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$\gamma\gamma$ background of the Drell-Yan process

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For lepton pair production in nucleon-nucleon collisions, the mechanism of $\gamma\gamma$ materialization is compared with that of quark-antiquark annihilation (Drell-Yan process). The predictions compared show that, whereas the $\gamma\gamma$ background of the Drell-Yan process should not exceed a few percent in present accelerator experiments, it might become an almost 100% effect with future proton-proton colliding beams of superhigh energies ($s \sim 10^6$ GeV^2) in the measurable range of lepton pair masses. It is also shown that the rate of the $\gamma\gamma$ effect should be almost independent of the particular model chosen for the inelastic structure function, and that the double equivalent-photon approximation works very well in computing that effect.

I. INTRODUCTION

The Drell-Yan process[1] (lepton pair production in hadron collisions, via quark-antiquark annihilation) is known to be of fundamental importance in view of analyzing the quark structure of hadrons. On the other hand, it has been noticed[2-4] that in hadron collisions producing lepton pairs another mechanism, namely, $\gamma\gamma$ materialization, is involved as well, and that the cross section of that process increases with energy. One may thus wonder whether, at the very high energies of future particle accelerators and—in particular—pp or $p\bar{p}$ colliding beams, the $\gamma\gamma$ mechanism might not become a serious background of the Drell-Yan process. This question shows some similarity—here considering quarks or partons instead of electrons—with the problem, raised many years ago,[5] of the $\gamma\gamma$ background in e^-e^+ annihilation.

In Sec. II, we show the formulas we use for computing the $\gamma\gamma$ effect in the double equivalent-photon approximation, and we check that approximation. In Sec. III, the model dependence of the $\gamma\gamma$ process is analyzed. Section IV contains the numerical comparison between the $\gamma\gamma$ and the Drell-Yan process. We finish with a brief conclusion.

II. CALCULATION OF THE $\gamma\gamma$ PROCESS

In the double equivalent-photon approximation,[6] one writes

$$d\sigma \simeq P(x)P'(x')\sigma_{\gamma\gamma}(M^2)dx\,dx' , \tag{2.1}$$

where $P(x)$ $(P'(x'))$ is the equivalent-photon spectrum (EPS) of either incoming particle, where x (x') is the fraction of energy of that particle—in the overall c.m. frame, for instance—taken off by the virtual photon; $\sigma_{\gamma\gamma}(M^2)$ is the cross section for $\gamma\gamma\to l^-l^+$, where M is the invariant mass of the lepton pair. We define $\mu = M/\sqrt{s}$, where s is the overall c.m. energy squared; one gets $xx'\simeq\mu^2$, insofar as both photons tend to be quasireal, i.e., their Q^2 values tend to be small with respect to M^2.

We are interested in computing the following quantity (using the rapidity parameter y):

$$\bar{\sigma} \equiv M^3\left(\frac{d^2\sigma}{dM\,dy}\right)_{y=0} \simeq 2\mu^4 sP(\mu)P'(\mu)\sigma_{\gamma\gamma}(\mu^2) . \tag{2.2}$$

The EPS of the proton (or antiproton) is computed by adding up an elastic and an inelastic part, which are given hereafter. Thus, on the whole, we are computing the sum of the four diagrams in Fig. 1.

A. Elastic part of the EPS

From Ref. 7, one gets (assuming $m_N \ll M$, where m_N is the nucleon mass)

$$P^e(x) = \frac{\alpha}{\pi\mu^2}\int_{\tau^e_{\min}}^{\tau^e_{\max}} d\tau\left[\frac{(\tau-\tau^e_{\min})(\tau^e_{\max}-\tau)}{\tau^2(1+\tau/\mu^2)^2}\,\frac{G_E{}^2+(\tau/4\epsilon^2)G_M{}^2}{1+\tau/4\epsilon^2}+\frac{\mu^2x}{2\tau}\,G_M{}^2\right], \tag{2.3}$$

where the superscript e stands for "elastic" and where we define

$$\tau = Q^2/s,\quad \epsilon^2 = m_N{}^2/s$$

noticing that we get

$$\tau^e_{\min} \simeq \epsilon^2x^2/(1-x),\quad \tau^e_{\max}\simeq(\mu^2/x)(1-x) .$$

B. Inelastic part of the EPS

The inelastic EPS of the proton is calculated (see Fig. 1) by convoluting the elastic EPS of a pointlike quark-parton of mass $m_q = z m_N$ (calling z the scaling variable) with the quark distribution function $F_2(z)/z$. One thus gets

$$P^i(x) = \frac{\alpha}{\pi\mu^2}\int_{z_{\min}}^{1} F_2(z)\,\frac{dz}{z}\int_{\tau^z_{\min}}^{\tau^z_{\max}} \frac{d\tau}{z^2}\left[\frac{z(\tau-\tau^z_{\min})(\tau^z_{\max}-\tau)}{\tau^2(1+\tau/\mu^2)^2}+\frac{\mu^2 x}{2\tau}\right], \tag{2.4}$$

where the superscript i stands for "inelastic", and the superscript z means "at z fixed"; one has

$$\tau^z_{\min} \simeq \epsilon^2 \frac{z x^2}{z-x}, \quad \tau^z_{\max} \simeq \frac{\mu^2}{x}(z-x)$$

$$z_{\min} \simeq x + \frac{\epsilon}{\mu^2}x^2 .$$

[We here notice that we may derive formula (2.4) as well from Ref. 7, independently of the quark model, by assuming $R \equiv \sigma_L/\sigma_T = Q^2/\nu^2$.] The integrand of formula (2.4) must still be multiplied by a correction factor, as one wants to extrapolate the structure function up to the $Q^2=0$ (photoproduction) limit; this factor is given by $\tau/(\tau+\beta)$ with $\beta \simeq F_2(0)s$, where s is in GeV^2, once one assumes $\sigma_{\gamma p} \simeq 100$ μb.

Let us now consider the central vertex in the diagrams of Fig. 1. The cross section for real photons would be

$$\sigma_{\gamma\gamma}(\mu^2) = \frac{4\pi\alpha^2}{\mu^2 s}\left(\ln\frac{\mu^2}{\epsilon_l^2}-1\right), \tag{2.5}$$

with $\epsilon_l = m_l/\sqrt{s}$, where m_l is the lepton mass. In order to make our predictions slightly more conservative, we shall take the virtuality of the photons into account to some extent. For the virtual $\gamma\gamma$ cross section of transversely polarized photons, one gets a good approximation by using

$$\sigma_{\gamma\gamma}(\mu^2,\tau,\tau') \simeq \frac{4\pi\alpha^2}{\mu^2 s}\left[\left(1+\frac{\tau+\tau'}{\mu^2}\right)^{-2}\ln\frac{\mu^4}{\epsilon_l^2\mu^2+\tau\tau'} - 1 - \frac{\tau\tau'}{\epsilon_l^2\mu^2+\tau\tau'}\right]. \tag{2.6}$$

Formula (2.6) should be used for electron pairs. For muon pairs, in the energy range considered, a sufficiently accurate and much handier approximation is obtained by taking simply

$$\sigma_{\gamma\gamma}(\mu^2,\tau,\tau') \simeq \sigma_{\gamma\gamma}(\mu^2)\left(1+\frac{\tau}{\mu^2}\right)^{-2}\left(1+\frac{\tau'}{\mu^2}\right)^{-2}, \tag{2.7}$$

with $\sigma_{\gamma\gamma}(\mu^2)$ given by (2.5)

We checked our formulas vs an exact computation (involving extensive computer work) of $d\sigma/dM$ in $p+p\to\mu^-+\mu^++X$, performed by Chen *et al.*[4]; we here selected two energies, $s=10^3$ and 10^5 GeV^2. We had to use the same physical input, i.e., the same inelastic structure function, as those authors; namely, an expression given by Bloom and Gilman[8] at 10^3 GeV^2 and a constant value ($F_2=0.2$) at 10^5 GeV^2, both with a correction factor $[Q^2/(Q^2+0.15\text{ GeV}^2)]$ for extrapolation to the photoproduction limit. Tables I and II show that our computations are accurate up to 10% at the lower energy and up to 50% at the higher one. The worsening of the approximation at $s=10^5$ GeV^2 should mainly be due to using the constant structure function (which does not exhibit the normal form-factor behavior at large Q^2).

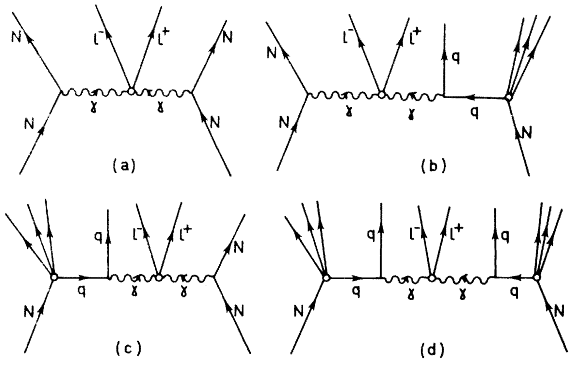

FIG. 1. Feynman diagrams for the $\gamma\gamma$ process, $N+N\to l^-+l^++X$: (a) elastic-elastic term; (b), (c) elastic-inelastic terms; (d) inelastic-inelastic term.

TABLE I. Comparison between an exact computation (Ref. 4) and our approximation for $d\sigma/dM$ (in nb/GeV) in the $\gamma\gamma$ process $p+p\to\mu^-+\mu^++X$, at $s=10^3$ GeV^2.

M (GeV)	Exact	Approx.
3	8.5×10^{-3}	7.7×10^{-3}
6	2.5×10^{-4}	2.3×10^{-4}
9	1.7×10^{-5}	1.5×10^{-5}
12	1.4×10^{-6}	1.3×10^{-6}
15	1.1×10^{-7}	1.0×10^{-7}
18	6.7×10^{-9}	6.7×10^{-9}

TABLE II. Same comparison as in Table I, at $s=10^5$ GeV^2.

M (GeV)	Exact	Approx.
3	1.5×10^{-1}	1.9×10^{-1}
6	1.4×10^{-2}	2.0×10^{-2}
9	3.4×10^{-3}	5.0×10^{-3}
12	1.2×10^{-3}	1.8×10^{-3}
15	5.0×10^{-4}	7.6×10^{-4}
20	1.6×10^{-4}	2.4×10^{-4}
30	2.9×10^{-4}	4.5×10^{-4}
40	8.2×10^{-4}	1.2×10^{-5}
50	2.8×10^{-4}	4.2×10^{-6}
60	1.2×10^{-6}	1.7×10^{-6}

TABLE IV. Same comparison as in Table III, at $s=2\times10^6$ GeV^2.

M^2/s	Model I	Model II
6×10^{-6}	2.7	2.5
2×10^{-5}	3.2	3.0
6×10^{-5}	3.4	3.1
2×10^{-4}	3.3	3.0
6×10^{-4}	2.9	2.6
2×10^{-3}	2.2	1.9
6×10^{-3}	1.4	1.2
2×10^{-2}	5.8×10^{-1}	4.7×10^{-1}
6×10^{-2}	1.5×10^{-1}	1.2×10^{-1}
2×10^{-1}	9.0×10^{-3}	5.5×10^{-3}

III. MODEL DEPENDENCE OF THE γγ PROCESS

The elastic part of the EPS is obviously model independent: Since very small transfer values are involved ($\langle Q^2\rangle \lesssim m_N^2$), the form factors are perfectly well determined.

In the inelastic part, on the other hand, much higher transfer values are involved. Indeed, a somewhat simplified calculation [based on formula (2.4) and including the correction factor $\tau/(\tau+\beta)$], shows that here one gets

$$\langle Q^2\rangle \simeq \frac{M^2}{3[\ln(\mu^2/\beta)-\frac{11}{6}]}$$

Since, from present deep-inelastic leptoproduction experiments, the structure function F_2 is well known up to $Q^2\sim10^2$ GeV^2, it appears that our predictions will be reliable up to $M^2\sim10^3$ GeV^2 (above that value they may become wrong, especially if scaling is badly violated at large Q^2).

With this restriction in mind, we may then use conventional models for $F_2(z)$. Since they are all based on experimental data, one should not expect our results to be strongly model dependent. Nevertheless, we preferred to check this expectation by comparing numerical predictions based on two different models[9,10] for quark distribution functions, which differ from each other mainly—and quite sharply—as far as sea quark distributions are concerned. As Tables III, and IV show, those predictions are indeed only weakly model dependent, even at very high energy.

TABLE III. Comparison between predictions for $(M^3d^2\sigma/dMdy)_{y=0}$ (in nb GeV^2) in the γγ process $p+p\to\mu^-+\mu^++X$, using the quark distribution functions of Ref. 9 (model I) and Ref. 10 (model II), respectively; $s=800$ GeV^2.

M^2/s	Model I	Model II
6×10^{-3}	1.1×10^{-1}	9.7×10^{-2}
2×10^{-2}	6.0×10^{-2}	5.2×10^{-2}
6×10^{-1}	1.8×10^{-2}	1.5×10^{-2}
2×10^{-1}	1.1×10^{-3}	9.0×10^{-4}

IV. COMPARISON OF THE γγ AND DRELL-YAN PROCESSES

In Fig. 2, we compare numerical predictions for the γγ and Drell-Yan processes, where the latter is computed with the model of Field and Feynman (including color). The rate $\bar{\sigma}$ shown is energy independent for the Drell-Yan process, whereas it steadily increases with energy as far as the γγ effect is concerned.

It is seen that in $p+p\to\mu^-+\mu^++x$, whereas the

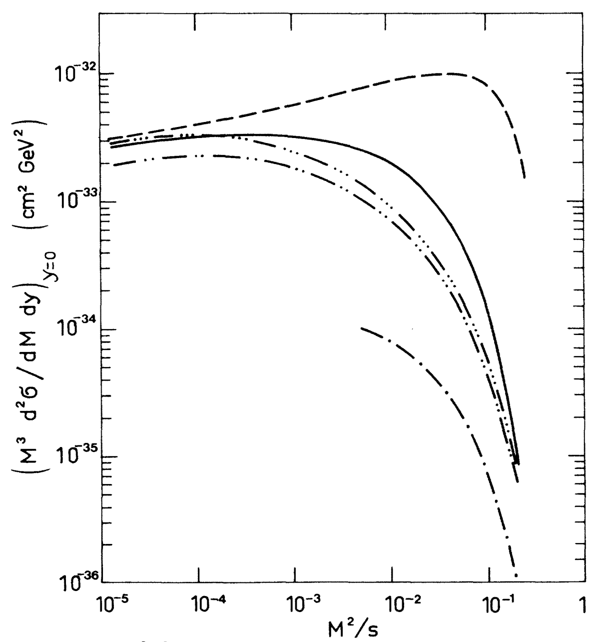

FIG. 2. $(M^3d^2\sigma/dM\,dy)_{y=0}$ computed for: ——— Drell-Yan process $p+p\to\mu^-+\mu^++X$; — — — Drell-Yan process $\bar{p}+p\to\mu^-+\mu^++X$; —·—·—· γγ process, $s=800$ GeV^2; —··—··— γγ process, $s=640\,000$ GeV^2; —···—···— γγ process, $s=2\times10^6$ GeV^2.

$\gamma\gamma$ background is only of the order of a few percent at present accelerator energies, it should become much larger (~100%) with future pp colliding beams of $s \sim 10^6$ GeV^2, if one sticks to the range $M^2 \lesssim 10^{-3}s$. This is precisely the invariant-mass range where predictions for the $\gamma\gamma$ effect should be very reliable, and also where counting rates should be high enough for measurement.

Actually, at high energy and small M^2/s, formulas (2.3) and (2.4) of Sec. II may be replaced by following simple expressions:

$$P^e(x) \simeq \frac{\alpha}{\pi x}\left(\ln\frac{1}{\mu^2} - \frac{17}{6}\right) , \tag{4.1}$$

$$P^i(x) \simeq \frac{\alpha}{\pi x}\left(\ln\frac{\mu^2}{\beta} - \frac{11}{6}\right) , \tag{4.2}$$

and one thus gets

$$\bar{\sigma}_{\gamma\gamma} \simeq \frac{8\alpha^4}{\pi}\left(\ln\frac{s}{F_2(0)(1\text{GeV}^2)} - \frac{14}{3}\right)^2\left(\ln\frac{\mu^2 s}{m_l^2} - 1\right) \tag{4.3}$$

and therefore

$$\frac{\bar{\sigma}_{\gamma\gamma}}{\bar{\sigma}_{\mathrm{DY}}} \simeq 6\,\frac{\alpha^2}{\pi^2}\frac{[\ln[s/F_2(0)(1\text{GeV}^2)] - \frac{14}{3}]^2[\ln(\mu^2 s/m_l^2) - 1]}{F_2(\mu)F_2(0)} , \tag{4.4}$$

which is indeed of the order of 1 for $s \simeq 10^6$ GeV^2 and $\mu^2 \simeq 10^{-5}$–10^{-3}.

It should be noticed that the asymptotic $\ln^3 s$ behavior of the $\gamma\gamma$ effect (at constant μ^2) is due to the inclusion of the inelastic EPS (otherwise, it would be only $\ln s$).

V. CONCLUSION

We here compared the $\gamma\gamma$ effect with the Drell-Yan process computed according to the naive model where quantum-chromodynamics corrections are not included. It seems to us that the rate of such corrections at superhigh energies is still an open question. We are aware of the possibility that those corrections might increase sharply with energy; in that case the $\gamma\gamma$ background might stay comparatively small at any s value. We also realize that, even where it is large, the $\gamma\gamma$ background can certainly be disentangled experimentally to a large extent from the Drell-Yan process, using the special characteristics of the former: forward-backward peaked lepton distributions, small hadron multiplicities. Nevertheless, we consider it worthwhile to draw the attention of experimentalists to that particular effect since, whatever happens at superhigh energies, it will be there.

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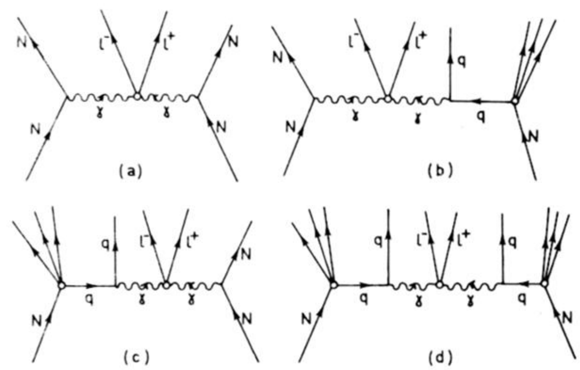

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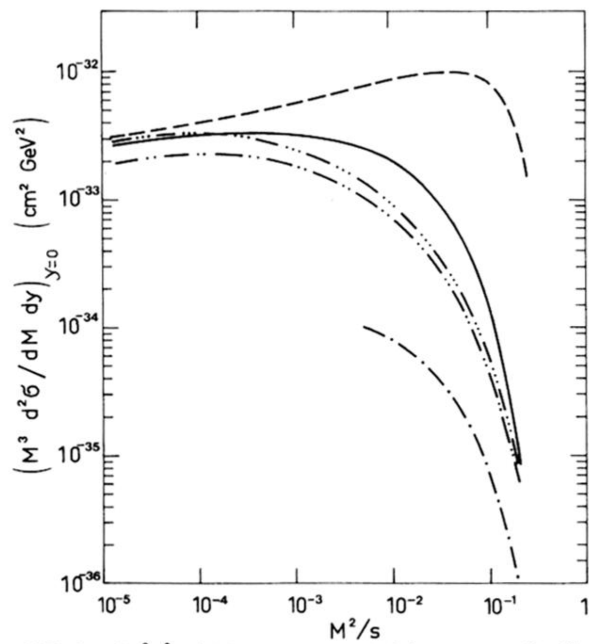

FIG. 2. $(M^3 d^2\sigma/dM\,dy)_{y=0}$ computed for: ——— Drell-Yan process $p+p\rightarrow\mu^-+\mu^++X$; — — — Drell-Yan process $\bar{p}+p\rightarrow\mu^-+\mu^++X$; — · — · — · $\gamma\gamma$ process, $s=800$ GeV^2; — ·· — ·· — $\gamma\gamma$ process, $s=640\,000$ GeV^2; — ··· — ··· — $\gamma\gamma$ process, $s=2\times10^6$ GeV^2.