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Contribution of second-class currents to $K \to 2\pi$

J. P. Krisch
Department of Physics, University of Michigan, Ann Arbor, Michigan 48109
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The effect of second-class currents on kaon nonleptonic, two-pion decays is discussed. Amplitude relations are derived. The relative magnitude of first- and second-class effects in $K_s \to \pi^\circ\pi^\circ$ is calculated in a phenomenological but rather general model. The results are compared to bag-model calculations of the first-class decays.

I. INTRODUCTION

Some recent experiments[1] have indicated the possible existence of currents with abnormal G parity. These second-class[2] vector and axial-vector currents have $G_V = (-1)^I$ and $G_A = (-1)^{I+1}$ in contrast to the conventional first-class currents. Although these currents have only been observed in nuclear β decay, it is possible that they would also act within a particle-physics context. In this paper we will examine the processes

$$K_1^0 \to \pi^+\pi^-,\ \pi^0\pi^0,\qquad K^\pm \to \pi^\pm\pi^0. \tag{1}$$

There is a problem of long standing associated with each of these reactions. First, both reactions are forbidden in exact SU(3) with the conventional choice of Hamiltonian[3]

$$H \sim F_1F_4 + F_4F_1 + F_2F_5 + F_5F_2. \tag{2}$$

Second, the K^+ decay violates the $\Delta I = \frac{1}{2}$ rule by an amount that is large to attribute to a symmetry-breaking effect.

Several authors[4,5] have pointed out that the introduction of second-class currents could solve the first problem. However, the strength of the $K^\pm$ violation of the $\Delta I = \frac{1}{2}$ rule still remains a puzzle. In this paper we shall look at the explicit effects of second-class currents on $K\pi\pi$ decays.

In Sec. II of the paper we find the SU(3) form of the second-class contribution to $K^\pm \to \pi^\pm\pi^0$ and $K_1^0 \to \pi^+\pi^-$, $\pi^0\pi^0$. Sum rules are derived identical in form to those found for first-class currents. In Sec. II C of the paper we relate first- and second-class effects in $K_1^0 \to \pi^0\pi^0$ using current algebra. We compare the result with estimates made from the bag model.

II. CALCULATIONS

A. Second-class Hamiltonian

To see the effects of second-class currents, define a new total current,[6]

$$J_i = F_i + iKS_i, \tag{3}$$

where F_i and S_i are respectively the first- and second-class currents. K is a strength parameter. Using this current the new Hamiltonian becomes

$$H = H_1 + H_2,$$
$$H_1 \sim F_1F_4 + F_4F_1 + F_2F_5 + F_5F_2 + K^2(S_1S_4 + S_4S_1 + S_2S_5 + S_5S_2), \tag{4}$$
$$H_2 \sim K(F_5S_1 - F_1S_5 + F_2S_4 - F_4S_2 + S_1F_5 - S_5F_1 + S_4F_2 - S_2F_4). \tag{5}$$

The second-class part of the Hamiltonian, H_2, has the SU(3) decomposition

$$H_2 \sim \frac{2K}{\sqrt{3}}\left[T^{10*}(\tfrac{3}{2},\tfrac{1}{2},-1) - T^{10}(\tfrac{3}{2},-\tfrac{1}{2},1) + T^{10}(\tfrac{1}{2},\tfrac{1}{2},-1) - T^{10*}(\tfrac{1}{2},-\tfrac{1}{2},1) + T^{8_2}(\tfrac{1}{2},-\tfrac{1}{2},1) + T^{8_2}(\tfrac{1}{2},\tfrac{1}{2},-1)\right], \tag{6}$$

where the argument labels (I, I_z, Y). The parity-violating form of the last term acts like the seventh component of a $C=+1$ octet (the sixth component of an 8^2 octet). This is the behavior needed to remove the SU(3) suppression of the $K\pi\pi$ decays. The $K^\pm$ problem still remains. The decay proceeds through a decuplet term with $T = \frac{3}{2}$ and the $\Delta I = \frac{1}{2}$ rule is still violated. Second-class effects will, however, provide another contribution of the $K^\pm$ amplitude, reducing the total amount of symmetry breaking needed to reproduce experiment.

B. Second-class amplitudes

We wish to calculate the amplitude

$$T(K^+ \to \pi^i\pi^j) = \langle \pi^i\pi^j | H_2 | K^k\rangle. \tag{7}$$

H_2 is given by Eq. (6). Since the two pions are in a relative s state we may symmetrize and write

$$\langle \pi^i\pi^j| = \sqrt{2}\sum_{\text{even}} \begin{pmatrix} 8 & 8 & \mu \\ \nu_i & \nu_j & \nu_l \end{pmatrix} \left\langle \begin{matrix}\mu \\ \nu_l\end{matrix}\right|, \tag{8}$$

where $\nu = (I, I_z, Y)$.

Substituting (6) and (8) into (7) and using the Wigner-Eckart theorem we obtain

$$
\begin{aligned}
T(K^+\to\pi^+\pi^0) &= \frac{3}{2\sqrt{3}}\langle 27\|S^{10*}\|8\rangle\,,\\
T(K^-\to\pi^-\pi^0) &= \frac{3}{2\sqrt{3}}\langle 27\|S^{10}\|8\rangle\,,\\
T(K_1^0\to\pi^0\pi^0) &= \frac{2}{\sqrt{6}}[-\tfrac{9}{20}\langle 27\|(S^{10}+S^{10*})\|8\rangle\\
&\qquad +\tfrac{1}{5}\langle 8\|(S^{10}-S^{10*})\|8\rangle\\
&\qquad -\frac{1}{\sqrt{5}}\langle 8\|S^8\|8\rangle_2]\,,\\
T(K_{10}\to\pi^+\pi^-) &= \frac{-2}{\sqrt{6}}[-\tfrac{3}{10}\langle 27\|(S^{10}+S^{10*})\|8\rangle\\
&\qquad -\tfrac{1}{5}\langle 8\|(S^{10}-S^{10*})\|8\rangle\\
&\qquad +\frac{1}{\sqrt{5}}\langle 8\|S^8\|8\rangle_2]\,.
\end{aligned}
\tag{9}
$$

These rates may be combined to give the relation

$$
\begin{aligned}
T(K_1^0\to\pi^+\pi^-)&-\sqrt{2}\,T(K_1^0\to\pi^0\pi^0)\\
&=T(K^+\to\pi^+\pi^0)+T(K^-\to\pi^-\pi^0)\,.
\end{aligned}
\tag{10}
$$

One may also calculate the $K_2^0\to\pi^+\pi^-, \pi^0\pi^0$ amplitudes. These are zero by CP invariance and provide two more amplitude relations,

$$\langle 27\|S^{10*}\|8\rangle=\langle 27|S^{10}|8\rangle\,, \tag{11}$$

$$
\begin{aligned}
\tfrac{1}{20}\langle 27\|S^8\|8\rangle_1&-\tfrac{1}{5}\langle 8\|(S^{10}+S^{10*})\|8\rangle\\
&+\tfrac{1}{5}\langle 8\|S^8\|8\rangle_1-\tfrac{1}{4}\langle 1\|S^8\|8\rangle=0\,.
\end{aligned}
\tag{12}
$$

Equations (10) and (11) combined are identical to the first-class sum rule,[7]

$$A(K_1^0\to\pi^+\pi^-)-\sqrt{2}A(K_1^0\to\pi^0\pi^0)=2A(K^+\to\pi^+\pi^0)\,. \tag{13}$$

Although this rule may be stated for any decay containing only $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$, the representation structure of the amplitudes A and T is of course very different. However, the right-hand side in each case measures the deviation from the $\Delta I=\frac{1}{2}$ rule.

C. Relation between first- and second-class effects

The first- and second-class amplitudes can be written

$$T_1=-\frac{\mu^2}{c}\langle\pi^0|[Q_5^3,H_1]|K_S\rangle\,, \tag{14}$$

$$T_2=-\frac{\mu^2}{c}\langle\pi^0|[Q_5^3,H_2]|K_S\rangle\,, \tag{15}$$

where H_1 and H_2 are defined in (4) and (5) and $c=g_{\pi NN}/M_N F_A(0)$. In order to relate these amplitudes, a specific definition for the second-class current must be chosen. Following Adler *et al.*,[8] using a σ model we define the vector current

$$S_a=\alpha d_{abc}(\pi^b F_5^c+F_5^c\pi^b)\,, \tag{16}$$

where $\alpha=(1/2\langle\sigma\rangle)(\frac{3}{2})^{1/2}$ and π^b is the pseudoscalar octet. Substituting (16) into H_1 and H_2 and explicitly calculating the commutators one obtains

$$
\begin{aligned}
[Q_5^3,H_2]\left(\frac{iK}{2}\right)^{-1} &= F^4F^2+F^2F^4-F^1F^5-F^5F^1-\tfrac{3}{2}(S^4F^1+S^5F^2+F^2S^5+F^1S^4)\\
&\quad+\alpha[d_{1cd}f_{3ce}(F^5X_A^{de}+X_A^{de}F^5)+d_{4cd}f_{3ce}(F^2X_A^{de}+X_A^{de}F^2)\\
&\quad -d_{5cd}f_{3ce}(F^1X_A^{de}+X_A^{de}F^1)-d_{2cd}f_{3ce}(F^4X_A^{de}+X_A^{de}F^4)]\,,
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
[Q_5^3,H_1]\left(\frac{i}{2}\right)^{-1} &= F^4F^2+F^2F^4-F^1F^5-F^5F^1+\frac{K^2}{2}(S^4S^2+S^2S^4-S^1S^5-S^5S^1)+K^2(S^2F^5+S^1F^4+F^5S^2+F^4S^1)\\
&\quad+\alpha K^2[d_{5cd}f_{3ce}(S^2X_A^{de}+X_A^{de}S^2)+d_{2cd}f_{3ce}(S^5X_A^{de}+X_A^{de}S^5)\\
&\quad+d_{4cd}f_{3ce}(S^1X_A^{de}+X_A^{de}S^1)+d_{1cd}f_{3ce}(S^4X_A^{de}+X_A^{de}S^4)]\,,
\end{aligned}
\tag{18}
$$

where $X_A^{de}=\pi^dF_5^e-\pi^eF_5^d+F_5^e\pi^d-F_5^d\pi^e$.

The next step is to calculate the matrix elements of these commutators between π^0 and K_S. We will evaluate them by saturating the intermediate states with the vacuum and pseudoscalar mesons. The amplitudes will be related in SU(3) using the Wigner-Eckart theorem. These assumptions provide a large simplification in the commutator matrix elements. If a specific S_i is calculated from (16), say S_{π^+}, one obtains

$$
\begin{aligned}
S_{\pi^+}&=\frac{1}{\sqrt{3}}(\pi^+F_5^\eta+\eta F_5^{\pi^+}+F_5^\eta\pi^++F_5^{\pi^+}\eta)\\
&\quad-\frac{1}{\sqrt{2}}(K^+F_5^{\overline{K}^0}+\overline{K}^0F_5^{K^+}+F_5^{\overline{K}^0}K^++F_5^{K^+}\overline{K}^0)
\end{aligned}
\tag{19}
$$

and one sees that $\langle\pi^0|S_{\pi^+}|\pi^-\rangle=0$. Similarly $\langle 0|S_5^{\pi^+}|\pi^-\rangle=0$. The same is true for other second-class currents sandwiched between states of appropriate quantum numbers. This means that in an SU(3)-symmetric theory there is no explicit second-class current contribution to the commutator matrix element. The only nonzero contribution of second-class origin is the term of the form $(F^i X_A^{de} + X_A^{de} F^i)$ in the H^2 commutator. This is an implicit second-class contribution coming from the nonlinear nature of the second-class-current–charge commutator. To obtain other second-class contributions would require second-class intermediate states which are expected to be of high mass and presumably unimportant. Using this fact one obtains

$$\langle \pi^0 | [Q_5^3, H_1] | K^0 - \bar{K}^0 \rangle = -\frac{1}{2\sqrt{2}} \sum_p \langle 8 \| F^8 \| 8_p \rangle \langle 8_p \| F^8 \| 8 \rangle \tag{20}$$

and

$$\begin{aligned}\langle \pi^0 | [Q_5^3, H_2] | K^0 - \bar{K}^0 \rangle &= K \langle \pi^0 | [Q_5^3, H_1] | K^0 - \bar{K}^0 \rangle \\ &+ \frac{i\alpha K}{8\sqrt{3}} \frac{5}{2} \sum_p \langle 8 \| F^8 \| 8_p \rangle \\ &\times (\langle 1 \| F_5^8 \| 8 \rangle + \langle 1 \| F_5^8 \| 8_p \rangle),\end{aligned} \tag{21}$$

where $|8\rangle$ denotes a soft-pion state. A symmetric form factor was used in expressing $\langle 8 \| F^8 \| 8_p \rangle$, the reduced matrix element. In order to evaluate the last term in the H_2 commutator without explicitly evaluating form-factor integrals, calculate the amplitude for $K^+ \to \pi^+ \pi^0$,

$$\langle (\pi^+\pi^0 + \pi^0\pi^+)/\sqrt{2} | H | K^+ \rangle = -(\mu^2/c\sqrt{2}) [\langle \pi^0 | [Q_5^+, H] | K^+ \rangle + \langle \pi^+ | [Q_5^3, H] | K^+ \rangle]. \tag{22}$$

The rate for this process is about 700 times smaller than the Ks decay rate. Setting it equal to zero gives

$$\frac{i2K}{16\sqrt{6}} \sum_p \langle 8 \| F^8 \| 8_p \rangle (\langle 1 \| F_5^8 \| 8 \rangle + \langle 1 \| F_5^8 \| 8_p \rangle) = \frac{(K+1)}{6} \sum_p \langle 8 \| F_8 \| 8_p \rangle \langle 8_p \| F_8 \| 8 \rangle - \frac{K-1}{16} \langle 1 \| F_5^8 \| 8 \rangle \langle 1 \| F_5^8 \| 8 \rangle. \tag{23}$$

Assuming a "universal" value of $K = 1^4$ and using Eq. (20), one finds

$$\frac{i\alpha K}{16\sqrt{6}} \sum_p \langle 8 \| F^8 \| 8_p \rangle (\langle 1 \| F_5^8 \| 8 \rangle + \langle 1 \| F_5^8 \| 8_p \rangle) = -\tfrac{2}{3}\sqrt{2} \langle \pi^0 | [Q_5^3, H_1] | K^+ \rangle, \tag{24}$$

or on substituting into (21),

$$T^2 = -\tfrac{17}{3} T^1. \tag{25}$$

III. DISCUSSION

The value calculated in the preceding section is a relatively large effect. As an example to use for comparison, consider the bag-model calculations of nonleptonic kaon rates. In Table I we list the ratio of theoretical to experimental amplitudes for two different calculations.[10-12] Both models quoted use the bag parameters of DeGrand *et al.*[9] corresponding to a bag radius of $R = 3.26$ GeV^{-1}. Column 4 gives the ratio of T^2 to T^1 calculated by assuming these are the only two contributions to the decay rate. The first value, corresponding to an unenhanced decay amplitude, is of the same order as our result. It is, however, misleading to compare specific numbers in this case. The result (24) assumes unbroken SU(3) relating all amplitudes. Table I, as a whole, indicates that first-class calculations of the kaon decay rates tend to predict values lower than experiment. Our result indicates that second-class effects could make a significant contribution to the decay rate. If the existence of second-class currents is established, they should not be neglected in trying to understand nonleptonic decays.

TABLE I. Ratios of theoretical to experimental amplitudes for two different calculations.

Model	Enhancement	$T^1_{\text{theory}}/T_{\text{expt}}$	$T^2_{\text{theory}}/T^2_{\text{theory}}$
Donoghue,	1	0.17	5.79
Golowich, and	2	0.25	3.87
Holstein,	3	0.35	2.68
Refs. 10 and 11	5	0.54	1.56
Katz and Tatur, Ref. 12	3	0.79	0.78

IV. CONCLUSION

We have discussed the influence of second-class effects in nonleptonic, two-pion kaon decays. The second-class Hamiltonian allows decays which are experimentally observed but first-class forbidden in SU(3). The amplitudes calculated from this Hamiltonian provide a sum rule identical to that obeyed by first-class amplitudes.

The relative magnitude of first- and second-class effects indicates that second-class effects may be important in nonleptonic decays.

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