of the sum rule (2.4). The new sum rule (2.2) is derived exactly, provided the isospin symmetry breaking is measured by proton-neutron mass difference.

V. CONCLUDING DISCUSSION

Predictions of parity-violating effects in nuclei do not depend only on the form of weak-interaction theories. Various dynamical assumptions made in the theoretical description of nonleptonic hyperon decays can lead to essentially different predictions for parity-violating but strangeness-conserving nonleptonic amplitudes. The values for the $A(p_+^+)$ amplitude calculated in this paper range from $\alpha(-)3.02\times10^{-7}$ to $\alpha(-)3.02\times10^{-9}$. The latter value with $\alpha = \tan\theta_C/\sqrt{2}$ also holds for the Weinberg-Salam model[22] of weak interactions under the assumption that the pion should be contracted before performing the spatial-coordinate integration over the intermediate vector-boson propagator.[10] If the integration leading to H^{eff} precedes the pion contraction, a considerable enhancement may appear.[8,14] Thus, even when nuclear physics calculations can be handled with confidence, the weak-interaction-model-dependent parameter α is not measured uniquely. The model parameters can be extracted only through studying and comparing various processes. In this way, the range, spin, and isospin dependence of the weak PV potential will probably emerge. This again strongly suggests the perfection of the experiments in which pion exchange, i.e., the isovector contribution, is dominant.

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APPENDIX

The measurement of A_γ in the $n+p\to d+\gamma$ reaction may give information on weak interactions and the dynamics of elementary-particle physics only if it is independent of uncertainties associated with nuclear theory.

This is true of the one-pion-exchange contribution [proportional to $A(p_+^+)$], which is of long-range nature. The early, rather crude estimates[25,26] already gave

$$|A_\gamma| \approx 0.5C_\pi\times10^{-8}. \tag{A1}$$

Here C_π measures the strength determined by the weak-interaction model (the first link in Fig. 1) and by the dynamical scheme chosen (loop in Fig. 1). For the standard pion-exchange potential,[12] $C_\pi = 1$. For the dynamical scheme of Sec. II, C_π is much smaller, i.e., $C_\pi = 10^{-2}$. The same value is obtained when the effective H_W from Sec. III is used. With the approximation defined in Sec. IV, $C_\pi = \frac{1}{6}$ would be obtained.

Newer and more sophisticated nuclear-physics calculations have resulted in no crucial changes in the value of A_γ, i.e.,

$$|A_\gamma| = 0.51C_\pi\times10^{-8}, \quad \text{Refs. 27 and 4} \tag{A2}$$

$$|A_\gamma| = 0.53C_\pi\times10^{-8}, \quad \text{Ref. 3}. \tag{A3}$$

On the basis of the sophisticated nuclear physics developed in Ref. 28, Craver obtained[29]

$$|A_\gamma| = 0.5977C_\pi\times10^{-8}. \tag{A4}$$

Weak-Hamiltonian models with neutral currents may also contain $|\Delta\vec{I}| = 1$ pieces of vector-meson-exchange contributions. Craver concludes[29] that vector-meson exchanges influence the result by less than 1%. His values for the d'Espagnat extra-current model[30] are

$A_\gamma = -12.95\times10^{-8}$ for pion exchange and

$$A_\gamma = -13.01\times10^{-8} \text{ with vector-meson exchanges included.}$$

Thus the effective strength C_γ is changed from 6.48 to 6.51.

However, extensive analyses of existing experiments indicate[3,4] that vector-meson-exchange potentials might be stronger by one order of magnitude than suggested by the naive factorization approximation[12] used in Refs. 28 and 29. As long as these potentials are of short-range nature, the results (A1)–(A4) might be uncertain by about 10%, which is still very promising.

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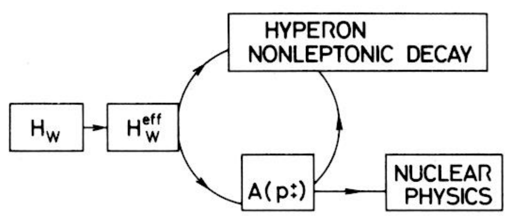

FIG. 1. Graphical chain for evaluating the $A(p_+^+)$ amplitude.