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Systematic approach to inclusive lepton pair production in hadronic collisions

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Strong-interaction dynamics as probed by lepton pair production in hadronic collisions is naturally separated from kinematics by using suitably defined structure functions. In the first part of this paper, general properties of invariant structure functions and a variety of "helicity" structure functions for this process are studied, and their use discussed. An exact parallelism to the case of deep-inelastic lepton-hadron scattering is set up. In the second part, a series of parton-model relations between the structure functions, reflecting the basic Drell-Yan on-shell quark-antiquark annihilation picture (but independent of details of parton distributions), is derived. These relations serve the dual purposes of (i) supplementing the model-independent structure-function formalism and rendering it useful for analyzing data of limited scope initially, and (ii) providing unambiguous tests of various aspects of the underlying quark-parton model when more detailed data become available.

I. INTRODUCTION

Much attention has been focused lately on lepton pair production in hadronic collisions.[1] Two main reasons for this surge of interest are the following: (i) Experimentally, this process has proved to furnish a powerful method of producing and detecting new heavy particles, and (ii) within the quark-parton picture, the continuum part of this process can be related to and, in fact, complements the much studied deep-inelastic lepton-hadron scattering. Studies of lepton pair production so far, both experimental and theoretical, rely heavily on the Drell-Yan quark-antiquark annihilation picture.[2] However, as more detailed experimental results are becoming available[3] and as certain initial experimental trends already indicate the inadequacy of the simplest version of the Drell-Yan model, an increasing amount of theoretical work is being directed toward the study of possible alternative or competing mechanisms for this process.[4,5] Most of these studies are based on perturbative QCD (quantum chromodynamics)—a theoretical approach still in its formative stage.[5]

The increasing sophistication of both experimental and theoretical work in this field demands an effective, model-independent language to describe the physics underlying this process. [Conventional discussions are based either on raw cross sections which represent a mixture of (essential but uninteresting) kinematics and dynamics, or on the parton model, the validity of which is one of the most interesting issues yet to be cleared up.] The natural language to describe lepton pair production is based on suitably defined structure functions—in complete analogy to inclusive lepton-hadron scattering. Two different but equivalent versions of this language complement each other: The invariant structure functions[6] represent pure dynamics of the hadron system without any kinematic constraints, whereas the "helicity" structure functions[7] allow the manifest factorization of hadronic and leptonic degrees of freedom. Both versions of this language are developed in some detail in Sec. II of this paper.

A purely model-independent approach to the phenomenology of lepton pair production, though always desirable, is hardly practical at the present stage. Simplifications must be sought initially through reasonable model considerations such as the Drell-Yan mechanism.[1-3] Here the conventional parton-model calculations suffer from several shortcomings: (i) The results do not clearly separate effects due to kinematics, the elementary quark-antiquark annihilation amplitude, and the choice of parton distribution functions; (ii) while the first two ingredients mentioned above are quite unambiguous, there is considerable uncertainty concerning the detailed form of the parton distribution functions; and (iii) this approach is rather inflexible in that the results usually appear in numerical form and are very hard to be stated succinctly or be adjusted when necessary. The cure to these shortcomings is again quite obvious once one remembers the lepton-hadron scattering analogy. After formulating the problem in terms of structure functions (which automatically separate out the kinematics), one must seek relations among the structure functions which are consequences of the elementary quark-antiquark annihilation assumption but independent of the details of the parton distribution functions. These relations reflect clearly defined physical features of the basic picture and help to reduce the complexity of the phenomenology to a manageable level. We have in mind here, of course, the example of the parton-model relations between F_1, F_2, and F_3 in eN, μN, and νN deep-inelastic scattering and the important role they played in the development of that field. A key feature of this approach, in addition to the advantages already mentioned, is that it is totally flexible: The various parton-model relations can be selectively withdrawn as the quality of experimental data improves. Under that circumstance, one can turn around and use the same relations as quantitative tests of the validity of the original assumptions. In Sec. III of this paper we derive a series of relations of this type based on simple but nontrivial parton-model considerations.[8] These relations are discussed both in the invariant- and "helicity"-structure-function formalisms. Section IV consists of a summary of this approach.

Because the paper is concerned with both theory and experiment, an effort is made to include enough details to render the presentation understandable to a wider audience than that for a specialized paper. Since the kinematics of this process goes beyond that of the familiar two-body-to-two-body case, some not-so-familiar shorthand notations were found useful (and even necessary) in keeping the calculations tractable. Although these notations are introduced along the way in the text, we assemble them, together with some kinematic details omitted in the text, in Appendix A for easy reference.

II. MODEL-INDEPENDENT CONSIDERATIONS

A. Basic formulas

If lepton pair creation in hadronic collisions is effected by one (virtual) photon production, the amplitude for this process can be written as[9]

$$f=\bar{u}(k_1)\gamma_\mu v(k_2)\frac{e^2}{q^2}\,{}_{\text{out}}\langle X|J^\mu(0)|p_1p_2\rangle_{\text{in}}\,. \tag{1}$$

The momentum labels are illustrated in Fig. 1: p_1 and p_2 refer to the colliding hadrons, k_1 and k_2 to the produced leptons (e^+e^- or $\mu^+\mu^-$), q to the virtual photon, and X to the unobserved hadronic final states. Spins are summed over (or averaged, as the case may be). Since we are primarily interested in studying the high-energy properties of this process, quantities of the order m^2/s are practically zero and will be systematically ignored.[10] Here m stands for all the masses, leptonic and hadronic, and $s=-(p_1+p_2)^2$ is the usual total center-of-mass energy squared variable.

It is useful to introduce certain combinations of momentum variables:

$$\begin{aligned} P&=p_1+p_2\,,\\ p&=p_1-p_2\,,\\ q&=k_1+k_2\,, \end{aligned} \tag{2}$$

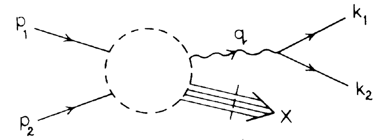

FIG. 1. Inclusive lepton pair production in hadronic collisions in the one-photon approximation.

and

$$k=k_1-k_2\,.$$

It is then trivial to show that

$$\begin{aligned} -P^2&=p^2\equiv s\,,\\ -q^2&=k^2\equiv M^2\,, \end{aligned} \tag{3}$$

and

$$p\cdot P=k\cdot q=0\,.$$

In the hadron c.m. frame, P has only a time component and p only a z component (by definition).

The fully differential cross section is given by

$$d\sigma=\left(\frac{\alpha}{sM}\right)^2 L^{\mu\nu}W_{\mu\nu}\frac{1}{(2\pi)^4}\frac{d^3k_1d^3k_2}{k_1k_2}\,, \tag{4}$$

where the lepton tensor $L^{\mu\nu}$ is well known

$$L^{\mu\nu}=g^{\mu\nu}-\frac{q^\mu q^\nu}{q^2}-\frac{k^\mu k^\nu}{k^2}\,, \tag{5}$$

and the hadron tensor $W_{\mu\nu}$ is given by

$$W_{\mu\nu}\equiv s\int d^4z\, e^{iq\cdot z}\langle p_1p_2|J_\mu(z)J_\nu(0)|p_1p_2\rangle\,. \tag{6}$$

$W^{\mu\nu}$ represents the square of the hadron matrix element in Eq. (1) (or the "blob" in Fig. 1) summed over the final hadron states X (the unitary sum). It can be represented graphically as in Fig. 2. All dynamical information—the "structure" of the hadron system as probed by the virtual photon—resides in this tensor.

A useful special case of Eq. (4) is obtained by integrating over the pure leptonic degrees of freedom (say, the lepton angles in the rest frame of

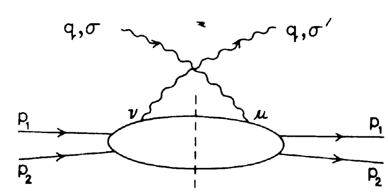

FIG. 2. Diagrammatic representation of the hadronic amplitude $W_{\mu\nu}$ or $W_{\sigma,\sigma'}$. The dashed line indicates that it is the absorptive part of this amplitude (or, equivalently, the sum over all intermediate states) that is being considered.

the virtual photon):

$$\frac{d\sigma}{d^4q}=\left(\frac{\alpha}{sM}\right)^2\frac{1}{12\pi^3}W^\mu{}_\mu . \tag{7}$$

This allows $W^\mu{}_\mu$ to be directly measured.

B. General properties of $W^{\mu\nu}$

The $W^{\mu\nu}$ tensor is a function of the momentum components (q^μ, p^μ, P^μ). To discuss its general properties, it is convenient also to consider "helicity" amplitudes $W_{\sigma,\sigma'}$ which are functions of the invariants

$$W_{\sigma,\sigma'}(s, M^2, q\cdot P, q\cdot p) \equiv \epsilon^\mu_{(\sigma)}(q)W_{\mu\nu}(P,p,q)\epsilon^{*\nu}_{(\sigma')}(q) . \tag{8}$$

Here $\epsilon^\mu_{(\sigma)}(q)$, $\sigma=-1,0,+1$, are a set of polarization vectors for the virtual photon defined with respect to some coordinate axes in the rest frame of the photon yet to be specified. (In this connection we are not using the word "helicity" in the strict Jacob-Wick sense, hence the quotation marks.) The physical interpretation of $W_{\sigma,\sigma'}$, Eq. (8), should be obvious as implied by its name (cf. also the graphical representation Fig. 2). We also note, by definition, that both $W_{\mu\nu}$ and $W_{\sigma,\sigma'}$ are dimensionless—a convenience for scaling and parton-language descriptions later.

Several general requirements on $W_{\mu\nu}$ restrict the number of independent components of this tensor.

(i) Symmetry. It is straightforward to show that

$$W_{\mu\nu}(q,p,P)=W^*_{\nu\mu}(q,p,P)$$

and (9)

$$W_{\sigma',\sigma}(s,M^2,q\cdot p,q\cdot P)=W^*_{\sigma,\sigma'}(s,M^2,q\cdot p,q\cdot P) .$$

This implies that the symmetric part of W is real while the antisymmetric part is imaginary. However, since the lepton tensor $L^{\mu\nu}$, Eq. (5), is explicitly symmetric (as a consequence of summing over polarizations of the leptons), only the symmetric part of W contributes to the measured cross sections. From now on, we shall treat $W_{\mu\nu}$ and $W_{\sigma',\sigma}$ as symmetric and real.

(iii) Gauge invariance. Current conservation, cf. Eq. (6), implies

$$q^\mu W_{\mu\nu}(q,p,P)=W_{\mu\nu}(q,p,P)q^\nu=0 . \tag{10}$$

(iii) Parity. In terms of $W^{\mu\nu}$, parity constraints imply it can only depend on (q,p,P) in "natural" combinations (i.e., no $\epsilon^{\mu\nu\lambda\sigma}$ symbol). For the "helicity" amplitudes, the same constraints imply

$$W_{\sigma,\sigma'}=(-1)^{\sigma+\sigma'}W_{-\sigma,-\sigma'} . \tag{11}$$

(iv) Positivity ("optical theorem", "unitarity"). The definition of $W^{\mu\nu}$, Eq. (6), together with the hermitian property of the electromagnetic current operator immediately imply that $W^{\mu\nu}$ is positive-semidefinite, i.e.,

$$V_\mu W^{\mu\nu}V^*_\nu \geq 0 \text{ for any four-vector } V . \tag{12}$$

C. The invariant structure functions

Since the gauge-invariance condition, Eq. (10), limits $W^{\mu\nu}$ to be a tensor in the three-space orthogonal to q^μ, it is useful to introduce the projection operator[11] to that space,

$$\tilde g^{\mu\nu}=g^{\mu\nu}-q^\mu q^\nu/q^2 , \tag{13}$$

which satisfies $\tilde g^{\mu\nu}q_\nu=q_\mu\tilde g^{\mu\nu}=0$. When contracted with any four-vector, it yields a vector orthogonal to q^μ. In particular, if we define

$$\tilde P^\mu=\tilde g^{\mu\nu}P_\nu/\sqrt{s} , \tag{14}$$

and

$$\tilde p^\mu=\tilde g^{\mu\nu}p_\nu/\sqrt{s} ,$$

then

$$q\cdot\tilde p=q\cdot\tilde P=0 . \tag{15}$$

Some useful results concerning $\tilde P$ and $\tilde p$ are listed in Appendix A.

We can now write down a representation of $W_{\mu\nu}$ that explicitly satisfies the general requirements (i)–(iii) above[6]:

$$W_{\mu\nu}(q,p,P)=W_1\tilde g_{\mu\nu}+W_2\tilde P_\mu\tilde P_\nu - W_3(\tilde P_\mu\tilde p_\nu+\tilde p_\mu\tilde P_\nu)/2+W_4\tilde p_\mu\tilde p_\nu . \tag{16}$$

$W_{1,2,3,4}$ are the invariant structure functions which depend on the invariant variables, say, $(s,q^2,q\cdot p,q\cdot P)$. Aside from the positivity requirement, (iv) above (which we shall come back to later), W_i are free from all kinematic constraints.[12] Hence structures in W_i are of purely dynamical origin. In this sense, they represent the "minimal" set[11,12] which is most suitable for theoretical and experimental study.

In this representation, the trace tensor which enters the cross-section formula, Eq. (7), is

$$W^\mu{}_\mu=3W_1+\left(\frac{q_P^2}{M^2}-1\right)W_2+\frac{q_Pq_p}{M^2}W_3+\left(1+\frac{q_p^2}{M^2}\right)W_4 . \tag{17}$$

Here we have used the notation

$$q_p=q\cdot p/\sqrt{s} ,$$

and (18a)

$$q_P=-q\cdot P/\sqrt{s} ,$$

which will recur frequently later. These are easily visualized as *the z and t components of the vector q in the hadron c.m. frame* with the beam direction defined as the z axis. We also define the

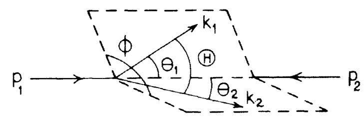

FIG. 3. Kinematic configuration in the hadron center-of-mass frame.

"perpendicular vector" $q_\perp^\mu$ by

$$q^\mu = q_P P^\mu/\sqrt{s} + q_p p^\mu/\sqrt{s} + q_\perp^\mu . \tag{18b}$$

It has the property that

$$\begin{aligned} &q_\perp^\mu P_\mu = q_\perp^\mu p_\mu = 0 , \\ &q_\perp{}^2 \equiv q_\perp^\mu q_{\perp\mu} = q_P{}^2 - q_p{}^2 - M^2 . \end{aligned} \tag{18c}$$

Again, intuitively, the *four-vector $q_\perp^\mu$ reduces to the two-vector $\vec{q}_\perp$ perpendicular to the beam axis in the hadron c.m. frame*. The covariant definitions given above are, however, useful in theoretical calculations. The same notation will be applied to other four-vectors (e.g. lepton and parton momenta) in subsequent discussions.

In order to write down the full differential cross section let us first define the kinematic variables. In the hadron c.m. frame, we can choose the following independent variables: k_1, k_2 as the magnitudes of the two lepton momenta θ_1 and θ_2 their polar angles, and ϕ the azmuthal angle between the two production planes. The kinematic picture is depicted in Fig. 3. We shall also need the opening angle Θ between the two lepton momenta,

$$\cos\Theta = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos\phi ,$$

and a related angle Θ' defined by (19)

$$\cos\Theta' = \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos\phi .$$

Substituting (16) into (4) and evaluating all the relevant scalar products, we obtain the basic formula

$$d\sigma = \frac{\alpha^2}{32\pi^3} \frac{1}{s^2} \frac{\cos^2(\Theta/2)}{\sin^4(\Theta/2)} dk_1 dk_2 d\cos\theta_1 d\cos\theta_2 d\phi \left(W_2 + 2\tan^2\frac{\Theta}{2} W_1 + \frac{\cos\theta_1 + \cos\theta_2}{1+\cos\Theta} W_3 + \frac{1+\cos\Theta'}{1+\cos\Theta} W_4 \right) . \tag{20}$$

One immediately notices the resemblance of this equation to that of lepton-hadron scattering. The similarity is, of course, just the manifestation of the underlying one-photon mechanism. The differing dynamical contents of the two types of processes lie in the number and behavior of the structure functions. Equation (20) suggests that even if the structure functions cannot be fully separated at the present stage, a minimal first step is to divide out the rapidly varying kinematic factors in front of the large parentheses in the presentation of experimental results.

One drawback (if it can be called that) of the description Eq. (20) is that the leptonic and hadronic degrees of freedom are not *manifestly* separated. For instance, for fixed arguments of the structure functions (i.e., hadron variables), only two of the lepton angle variables θ_1, θ_2, and ϕ are independent. There is an implicit relation between them, making the simple angular factors in Eq. (20) slightly deceiving in appearance. An alternative description which explicitly displays the factorization of the leptonic and hadronic degrees of freedom can be formulated in terms of the "helicity structure functions" to which we turn our attention next.

D. The "helicity" structure functions

These are simply the linearly independent "helicity" amplitudes defined by Eq. (8). We denote them by the following[13]:

$$\begin{aligned} &W_T \equiv W_{1,1} , \\ &W_L \equiv W_{0,0} , \\ &W_\Delta \equiv (W_{1,0} + W_{0,1})/\sqrt{2} , \\ &W_{\Delta\Delta} \equiv W_{1,-1} . \end{aligned} \tag{21}$$

Thus, W_T, W_L are structure functions for transversely and longitudinally polarized virtual photons, respectively, W_Δ is the single-spin-flip structure function, while $W_{\Delta\Delta}$ is the double-spin-flip one.

In order to define these structure functions uniquely, one has to specify the choice of the polarization vectors $\epsilon_{(\sigma)}$ in Eq. (8). These are related to the unit vectors (X, Y, Z) of a Cartesian system in the lepton-pair c.m. frame in the usual way: $\epsilon^\mu_{(0)} = Z^\mu$, $\epsilon^\mu_{(\pm1)} = (\mp X - iY)^\mu/\sqrt{2}$. We still have to define (X, Y, Z) in terms of the physical momenta (q, p, P). This is done in three steps: (i) Pick Z^μ to be a linear combination of $\tilde{P}^\mu$ and $\tilde{p}^\mu$ (there is a one-parameter freedom in this choice; (ii) define X to be also in the $(\tilde{P}, \tilde{p})$ plane and orthogonal to Z (only the choice of sign is arbitrary); and (iii) define Y to complete a right-handed system. The vectors $\tilde{P}, \tilde{p}, X, Y, Z$ are simply visualized as ordinary three-vectors in the lepton-pair c.m. frame; although, again, the covariant four-vector notation is convenient for calculational purposes. Among the (infinite number of) possible choices of Z axis each of which defines a corresponding set of "helicity" structure functions, we mention a few simpler ones:

(a) $Z^\mu \propto \tilde{P}^\mu$: "$s$-channel helicity" or simply "helicity" (in the sense of Jacob-Wick);

(b) $Z^\mu \propto \tilde{p}_1^\mu$: "$t$-channel helicity" or "Gottfried-Jackson";

(c) $Z^\mu \propto \tilde{p}^\mu$: (no official name) referred to as p-helicity in the following;

(d) $Z^\mu \propto q_p \tilde{P} + q_P \tilde{p} = 2[-(q\cdot p_2)\tilde{p}_1 + (q\cdot p_1)\tilde{p}_2]$: Collins-Soper.[4]

We shall comment on features of these specific choices later, but first let us discuss the common features.

A useful definition of the helicity structure functions directly in terms of the (X, Y, Z) vectors is

$$W^{\mu\nu} = \tilde{g}^{\mu\nu}(W_T + W_{\Delta\Delta}) - 2X^\mu X^\nu W_{\Delta\Delta} + Z^\mu Z^\nu (W_L - W_T - W_{\Delta\Delta}) - (X^\mu Z^\nu + Z^\mu X^\nu) W_\Delta . \tag{22}$$

From this, it is easy to obtain

$$W^\mu{}_\mu = 2W_T + W_L \tag{23}$$

which enter into the partially integrated cross-section formula Eq. (7). In order to write the full differential cross section in terms of the "helicity" structure functions, we use the hadronic variables $(s, q_P, q_p, q_\perp)$ and lepton "decay" angles θ^*, ϕ^* in the photon rest frame specified by the (X, Y, Z) axes, Fig. 4. (Notice that the choice of X and Z axes is left open in Fig. 4.) Substituting Eq. (22) into Eq. (4) and evaluating the scalar products, one obtains[7]

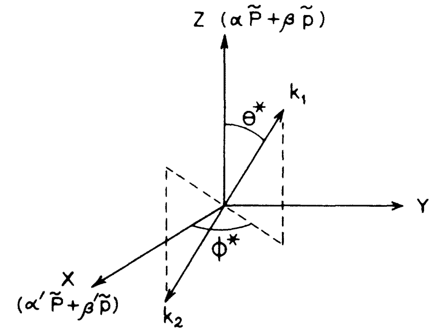

FIG. 4. Kinematic configuration in the lepton pair center-of-mass frame. The X-Z plane is defined by the hadronic momenta $\tilde{P}$ and $\tilde{p}$ but the choice of Z axis is left open. The coefficients $(\alpha, \beta, \alpha', \beta')$ corresponding to some specific "helicity" frame are listed in Table I. A typical configuration for the various Z axes is depicted in Fig. 5.

$$\frac{d\sigma}{d^4q\, d\Omega_k^*} = \frac{1}{2}\frac{1}{(2\pi)^4}\left(\frac{\alpha}{Ms}\right)^2 [W_T(1+\cos^2\theta^*) + W_L(1-\cos^2\theta^*) + W_\Delta \sin 2\theta^* \cos\phi^* + W_{\Delta\Delta}\sin^2\theta^* \cos 2\phi^*] . \tag{24}$$

In contrast to Eq. (20), here the dependences on the leptonic variables (θ^*, ϕ^*) are completely *manifest* and those on the hadronic variables $(s, q_P, q_p, q_\perp)$ are completely hidden inside the structure functions. Hence, if very accurate data are available, Eq. (24) furnishes the most efficient means to separate the various structure functions. For example, a conceptually simple procedure to separate all the structure functions is (for a given set of hadronic variables) to use

$$\text{(I)}\ 2W_T + W_L = 12\pi^3\left(\frac{sM}{\alpha}\right)^2 \frac{d\sigma}{d^4q}$$

$$\text{(II)}\ W_T - W_L = 32\pi^3\left(\frac{sM}{\alpha}\right)^2\left[\frac{d\sigma}{d^4q}\left(|\cos\theta^*| > \frac{1}{2}\right) - \frac{d\sigma}{d^4q}\left(|\cos\theta^*| < \tfrac{1}{2}\right)\right],$$

$$\text{(III)}\ W_\Delta = 6\pi^4\left(\frac{sM}{\alpha}\right)^2\left[\frac{d\sigma}{d^4q}(\sin 2\theta^* \cos\phi^* > 0) - \frac{d\sigma}{d^4q}(\sin 2\theta^* \cos\phi^* < 0)\right],$$

and

$$\text{(IV)}\ W_{\Delta\Delta} = 6\pi^4\left(\frac{sM}{\alpha}\right)^2\left[\frac{d\sigma}{d^4q}(\cos 2\phi^* > 0) - \frac{d\sigma}{d^4q}(\cos 2\phi^* < 0)\right].$$

These expressions give all the structure functions as integrated cross sections and asymmetries. [One trivial remark concerning Eqs. (24), and (I) to (IV) above is that there is one redundant variable in d^4q—the azimuthal angle of $\vec{q}$. Integrating over this variable results in replacing (d^4q) by $(\pi dq_P dq_p dq_\perp^2)$ in all these formulas.]

The elegant factorization of leptonic and hadronic degrees of freedom in Eq. (24) is obtained at the expense of partially sacrificing the factorization of kinematics and dynamics: *the "helicity" structure functions*, unlike their invariant cousins, *have a number of kinematic singularities and zeros* (henceforth collectively referred to as "constraints") which may lie close to the region of interest. In other words, the behavior of these structure functions is controlled by both dynamics and kinematics. Care must be exercised in interpreting these behaviors. In addition, one should point out that θ^* and ϕ^* are not directly measured; they are calculated from quantities measured effectively (if not actually) in the hadron c.m. frame. *The transformation formulas are explicitly dependent on the hadronic variables*, and usually have nonsmooth behavior at the same places where the structure functions have kinematic constraints. This fact demands additional care in applying Eq. (24).

In order to illustrate some of these points, let us consider a concrete example. The *s-channel helicity* is defined in terms of a coordinate system with

$$Z^\mu = \frac{M}{(q_p{}^2+q_\perp{}^2)^{1/2}}\,\tilde{P}^\mu ,$$
$$X^\mu = \frac{(q_p{}^2+q_\perp{}^2)^{1/2}}{q_\perp}\,\tilde{p}^\mu + \frac{q_p q_P}{q_\perp(q_p{}^2+q_\perp{}^2)^{1/2}}\,\tilde{P}^\mu . \tag{25}$$

Notice that X^μ is singular as $q_\perp \to 0$ and both X^μ and Z^μ are singular as $(q_\perp{}^2+q_p{}^2)\to 0$. These points are close to the region of interest. (Remember that q_p is q_z in the hadron c.m. frame.) Using Eq. (25) in Eq. (22) and comparing with Eq. (16), one can derive the relations between the s-channel helicity structure functions (distinguished by the superscript s) and the invariant ones:

$$W_T^s = W_1 + \frac{1}{2}\frac{q_\perp{}^2}{q_p{}^2+q_\perp{}^2}W_4 ,$$
$$W_L^s = W_1 + \frac{q_p{}^2+q_\perp{}^2}{M^2}W_2 + \frac{q_p q_P}{M^2}W_3 + \frac{q_p{}^2 q_P{}^2}{M^2(q_p{}^2+q_\perp{}^2)}W_4 , \tag{26}$$
$$W_\Delta^s = \frac{q_\perp}{M}\left(\frac{1}{2}W_3 + \frac{q_p q_P}{q_p{}^2+q_\perp{}^2}W_4\right),$$
$$W_{\Delta\Delta}^s = -\frac{1}{2}\frac{q_\perp{}^2}{q_p{}^2+q_\perp{}^2}W_4 .$$

Since W_i are free from kinematic constraints,[12] we make the following conclusions:

(i) All four W^s-structure functions have a *kinematic singularity* at $q_p{}^2+q_\perp{}^2=0$.

(ii) W_Δ^s has a *kinematic-zero* factor $q_\perp$ as $q_\perp\to 0$ (this is the familiar zero of spin-flip amplitudes for forward scattering due to conservation of angular momentum along the beam axis).

(iii) $W_{\Delta\Delta}^s$ has a *kinematic-zero* factor $q_\perp{}^2$ as $q_\perp \to 0$ [same interpretation as (ii)].

(iv) There is a nontrivial *kinematic-constraint equation*

$$\left[W_L^s - W_T^s - 2\frac{q_p q_P}{q_\perp M}W_\Delta^s - \left(1+2\frac{q_p{}^2 q_P{}^2}{q_\perp{}^2M^2}\right)W_{\Delta\Delta}^s\right] \xrightarrow[(q_p{}^2+q_\perp{}^2)\to 0]{} 0 . \tag{27}$$

This equation can be derived by examining the inverse equations to Eq. (26).

These kinematic constraints close to the physical region of interest make the s-channel helicity structure functions quite delicate to handle.[14]

The kinematic constraints on other sets of "helicity" structure functions can be studied in exactly the same way. The details are presented in Appendix B. For now, we give in Table I the relevant X^μ and Z^μ vectors and depict in Fig. 5 a typical configuration of the various Z vectors. This information is helpful in conveying some idea of the geometric definition of the various "helicity" frames. Also, Table I explicitly gives the locations, if not the details, of the associated kinematic constraints. We see that all "helicity-flip" structure functions will have a (trivial) kinematic zero at $q_\perp=0$ [due to angular momentum conservation, cf (ii) and (iii) above]. Otherwise, the location of the constraints differs from one set to another. In the large M^2 region, the only closeby singularity (among the sets displayed here) is the one in the s-channel helicity case as explicitly given above.

Aside from these kinematic considerations the relative convenience of choosing one frame vs another depends essentially on the dynamics of the system. For instance, as we shall explain in Sec. III, if the quark-parton picture is the dominant mechanism, the Collins-Soper frame[4] offers a special advantage. On the other hand, if s-channel or t-channel exchange mechanisms are important then the corresponding helicity frames will become particularly convenient.

E. Positivity constraints

The positivity requirement on $W^{\mu\nu}$, Eq. (12), can be expressed in terms of the structure functions. Requiring all three eigenvalues of the matrix $W^{\mu\nu}$ (in the three-space orthogonal to q^μ) to be positive, we obtain

$$W_L \geq 0 , \tag{28a}$$
$$W_T \geq |W_{\Delta\Delta}| \geq 0 , \tag{28b}$$
$$W_L(W_T - W_{\Delta\Delta}) \geq W_\Delta{}^2 \tag{28c}$$

in any "helicity" basis. These conditions can, of course, be translated into statements on the invariant structure functions by using relations such as Eq. (26). We have not been able to reduce such statements to a simple and transparent form.

TABLE I. Definition of the coordinate axes of the various helicity frames in terms of the physical momenta $\tilde{P}$ and $\tilde{p}$; listed are the coefficients which occur in the expansions $Z=\alpha\tilde{P}+\beta\tilde{p}$ and $X=\alpha'\tilde{P}+\beta'\tilde{p}$. A typical configurations of the various Z vectors is depicted in Fig. 5.

	Z^μ		X^μ	
	α	β	α'	β'
s-channel helicity	$\dfrac{M}{(q_p^2+q_\perp^2)^{1/2}}$	0	$\dfrac{q_P q_p}{q_\perp(q_p^2+q_\perp^2)^{1/2}}$	$\dfrac{(q_p^2+q_\perp^2)^{1/2}}{q_\perp}$
p helicity	0	$\dfrac{M}{(q_P^2-q_\perp^2)^{1/2}}$	$\dfrac{(q_P^2-q_\perp^2)^{1/2}}{q_\perp}$	$\dfrac{q_P q_p}{q_\perp(q_P^2-q_\perp^2)^{1/2}}$
Collins-Soper frame	$\dfrac{q_p}{(M^2+q_\perp^2)^{1/2}}$	$\dfrac{q_P}{(M^2+q_\perp^2)^{1/2}}$	$-\dfrac{Mq_P}{q_\perp(M^2+q_\perp^2)^{1/2}}$	$-\dfrac{Mq_p}{q_\perp(M^2+q_\perp^2)^{1/2}}$
t-channel helicity	$\dfrac{M}{q_P-q_p}$	$\dfrac{M}{q_P-q_p}$	$\dfrac{-M^2q_P+q_\perp^2q_p}{q_\perp(M^2+q_\perp^2)}$	$\dfrac{-M^2q_p+q_\perp^2q_P}{q_\perp(M^2+q_\perp^2)}$

F. Scaling

In complete analogy to deep-inelastic lepton-hadron scattering, scaling behavior of the structure functions (in the manner of Bjorken) can be introduced without reference to any specific model. The following discussion applies to the invariant as well as to the "helicity" structure functions—all of which have been chosen to be dimensionless. Consider the behavior of W_i as functions of, say, s, q_P, q_p, and $q_\perp$, at very high energies. Neglecting possible mild scaling-violating effects for the moment, the following alternatives suggest themselves.

(i) If, as suggested by usual hadronic collisions, $q_\perp$ remains constant and can be neglected along with the masses as the other variables become very large, W_i will be functions of ratios of the latter, e.g.

$$W_i(s,q_p,q_P,q_\perp)\to W_i(x_1,x_2,q_\perp\approx 0)\,, \tag{29}$$

where $x_{1,2}$ are the usual scaling variables[2]

$$x_{1,2}=(q_P\pm q_p)/\sqrt{s}\ . \tag{30}$$

The Drell-Yan model[2] is, of course, a concrete example of this type of behavior.

(ii) On the other hand, if $q_\perp$ can become large along with the other variables (as occurs in perturbative solutions to all nontrivial field theories) and again the masses are neglected, then

$$W_i(s,q_P,q_p,q_\perp)\to W_i(x_1,x_2,x_\perp)\,, \tag{31}$$

where

$$x_\perp=q_\perp/\sqrt{s}\ . \tag{32}$$

Apart from logarithmic-correction factors, asymptotically free gauge theories furnish examples of this second kind.

Preliminary experimental evidence[1,3] on the average transverse momentum of the virtual photon ($\langle q_\perp\rangle$) does not give a clear-cut edge to either of these possibilities: The fact that $\langle q_\perp\rangle$ approaches a constant as M increases seems to favor (i), but the asymptotic value $\langle q_\perp\rangle\simeq 1$ GeV is considerably higher than that seen in typical hadron collisions; conversely, this relatively large value of $\langle q_\perp\rangle$ may suggest the onset of possibility (ii), but other factors have to be introduced to explain the relative constancy of $\langle q_\perp\rangle$ over the observed range.[5] Obviously, this issue is yet to be settled.

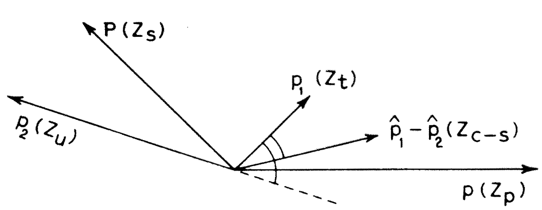

FIG. 5. A typical configuration of the Z axes corresponding to various "helicity" frames listed in Sec II D (and Table I) is depicted here. For the region $q_\perp^2/M^2 \ll 1$, which is of practical interest, the transverse dimension should be further contracted. (This is not done in this picture in order to avoid crowding.)

G. Discussions

The model-independent considerations described up to this point help to separate explicitly the known kinematical characteristics from the unknown dynamics, the latter being represented by the structure functions. These structure functions furnish the most natural arena for the confrontation of theory and experiment.

Unfortunately, complete separation of the structure functions based on Eqs. (20) or (24) represents a rather formidable experimental task. For practical purposes, it is useful to seek simplifications

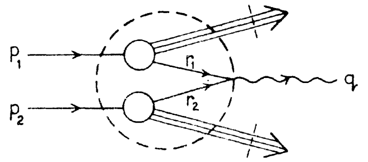

FIG. 6. The Drell-Yan on-shell quark-antiquark annihilation picture for lepton pair production. The dashed circle corresponds to that of Fig. 1.

by making use of the qualitatively successful quark-parton model. For that purpose we shall derive in Sec. III a series of relations between the structure functions based on the quark-parton model. These will reduce the complexity of Eqs. (20) and (24), and render them tractable for phenomenological study. As is well known, similar relations (between F_1, F_2, and F_3) in deep-inelastic eN, μN, and μN scattering have played an important role in the evolution of that field.

Since the quark-parton picture can be viewed as the simplest among all possible interaction mechanisms underlying the lepton pair production process at high energies, the above-mentioned procedure offers an ordered approach which should be useful even if some of the simple relationships eventually turn out to be not fully accurate in certain kinematic regions. The reason is that the parton-model relations can be selectively relaxed and tested against experimental data as the latter become available. (This has been precisely the case in eN, μN, and νN deep-inelastic scattering.) The relative importance and the setting in of alternative mechanisms can be determined phenomenologically by studying their characteristic features expressed in the same convenient structure-function language and by comparing them with observations.

III. THE QUARK-PARTON-MODEL RELATIONS

A. The Drell-Yan formula

The Drell-Yan quark-parton annihilation picture for lepton pair production is depicted in Fig. 6. The parton momenta are labeled r_1 and r_2, respectively. The basic ingredients of this picture are as follows: (i) The partons are on-shell with negligible mass, (ii) they have spin $\frac{1}{2}$, being unpolarized if the parent particles are not polarized, and (iii)

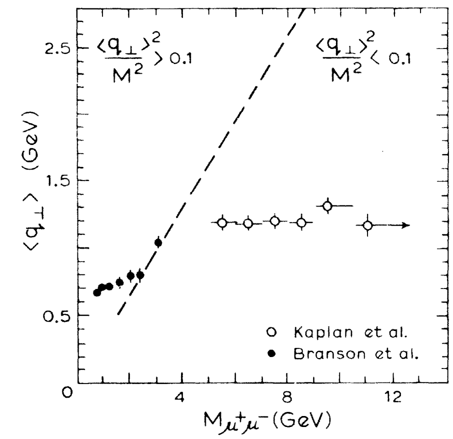

FIG. 7. Reproduction of the experimental data on $\langle q_\perp\rangle^2$ by Kaplan *et al.* (Ref. 3) with the dashed line $\langle q_\perp\rangle^2/M^2 = 0.1$ superimposed. The region to the right of the line corresponds to $\langle q_\perp\rangle^2/M^2 < 0.1$.

the coupling to the virtual photon is that of a pointlike Dirac particle. Both experimental evidence[1,3] and theoretical considerations[15] suggest that transverse momentum for the partons should not be ignored. The nonzero transverse momentum of the virtual photon serves the important function of defining the hadronic reaction plane (Fig. 4) without which the full lepton angular distribution in (θ^*, ϕ^*) cannot manifest itself. We shall not delve into the deep question of the origin of the parton transverse momentum.[15] Suffice it to note that traditionally the parton-model picture is taken to be valid[16] where $q_\perp$ is relatively small compared to M.[17] As it turns out, this is precisely the region where useful relations among the structure functions can be derived. On the experimental side, the average $q_\perp$ is observed to stay relatively constant at ~1 GeV/c for $M > 4$ GeV. Although the value of $\langle q_\perp\rangle$ is larger than previous naive expectations, the condition $\langle q_\perp^2/M^2\rangle \ll 1$ is very well satisfied throughout the high-energy range. Figure 7 illustrates this point. This condition is quite sufficient for our purpose.

The Drell-Yan picture, Fig. 6, assumes that the hadronic tensor amplitude $W^{\mu\nu}$ is given as an incoherent sum over the corresponding elementary amplitude for quark-antiquark annihilation $W^{\mu\nu}$. We use the notation

$$W^{\mu\nu}(P, p, q) = \langle \omega_{\mu\nu}\rangle , \tag{33}$$

$$\omega_{\mu\nu} = \tilde{g}_{\mu\nu} - r_\mu r_\nu / r^2 , \tag{34}$$

and

$$\langle A\rangle = \tfrac{1}{6} sM^2 \int \frac{d\xi_1}{\xi_1} d^2 r_{1\perp} \int \frac{d\xi_2}{\xi_2} d^2 r_{2\perp} (2\pi)^4 \delta^4(q - r_1 - r_2) \times A \sum_i Q_i^2 [u_1^i(\xi_1, r_{1\perp}^2) \bar{u}_2^i(\xi_2, r_{2\perp}^2) + \bar{u}_1^i(\xi_1, r_{1\perp}^2) u_2^i(\xi_2, r_{2\perp}^2)] , \tag{35}$$

where

$$r^\mu = r_1^\mu - r_2^\mu\,,$$
$$\xi_\alpha = (r_\alpha^0 + r_\alpha^z)/(p_\alpha^0 + p_\alpha^z)\,, \quad \alpha = 1, 2 \qquad (36)$$

i is the flavor index of the quark, and $u_\alpha^i(\xi_\alpha, r_{\alpha\perp}^2)$ $[\bar{u}_\alpha^i(\xi_\alpha, r_{\alpha\perp}^2)]$ is the probability of finding a quark (antiquark) of flavor i inside the parent particle α with "longitudinal fraction" of momentum ξ_α and transverse momentum squared $r_{\alpha\perp}^2$.

Many calculations based on this ansatz have been performed in the literature.[1,4] The following study differs from the previous ones in that we seek to single out those features of $W^{\mu\nu}$ which characterize the basic on-shell quark-parton annihilation hypothesis, Eq. (34), independent of details of the relatively uncertain parton distribution functions u_α^i and $\bar{u}_\alpha^i$ in Eq. (35). To be more specific: It is reasonable to expect that the simple tensor structure of $\omega_{\mu\nu}$, Eq. (34) (it is a projection operator onto the two-plane perpendicular to q^μ and r^μ), will be transcribed through Eq. (33) into a simple tensor structure for $W_{\mu\nu}$, thus yielding definite relationships between the structure functions W_i. (Note that W_i are just independent components of $W_{\mu\nu}$.) These relationships intrinsic to the basic parton picture are just what is needed to supplement the general formalism of Sec. II. In contrast, the dependence of $W_{\mu\nu}$ (hence W_i) on q^μ must be model dependent as it arises almost solely from the convolution integral, Eq. (35).

B. Derivation of parton-model relations

We shall now concentrate on the elementary amplitude $\omega_{\mu\nu}$. We decompose $\omega_{\mu\nu}$ into the same tensors that define the invariant-structure functions [(Eq. (16)] and denote the coefficients by $\omega_1 \cdots \omega_4$,[18]

$$\omega_{\mu\nu} = \tilde{g}_{\mu\nu}\omega_i + \tilde{P}_\mu \tilde{P}_\nu \omega_2 - \tfrac{1}{2}(\tilde{P}_\mu \tilde{p}_\nu + \tilde{p}_\mu \tilde{P}_\nu)\omega_3 + \tilde{p}_\mu \tilde{p}_\nu \omega_4\,. \qquad (37)$$

It is then easy to derive the trace relation

$$\omega_\mu^\mu = 3\omega_1 + \left(\frac{q_P^2}{M^2} - 1\right)\omega_2 + \frac{q_p q_P}{M^2}\,\omega_3 + \left(1 + \frac{q_p^2}{M^2}\right)\omega_4 = 2\,. \qquad (38)$$

This implies, according to the notation (35),

$$W_\mu^\mu = 2\langle 1\rangle\,. \qquad (39)$$

In order to evaluate the individual ω_i, we first decompose the vector r^μ in a way similar to Eq. (18b),

$$r^\mu = r_P P^\mu/\sqrt{s} + r_p p^\mu/\sqrt{s} + r_{q\perp} q_\perp^\mu/q_\perp + r_n \hat{n}^\mu\,, \qquad (40)$$

where $\hat{n}$ is the unit vector normal to the hadron reaction plane. (These kinematic details are summarized in Appendix A.) In the hadronic c.m. frame (which is the natural frame to formulate the parton picture), r_P, r_p, $r_{q\perp}$, and r_n are simply r^0, r_z, r_x, and r_y, respectively. Since r^μ must satisfy

$$r\cdot q = 0\,,$$
$$r^2 = -q^2 = M^2\,, \qquad (41)$$

its components are related to each other by

$$q_\perp r_{q\perp} + q_p r_p - q_P r_P = 0\,,$$
$$r_n^2 + r_{q\perp}^2 + r_p^2 - r_P^2 = M^2\,. \qquad (42)$$

In the following, we shall regard $r_{q\perp}$ and r_n as the independent components. From Eqs. (34) and (37) we get

$$\hat{n}_\mu \omega^{\mu\nu} \hat{n}_\nu = 1 - \frac{r_n^2}{M^2} = \omega_1\,. \qquad (43)$$

Similarly, by evaluating $\tilde{P}_\mu\omega^{\mu\nu}\tilde{P}_\nu, \tilde{p}_\mu\omega^{\mu\nu}\tilde{p}_\nu, \tilde{p}_\mu\omega^{\mu\nu}\tilde{P}_\nu$ and then taking, in turn, an appropriate linear combination with (38) and (43), we get

$$\omega_2 = (r_p^2 - q_P^2 + q_\perp^2)/q_\perp^2 + 2(q_P^2 - q_\perp^2) r_n^2/M^2 q_\perp^2\,,$$
$$-\tfrac{1}{2}\omega_3 = (r_p r_P - q_p q_P)/q_\perp^2 + 2 q_p q_P r_n^2/M^2 q_\perp^2\,, \qquad (44)$$
$$\omega_4 = (r_P^2 - q_p^2 - q_\perp^2)/q_\perp^2 + 2(q_p^2 + q_\perp^2) r_n^2/M^2 q_\perp^2\,.$$

The invariant structure functions $W_1, \ldots, W_4$ are obtained from Eqs. (43) and (44) by taking the convolution integral, Eq. (35), on both sides of the equations. They are, thereby, expressed in terms of $\langle 1\rangle$, $\langle r_n^2\rangle$, $\langle r_p^2\rangle$, $\langle r_p r_P\rangle$, and $\langle r_P^2\rangle$. All these quantities depend on the form of the parton distribution functions. Without commitment to any particular set of parton functions, all these quantities are unrelated and no model-independent predictions on the structure function can be made. However, since we are interested in the region where $q_\perp^2/M^2$ is small, much simplification can be brought out. In particular, we show in Appendix A that

$$r_P = q_p\left(1 - \frac{r_n^2 + r_{q_\perp}^2 + q_\perp^2}{2M^2}\right) + q_P\,\frac{q_\perp r_{q_\perp}}{M^2} + O\!\left(\frac{q_\perp^4}{M^4}\right)\,, \qquad (45)$$
$$r_p = q_P\left(1 - \frac{r_n^2 + r_{q_\perp}^2 + q_\perp^2}{2M^2}\right) + q_p\,\frac{q_\perp r_{q_\perp}}{M^2} + O\!\left(\frac{q_\perp^4}{M^4}\right)\,.$$

Substituting into Eq. (44), and taking the convolution integral, we obtain

$$W_2 = \langle 1\rangle - \frac{q_P^2}{M^2}\left(\langle 1\rangle + \left\langle\frac{r_{q_\perp}^2 - r_n^2}{q_\perp^2}\right\rangle\right) + 2\,\frac{q_p q_P}{M^2}\left\langle\frac{r_{q_\perp}}{q_\perp}\right\rangle\,,$$
$$W_3 = 2\,\frac{q_p q_P}{M^2}\left(\langle 1\rangle + \left\langle\frac{r_{q_\perp}^2 - r_n^2}{q_\perp^2}\right\rangle\right) - 2\,\frac{q_p^2 + q_P^2}{M^2}\left\langle\frac{r_{q_\perp}}{q_\perp}\right\rangle\,, \qquad (46)$$
$$W_4 = \langle 1\rangle - \frac{q_p^2}{M^2}\left(\langle 1\rangle + \left\langle\frac{r_{q_\perp}^2 - r_n^2}{q_\perp^2}\right\rangle\right) + 2\,\frac{q_p q_P}{M^2}\left\langle\frac{r_{q_\perp}}{q_\perp}\right\rangle\,.$$

All terms exhibited here are of order unity, those neglected are of order $q_\perp^2/M^2$. To the same accuracy, Eqs. (38) and (43) yield

$$W_1 = \langle 1 \rangle = \tfrac{1}{2} W^\mu{}_\mu . \tag{47}$$

A little scrutiny of these results will reveal that they are not all linearly independent; the structure functions are related by

$$\text{(A)} \quad W_1 + \frac{q_p^2}{M^2} W_2 + \frac{q_p q_P}{M^2} W_3 + \frac{q_P^2}{M^2} W_4 = 0 .$$

This is the first of the several parton-model relations we shall derive.

Equation (46), as it stands, cannot be further reduced. The three structure functions are expressed in terms of three unknowns $\langle 1 \rangle$, $\langle r_{q_\perp}{}^2 - r_n{}^2 \rangle / q_\perp{}^2$, and $\langle r_{q_\perp} \rangle / q_\perp$. However, we now argue that the last two quantities are expected to be zero or, at least, small as compared to the first. First, in the hadronic c.m. frame where the parton variables are most naturally defined,

$$r_{q_\perp} = r_x = r_{1x} - r_{2x} = (r_{1\perp}{}^2 - r_{2\perp}{}^2)/q_\perp , \tag{48}$$

cf. Appendix A; hence,

$$\left\langle \frac{r_{q_\perp}}{q_\perp} \right\rangle = \frac{sM^2}{6 q_\perp^2} \int \frac{d\xi_1}{\xi_1} d^2 r_{1\perp} \int \frac{d\xi_2}{\xi_2} (2\pi)^4 \delta^4(q - r_1 - r_2) d^2 r_{2\perp} (r_{1\perp}{}^2 - r_{2\perp}{}^2) \times \sum_i Q_i{}^2 [u_1^i(\xi_1, r_{1\perp}{}^2) \bar{u}_2^i(\xi_2, r_{2\perp}{}^2) + \bar{u}_1^i(\xi_1, r_{1\perp}{}^2) u_2^i(\xi_2, r_{2\perp}{}^2)] . \tag{49}$$

This quantity vanishes if r_x is equally likely to be positive as negative or, mathematically, if the square bracket containing products of the parton distribution functions is *even* under the exchange $r_1{}^2 \leftrightarrow r_2{}^2$. This is certainly the case if (i) the transverse-momentum distribution is the same for the two factors, and (ii) it is not strongly coupled to the longitudinal-momentum distribution. For N-N and N-$\bar{N}$ scattering, condition (ii) is all that is needed since in these cases either $u_1 = u_2$ or $u_1 = \bar{u}_2$. Also, for any initial state, near the region $q_p \simeq 0 (x_1 \simeq x_2)$, where much data exist, condition (i) alone is sufficient. In general, even if the symmetry is not exact, it is reasonable to expect that

$$\left\langle \frac{r_{q_\perp}}{q_\perp} \right\rangle = \left\langle \frac{r_{1\perp}{}^2 - r_{2\perp}{}^2}{q_\perp^2} \right\rangle \ll 1 . \tag{50}$$

Equation (50), when used in conjunction with (46), implies

$$\text{(B)} \quad q_p^2 W_2 - q_P^2 W_4 = (q_p^2 + q_P^2) W_1 .$$

Similarly, in terms of hadronic c.m. variables,

$$\left\langle \frac{r_{q_\perp}{}^2 - r_n{}^2}{q_\perp^2} \right\rangle = \left\langle \frac{r_x{}^2 - r_y{}^2}{q_\perp^2} \right\rangle . \tag{51}$$

This quantity vanishes if the independent variables $r_x{}^2$ and $r_y{}^2$ have the same expection value or, equivalently, if the square bracket of (49) is even under the interchange $r_x{}^2 \leftrightarrow r_y{}^2$. This is exactly true, for example, if the transverse momentum distribution is represented by a Gaussian curve. For other reasonable choices, one can only say that one expects

$$\left\langle \frac{r_x{}^2 - r_y{}^2}{q_\perp^2} \right\rangle \ll \langle 1 \rangle . \tag{52}$$

Equation (52), together with (46), implies

$$\text{(C)} \quad W_2 - W_4 \doteq W_1 .$$

Finally, combining (A), (B), and (C) we get the simple relations

$$\text{(D)} \quad 2W_1 = -\frac{2M^2}{q_p^2} W_2 = \frac{M^2}{q_p q_P} W_3 = \frac{2M^2}{q_P^2} W_4 = W^\mu{}_\mu .$$

These relations specify all the structure functions in terms of the most readily measurable $W^\mu{}_\mu$, cf. Eq. (7). Utilizing (D), the full differential cross-section formula, Eq. (20), can be written as

$$d\sigma = \frac{\alpha^2}{32\pi^3} \frac{1}{s^2} \frac{\cos^2(\Theta/2)}{\sin^4(\Theta/2)} dk_1 dk_2 d\cos\theta_1 d\cos\theta_2 d\phi \times W^\mu{}_\mu \left(\tan^2\frac{\Theta}{2} - \frac{q_p^2}{2M^2} + \frac{q_p q_P}{M^2} \frac{\cos\theta_1 + \cos\theta_2}{1 + \cos\Theta} - \frac{q_P^2}{2M^2} \frac{1 + \cos\Theta'}{1 + \cos\Theta} \right) , \tag{53}$$

where $W^\mu{}_\mu$ itself is determined from Eq. (7).

C. Alternative derivation in terms of helicity structure functions

We derived the parton-model relations (A)–(D) in terms of invariant functions because they reflect the full dynamical structure of the system without any complications from kinematics. Additional insight into the nature of these relations can be gained by examining the alternative derivation of these relations in terms of the helicity structure functions. Some care has to be exercised in this approach because these structure functions have kinematic constraints close to the region of interest, $q_\perp^2/M^2 \ll 1$ (cf. Sec. II D).

Let (X,Y,Z) be one of the sets of unit vectors which define the "helicity" frame, and r_X, r_Y, r_Z be the components of r^μ along these axes (i.e., $r_X = r^\mu X_\mu \cdots$ etc.). Then, by contracting $W_{\mu\nu}$, Eq. (34), with the vectors Z^μ and $(\mp X^\mu - iY^\mu)/\sqrt{2}$ on both sides, we can get

$$\begin{aligned} W_T &= \langle 1\rangle - \frac{1}{2}\left\langle \frac{r_X^2+r_Y^2}{M^2}\right\rangle ,\\ W_L &= 0 + \left\langle \frac{r_X^2+r_Y^2}{M^2}\right\rangle ,\\ W_\Delta &= \left\langle \frac{r_X}{M}\left(1-\frac{r_X^2+r_Y^2}{M^2}\right)^{1/2}\right\rangle ,\\ W_{\Delta\Delta} &= \frac{1}{2}\left\langle \frac{r_X^2-r_Y^2}{M^2}\right\rangle . \end{aligned} \tag{54}$$

Since the normal to the hadronic reaction plane is an invariant under all coordinate transformations that we are concerned with, r_Y is the same as r_n (the y component of $\vec{r}$ in the hadron c.m. frame). On the other hand, r_X is, in general, not the same as $r_{q_\perp}$ (the x component of $\vec{r}$ in the hadron c.m. frame). Let us now restrict our attention to the $q_\perp^2/M^2 \ll 1$ region and consider only those "helicity" frames in which r_X is of order $q_\perp$. These include all the examples cited in Sec. II D (plus any others for which the coefficients α, β in Table I do not contain $q_\perp$ in the denominator). Recognizing that W_Δ and $W_{\Delta\Delta}$ have kinematic zeros of order $q_\perp$ and $q_\perp^2$ respectively (Sec. II D), one should consider the quantities W_T, W_L, $(M/q_\perp)W_\Delta$, and $(M^2/q_\perp^2)W_{\Delta\Delta}$ before dropping small terms. Equation (54) then implies[19]

$$\begin{aligned} W_T &= \langle 1\rangle ,\\ W_L &= 0 ,\\ \frac{M}{q_\perp} W_\Delta &= \left\langle \frac{r_X}{q_\perp}\right\rangle ,\\ \frac{M^2}{q_\perp^2} W_{\Delta\Delta} &= \frac{1}{2}\left\langle \frac{r_X^2-r_Y^2}{q_\perp^2}\right\rangle . \end{aligned} \tag{55}$$

It can be shown that the second of these relations is equivalent to Eq. (A) of Sec. III B section. This is therefore the analog of the Callan-Gross relation[20] in deep-inelastic lepton-hadron scattering.

With regard to the last two equations of (55), one might be tempted to set the right-hand side equal to zero in analogy with what was done in the previous section. But that would be wrong. The reason is the variable r_X does not have a direct parton-model interpretation in general. For each choice of "helicity" frame, one has to reexpress r_X in terms of the hadronic c.m. frame variables $(r_{q_\perp}, r_n)$ before one can proceed any further. Working to the leading order in an expansion in $q_\perp^2/M^2$, there is a "preferred" frame for which

$$r_X \simeq r_{q_\perp} . \tag{56}$$

This turns out to be the Collins-Soper frame[4] (as can be verified by using the formulas given in Appendix A and Table I). Denoting the structure functions corresponding to this choice of frame by $W^{\rm CS}$, arguments of the previous sections lead to the expectation

$$\frac{M}{q_\perp} W_\Delta^{\rm CS} \simeq 0 , \tag{57}$$

$$\frac{M^2}{q_\perp^2} W_{\Delta\Delta}^{\rm CS} \simeq 0 . \tag{58}$$

Equation (57) is equivalent to Eq. (B) and Eq. (58) to Eq. (C) of Sec. III B, respectively.

The situations in other "helicity" frames are rather different. For instance, in the "t-channel helicity" frame (Sec. II D and Table I)

$$\begin{aligned} r_Z &\simeq M ,\\ r_X &\simeq r_{q_\perp} + q_\perp ,\\ r_X^2 - r_Y^2 &\simeq r_{q_\perp}^2 - r_n^2 + q_\perp^2 + 2q_\perp r_{q_\perp} . \end{aligned} \tag{59}$$

Assuming $\langle r_{q_\perp}\rangle = \langle r_{q_\perp}^2 - r_n^2\rangle = 0$ as before, we get for the corresponding structure functions W^t

$$\frac{M}{q_\perp} W_\Delta^t \simeq \langle 1\rangle = W_T^t , \tag{60}$$

$$\frac{M^2}{q_\perp^2} W_{\Delta\Delta}^t = \tfrac{1}{2}\langle 1\rangle = \tfrac{1}{2} W_T^t . \tag{61}$$

Equation (60) is equivalent to Eq. (B), but Eq. (61) represents a combination of Eq. (B) and Eq. (C) of Sec. III B.

These results inspire the following observation: If the on-shell quark-parton annihilation picture and the assumed symmetries for the products of parton wave functions are valid, the lepton angular distribution should be particularly simple (very close to $1+\cos^2\theta^*$) in the Collins-Soper frame; conversely, *the best way to test these features of the parton model is to contrast the angular distribution of the leptons in the Collins-Soper frame with those in other frames*. For this latter purpose, the t-channel helicity frame seems to be a good choice. (The "s-channel helicity" structure functions are singular for small $q_p{}^2+q_\perp{}^2$; the "p-helicity" frame nearly coincides with the Collins-Soper frame if q_p/q_P is small as is usually the case, cf. Fig. 5.) If the basic interaction mechanism were different from that of the on-shell quark-antiquark annihilation, one would not expect this distinctive role played by the Collins-Soper frame.

D. Summary

(i) We presented the derivation of the parton-model relations in terms of the invariant structure functions first, because it is free from possible ambiguities caused by kinematic constraints.

(ii) The restatement of these results in terms of helicity-structure functions brings out more clearly the physical meaning of these relations and singles out the Collins-Soper frame as the most "natural" one for these parton-model considerations (provided $q_\perp{}^2/M^2$ is small).

(iii) The best way to check these parton-model relations is to determine if the lepton angular distribution is closer to $1+\cos^2\theta^*$ in the Collins-Soper frame than any others.

(iv) Deviations of this angular distribution from $1+\cos^2\theta^*$ in other frames are expected to show up most prominently in the ϕ^* distribution (W_Δ term) rather than in the θ^* distribution integrated over ϕ^*; the former is of order $q_\perp/M$, the latter of order $q_\perp{}^2/M^2$ (counting theoretical factors only).

IV. CONCLUDING REMARKS

(i) The structure-function language presented in some detail in Sec. II should be useful as the common meeting ground of theory and experiment because it is model independent and because it clearly separates dynamics and kinematics.

(ii) The invariant structure functions $W_1, \ldots, W_4$ are unique in being almost totally free from kinematic constraints; their behavior reflects the dynamics of the hadron system in unadorned form.

(iii) There are a large number of possible choices of "helicity" structure functions; one may be more suitable than the others in considering a particular dynamical model, and all have some well defined kinematic singularities and zeros which should be taken explicitly into account in these considerations.

(iv) The quark-parton picture is the "zeroth-order" interaction mechanism for a large class of dynamical models. Its usual domain of applicability is where M is large and $q_\perp{}^2/M^2$ is small. The parton-model relations (A)–(D) derived in Sec. IIIB for this region serve as the starting point of a systematic approach to the lepton pair production process in a way which is familiar in lepton-hadron scattering.

(v) Important virtues of this approach over conventional parton-model calculations are the following: (a) The explicit form for the parton distributions (of which there is considerable uncertainty) is never needed; the main results reflect characteristics of the basic on-shell quark-antiquark annihilation mechanism. (b) This approach is flexible; each of the parton-model relations can be individually withdrawn when not needed. (c) As these relations are tested by improved data, they help to define quantitatively the region of validity of the Drell-Yan mechanism.

(vi) More sophisticated models can be studied in the same way as done in Sec. III for the "minimal" model. The characteristics of the former can then be clearly contrasted with those of the latter.

ACKNOWLEDGMENT

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APPENDIX A

This appendix consists of a collection of notations and kinematic details.

(1) *The metric* is given by $-g^{00}=g^{ii}=1$.

(2) *Hadronic c.m. frame variables*:

(a) The vectors $P=p_1+p_2$ and $p=p_1-p_2$ satisfy $-P^2=p^2=s$ and $p\cdot P=0$. In the hadronic c.m. frame $P^\mu/\sqrt{s}=(1,0,0,0)$ and $p^\mu/\sqrt{s}=(0,0,0,1)$. They define the t and z axes for this frame.

(b) For any vector v (e.g., q, k, r_1, r_2, r) define

$$v^\mu = v_P\frac{P^\mu}{\sqrt{s}} + v_p\frac{p^\mu}{\sqrt{s}} + v_\perp^\mu, \tag{A1}$$

where $v_\perp\cdot P = v_\perp\cdot p = 0$, then

$$v_P = -v\cdot P/\sqrt{s}\,, \tag{A2}$$

$$v_p = v\cdot p/\sqrt{s}\,, \tag{A3}$$

and

$$v^2 = -v_P{}^2 + v_p{}^2 + v_\perp{}^2. \tag{A4}$$

(c) If we pick the x-z plane to be that of the hadronic reaction plane, the x axis is along the vector $q_\perp^\mu$. Hence for an arbitrary vector (e.g., k, r_1, r_2, r) we can further decompose $v_\perp^\mu$ into

$$v_\perp^\mu = v_{q_\perp}\frac{q_\perp^\mu}{q_\perp} + v_n\hat{n}^\mu, \tag{A5}$$

where $\hat{n}^\mu$ is the unit vector normal to the hadronic reaction plane. It is easy to see that

$$v_{q_\perp}=(v_\perp\cdot q_\perp)/q_\perp=(v\cdot q_\perp)/q_\perp\,,\qquad v_n=v_\perp\cdot\hat{n}=v\cdot\hat{n}\,, \tag{A6}$$

and

$$v^2=-v_P{}^2+v_p{}^2+v_{q_\perp}{}^2+v_n{}^2\,. \tag{A7}$$

These covariant definitions are convenient for theoretical calculations. In practical terms, the four Cartesian components of v^μ in the hadronic c.m. frame are simply

$$v^\mu=(v_P,v_{q_\perp},v_n,v_p)\,. \tag{A8}$$

(3) *The lepton-pair c.m. frame*:

(a) The time axis is specified by q^μ. The three-space consists of vectors orthogonal to q^μ. The hadronic vectors which serve to define the spatial coordinate axes are linear combinations of

$$\tilde{P}_\mu=\tilde{g}_{\mu\nu}P^\nu/\sqrt{s} \tag{A9}$$

and

$$\tilde{p}_\mu=\tilde{g}_{\mu\nu}p^\nu/\sqrt{s}\,, \tag{A10}$$

where

$$\tilde{g}_{\mu\nu}=g_{\mu\nu}-q_\mu q_\nu/q^2=g_{\mu\nu}+q_\mu q_\nu/M^2\,. \tag{A11}$$

$\tilde{g}$ has the property that, when contracted with any four-vector v^μ, it yields another vector $\tilde{v}^\mu$ (the projection of v) which is orthogonal to q^μ.

(b) $\tilde{P}$ and $\tilde{p}$ has the following properties:

$$\tilde{P}^2=-1+q_P{}^2/M^2=(q_p{}^2+q_\perp{}^2)/M^2\,, \tag{A12}$$

$$\tilde{p}^2=1+q_p{}^2/M^2=(q_P{}^2-q_\perp{}^2)/M^2\,, \tag{A13}$$

$$\tilde{p}\cdot\tilde{P}=-q_pq_P/M^2\,. \tag{A14}$$

A very useful identity is

$$\tilde{P}^2\tilde{p}^2-(\tilde{p}\cdot\tilde{P})^2=q_\perp{}^2/M^2\,. \tag{A15}$$

(4) *Parton-model kinematics*:

(a) In the hadronic c.m. frame, one can parametrize the parton momenta as follows:

$$r_1^\mu=\left(\tfrac{1}{2}\xi_1\sqrt{s}+\frac{1}{2}\frac{r_{1\perp}{}^2}{\xi_1\sqrt{s}},\vec{r}_{1\perp},\tfrac{1}{2}\xi_1\sqrt{s}-\frac{1}{2}\frac{r_{1\perp}{}^2}{\xi_1\sqrt{s}}\right),\qquad r_2^\mu=\left(\tfrac{1}{2}\xi_2\sqrt{s}+\frac{1}{2}\frac{r_{2\perp}{}^2}{\xi_2\sqrt{s}},\vec{r}_{2\perp},-\tfrac{1}{2}\xi_2\sqrt{s}+\frac{1}{2}\frac{r_{2\perp}{}^2}{\xi_2\sqrt{s}}\right), \tag{A16}$$

which satisfy

$$\left.\begin{aligned} &r_\alpha{}^2=0\\ &\frac{r_\alpha^0+r_\alpha^z}{p_\alpha^0+p_\alpha^z}=\xi_\alpha\end{aligned}\right\}\ \alpha=1,2\,. \tag{A17}$$

Here ξ_α is the "longitudinal fraction momentum" of the parton with respect to the parent particle. [See Eqs. (A23 and A24) below.]

(b) The energy-momentum-conservation condition

$$q^\mu=r_1^\mu+r_2^\mu \tag{A18}$$

yields four constraints among the six parton parameters $(\xi_1,\vec{r}_{1\perp},\xi_2,\vec{r}_{2\perp})$. Since the parton wave functions depend on $r_{1\perp}{}^2$ and $r_{2\perp}{}^2$, let us express all other parameters in terms of these two. It suffices to specify the components of $r=r_1-r_2$, since $r_1=\frac{1}{2}(q+r)$ and $r_2=\frac{1}{2}(q-r)$. To order $q_\perp{}^2/M^2$, the various components of r can be calculated. The results are

$$r_P=\tfrac{1}{2}\sqrt{s}\left(\xi_1-\xi_2+\frac{r_{1\perp}{}^2}{\xi_1 s}-\frac{r_{2\perp}{}^2}{\xi_2 s}\right)\simeq q_p-q_p\frac{r_{1\perp}{}^2+r_{2\perp}{}^2}{M^2}+q_P\frac{r_{1\perp}{}^2-r_{2\perp}{}^2}{M^2}\,, \tag{A19}$$

$$r_p=\tfrac{1}{2}\sqrt{s}\left(\xi_1+\xi_2-\frac{r_{1\perp}{}^2}{\xi_1 s}-\frac{r_{2\perp}{}^2}{\xi_2 s}\right)\simeq q_P-q_P\frac{r_{1\perp}{}^2+r_{2\perp}{}^2}{M^2}+q_p\frac{r_{1\perp}{}^2-r_{2\perp}{}^2}{M^2}\,, \tag{A20}$$

$$r_{q_\perp}=r_{1x}-r_{2x}=(r_{1\perp}{}^2-r_{2\perp}{}^2)/q_\perp\,, \tag{A21}$$

$$r_n=r_{1y}-r_{2y}=(M^2+r_P{}^2-r_p{}^2-r_{q_\perp}{}^2)^{1/2}\,. \tag{A22}$$

One can also see that

$$\xi_1\simeq x_1-\frac{r_{2\perp}{}^2}{x_2 s}\,,\qquad \xi_2\simeq x_2-\frac{r_{1\perp}{}^2}{x_1 s}\,, \tag{A23}$$

where x_1 and x_2 are the conventional longitudinal fractional momenta in the absence of transverse momentum:

$$x_{1,2}=(q_P\pm q_p)/\sqrt{s}\,. \tag{A24}$$

(c) Since the components of the elementary hadron tensor $\omega^{\mu\nu}$ are expressed in terms of components of r^μ, it is also useful to use as independent variables $r_{q_\perp}$ and r_n (instead of $r_{1\perp}{}^2$ and $r_{2\perp}{}^2$). From $r_{1,2}=\frac{1}{2}(q\pm r)$, one directly derives

$$r_{1\perp}{}^2=\tfrac{1}{4}[(q_\perp+r_{q_\perp})^2+r_n{}^2]\,,\qquad r_{2\perp}{}^2=\tfrac{1}{4}[(q_\perp-r_{q_\perp})^2+r_n{}^2]\,, \tag{A25}$$

or

$$r_{1\perp}{}^2+r_{2\perp}{}^2=\tfrac{1}{2}(q_\perp{}^2+r_{q_\perp}{}^2+r_n{}^2)\,,\qquad r_{1\perp}{}^2-r_{2\perp}{}^2=q_\perp r_{q_\perp}\,. \tag{A26}$$

These results can be substituted into Eqs. (A19 and A20) to obtain Eq. (45) of the text.

APPENDIX B

For various theoretical and practical considerations, one may want to transform from the invariant structure functions to the various "helicity" structure functions and vice versa. It is therefore useful to have ready at hand the transformation formulas. One set of such formulas, for the s-channel helicity, is already given in the text [Eq. (26)]. Here we give the results for the other sets of "helicity" structure functions listed in Table I.

Using the definition of Z^μ and X^μ vectors given in Table I and comparing the two equivalent expansions Eq. (16) and Eq. (22), one can obtain by straightforward calculations the following results:

(i) for the p-helicity structure functions,

$$\begin{aligned} W_T^p &= W_1 + \frac{1}{2}\frac{q_\perp^2}{q_P^2 - q_\perp^2} W_2 , \\ W_L^p &= W_1 + \frac{q_p^2 q_P^2}{M^2(q_P^2 - q_\perp^2)} W_2 + \frac{q_p q_P}{M^2} W_3 + \frac{q_P^2 - q_\perp^2}{M^2} W_4 , \\ W_\Delta^p &= \frac{q_\perp}{M}\left[\frac{q_p q_P}{q_P^2 - q_\perp^2} W_2 + \tfrac{1}{2} W_3\right], \\ W_{\Delta\Delta}^p &= -\tfrac{1}{2}\frac{q_\perp^2}{q_P^2 - q_\perp^2} W_2 . \end{aligned} \tag{B1}$$

(ii) For the structure functions in the Collins-Soper frame,

$$\begin{aligned} W_T^{CS} &= W_1 + \frac{q_\perp^2}{2M^2}\frac{q_P^2 W_2 + q_p q_P W_3 + q_p^2 W_4}{M^2 + q_\perp^2} , \\ W_L^{CS} &= W_1 + \frac{q_p^2 W_2 + q_p q_P W_3 + q_P^2 W_4}{M^2 + q_\perp^2} , \\ W_\Delta^{CS} &= -\frac{q_\perp}{M}\frac{q_p q_P (W_2 + W_4) + \frac{1}{2}(q_p^2 + q_P^2) W_3}{M^2 + q_\perp^2} , \\ W_{\Delta\Delta}^{CS} &= -\frac{q_\perp^2}{2M^2}\frac{q_P^2 W_2 + q_p q_P W_3 + q_p^2 W_4}{M^2 + q_\perp^2} . \end{aligned} \tag{B2}$$

(iii) For the t-channel helicity structure functions,

$$\begin{aligned} W_T^t &= W_1 + \frac{1}{2}\frac{q_\perp^2}{(q_P - q_p)^2}(W_2 + W_3 + W_4) , \\ W_L^t &= W_1 + \left(\frac{q_P}{M} - \frac{M}{q_P - q_p}\right)^2 W_2 + \left(\frac{q_p}{M} - \frac{M}{q_P - q_p}\right)\left(\frac{q_p}{M} - \frac{M}{q_P - q_p}\right) W_3 + \left(\frac{q_p}{M} - \frac{M}{q_P - q_p}\right)^2 W_4 , \\ W_\Delta^t &= \frac{q_\perp}{M}\left[\left(\frac{q_p}{M} - \frac{M}{q_P - q_p}\right)(W_2 + \tfrac{1}{2} W_3) + \left(\frac{q_p}{M} - \frac{M}{q_P - q_p}\right)(W_4 + \tfrac{1}{2} W_3)\right] . \\ W_{\Delta\Delta}^t &= -\frac{1}{2}\frac{q_\perp^2}{(q_P - q_p)^2}(W_2 + W_3 + W_4) . \end{aligned} \tag{B3}$$

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*On leave from Illinois Institute of Technology, Chicago, Ill.

[1]For recent reviews and extensive references see, for example, T.-M. Yan, talk given at 8th International Symposium on Multiparticle Dynamics, Kaysersberg, France, 1977, Cornell Report No. CLNS 369 (unpublished); L. M. Lederman, in *Proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg*, 1977, edited by F. Gutbrod (DESY, Hamburg, 1977).

[2]S. Drell and T.-M. Yan, Phys. Rev. Lett. 25, 316 (1970); Ann. Phys. (N.Y.) 66, 578 (1971).

[3]D. M. Kaplan *et al.*, Phys. Rev. Lett. 40, 435 (1978); J. G. Branson *et al.*, *ibid.* 38, 457 (1977); 38, 1334 (1977); and recent results reported at XIIIth Rencontre de Moriond, 1978 (unpublished).

[4]K. V. Vasavada, Phys. Rev. D 16, 146 (1977); M. Duong-van, K. Vasavada, and R. Blankenbecler, *ibid.* 16, 1389 (1977); J. C. Collins and D. E. Soper, *ibid.* 16, 2219 (1977); E. L. Berger, J. T. Donohue, and S. Wolfram, *ibid.* 17, 858 (1978).

[5]An incomplete list is J. Kogut and J. Shigemitsu, Phys. Lett. 71B, 165 (1977); I. Hinchliffe and C. H. Llewellyn Smith, *ibid.* 66B, 281 (1977); D. Politzer, Nucl. Phys. B129, 301 (1977); C. S. Lam and T.-M. Yan, Phys. Lett. 71B, 173 (1977); K. Kajantie and R. Raitio, Helsinki Report No. HU-TFT-77-21 (unpublished); K. Kajantie, J. Lindfors, and R. Raitio, Phys. Lett. 74B, 384 (1978); G. Altarelli, G. Parisi, and R. Petronzio, *ibid.* 76B, 351 (1978); 76B, 356 (1978); H. Fritzsch and P. Minkowski, *ibid.* 73B, 80 (1978); C. T. Sachrajda, *ibid.* 73B, 185 (1978).

[6]H. Terazawa, Phys. Rev. D 8, 3026 (1973).

[7]R. J. Oakes, Nuovo Cimento 44, 440 (1966).

[8]The model considerations are similar in spirit to those of Collins and Soper (Ref. 4); these authors, however, stopped short of arriving at the parton-model relations. Details are given in Sec. IIIC and Ref. 19.

[9]We use the metric $-g^{00} = g^{ii} = 1$ and the covariant normalization of states $\langle p | p' \rangle = (2\pi)^3 2p^0 \delta^3(p - p')$. Dirac spinors are normalized to $\bar{u}(k)u(k) = 2m$.

[10]This approximation is in no way essential to any of the derivations contained in Sec. II. Since it is a very good approximation, the simplifications brought about by its adoption are obtained with no loss in practice.

[11]W. Bardeen and W.-K. Tung, Phys. Rev. 173, 1423 (1968).

[12]Strictly speaking, W_i have trivial kinematic zeros at $q^2 = 0$ and $s = 0$, forced upon them by the denominators of Eqs. (13) and (14). But these are far from the physical region of interest and hence quite harmless. A set of "minimal" invariant amplitudes (cf. Ref. 11) which are free from *all* kinematic singularities and

zeros can be defined as the coefficients of $q^2g^{\mu\nu}-q^\mu q^\nu$; $q^2P^\mu P^\nu-(q\cdot P)(P^\mu q^\nu+q^\mu P^\nu)+(q\cdot P)^2g^{\mu\nu}$; $q^2(P^\mu p^\nu+p^\mu P^\nu)-(q\cdot P)(q^\mu p^\nu+p^\mu q^\nu)-(q\cdot p)(q^\mu P^\nu+P^\mu q^\nu)+2g^{\mu\nu}(q\cdot p)(q\cdot P)$, and $q^2p^\mu p^\nu-(q\cdot p)(p^\mu q^\nu+q^\mu p^\nu)+g^{\mu\nu}(q\cdot p)^2$. However, this set offers no practical advantage over the more convenient set $\{W_i\}$.

[13]These are also the density matrix elements for the virtual photon in traditional terminology, cf. Ref. 7.

[14]This is the basic reason why numerical calculation, done in this frame exhibit nonsmooth behavior; cf. Berger *et al.*, Ref. 4.

[15]Uncertainty principle alone would imply a nonvanishing average transverse momentum. The precise nature of the transverse momentum distribution is, of course, a deep theoretical problem involving the quark-confinement and other interaction mechanisms.

[16]R. F. Feynman, *Photon-Hadron Interactions* (Benjamin, N.Y., 1972).

[17]A recent proposal for an alternative viewpoint is given by R. C. Hwa, S. Matsuda, and R. G. Roberts, Rutherford Report No. RL-77-117/A and CERN Report No. TH-2456, 1978 (unpublished).

[18]Strictly speaking two other tensors of the form $\tilde{P}_\mu n_\nu+n_\mu\tilde{P}_\nu$ and $\tilde{p}_\mu n_\nu+n_\mu\tilde{p}_\nu$ also come in. But the coefficients average out to zero when inserted into Eq. (33) as a consequence of parity conservation. Hence, we ignore them from the beginning. (A term of the form $n_\mu n_\nu$ can always be rewritten as a linear combination of the others.)

[19]Results eqiuvalent to these were obtained previously by Collins and Soper (Ref. 4), in a particular frame (referred to here as the Collins-Soper frame). Differences in details are (i) they kept the next-to-leading terms in the first two equations of Eq. (54), (ii) their expressions for $W_{\Delta\Delta}$ (A_2, in their notation) are more complicated in appearance, hence they could not give it a physical interpretation, and (iii) they did not suggest that $(M/q_\perp)W_\Delta$ and $(M^2/q_\perp^2)W_{\Delta\Delta}$ may be small; cf. Eq. (57) and (58). The last point is, of course, what enabled us to derive the parton-model relations (B) and (C). Concerning (i), we remark that although W_L and $W_{\Delta\Delta}$ are both of order $q_\perp^2/M^2$ in this model, the two are of very different origin: The first is dynamical (reflecting the spin-$\frac{1}{2}$ nature of the parton) and the second is purely kinematical (shared by all models). The order counting is less tricky in terms of the invariant structure functions where only dynamics counts. The $q_\perp^2/M^2$ term in W_L is of interest on its own, as pointed out by Collins and Soper.

[20]C. Callan and D. Gross, Phys. Rev. Lett. 22, 156 (1969).

[21]See, for example, Berger *et al.*, Ref. 4.

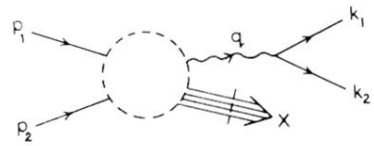

FIG. 1. Inclusive lepton pair production in hadronic collisions in the one-photon approximation.

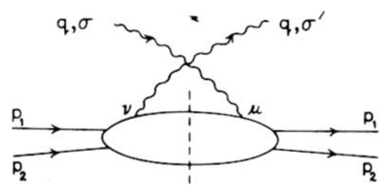

FIG. 2. Diagrammatic representation of the hadronic amplitude $W_{\mu\nu}$ or $W_{\sigma,\sigma'}$. The dashed line indicates that it is the absorptive part of this amplitude (or, equivalently, the sum over all intermediate states) that is being considered.

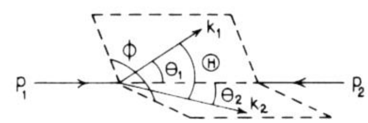

FIG. 3. Kinematic configuration in the hadron center-of-mass frame.

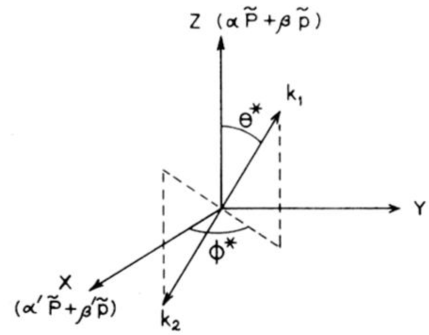

FIG. 4. Kinematic configuration in the lepton pair center-of-mass frame. The X-Z plane is defined by the hadronic momenta $\tilde{P}$ and $\tilde{p}$ but the choice of Z axis is left open. The coefficients $(\alpha, \beta, \alpha', \beta')$ corresponding to some specific "helicity" frame are listed in Table I. A typical configuration for the various Z axes is depicted in Fig. 5.

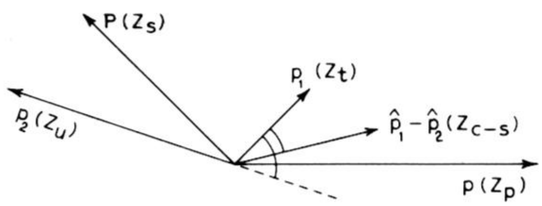

FIG. 5. A typical configuration of the Z axes corresponding to various "helicity" frames listed in Sec II D (and Table I) is depicted here. For the region $q_\perp{}^2/M^2 \ll 1$, which is of practical interest, the transverse dimension should be further contracted. (This is not done in this picture in order to avoid crowding.)

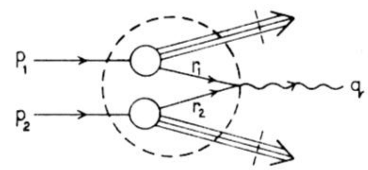

FIG. 6. The Drell-Yan on-shell quark-antiquark annihilation picture for lepton pair production. The dashed circle corresponds to that of Fig. 1.

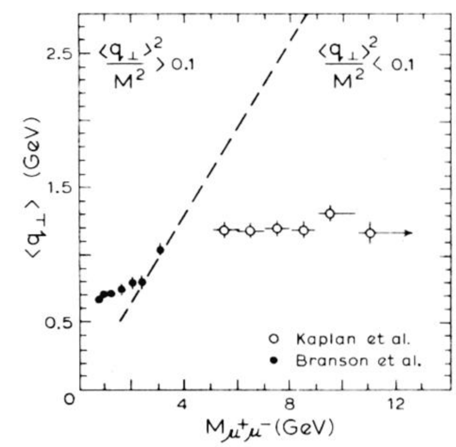

FIG. 7. Reproduction of the experimental data on $\langle q_\perp \rangle^2$ by Kaplan *et al.* (Ref. 3) with the dashed line $\langle q_\perp \rangle^2/M^2 = 0.1$ superimposed. The region to the right of the line corresponds to $\langle q_\perp \rangle^2/M^2 < 0.1$.