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Confined quarks and the decays of "old" and "new" vector and tensor mesons

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The decays of vector and tensor mesons into two pseudoscalar mesons and into a vector and a pseudoscalar meson are considered in a model of relativistic confined quarks. The calculation is based on the quark-loop graph. The question of internal-symmetry breaking is investigated in detail.

I. INTRODUCTION

The strong decays of hadronic resonances are constant objects of extensive experimental and theoretical investigations. The rich experimental material in this field is collected in the Particle Data Group table[1] and in the coupling-constant compilation.[2] Concerning the theoretical interpretation of the data the situation is less unique although some basic ideas such as the underlying quark structure, the Okubo-Zweig-Iizuka rule[3] etc. are commonly accepted.

One of the families of theoretical approaches is based on the "quark-loop coupling"[4] arising naturally in any relativistic quantum field-theoretical framework. It was extensively studied in the papers of Böhm, Joos, and Krammer[5] starting from a Wick-rotated Bethe-Salpeter equation for the meson bound-state wave functions. The approach of Kim and Noz[6] does not assume the possibility of Wick rotation and works in the Minkowski space of relative positions and momenta of the quarks emphasizing the relativistic quantum mechanical aspects of the problem. Recently, the coupling constants of resonances were calculated by Preparata in his quark-confinement model[7] assuming the quark-loop coupling.

In the present paper we shall calculate the two-body strong decays of the vector and tensor mesons from the quark-loop coupling graph. Compared to Ref. 7 (which is nearest to our model among the aforementioned papers) the difference is that we shall also calculate the decays of the mesons containing strange quarks and consider, therefore, the interesting problem of SU(3) breaking. In addition, the actual (oscillator-type) wave functions we use are also different from those in Ref. 7. Our model was described in detail previously in Ref. 8, where the "direct terms" to the meson form factors were calculated and applied to some weak decays of the new particles. In summary, our main assumptions are[8] (a) confinement in the Minkowski space of relative positions (and momenta), (b) an effective-quark-mass approximation for quark propagation inside hadrons, and (c) the quark-diagram structure of hadron interactions. In addition, we (i) use oscillator-type (Gaussian) wave functions, (ii) retain only the terms corresponding to partial conservation of axial-vector current (PCAC) and vector-dominance model (VDM) in the spin structure of the pseudoscalar and vector-meson wave functions respectively, and (iii) assume that the tensor-meson wave function can be obtained from the vector meson's one via a multiplication by an "orbital excitation factor" (taken from the spinless relativistic oscillator model). For the calculation of quark-loop graph we use the Feynman rules given in Ref. 8 and, for the reader's convenience, also in Appendix A.

The plan of the paper is as follows: Section II contains the calculation of the "old" vector-meson decay coupling constants. In Sec. III the "old" tensor-meson decays are considered and the known experimental data are fitted varying the free parameters of the model. The analogous decays of the new mesons [such as J (ψ), χ, etc.] are treated in Sec. IV. The conclusions are summarized in Sec. V. Appendix B contains some details of the lengthy calculations.

II. VECTOR-MESON DECAYS

The explicit form of the Bethe-Salpeter (BS) wave functions we shall use in the calculation of the decay coupling constants is, for pseudoscalar mesons ($J^{PC}=0^{-+}$),

$$\chi_{0^{-+}}(p,q)=cM\gamma\cdot p\gamma_5\exp\{\alpha[q^2-(2/m^2)(p\cdot q)^2]\}, \tag{2.1}$$

for vector mesons ($J^{PC}=1^{--}$),

$$\chi_{1^{--}}(p,q)=idM\gamma_\rho\epsilon^\rho\exp\{\alpha[q^2-(2/m^2)(p\cdot q)^2]\}, \tag{2.2}$$

and, for tensor mesons ($J^{PC}=2^{++}$),

$$\chi_{2^{++}}(p,q)=id\sqrt{4\alpha}\,M\gamma_\rho q_\sigma\epsilon^{\rho\sigma}\exp\{\alpha[q^2-(2/m^2)(p\cdot q)^2]\}. \tag{2.3}$$

Here p is the total, q the relative four-momentum, c and d are normalization factors, m is the meson mass, α is a parameter proportional to the size of the region (in the relative-position space) where the wave function is concentrated. (It is important to note that the extension of the confinement region determined by α is not directly the same as the physical size of the meson as seen, for instance, from the charge distribution. The reason is that in the electromagnetic form factor of mesons the "direct" graph built from the wave functions only is generally not dominating. As was discussed in Ref. 8 the "indirect" terms, e.g., the vector-meson pole term, dominate. In other words, the charge distribution is given predominantly by the virtual vector mesons flying around, and not directly by the "bare" wave function of the meson.) Coming back to the notations in Eqs. (2.1)–(2.3), ϵ is the polarization vector (tensor) of vector (tensor) mesons, and M is the usual SU(3) matrix of mesons. The latter for the pseudoscalar mesons is

$$M=\begin{bmatrix} (1/\sqrt{2})(\pi^0+\eta\cos\varphi_P+\eta'\sin\varphi_P) & \pi^- & K^- \\ \pi^+ & (1/\sqrt{2})(-\pi^0+\eta\cos\varphi_P+\eta'\sin\varphi_P) & \bar{K}^0 \\ K^+ & K^0 & \eta'\cos\varphi_P-\eta\sin\varphi_P \end{bmatrix}. \tag{2.4}$$

For the vector (tensor) mesons the substitutions $\eta\to\omega$, $\eta'\to\phi$, $\pi\to\rho$, $K\to K^*$ ($\eta\to f$, $\eta'\to f'$, $\pi\to A_2$, $K\to K^{**}$) have to be made and the mixing angle is denoted by φ_V (φ_T). From the Gell-Mann–Okubo mass formula, for mass squared, the mixing angles are[1] $\varphi_P=44°$, $\varphi_V=5°$, $\varphi_T=-4°$. (Note that these angles are measured from the ideal mixing and our ϕ and f' have an overall opposite sign compared to the usual convention.)

The normalization of the wave functions was discussed in Ref. 8. Following the parametrization introduced there, we write for pseudoscalar mesons ($P=\pi, K, \eta, \eta'$):

$$C_P=2\alpha_P\delta_P/\pi m_P\,, \tag{2.5}$$

and for the vector mesons ($V=\rho, K^*, \omega, \phi$):

$$d_V=2\alpha_V\delta_V/\pi\,, \tag{2.6}$$

where δ is a dimensionless parameter determining the relative amount of the direct term to the indirect terms at $q^2=0$ in the form factor. The tensor-meson wave function was obtained from that of the vector meson by the substitution

$$\epsilon^\rho\to\sqrt{4\alpha}\,\epsilon^{\rho\sigma}q_\sigma\,. \tag{2.7}$$

This corresponds to the rule for obtaining the first excited state from the ground state in a relativistic-spinless-oscillator model (see, for instance, Ref. 6 or 9). In a model with spin this substitution may be considered as the definition of the orbital excitation.

The simplest quark graph contributing to the decay of a vector meson to two pseudoscalar mesons ($V\to PP$) is the single-quark-loop graph in Fig. 1. According to the Feynman rules in Appendix A we have

$$\begin{aligned} {}_{\text{out}}\langle p_2P_2, p_3P_3|p_1\sigma_1V_1\rangle_{\text{in}} = (2\pi)^6\delta^4(p_1-p_2-p_3)\int d^4q\,\exp\Big\{&\alpha_1\Big[q^2-\frac{2}{m_1^{\,2}}(p_1\cdot q)^2\Big] \\ &+\alpha_2\Big[\Big(q+\frac{p_3}{2}\Big)^2-\frac{2}{m_2^{\,2}}\Big(p_2\cdot q+\frac{p_2\cdot p_3}{2}\Big)^2\Big] \\ &+\alpha_3\Big[\Big(q-\frac{p_2}{2}\Big)^2-\frac{2}{m_3^{\,2}}\Big(p_3\cdot q-\frac{p_2\cdot p_3}{2}\Big)^2\Big]\Big\} \\ &\times \mathrm{Tr}_{SU,D}[M(V_1)id_1\epsilon_1\cdot\gamma(M_q-k_3\cdot\gamma)c_2\gamma\cdot p_2\gamma_5\bar{M}(P_2) \\ &\times(M_q-k_1\cdot\gamma)c_3\gamma\cdot p_3\gamma_5\bar{M}(P_3)(M_q-k_2\cdot\gamma)]\,. \end{aligned} \tag{2.8}$$

The calculation of this expression is straightforward. Nevertheless it requires some effort due especially to the rather lengthy expressions resulting from the loop integration. An economic way of performing the necessary algebraic operations is described in Appendix B. Using the notations of Eqs. (B6), (B12), and (B18), the coupling constant g defined by

$$ {}_{\text{out}}\langle p_2P_2, p_3P_3|p_1\sigma_1V_1\rangle_{\text{in}} = -i(2\pi)^4\delta^4(p_1-p_2-p_3)\epsilon_1\cdot(p_2-p_3)g_{V_1P_2P_3} \tag{2.9}$$

is given like

$$\begin{aligned}g_{V_1P_2P_3} = (S_{123}/m_2m_3)\{&(T_0 - T_0')[(\alpha_1+\alpha_2+\alpha_3)^{-1}(2m_2m_3\beta(z_2-z_3) - m_2^2(1+z_2) - m_3^2(1-z_3))\\ &+ m_2^2m_3^2(z_3 - z_2 - z_{22} - z_{33} + 4z_{23} + 3z_{223} - 3z_{233}) + m_2^2(m_2^2+2m_2m_3\beta)(z_{22}+z_{222})\\ &+ m_3^2(m_3^2+2m_2m_3\beta)(z_{33}-z_{333})]\\ &+(T_{12}-T_{31}')[m_2^2(1+z_2-z_3)+m_1^2z_3] + (T_{23}-T_{23}')(m_3^2z_3 - m_2^2z_2)\\ &+(T_{31}-T_{12}')[m_3^2(1+z_2-z_3) - m_1^2z_2]\}\,. \end{aligned} \tag{2.10}$$

In addition to the notations introduced in Appendix B, we also used

$$S_{123} = 64\pi\frac{\alpha_1\alpha_2\alpha_3\delta_1\delta_2\delta_3}{(\alpha_1+\alpha_2+\alpha_3)(A_+A_-)^{1/2}}\exp\left(\frac{a_+^2}{8A_+}+\frac{a_-^2}{8A_-}-E\right), \tag{2.11}$$

and for the occurring SU(3) traces

$$\begin{aligned}T_0 &= \mathrm{Tr}_{SU}[M(V_1)\bar{M}(P_2)\bar{M}(P_3)]\,,\\ T_{12} &= \mathrm{Tr}_{SU}[M(V_1)\bar{M}(P_2)M_q\bar{M}(P_3)M_q]\,,\\ T_{23} &= \mathrm{Tr}_{SU}[M(V_1)M_q\bar{M}(P_2)\bar{M}(P_3)M_q]\,,\\ T_{31} &= \mathrm{Tr}_{SU}[M(V_1)M_q\bar{M}(P_2)M_q\bar{M}(P_3)]\,.\end{aligned} \tag{2.12}$$

Note that in Eq. (2.10) the graph obtained from Fig. 1 by the interchange of the two final-state mesons is also included. The SU(3) factors with primes (like T_0', T_{12}', etc.) belong to these graphs and are, therefore, obtained from the unprimed ones by the exchange $P_2 \leftrightarrow P_3$. M_q denotes the quark mass matrix:

$$M_q = \begin{bmatrix} M_n & 0 & 0\\ 0 & M_n & 0\\ 0 & 0 & M_s\end{bmatrix}. \tag{2.13}$$

It can be seen that we allow for SU(3) breaking in the quark masses. (The nonstrange quark mass M_n is in general unequal to the strange quark mass M_s.) Other SU(3)-breaking effects in Eq. (2.10) arise in the external masses $m_{1,2,3}$, the confinement-region-size parameters $\alpha_{1,2,3}$, and the wave-function-normalization parameters $\delta_{1,2,3}$.

The $V\to VP$ coupling constant defined by

$$_{\rm out}\langle p_2\sigma_2V_2, p_3P_3|p_1\sigma_1V_1\rangle_{\rm in} = -i(2\pi)^4\delta^4(p_1-p_2-p_3)\epsilon_{\mu\nu\rho\sigma}\epsilon_1^{\mu}\epsilon_2^{*\nu}p_2^{\rho}p_3^{\sigma}g_{V_1V_2P_3}\,, \tag{2.14}$$

can be calculated in exactly the same way. The result is

$$\begin{aligned}g_{V_1V_2P_3} = (2S_{123}/m_3)\{&(T_0+T_0')[m_3^2(z_3-z_{33}+z_{233}) + m_2^2(z_{22}+z_{222}) + 2m_2m_3\beta z_{223} - (1+3z_2)/(\alpha_1+\alpha_2+\alpha_3)]\\ &+T_{12}+T_{31}' - z_2(T_{23}+T_{31}-T_{12}+T_{12}'+T_{23}'-T_{31}')\}\,.\end{aligned} \tag{2.15}$$

The decay widths for $V\to PP$ and $V\to VP$ are given by the coupling constants like

$$\begin{aligned}\Gamma_{V_1\to P_2P_3} &= g^2_{V_1P_2P_3}\frac{w^3}{6\pi m_1^2}\\ \Gamma_{V_1\to V_2P_3} &= g^2_{V_1V_2P_3}\frac{w^3}{12\pi}\,.\end{aligned} \tag{2.16}$$

Here w is the c.m. three-momentum defined in Eq. (B9). Note that in the present paper we do not consider off-mass-shell extrapolations at all; therefore the only $V\to VP$ process we use is $\phi\to\rho\pi$. The often advocated coupling $\omega\to\rho\pi$ is, however, connected to $\phi\to\rho\pi$ by SU(3) symmetry; hence it can be determined indirectly.

The SU(3) symmetry structure (eventually the SU(3)-breaking structure) is a very interesting general question which, according to our knowledge, was not considered previously in the "quark-loop coupling" scheme[4-7] of the meson coupling constants. We saw that there are lots of sources of SU(3) violation in Eqs. (2.10) and (2.15). Experimentally, on the other hand, the SU(3) works rather well for the $V\to PP$ (and $V\to VP$) coupling constants. The SU(3) symmetry of the parameters α and δ may, perhaps, seem at the first sight natural. These are, however, the parameters not directly accessible for measurements. The well-known values of the masses, however (especially the pseudoscalar-meson masses ranging from $m_\pi = 138$ MeV to $m_{\eta'} = 958$ MeV) introduce a tremendously large SU(3) breaking. This seems at first sight disastrous not only for

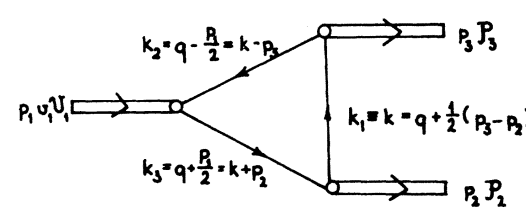


FIG. 1. The quark-loop graph contributing to the $V\to PP$ decay (p denotes four momenta, σ the spin index, V and P the SU(3) indices for vector and pseudoscalar mesons, respectively).

our model but also for the whole class of models based on the quark-loop coupling, showing quark confinement in the Minkowski space and possessing wave functions which do Lorentz contract at boosts. (For general discussions of the Lorentz-contraction properties of the relativistic wave functions see Ref. 6 and other references quoted there.) The point is that due to the mass differences such decays like, for instance, $\rho \to \pi\pi$ and $\phi \to KK$ are kinematically rather different, and consequently the Lorentz contraction of the wave functions of decay products are very much different in the two cases. The c.m. rapidity of the pions in $\rho \to \pi\pi$ is 1.7, whereas that of the kaons in $\phi \to KK$ is only 0.25; therefore the pion wave function is much more Lorentz contracted than the kaon wave function. The value of the overlap integral is, of course, decreased by the Lorentz contraction. As a consequence, the overlap integral is much smaller for $\rho \to \pi\pi$ than for $\phi \to KK$ if the (rest) size of the regions where the pion and kaon wave functions are concentrated is the same. This issue follows, in fact quite generally, from the Lorentz contraction and does not depend on many details like the spin structure or the actual Gaussian shape of the wave functions we use. [The factor S_{123} occurring in the Eq. (2.10) is roughly equal to the coupling constant for spinless mesons and quarks.] A similar conclusion holds also for the comparison of the decays $\rho \to \pi\pi$ and $\phi \to \rho\pi$, where again in the former case the final-state particles are much more relativistic in the c.m. frame than in the latter case. The only way of obtaining coupling constants consistent with SU(3) symmetry (and with experiment) is to take the size of the region where the pion wave function is confined much larger than those of the K and ρ mesons, that is $\alpha_\pi \gg \alpha_K$ and $\alpha_\pi \gg \alpha_\rho$. In other words, the anomalously small pion mass involves an anomalously large α_π compared to the α's of other hadrons.

Before concluding this section, we give a few relations among the parameters of the model. In Ref. 8 we have calculated the P_{12} and V_{12} decay constants, which are

$$f_P = \frac{c_P}{\alpha_P^2} 2\delta_P/\pi\alpha_P m_P \quad (P=\pi, K) \tag{2.17}$$

and

$$f_{\rho^0} = \frac{\alpha_{\rho^0}^2 m_{\rho^0}^2\sqrt{2}}{d_{\rho^0}} = \frac{\alpha_{\rho^0} m_{\rho^0}^2\pi}{\delta_{\rho^0}\sqrt{2}},$$

$$f_\omega = \frac{3\pi\alpha_\omega m_\omega^2}{\sqrt{2}\delta_\omega(\cos\varphi_V + \sqrt{2}\sin\varphi_V)}, \tag{2.18}$$

$$f_\phi = \frac{3\pi\alpha_\phi m_\phi^2}{2\delta_\phi[-\cos\varphi_V + (1/\sqrt{2})\sin\varphi_V]}.$$

Note that in the last two expressions the vector-meson mixing angle φ_V was introduced contrary to Ref. 8 where, for simplicity, $\varphi_V = 0$ was taken. Numerically, from the experimental values of f_P and f_V (Ref. 2) (and $\varphi_V = 5°$) we have

$$\alpha_\pi\delta_\pi^{-1} = 34.6\ \text{GeV}^{-2}, \quad \alpha_K\delta_K^{-1} = 8.41\ \text{GeV}^{-2},$$
$$\alpha_\rho\delta_\rho^{-1} = 4.02\ \text{GeV}^{-2}, \quad \alpha_\omega\delta_\omega^{-1} = 4.17\ \text{GeV}^{-2}, \tag{2.19}$$
$$\alpha_\phi\delta_\phi^{-1} = 2.54\ \text{GeV}^{-2}.$$

III. TENSOR-MESON DECAYS

The $J^{PC} = 2^{++}$ mesons (T) decay most frequently to final states involving two pseudoscalar mesons ($T \to PP$) or a vector and a pseudoscalar meson ($T \to VP$). The calculation of the $T \to PP$ and $T \to VP$ coupling constants is very similar to the $V \to PP$ and $V \to VP$ case, respectively. In the wave functions the only change is the substitution in Eq. (2.7). The additional q factor makes the loop integration $\int d^4q$ somewhat more involved but the rest is the same.

The coupling constants $g_{T_1P_2P_3}$ and $g_{T_1V_2P_3}$ are defined as usual by

$$_{\text{out}}\langle p_2P_2, p_3P_3 | p_1\sigma_1T_1\rangle_{\text{in}} = -i(2\pi)^4\delta^4(p_1-p_2-p_3)\epsilon_{1\mu\nu}(p_2-p_3)^\mu(p_2-p_3)^\nu g_{T_1P_2P_3},$$
$$_{\text{out}}\langle p_2\sigma_2V_2, p_3P_3 | p_1\sigma_1T_1\rangle_{\text{in}} = -i(2\pi)^4\delta^4(p_1-p_2-p_3)\epsilon^{\mu\nu\rho\sigma}\epsilon_{1\mu\lambda}(p_2-p_3)^\lambda\epsilon^*_{2\nu}p_{2\rho}p_{3\sigma}g_{T_1V_2P_3}. \tag{3.1}$$

This leads to the following expressions for the widths:

$$\Gamma_{T_1\to P_2P_3} = g^2_{T_1P_2P_3}(4w^5/15\pi m_1^2), \quad \Gamma_{T_1\to V_2P_3} = g^2_{T_1V_2P_3}(w^5/10\pi). \tag{3.2}$$

Starting from the expressions like Eq. (2.8) and using the method given in Appendix B to perform the necessary integrations and algebraic transformations, the result for the coupling constants is

$$
\begin{aligned}
g_{T_1P_2P_3} = (\sqrt{\alpha_1}\,S_{123}/m_2m_3)\{(T_0+T_0')[&m_2^2m_3^2(z_3-z_2-2z_{22}-2z_{33}+6z_{23}-z_{222}+8z_{223}\\
&-8z_{233}+z_{333}+3z_{2223}-6z_{2233}+3z_{2333})\\
&+m_2^2(m_2^2+2m_2m_3\beta)(z_{22}+2z_{222}-z_{223}+z_{2222}-z_{2223})\\
&+m_3^2(m_3^2+2m_2m_3\beta)(z_{33}+z_{233}-2z_{333}-z_{2333}+z_{3333})\\
&+(\alpha_1+\alpha_2+\alpha_3)^{-1}((\alpha_1+\alpha_2+\alpha_3)^{-1}+m_2^2(-1-2z_2+z_3-z_{22})\\
&+m_3^2(-1-z_2+2z_3-z_{33})+2m_2m_3\beta(z_2-z_3+z_{22}-z_{23}+z_{33}))]\\
&+(T_{12}+T_{31}')[m_2^2(1+2z_2-z_3+z_{22}-z_{23})+(m_3^2+2m_2m_3\beta)(z_3+z_{23}-z_{33})]\\
&+(T_{23}+T_{23}')[(\alpha_1+\alpha_2+\alpha_3)^{-1}+m_2^2(-z_2-z_{22}+z_{23})+m_3^2(z_3+z_{23}-z_{33})]\\
&+(T_{31}+T_{21}')[m_3^2(1+z_2)+m_3^2(-2z_3-z_{23}+z_{33})+(m_2^2+2m_2m_3\beta)(-z_2-z_{22}+z_{23})]\},\\
g_{T_1V_2P_3} = (2\sqrt{\alpha_1}\,S_{123}/m_3)\{(T_0-T_0')[&m_2^2(z_{22}-z_{223}+2z_{222}+z_{2222}-z_{2223})+2m_2m_3\beta(z_{223}+z_{2223}-z_{2233})\\
&+m_3^2(z_3+z_{23}-2z_{33}+z_{333}+z_{2233}-z_{2333})-(\alpha_1+\alpha_2+\alpha_3)^{-1}(1+5z_2-z_3+4z_{22}-4z_{23})]\\
&+(T_{12}-T_{31}')(1+z_2-z_3)+(T_{23}+T_{31}-T_{12}+T_{31}'-T_{12}'-T_{23}')(-z_2-z_{22}+z_{23})\}.
\end{aligned} \tag{3.3}
$$

Here the same notations were used as in Eqs. (2.10) and (2.15).

The parameters involved in the coupling constants in Eqs. (2.10), (2.15), and (3.3) are the nonstrange- and strange-quark masses (M_n, M_s), and for each isomultiplet of particles the parameters α and δ specifying the size of the region where the wave function is concentrated and the wave-function normalization, respectively. From the leptonic decays of π, K, ρ, ω, and ϕ we have the constraints in Eq. (2.19). Moreover, the vector and tensor mesons lie on very nice straight-line trajectories consistent with the excitations of the relativistic harmonic oscillator.[6] This means that it makes sense to determine the confinement-region-size parameters α_V and α_T from the measured trajectory slopes. (The situation for the pseudoscalar trajectory is much less clear; therefore we leave α_P as a free parameter.) For the ρ and ω mesons, the g-meson mass $m_g = 1690$ MeV and the $\omega(1675)$ mass $m_{\omega^*} = 1667$ MeV give, respectively

$$
\begin{aligned}
\alpha_\rho &= 8/(m_g^2 - m_\rho^2) = 3.54\ \text{GeV}^{-2},\\
\alpha_\omega &= 8/(m_{\omega^*}^2 - m_\omega^2) = 3.69\ \text{GeV}^{-2}.
\end{aligned} \tag{3.4}
$$

Together with Eq. (2.19) this involves $\delta_\rho = 0.88$ and $\delta_\omega = 0.89$. For the ϕ and K^* mesons the 3^- excitation is not yet known; therefore one has to rely on (approximate) exchange degeneracy: $\alpha_{K^*} \cong \alpha_{K^{**}}$ and $\alpha_\phi \cong \alpha_{f'}$. Similarly, we assume $\alpha_\omega \cong \alpha_f$ and $\alpha_\rho \cong \alpha_{A_2}$. (For the $I = 0$ states the octet-singlet mixing is not the same for vector and tensor mesons, showing the approximate nature of exchange degeneracy and hence allowing for more departure from these rough equalities.)

Under these constraints one can try to fit the measured values of coupling constants. In order to get reasonable values for the parameters in our fit we constrained the ranges of α_π, α_{K^*}, and α_f by: $\alpha_\pi \leq 120\ \text{GeV}^{-2}$, $\alpha_{K^*} \leq 5\ \text{GeV}^{-2}$, and $\alpha_f \geq 2\ \text{GeV}^{-2}$. The latter two values are dictated by approximate exchange degeneracy (exact exchange degeneracy would give $\alpha_{K^*} = 3.3$ and $\alpha_f = 3.5$). The upper limit of the parameter α_π is roughly equal to the value obtained from $\alpha_\pi m_\pi^2 \cong \alpha_\rho m_\rho^2 \cong 2.1$, that is we allow for the same amount of SU(6)-symmetry breaking in α as is observed in the mass squared m^2.

The best fit under these constraints is given in Table I, together with the experimental values of coupling constants and kinematical variables. The best-fit values of the parameters are given in Table II. We consider the fit satisfactory although it is true that the number of parameters is relatively large. It must be emphasized, however, that in the light of the large number of approximations we made (Gaussian wave functions, PCAC and VMD terms only in the spin structure, neglection of more complicated graphs and of the meson widths, etc.) the essential thing is the qualitative agreement with experimental data and not the exact values of parameters belonging to the best fit.

The value of the qualitative picture is shown, for instance, by the value of f_η corresponding to α_η and δ_η in the fit. As shown in Ref. 8 we have in general (2.17) which gives $f_\eta = 150$ MeV in nice agreement with the measured values $f_\pi = 132$ MeV and $f_K = 153$ MeV.

IV. NEW-PARTICLE DECAYS

The results of the preceding sections can be applied without further ado to predict the decays of the recently discovered charmed-meson states.

TABLE I. The kinematics of the decays and the best fit to the coupling constants.

Decay	$\beta=\cosh\eta$ (η = relative rapidity of decay products)	w (GeV) (c.m. momentum)	Γ (MeV) (partial width)	$\lvert g\rvert_{exp}$ (in GeV units)	$\lvert g\rvert_{theor}$ (in GeV units)
$\rho^0\to\pi^+\pi^-$	14.4	0.361	152	6.04 ± 0.06	6.14
$\phi\to K^+K^-$	1.13	0.126	1.91	4.30 ± 0.25	3.44
$K^{*0}\to K^+\pi^-$	3.86	0.288	32.9	4.54 ± 0.08	4.42
$f\to\pi^+\pi^-$	40.5	0.621	97.2	4.4 ± 0.3	2.72
$f\to\eta\eta$	1.69	0.322	<3.6	<4.4	2.67
$f\to K^+K^-$	2.32	0.401	2.43	2.1 ± 0.4	3.05
$f'\to\pi^+\pi^-$	59.0	0.758	<8.0	<0.93	0.08
$f'\to\eta\eta$	2.82	0.523	<20	<3.7	3.27
$f'\to K^+K^-$	3.71	0.575	≤20	≤2.9 ± 0.4	2.20
$A_2^0\to K^+K^-$	2.52	0.431	2.4	1.80 ± 0.15	1.86
$A_2^+\to\eta\pi^+$	9.13	0.530	15.3	2.7 ± 0.2	2.70
$A_2^+\to\eta'\pi^+$	2.93	0.281	<1.0	<3.4	3.38
$K^{**0}\to K^+\pi^-$	12.7	0.616	40.4	3.30 ± 0.25	2.96
$K^{**+}\to K^+\eta$	2.72	0.483	2.16	1.4 ± 1.0	1.40
$\phi\to\rho^+\pi^-$	1.94	0.177	≤0.22	≤1.2 ± 0.1	2.00
$A_2^0\to\rho^+\pi^-$	5.09	0.411	36.2	9.8 ± 0.5	9.59
$K^{**0}\to K^{*+}\pi^-$	4.81	0.413	22.3	7.6 ± 0.7	8.51
$K^{**0}\to\rho^-K^+$	1.54	0.314	4.75	7.0 ± 1.2	7.55
$K^{**+}\to\omega K^+$	1.50	0.306	4.86	7.6 ± 2.0	5.62
$f'\to K^{*+}K^-$	1.42	0.294	<7	<10	1.50

The only change is in the internal-symmetry factors $T_0, T_{12}, \ldots$ which now become SU(4) factors instead of SU(3) factors. The meson matrix M in Eq. (2.4) has to be extended by a fourth row and column and the quark mass matrix M_q in Eq. (2.13) has to include also the charmed-quark mass M_c.

The decays like $D^*\to D\pi$, $F^*\to F\pi$, etc. are exactly the same as $K^*\to K\pi$ and presently there are almost no experimental results on them (except for some hint on $D^*\to D\pi$); therefore we shall not consider these decays here. We shall concentrate on the VP and PP decays of the $\psi(3100)$ ($J^{PC}=1^{--}$) and $\chi(3550)$ ($J^{PC}=2^{++}$) states.[10] These Zweig-rule-violating strong decays are displayed schematically on Fig. 2.[11] Experimentally, the Zweig-rule-violating I_{ZRV} is small; therefore this graph is equivalent (to first order in I_{ZRV}) to an ordinary quark-loop coupling (like on Fig. 1) if some admixture of ordinary quarks is introduced in the ψ (and χ) matrix M.[12] We need only the SU(3) part of $M(\psi)$ and $M(\chi)$. Denoting the two relevant mixing angles (measured from ideal mixing) by

TABLE II. The best-fit values of the parameters α and δ. $M_n = 0.329$ GeV and $M_s = 0.350$ GeV.

	π	K	η	η'	ρ	ϕ	K^*	ω	A_2	f	f'	K^{**}
α (GeV^{-2})	120	11.6	18.3	6	3.5 [a]	2.7	5	3.7 [a]	3.5 [a]	2	3.9	3.6
δ	3.47 [a]	1.38 [a]	2.37	1 (fixed)	0.88 [a]	1.05 [a]	0.45	0.89 [a]	0.49	1.05	1 (fixed)	1.4

[a] Calculated from Eqs. (2.19) and (3.4).

θ_V and ψ_V we have for ψ

$$M_{SU(3)}(\psi)=\begin{bmatrix}(1/\sqrt{2})\sin\theta_V\sin\psi_V & 0 & 0\\ 0 & (1/\sqrt{2})\sin\theta_V\sin\psi_V & 0\\ 0 & 0 & \sin\theta_V\cos\psi_V\end{bmatrix} \tag{4.1}$$

$M(\chi)$ is the same with generally different mixing angles θ_T and ψ_T.

Using Eq. (4.1) the expressions for the coupling constants are given by Eqs. (2.10), (2.15), and (3.3). The new parameters appearing in the expressions are α and δ for ψ and χ. We shall assume $\alpha_\psi=\alpha_\chi=1.33\ \mathrm{GeV}^{-2}$ calculated from the masses (and exchange degeneracy for the trajectory slopes) and, for simplicity, we take the mixing angles $\theta_V=\theta_T\equiv\theta$, $\psi_V=\psi_T\equiv\psi$. As the parameters δ and θ appear in the formulas only in the combination $\delta\sin\theta$ which is canceled from ratios, we can fit ψ to the known experimental results on $\psi\to VP$ coupling constants[10]

$$\begin{aligned} g_{\psi\rho^+\pi^-}&=(1.7\pm0.4)\times10^{-3}\ \mathrm{GeV}^{-1},\\ g_{\psi K^{*+}K^-}&=(1.3\pm0.3)\times10^{-3}\ \mathrm{GeV}^{-1},\\ g_{\psi\phi\eta}&=(0.9\pm0.4)\times10^{-3}\ \mathrm{GeV}^{-1},\\ g_{\psi\phi\eta'}&=(0.25\pm0.15)\times10^{-3}\ \mathrm{GeV}^{-1}. \end{aligned} \tag{4.2}$$

The fit to the ratios gives

$$\psi=(50\pm15)^\circ, \tag{4.3}$$

which leads to $\delta_\psi\sin\theta\cong0.023$ or, assuming $\delta_\psi=1$,

$$\sin\theta=(2.3\pm0.5)\times10^{-2}. \tag{4.4}$$

The values in Eqs. (4.3) and (4.4) are reasonable. The admixture of the ordinary quarks is about 2% in ψ (compared to the 10% mixing of nonstrange quarks in ϕ and f'). This is roughly consistent also with SU(4) mass formulas.[12] The SU(3) character of ψ is determined by the mixing angle ψ. A pure SU(3) singlet would correspond to $\psi=\arctan\sqrt{2}\cong55^\circ$. This is consistent with the value in Eq. (4.3) due to the large error.

Taking the values in Eqs. (4.3) and (4.4) the other VP and PP decays of ψ and χ (with $\delta_\chi=1$) can easily be calculated. The results are summarized in Table III. The present situation seems rather satisfactory but due to the large experimental errors we did not look for the best fit in θ and ψ. This can however be easily done if more accurate data will become known.

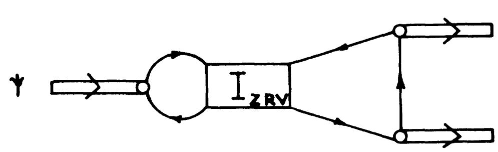


FIG. 2. The quark diagram for the two-body strong decays of ψ. I_{ZRV} is some Zweig-rule-violating interaction transforming the charmed-quark pair into an ordinary-quark pair.

V. CONCLUSIONS

The quark-loop coupling for the calculation of meson decays is natural in any relativistic quark model based on Bethe-Salpeter-type amplitudes or on bilocal fields.[4-7] In the present paper we calculated the coupling constants in a version of the model[8] which can be used as a general phenomenological basis for the understanding of the properties of low-lying meson states. (For highly excited states a more complete theory is needed as in Ref. 7.) Our main interest was in the internal-symmetry-breaking structure of the model.

The description of the decays of "old" vector and tensor mesons leads to a consistent qualitative picture with small effective quark masses (about 300 MeV) and considerable differences in the size of the quark-confinement region for different mesons.

The smallness of the effective quark masses inside hadrons (with the strange quark somewhat heavier) is a persistent feature of the quark models. In our case it can be shown that in the loop integration the integrand is sharply peaked in the region where on the quark lines the four-momentum squared is between 0 and $(400\ \mathrm{MeV})^2$, depending on the external kinematics of the graph. This shows that our effective quark masses are not necessarily the same as the effective quark-parton masses in the deep Euclidean region.

The other general feature, the large differences in the sizes, seems to us quite general, too, as it follows from the Lorentz contraction of the wave functions (depending on the mass differences). In particular, the differences in the parameter α (characterizing the size of the confinement region in the wave function) are roughly the same for the ground-state mesons as the differences in the mass squared, m^2. The relation is such that αm^2 is in the same order of magnitude for all the states. This leads to $\alpha_\pi\gg\alpha_\rho$ and $\alpha_\pi\gg\alpha_K$.

One can also go beyond the qualitative picture and try to fit the measured coupling constants. The best fit is good, but the values of the parameters belonging to it are somewhat questionable

TABLE III. The values of ψ and $\chi(3550)$ decay coupling constants (in GeV units).

Decay	$\lvert g\rvert_{\rm exp}$	$\lvert g\rvert_{\rm theor}$	Decay	$\lvert g\rvert_{\rm theor}$
$\psi\to\rho^+\pi^-$	$(1.7\ \pm0.4)\ \times10^{-3}$	$1.7\ \times10^{-3}$	$\chi\to\pi^+\pi^-$	0.11×10^{-3}
$\psi\to K^{*+}K^-$	$(1.3\ \pm0.3)\ \times10^{-3}$	$0.5\ \times10^{-3}$	$\chi\to K^+K^-$	0.19×10^{-3}
$\psi\to\varphi\eta$	$(0.9\ \pm0.4)\ \times10^{-3}$	$1.4\ \times10^{-3}$	$\chi\to\eta'\eta$	0.04×10^{-3}
$\psi\to\phi\eta'$	$(0.25\pm0.15)\times10^{-3}$	0.26×10^{-3}	$\chi\to\eta'\eta'$	0.13×10^{-3}
$\psi\to\omega\eta$	$\cdots$	0.83×10^{-3}	$\chi\to\eta\eta$	0.31×10^{-3}
$\psi\to\omega\eta'$	$\cdots$	0.10×10^{-3}		
$\psi\to K^0\bar{K}^0$	$<0.6\times10^{-3}$	0.02×10^{-3}	$\chi\to K^{*+}K^-$	0.01×10^{-3}

mainly in the normalization parameter δ of the wave functions. The most problematic value is that of δ_π, which comes out rather large. This seems to be a problem if one tries to calculate the pion form factor. A rather miraculous cancellation between the "direct" and "indirect" terms in the form factor is required in order to be consistent with the mean squared charge radius, for instance. A possible solution is to allow for different extensions of the wave function in space and time [change the factor 2 in the exponent of Eqs. (2.1)–(2.3)]. It is possible that the large extension of the pion is only in the time direction. This may be enough for the overlap integrals in the quark-loop graph, leaving the direct term to the form factor reasonable. (We hope to return to this problem in a later publication.) We emphasize again that for the moment we consider the emerging qualitative picture more important than the actual values of the best-fitting parameters. (This is natural if one tries to fit an approximate formula to the measured values.)

After fitting the old-particle decays, a very good independent test of the model is its application to the new-particle decays where the physical circumstances are rather different. For the moment the situation is very promising in ψ decays. It would be very interesting to have accurate data also for the decays of the 2^{++} charmonium state and compare it to our predictions.

The detailed comparison of our model with the related approaches worked out previously is not an easy task, and is outside of the scope of the present paper. The main source of difficulties is that the models in general contain a lot of different assumptions, approximations, etc.

As far as the main features are concerned, the essential difference compared, for instance, to the model of Böhm, Joos, and Krammer[5] is that we have quark confinement in real Minkowski space, whereas the model in Ref. 5 is based on wave functions confined in the Euclidean region (after Wick-rotation). In the Euclidean region the Lorentz-boosts go over into rotations; therefore there is no Lorentz contraction. This is the reason why in the Euclidean wave functions there is not so much symmetry breaking as in our (Minkowski) case.

Another related, physically very appealing, model is the so-called quark-pair-creation model.[13-15] Compared to ours the main difference is that the model in Refs. 13–15 is noncovariant. For the detailed discussion of the covariant versus noncovariant approaches see Ref. 16. The quark pair with the quantum numbers of the vacuum "created from the hadronic vacuum" can be looked upon as the nonrelativistic picture of the relativistic Feynman graph giving rise to the quark-loop coupling in relativistic approaches.

APPENDIX A: THE FEYNMAN RULES FOR SINGLE-QUARK-LOOP GRAPHS

The Feynman rules for single-quark-loop graphs are as follows.

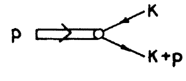
1. Label the internal-quark-line four-momenta taking into account four-momentum conservation.

2. Write a factor

$$(2\pi)^{2m} i^{1+l}\delta^4\Big(\sum_{\rm in} p-\sum_{\rm out} p\Big)\int d^4k\,{\rm Tr}_{\rm SU}\,{\rm Tr}_D\,,$$

where k is the four-momentum of some quark line, l is the number of quark lines, m is the number of BS vertices (small circles), $\sum_{\rm in(out)} p$ is the sum of four-momenta of incoming (outgoing) mesons, and ${\rm Tr}_{\rm SU}$ and ${\rm Tr}_D$ denote traces over internal symmetry and Dirac (quark) indices, respectively.

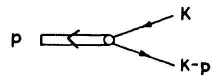
3. For incoming mesons



$$= M\chi(p, K+\tfrac{p}{2})$$

where M is the internal-symmetry matrix of the meson and χ is the momentum-dependent part of its BS wave function.

4. For outgoing mesons



$$= \bar{M}\bar{\chi}(p, K-\tfrac{p}{2})$$

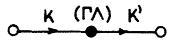
where $\bar{M}$ and $\bar{\chi}$ are the conjugate wave functions.

5.

$= (M_q - K\cdot\gamma)$

where M_q is the quark mass matrix.

6.



$\Rightarrow \Gamma\Lambda\left\{(2\pi)^4\delta^4\left(\sum_{in} p - \sum_{out} p\right)\right\}^{-1}$

where $\bullet(\Gamma\Lambda)$ denotes the (single) local current operator with Dirac part Γ and internal-symmetry part Λ.

APPENDIX B: THE EVALUATION OF INTEGRALS

The Gaussian integrals entering the loop integrations are in principle easy to evaluate. Introducing suitable integration variables the necessary algebra with the trace factors becomes transparent and simple.

The one-dimensional structure of two-body decays leads naturally to decompose the four-momentum vectors into longitudinal and transverse components. For the longitudinal variables let us use the "light-cone" components $p_\pm$ of the four-vector $p = (p_0, \vec{p})$ introduced by

$$p_\pm = p_0 \pm p_\parallel\,, \tag{B1}$$

where $p_\parallel$ is the longitudinal component of the three-momentum $\vec{p}$. If $\vec{p}_\perp$ is the transverse part of $\vec{p}$, then the scalar product of two vectors is

$$p_1\cdot p_2 = \tfrac{1}{2}(p_{1+}p_{2-} + p_{1-}p_{2+}) - \vec{p}_{1\perp}\cdot\vec{p}_{2\perp}\,, \tag{B2}$$

and we have

$$d^4p = \tfrac{1}{2}dp_+dp_-d^2\vec{p}_\perp\,. \tag{B3}$$

The rapidity y is defined by

$$p_\pm = e^{\pm y}(m^2 + p_\perp^2)^{1/2}\,, \tag{B4}$$

where m is the mass belonging to p and $p_\perp$ is the length of the transverse component: $\vec{p}_\perp^{\,2} = p_\perp^2$.

Let us denote the four-momenta of the decay products by p_2 and p_3, their masses by m_2 and m_3, respectively (the same variables for the decaying particle are p_1 and m_1). We have, of course, $p_1 = p_2 + p_3$ and we choose p_2 and p_3 to be longitudinal, that is $\vec{p}_{2\perp} = \vec{p}_{3\perp} = 0$. In this case the relative rapidities can be expressed from

$$\begin{aligned} \frac{p_2\cdot p_3}{m_2m_3} &= \frac{m_1^2 - m_2^2 - m_3^2}{2m_2m_3} = \cosh(y_2 - y_3)\,,\\ \frac{p_1\cdot p_2}{m_1m_2} &= \frac{m_1^2 + m_2^2 - m_3^2}{2m_1m_2} = \cosh(y_2 - y_1)\,,\\ \frac{p_1\cdot p_3}{m_1m_3} &= \frac{m_1^2 + m_3^2 - m_2^2}{2m_1m_3} = \cosh(y_3 - y_1)\,. \end{aligned} \tag{B5}$$

It is convenient to introduce the notations

$$\begin{aligned} &\eta_2 = y_2 - y_1, \quad \eta_3 = y_3 - y_1,\\ &\eta = y_2 - y_3 = \eta_2 - \eta_3, \quad \beta = \cosh\eta\,. \end{aligned} \tag{B6}$$

In these variables the four-momentum conservation means

$$m_1 = m_2e^{\pm\eta_2} + m_3e^{\pm\eta_3}\,. \tag{B7}$$

The usual function λ defining the phase-space boundary is given by

$$\begin{aligned} \lambda^{1/2} &= (m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2\\ &\quad - 2m_1^2m_3^2 - 2m_2^2m_3^2)^{1/2}\\ &= 2m_2m_3\sinh\eta = 2m_1m_2\sinh\eta_2 = -2m_1m_3\sinh\eta_3\,, \end{aligned} \tag{B8}$$

and the magnitude of the c.m. three-momentum is

$$w = \frac{m_2m_3}{m_1}\sinh\eta\,. \tag{B9}$$

The four-momenta in the loop integral are defined in Fig. 1. The basic integral entering the one-loop calculations is

$$\begin{aligned} J = \int d^4q \exp\Bigg\{&\alpha_1\left[q^2 - \frac{2}{m_1^2}(p_1\cdot q)^2\right]\\ &+ \alpha_2\left[\left(q + \frac{p_3}{2}\right)^2 - \frac{2}{m_2^2}\left(p_2\cdot q + \frac{p_2\cdot p_3}{2}\right)^2\right] + \alpha_3\left[\left(q - \frac{p_2}{2}\right)^2 - \frac{2}{m_3^2}\left(p_3\cdot q - \frac{p_2\cdot p_3}{2}\right)^2\right]\Bigg\}. \end{aligned} \tag{B10}$$

In our variables this is

$$J = \tfrac{1}{2}\int dq_+dq_-d^2\vec{q}\,\exp\left[-(\alpha_1 + \alpha_2 + \alpha_3)\vec{q}^{\,2} - \frac{A_-}{2}q_+^2 - \frac{A_+}{2}q_-^2 + \frac{a_-}{2}q_+ + \frac{a_+}{2}q_- - E\right], \tag{B11}$$

where

$$A_\pm = \alpha_1 + \alpha_2e^{\pm 2\eta_2} + \alpha_3e^{\pm 2\eta_3}\,, \quad a_\pm = \alpha_3m_2e^{\pm\eta_3\mp\eta} - \alpha_2m_3e^{\pm\eta_2\pm\eta}\,, \quad E = \frac{\cosh 2\eta}{4}(\alpha_2m_3^2 + \alpha_3m_2^2)\,. \tag{B12}$$

The result of the integration is

$$J=\frac{\pi^2}{(\alpha_1+\alpha_2+\alpha_3)(A_+A_-)^{1/2}}\exp\left(\frac{a_+^2}{8A_+}+\frac{a_-^2}{8A_-}-E\right). \tag{B13}$$

The trace factors result in integrals like

$$\int d^4q\,q_{\rho_1}\cdots q_{\rho_n}\exp(\cdots)\,, \tag{B14}$$

where the exponent is the same as in Eq. (B10). For us it will be more convenient to work with the four-vector

$$k=q+\tfrac{1}{2}(p_3-p_2)\,, \tag{B15}$$

hence with the integrals

$$J_{\rho_1\cdots\rho_n}=\int d^4k\,k_{\rho_1}\cdots k_{\rho_n}\exp(\cdots)\,. \tag{B16}$$

This integral can be expressed generally for any n. Actually, we shall need only $n=1$, 2, 3, and 4. A suitable form of the result is

$$\begin{aligned}
J_\rho&=J(z_2p_{2\rho}+z_3p_{3\rho})\,,\\
J_{\rho\sigma}&=J\left[z_{22}p_{2\rho}p_{2\sigma}+z_{23}(p_{2\rho}p_{3\sigma}+p_{2\sigma}p_{3\rho})+z_{33}p_{3\rho}p_{3\sigma}-\frac{g_{\rho\sigma}}{2(\alpha_1+\alpha_2+\alpha_3)}\right],\\
J_{\rho\sigma\tau}&=J\Big\{z_{222}p_{2\rho}p_{2\sigma}p_{2\tau}+z_{223}(p_{2\rho}p_{2\sigma}p_{3\tau}+p_{2\rho}p_{2\tau}p_{3\sigma}+p_{2\sigma}p_{2\tau}p_{3\rho})+z_{233}(p_{2\rho}p_{3\sigma}p_{3\tau}+p_{2\sigma}p_{3\rho}p_{3\tau}+p_{2\tau}p_{3\rho}p_{3\sigma})\\
&\qquad+z_{333}p_{3\rho}p_{3\sigma}p_{3\tau}-\frac{1}{2(\alpha_1+\alpha_2+\alpha_3)}[z_2(g_{\rho\sigma}p_{2\tau}+g_{\sigma\tau}p_{2\rho}+g_{\tau\rho}p_{2\sigma})+z_3(g_{\rho\sigma}p_{3\tau}+g_{\sigma\tau}p_{3\rho}+g_{\tau\rho}p_{3\sigma})]\Big\}\,,\\
J_{\rho_1\rho_2\rho_3\rho_4}&=J\sum_{\pi(!)4}\Big\{\frac{1}{4!}z_{2222}p_{2\rho_{\pi(1)}}p_{2\rho_{\pi(2)}}p_{2\rho_{\pi(3)}}p_{2\rho_{\pi(4)}}+\frac{1}{3!}z_{2223}p_{2\rho_{\pi(1)}}p_{2\rho_{\pi(2)}}p_{2\rho_{\pi(3)}}p_{3\rho_{\pi(4)}}\\
&\qquad+\frac{1}{2!2!}z_{2233}p_{2\rho_{\pi(1)}}p_{2\rho_{\pi(2)}}p_{3\rho_{\pi(3)}}p_{3\rho_{\pi(4)}}+\frac{1}{3!}z_{2333}p_{2\rho_{\pi(1)}}p_{3\rho_{\pi(2)}}p_{3\rho_{\pi(3)}}p_{3\rho_{\pi(4)}}\\
&\qquad+\frac{1}{4!}z_{3333}p_{3\rho_{\pi(1)}}p_{3\rho_{\pi(2)}}p_{3\rho_{\pi(3)}}p_{3\rho_{\pi(4)}}-\frac{1}{2(\alpha_1+\alpha_2+\alpha_3)2!2!}g_{\rho_{\pi(1)}\rho_{\pi(2)}}\\
&\qquad\times[z_{22}p_{2\rho_{\pi(3)}}p_{2\rho_{\pi(4)}}+z_{23}(p_{2\rho_{\pi(3)}}p_{3\rho_{\pi(4)}}+p_{3\rho_{\pi(3)}}p_{2\rho_{\pi(4)}})+z_{33}p_{3\rho_{\pi(3)}}p_{3\rho_{\pi(4)}}]+\frac{g_{\rho_{\pi(1)}\rho_{\pi(2)}}g_{\rho_{\pi(3)}\rho_{\pi(4)}}}{8(\alpha_1+\alpha_2+\alpha_3)^2 2!2!}\Big\}.
\end{aligned} \tag{B17}$$

In the last expression $\sum_{\pi(!)n}$ means ($n=4$) a summation over the permutations $\pi(1),\ldots,\pi(n)$ of the numbers $1,2,\ldots,n$.

The coefficients $z_2, z_3, \ldots$ are given by

$$\begin{aligned}
z_2&=\frac{1}{2m_2\sinh\eta}\left(\frac{a_-}{2A_-}e^{-\eta_3}-\frac{a_+}{2A_+}e^{\eta_3}\right)-\tfrac{1}{2}\,,\qquad z_3=\frac{1}{2m_3\sinh\eta}\left(\frac{a_+}{2A_+}e^{\eta_2}-\frac{a_-}{2A_-}e^{-\eta_2}\right)+\tfrac{1}{2}\,,\\
z_{22}&=z_2^2+\frac{1}{4m_2^2\sinh^2\eta}\left(\frac{e^{2\eta_3}}{A_+}+\frac{e^{-2\eta_3}}{A_-}-\frac{2}{\alpha_1+\alpha_2+\alpha_3}\right),\\
z_{23}&=z_2z_3-\frac{1}{4m_2m_3\sinh^2\eta}\left(\frac{e^{\eta_2+\eta_3}}{A_+}+\frac{e^{-\eta_2-\eta_3}}{A_-}-\frac{2\cosh\eta}{\alpha_1+\alpha_2+\alpha_3}\right),\\
z_{33}&=z_3^2+\frac{1}{4m_3^2\sinh^2\eta}\left(\frac{e^{2\eta_2}}{A_+}+\frac{e^{-2\eta_2}}{A_-}-\frac{2}{\alpha_1+\alpha_2+\alpha_3}\right),\\
z_{222}&=3z_2z_{22}-2z_2^3\,,\qquad z_{223}=z_{22}z_3+2z_{23}z_2-2z_2^2z_3\,,\\
z_{233}&=z_{33}z_2+2z_{23}z_3-2z_3^2z_2\,,\qquad z_{333}=3z_3z_{33}-2z_3^3\,,\qquad z_{2222}=3z_{22}^2-2z_2^4\,,\\
z_{2223}&=3z_{22}z_{23}-2z_2^3z_3\,,\qquad z_{2233}=z_{22}z_{33}+2z_{33}^2-2z_2^2z_3^2\,,\qquad z_{2333}=3z_{33}z_{23}-2z_3^3z_2\,,\\
z_{3333}&=3z_{33}^2-2z_3^4\,.
\end{aligned} \tag{B18}$$

In the case of equal masses $m_2=m_3\equiv m$ (and $\alpha_2=\alpha_3\equiv\alpha$) the above expressions simplify considerably. We have, namely

$$\beta \equiv \cosh\eta = \frac{m_1^2}{2m^2} - 1\,,$$

$$A_+ = A_- = \alpha_1 + 2\alpha\beta\,,$$

$$-a_+ = a_- = 2\alpha m(2\beta+1)\left(\frac{\beta-1}{2}\right)^{1/2},$$

$$J = \frac{\pi^2}{(\alpha_1+2\alpha)(\alpha_1+2\alpha\beta)}\exp\left(-\frac{\alpha m^2}{2}\frac{\alpha(\beta+1)+\alpha_1(2\beta^2-1)}{\alpha_1+2\alpha\beta}\right), \tag{B19}$$

$$z_2 = -z_3 = \frac{\alpha-\alpha_1}{2(\alpha_1+2\alpha\beta)}\,,\quad z_{22} = z_{33} = z_2^2 + \frac{\alpha_1}{m_1^2(\alpha_1+2\alpha)(\alpha_1+2\alpha\beta)}\,,\quad z_{23} = -z_{22} + \frac{2}{m_1^2}\frac{\alpha_1+\alpha(\beta+1)}{(\alpha_1+2\alpha)(\alpha_1+2\alpha\beta)},$$

$$z_{222} = -z_{333} = 3z_{22}z_2 - 2z_2^3\,,\quad z_{223} = -z_{233} = 2z_{23}z_2 - z_{22}z_2 + 2z_2^3\,,$$

$$z_{2222} = z_{3333} = 3z_{22}^2 - 2z_2^4\,,\quad z_{2223} = z_{2333} = 3z_{22}z_{23} + 2z_2^4\,,\quad z_{2233} = z_{22}^2 + 2z_{23}^2 - 2z_2^4\,.$$

[1]Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[2]M. M. Nagels *et al.*, Nucl. Phys. B109, 1 (1976).

[3]S. Okubo, Phys. Lett. 5, 165 (1963); G. Zweig, 1964 (unpublished); J. Iizuka, Prog. Theor. Phys. Suppl. 37-38, 21 (1966).

[4]R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971).

[5]M. Böhm, H. Joos, and M. Krammer, Nucl. Phys. B51, 397 (1973); M. Böhm, J. Joos, and M. Krammer, CERN Report No. Th-1715, 1973 (unpublished); H. Joos, in *Current Induced Interactions*, proceedings of the Summer School on Theoretical Particle Physics, Hamburg, 1975, edited by J. G. Körner, G. Kramer, and D. Schildknecht (Springer, New York, 1976), p. 428.

[6]Y. S. Kim and M. E. Noz, Phys. Rev. D 8, 3521 (1973); 12, 129 (1975).

[7]G. Preparata, CERN Report No. Th-2271, 1977 (unpublished).

[8]I. Montvay and L. Urbán, KFKI Report No. 1977-5, 1977 (unpublished).

[9]I Montvay, in *Proceedings of the JINR-CERN School on High Energy Physics, Varna, 1971* (JINR, Dubna, 1972), Vol. II, p. 208.

[10]B. Wiik and G. Wolf, DESY Report No. 77/01, 1977 (unpublished).

[11]W. J. Wilson, Nucl. Phys. B116, 205 (1976).

[12]V. S. Mathur, S. Okubo, and S. Borchardt, Phys. Rev. D 11, 2572 (1975); H. Harari, Phys. Lett. 60B, 172 (1976); D. H. Boal and R. Torgerson, Phys. Rev. D 15, 327 (1977).

[13]L. Micu, Nucl. Phys. B10, 521 (1969).

[14]R. Carlitz and M. Kislinger, Phys. Rev. D 2, 336 (1970).

[15]A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal, Phys. Rev. D 8, 2223 (1973).

[16]I. Kumari and A. N. Mitra, Nuovo Cimento 38A, 65 (1977); 38A, 89 (1977).

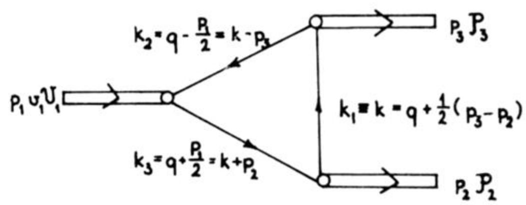


FIG. 1. The quark-loop graph contributing to the $V \rightarrow PP$ decay (p denotes four momenta, σ the spin index, V and P the SU(3) indices for vector and pseudoscalar mesons, respectively).

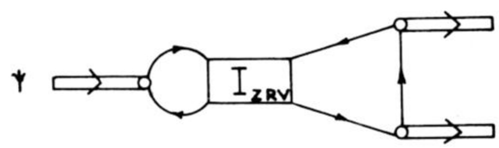


FIG. 2. The quark diagram for the two-body strong decays of ψ. I_{ZRV} is some Zweig-rule-violating interaction transforming the charmed-quark pair into an ordinary-quark pair.