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Low-energy manifestations of heavy particles: Application to the neutral current

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It is argued that in broken-symmetry theories the decoupling of heavy particles is incomplete. Some possible manifestations of this effect in neutral-current processes and the decay $K_L \to \mu\mu$ are discussed. In the case of the neutral current, heavy-particle effects may be calculated accurately including strong-interaction corrections. A new method of making such calculations is explained and its application to the neutral-current process discussed in detail. The method is more efficient than previous ones and is especially useful in calculations involving several mass scales.

I. INTRODUCTION: GENERAL ANALYSIS

In an interesting paper,[1] Appelquist and Carazzone have argued that one practical advantage of renormalizable theories in a world with widely different mass scales is that heavy particles "decouple" at low energies. More precisely, it was argued that all processes involving only light particles at small energies $E \ll M$, where M denotes the mass of the heavy particles, can be summarized in an approximate renormalizable effective theory involving only light particles, correct to order E/M.

Roughly speaking, the argument for this goes as follows. Consider a process involving virtual-heavy-particle exchange. If all subgraphs containing the heavy particles are convergent, then the process is suppressed by inverse powers of M (up to logarithms). If, on the other hand, the heavy particle occurs in a primitively divergent subgraph, then its effect can be absorbed in the counterterm associated with this subgraph. In this way, all effects of the heavy particle are either suppressed by its mass or absorbed into renormalizations of couplings involving only light particles.

In gauge theories, however, there is an important limitation to this reasoning. Gauge invariance may forbid counterterms corresponding to certain primitively divergent graphs. The finiteness of the theory results from delicate cancellations between different graphs, sometimes between graphs containing heavy virtual particles and graphs containing only light virtual particles. In this situation the heavy particles certainly do not decouple; on the contrary we should expect effects which *grow* with the heavy-particle mass (since the cancellation between light and heavy becomes less accurate), at least in perturbation theory.

From this analysis, we would expect that heavy-particle effects might be large in some interactions of dimension 4 or less forbidden by gauge invariance. As a result, the low-energy effective theory involving only the light particles need not look renormalizable; the contraints on vector-meson couplings required for renormalizability are in general not satisfied to order $(1/M)^0$.

We expect these effects to occur in general when the effective gauge group contains multiplets whose members vary widely is mass (or for axial couplings if any member of the multiplet is very massive). Then the cancellations guaranteed by gauge invariance which make the theory finite occur only at very large virtual momentum, and our effect, although finite, can be large. Similarly, we can expect large effects if anomaly cancellation occurs between very heavy and light particles.

Indeed, examples related to this phenomenon, although not explicitly regarded as such, have been calculated in the literature. Adler[2] in his classic paper on anomalies found effects in νe elastic scattering of this kind. Veltman[3] has recently calculated large corrections to the vector-boson mass matrix due to heavy leptons in gauge theories. Marciano[4] calculated one-loop corrections to the relation $M_W = \cos\theta M_Z$ in the Weinberg-Salam model and showed that it grows with large Higgs-boson mass.

In Sec. II we will illustrate the preceding remarks with some low-energy processes accessible to experiment. For the sake of simplicity we specialize to the standard sequential Weinberg–Salam–Glashow-Iliopoulos-Maiani[5] model of weak interactions and the color gauge theory of strong interactions[6]; it will be obvious that our remarks apply to any renormalizable gauge theory of the weak interaction. In Sec. III the heavy-quark contribution to the neutral current is analyzed with strong-interaction renormalization effects taken carefully into account, following the method of Witten.[7] A formalism is developed to handle situations involving more than one large mass scale.

II. NEUTRAL CURRENTS AND OTHER APPLICATIONS

Consider the class of graphs in Fig. 1(a) depicting strong-interaction corrections to the neutral-current vertex. The axial coupling of the neutral boson is proportional to the weak isospin T_3^w. Now note the following:

(i) The value of the graph is essentially independent of the external light quark q. This reflects the fact that the gluon coupling is flavor independent. Hence the strong interaction induces an effective axial-vector baryon-number neutral current coupling to neutrinos.

(ii) All quarks in the world, no matter how heavy, participate in the internal quark loop in Fig. 1(a).

(iii) Gauge invariance of the weak interaction forbids any coupling to the axial baryon number current at the Lagrangian level (i.e., a local coupling); indeed, since this current is anomalous,[2,8] such a coupling would spoil renormalizability. Because of this, the induced coupling must be finite. The mechanism ensuring this is, of course, a cancellation between graphs involving internal quarks of opposite T_3^w.

(iv) Considering for a moment the strong interactions perturbatively, we may simply compute the lowest-order graph, shown in Fig. 1(b). At small momentum transfer the axial neutral current is modified to read[9] (for notational simplicity we write only the part involving u and d quarks)

$$J_\mu^5 = \tfrac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) + (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)\tfrac{1}{4}\left(\frac{g^2}{4\pi^2}\right)^2 \ln \frac{\prod_i m_{+i}^2}{\prod_i m_{-i}^2}, \tag{2.1}$$

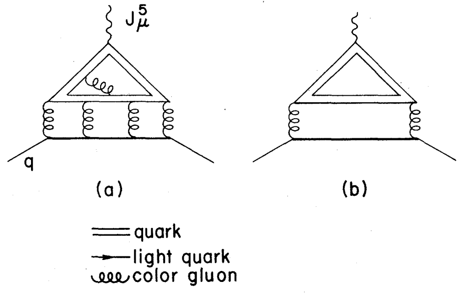

FIG. 1. Virtual-quark loop inducing an axial-vector-baryon-number neutral-current coupling.

where $\prod_i m_{+i}$ denotes the product of the masses of all the heavy charge $+\frac{2}{3}$ quarks, similarly $\prod_i m_{-i}$ denotes the product of the masses of all the heavy charge $-\frac{1}{3}$ quarks. ("Heavy" means "heavy on the scale of the momentum transfer considered.") This result illustrates our analysis in Sec. I: Evidently heavy quarks play a role in determining the coupling; indeed in perturbation theory their effect increases with their mass. There is a similar contribution to the axial-vector lepton-number neutral current, in which the gluons of Fig. 1 are replaced by photons. This effect is of order $(\alpha/\pi)^2$ and negligibly small, but is interesting that the logarithm in the analog of Eq. (2.1) now involves ratios of quark to lepton masses. Taking the known fermions one obtains a large numerical value for the logarithm.

(v) A more realistic evaluation of the strong-interaction effects is attempted in Sec. III. Roughly speaking, the outcome of this analysis is that for each doublet q_+, q_- of charge $\frac{2}{3}, -\frac{1}{3}$ heavy quarks there is a contribution to the axial baryon number current

$$\Delta J_\mu^5 = (\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)\frac{1}{4}\left[\frac{\bar{g}^2(m_{q+})}{4\pi^2}\right] \times \left[\frac{\bar{g}^2(m_{q-})}{4\pi^2}\right]\ln\frac{m_{q+}^2}{m_{q-}^2}, \tag{2.2}$$

where $\bar{g}$ is the running strong-interaction coupling. For a numerical example, if we take $m_{q+} = 1500$ MeV, $m_{q-} = 300$ MeV, $\bar{g}^2/4\pi^2 = \frac{1}{4}$ to estimate the contribution of the charmed and strange quarks, we get

$$J_\mu^5 \simeq \tfrac{1}{2}(\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d) + 0.05(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d). \tag{2.3}$$

It is certainly very questionable whether the strange quark is heavy enough for the analysis of Sec. III to apply; furthermore, we have ignored renormalization effects due to chiral-symmetry breaking and confinement which are certainly significant. Nevertheless, we consider Eq. (2.3) as an indication that a very substantial axial-vector baryon-number piece of the neutral current is induced, with the indicated sign. Note that the matrix element of the axial-vector baryon-number neutral current may be determined from spin-dependent electroproduction.[10] Two further qualitative features of this result are worthy of note. The first is that the induced axial baryon number current is strongly momentum dependent; roughly speaking for momentum transfers of order Q the quark masses squared m_q^2 in Eqs. (2.1) and (2.2) get replaced by $m_q^2 \to m_q^2 + Q^2$. Second, no similarly large effect occurs for the vector piece of the neutral current. All these features can be compared with experiment when low-energy neutral-current data become more precise.[11]

(vi) When $Q^2 \approx 0$, one should not expect to apply our argument to the u and d quarks. First of all, there are no lighter quarks to provide an effective axial-vector baryon current. Second, one cannot apply the perturbative argument of Sec. III when the effective coupling constant $g(m_u)$ is large. Nevertheless one might still ask whether diagrams such as Fig. 1 give a substantial violation of isospin symmetry, since m_d/m_u is probably 1.8 or 1 larger.[12] This problem is common to all of low-energy hadron physics, and the natural way out is to say that confinement is the dominant infrared effect. In particular one might expect from this an isospin-conserving cutoff at hundreds of MeV compared to an infrared cutoff coming from quark masses[12] of a few MeV.

(vii) Another way of looking at the effect above is to note that the axial-vector part of the neutral current looks like (schematically)

$$\bar{u}u - \bar{d}d + (\bar{c}c - \bar{s}s + \bar{t}t - \bar{b}b) + \cdots . \tag{2.4}$$

In most phenomenological analyses one discards the terms in parentheses on the grounds that their matrix element between nucleons is small.

Our discussion indicates that the application of Zweig's rule[13] in this case is vindicated in principle: As m_t and $m_b \to \infty$, the matrix element of $\bar{t}t - \bar{b}b$ between nucleon states indeed vanishes, albeit logarithmically slowly. In practice, however, Zweig's rule may be violated substantially; witness the ~10% correction in Eq. (2.3).

We note that our discussion points up another feature of asymptotically free gauge theory of strong interaction. In a non-asymptotically-free world experiments at low energy would in general be strongly affected by arbitrarily heavy particles.

(viii) In an interesting application of Zweig's rule Cheng[14] determined the nucleonic σ term by arguing that the scalar density $\bar{s}s$ should have small nucleon matrix elements (compared to $\bar{u}u$, $\bar{d}d$). Our considerations do not affect his work, since the graph analogous to Fig. 1 with a scalar density replacing the axial current will be proportional to the light-quark mass by a chirality argument. More precisely, one finds that to lowest nontrivial order in the strong interaction.

$$\langle N|\bar{s}s|N\rangle \sim \left(\frac{g^2}{4\pi^2}\right)^2 \frac{m_u}{m_s} \ln\frac{m_s}{m_u} \langle N|\bar{u}u|N\rangle . \tag{2.5}$$

(ix) There is a contribution to the real part of the $K_L \to \mu\mu$ amplitude proportional to $\ln m_c$, from the graph depicted in Fig. 2. In contrast, the contribution of the charmed quark to the imaginary part of the amplitude (i.e., to $K_L \to \gamma\gamma$) vanishes as $1/m_c^2$.

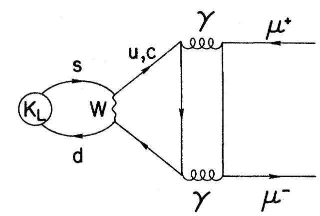

FIG. 2. Contribution to $\mathrm{Re}(K_L \to \mu\mu)$ proportional to $\ln m_c$.

III. EXPANSION OF THE AXIAL-VECTOR CURRENT

A. Strategy

In this section we set ourselves the problem of expanding the axial-vector current, which includes heavy-quark contributions, in a sum of operators for which the heavy-quark contributions are expected to be small. To be precise, consider the case of two heavy quarks, t and b, with masses m_t and m_b, $m_t \geq m_b$. Our results will be an exact expansion of the operator

$$H = \bar{t}\gamma_\mu\gamma_5 t - \bar{b}\gamma_\mu\gamma_5 b \tag{3.1}$$

of the form

$$H = \sum_i A_i \overline{L}_i + \sum_i B_i \overline{H}_i , \tag{3.2}$$

where $\overline{L}_i$ and $\overline{H}_i$ are respectively light- and heavy-particle operators renormalized in such a way as to exhibit decoupling: As the heavy-quark masses grow the light-particle Green's functions of $\overline{H}_i$ go to zero as a power of the heavy-quark masses, while those of $\overline{L}_i$ have their values in the theory with heavy quarks removed up to power-law corrections.

Our work here owes much to that of Witten[7] and of Georgi and Politzer,[15] who addressed some related problems. We have organized the calculation in a different way which seems to have important technical advantages. Since we believe our method of calculation will be useful in other problems involving more than one large mass scale or nonasymptotic masses, we will give a detailed description.

Our main technical innovation is the use of a set of renormalization conditions, each of which is used in a different region of the subtraction point μ. Our procedure has the advantages that no large logarithms of mass ratios ever appear in our perturbation theory, that the renormalization group is particularly simple, that we always work from formally exact equations so that subsequent approximations can be systematically improved, and that the partially conserved currents are appropriately renormalized at all stages (i.e., symmetries broken only softly by generalized mass terms with scale $m \lesssim \mu$ are respected by the renormalization procedure).

The procedure is, heuristically, to subtract at zero mass for light particles but at zero momentum for heavy particles. A particle is defined as light or heavy according to whether its mass is less than or greater than the subtraction point mass μ.

More precisely, we define a heavy graph recursively as one that contains a heavy-quark line or a counterterm to a heavy graph. Subtractions to a heavy graph are made at zero momentum with the heavy masses at their actual values and light-quark masses set equal to zero.

Subtractions to light graphs (i.e., graphs that are not heavy graphs) can be made by a zero-mass scheme.[16] The scheme of 't Hooft[16] is most convenient because it automatically preserves gauge invariance.

In our present problem we are interested in relating the axial current H, which is the heavy-quark part of the coupling to the Z boson, to axial currents renormalized at $\mu^2 = |q^2|$, the momentum transfer typical of the physical process under consideration. Finally we will want to work with μ^2 much less than m_t^2 and m_b^2, so that we can eliminate the heavy quarks. Then we can compare the currents at different q^2 and perhaps join on to some quark-model prediction or relate one process to another.

As we vary the subtraction point through a quark mass the definition of "light" and "heavy" will switch and some simple calculations must be done to join one region to the other. No large logarithms occur in this joining, so it can be done perturbatively. In the regions between masses we can use the renormalization group to trace the evolution of operators as μ varies. Note that since heavy graphs are subtracted at zero momentum, the renormalization-group coefficients are mass independent and heavy operators are renormalization-group independent. This makes our renormalization-group calculations particularly simple.

In our case we need three renormalization prescriptions:

(a) When $\mu > m_t$, we renormalize everything using a zero-mass method.[16] Chiral symmetries are respected (e.g., H is not renormalized). We denote renormalized quantities in this scheme by unadorned symbols g, H, etc.

(b) When $m_t > \mu > m_b$, the t quark is treated as heavy but the b quark as light. We use a caret to indicate renormalized quantities: $\hat{g}$, $\hat{H}$, etc. Decoupling of t and of b as $m_t \to \infty$, $m_b \to \infty$ is manifest.

(c) When $\mu < m_b$, both t and b quarks are considered heavy. We use an overbar to indicate renormalized quantities: $\bar{g}$, $\bar{H}$, etc. Decoupling of t and b as $m_t \to \infty$, $m_b \to \infty$ is manifest.

B. Computation

Let $H_t = \bar{t}\gamma_\mu\gamma_5 t$, $H_b = \bar{b}\gamma_\mu\gamma_5 b$, and $L = \sum_1^n \bar{q}\gamma_\mu\gamma_5 q$, where the q's are the light quarks. These axial-vector operators, which are singlets under the color group and under the chiral $SU(n)\times SU(n)$ of light quarks, form a closed set under renormalization.

Following the strategy outlined above, we will express $H = H_t - H_b$, which gets no renormalization. in terms of $\bar{H}_t$, $\bar{H}_b$, and $\bar{L}$ renormalized at some $\mu_0 < m_t, m_b$. This will require shifts in the definitions of operators at $\mu = m_t$ and $\mu = m_b$, and renormalization-group (RG) transformations in the regions $m_b \leq \mu \leq m_t$. (There is no renormalization of the current H in the region $m_t < \mu$, since in this region we renormalize in the massless theory where H is conserved.) So we have four steps:

(a) At $\mu = m_t$, express H in terms of careted operators $\hat{H}_t$, $\hat{H}_b$, and $\hat{L}$, whose renormalizations treat t as a heavy particle.

(b) Transform from $\mu = m_t$ to $\mu = m_b$ using the renormalization group.

(c) At $\mu = m_b$ express the careted operators in terms of the barred operators $\bar{H}_t, \bar{H}_b, \bar{L}$.

(d) Use the renormalization group to transform from $\mu = m_b$ to $\mu = \mu_0$. At this stage we will have the desired result, Eq. (3.2).[17]

To carry out step (a), it is sufficient to order $g^4(m_t) \equiv g_t^4$ to calculate the Feynman diagram of Fig. 1(b). Let this diagram, as a function of (common) momentum squared of the external quarks, the mass of the loop quark, and the mass of the line quark be $g^4\Gamma(p^2, m_{\mathrm{loop}}^2, m_{\mathrm{line}}^2)$. Tracing through the definitions, we find

$$\begin{bmatrix} H_t \\ H_b \\ L \end{bmatrix}_{\mu=m_t} = \mathfrak{M}_1 \begin{bmatrix} \hat{H}_t \\ \hat{H}_b \\ \hat{L} \end{bmatrix}_{\mu=m_t}, \tag{3.3}$$

where

$$\mathfrak{M}_1 = \begin{bmatrix} 1+g_t^4\kappa & g_t^4\lambda & g_t^4\lambda \\ g_t^4\nu & 1 & 0 \\ ng_t^4\nu & 0 & 1 \end{bmatrix}, \tag{3.4}$$

$$\kappa \equiv \Gamma(0, m_t^2, m_t^2) - \Gamma(-m_t^2, 0, 0), \tag{3.5}$$

$$\lambda \equiv \Gamma(0, m_t^2, 0) - \Gamma(-m_t^2, 0, 0), \tag{3.6}$$

$$\nu \equiv \Gamma(0, 0, m_t^2) - \Gamma(-m_t^2, 0, 0) \tag{3.7}$$

(recall that n is the number of light quarks).[18]

For step (b) notice that $\hat{H}_t$ and $n\hat{H}_b - \hat{L}$ are RG invariant (in the sense that they have no anomalous dimension). This happens for $\hat{H}_t$ because its renormalization conditions nowhere involve μ, and for $n\hat{H}_b - \hat{L}$ because of chirality conservation (respected by our renormalization procedure). Owing to the anomaly, $\hat{H}_b + \hat{L}$ is multiplicatively renormalized; its anomalous dimension is of order g^4, arising from (what else?) the diagram in Fig. 1(b). Using standard methods[19] we find the multiplicative renormalization factor

$$\hat{z} \equiv \exp\int_{\hat{g}_b}^{\hat{g}_t} \frac{\hat{\gamma}(g)}{\hat{\beta}(g)}\, dg$$

$$\simeq \exp\int_{\hat{g}_b}^{\hat{g}_t} \frac{cg^4}{g^3}\, dg = 1 + \tfrac{1}{2}c(\hat{g}_t{}^2 - \hat{g}_b{}^2) + O(g^4) \quad (3.8)$$

with $\hat{\beta}$ the β function appropriate to our renormalization prescription (i.e., with $n+1$ massless quarks) and c a computable constant.

Steps (c) and (d) are similar to steps (a) and (b), respectively. The final result is

$$\begin{bmatrix} H_t \\ H_b \\ L \end{bmatrix}_{\mu=m_t} = \mathfrak{M}_1\mathfrak{M}_2\mathfrak{M}_3\mathfrak{M}_4 \begin{bmatrix} \overline{H}_t \\ \overline{H}_b \\ \overline{L} \end{bmatrix}_{\mu=\mu_0}, \quad (3.9)$$

where $\mathfrak{M}_1$ is defined by Eq. (3.4), and

$$\mathfrak{M}_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & (\hat{z}+n)/(n+1) & (\hat{z}-1)/(n+1) \\ 0 & n(\hat{z}-1)/(n+1) & (n\hat{z}+1)/(n+1) \end{bmatrix},$$

$$\mathfrak{M}_3 = \begin{bmatrix} 1 & \hat{g}_b{}^4\sigma(m_t/m_b) & 0 \\ g_b{}^4\tau(m_t/m_b) & 1+\hat{g}_b{}^4\kappa & \hat{g}_b{}^4\lambda \\ 0 & n\hat{g}_b{}^4\nu & 1 \end{bmatrix},$$

$$\mathfrak{M}_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \overline{z} \end{bmatrix},$$

$$\sigma(m_t/m_b) \equiv \Gamma(0, m_t{}^2, m_b{}^2) - \Gamma(0, m_t{}^2, 0),$$

$$\tau(m_t/m_b) \equiv \Gamma(0, 0, m_t{}^2) - \Gamma(0, m_b{}^2, m_t{}^2),$$

and

$$\overline{z} \equiv \exp\int_{\overline{g}(\mu_0)}^{\overline{g}_b} \frac{\overline{\gamma}(g)}{\overline{\beta}(g)}\, dg.$$

We have ignored corrections of order g^6.

We are interested in the corrections appearing with the smallest power of the coupling constant, and these appear in the multiplicative renormalization factor (3.8). Matching the *form* of this correction to the perturbation theory expression Eq. (2.1), we arrive at the expression Eq. (2.2). The next term in the expansion has a much more complicated appearance, since order g^4 corrections appear at each stage (a)–(d), but it is in principle computable.

When we consider μ_0 near typical light-hadron scales, there are important renormalization effects due to purely light-quark effects—chiral symmetry breaking, instantons, confinement, etc. Obviously our method sheds little on these questions; it merely allows a clean separation of those effects intrinsic to heavy quarks.

C. Gauge invariance and proof

It can be proved from the Ward-Takahashi identities that our subtraction of certain graphs at zero momentum preserves gauge invariance: The Ward identities relate graphs with the same number of heavy-quark loops. Zero-mass subtractions using 't Hooft's method[16] automatically preserve gauge invariance.

The proof of infrared finiteness and our decoupling statements can be carried out inductively, very similarly to Witten's paper.[7] We will only remark that zero-momentum subtraction improves the infrared behavior of higher-order corrections to heavy-quark graphs; this improvement renders the zero-momentum subtraction scheme self-consistent (i.e., infrared finite).

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[1]T. Appelquist and J. Carazzone, Phys. Rev. D 11, 2856 (1975).
[2]S. Adler, Phys. Rev. 177, 2426 (1969).
[3]M. Veltman, Nucl. Phys. B123, 89 (1977), also Acta Phys. Polonica B8, 475 (1977).
[4]W. Marciano, Nucl. Phys. B84, 132 (1975).
[5]S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); A. Salam, in *Elementary Particle Theory: Relativistic Groups and Analyticity* (Nobel Symposium No.

8), edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367; S. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[6]D. Gross and F. Wilczek, Phys. Rev. Lett. 30, 1343 (1973); H. D. Politzer, *ibid.* 30, 1346 (1973); S. Weinberg, *ibid.* 31, 494 (1973).

[7]E. Witten, Nucl. Phys. B104, 445 (1976).

[8]J. Bell and R. Jackiw, Nuovo Cimento 60A, 47 (1969).

[9]The calculation is very similar to the one carried out by Adler, Ref. 2. See also L. Rosenberg, Phys. Rev. 129, 2786 (1963).

[10]J. Bjorken, Phys. Rev. 148, 1467 (1966).

[11]Equation (2.3) shows that the induced axial-vector baryon current can easily be 10% of the axial-vector isospin current. This is not much smaller than the error in the axial-vector baryon current determined from present experiments. See P. Q. Hung and J. J. Sakurai, Phys. Lett. 72B, 208 (1977), where, of the two solutions for neutral-current couplings, the one compatible with the Weinberg-Salam model has an axial-vector baryon coupling equal to $(12 \pm 15)\%$ of the Weinberg-Salam axial-vector isospin coupling.

[12]S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964); H. Pagels, Phys. Rep. 16, 219 (1975).

[13]G. Zweig report, 1964 (unpublished) and in *Symmetries in Elementary Particle Physics* (Academic, N.Y., 1965), p. 192; S. Okubo, Phys. Lett. 5, 165 (1963); J. Iizuka, Prog. Theor. Phys. Suppl. 37–38, 21 (1966).

[14]T. P. Cheng, Phys. Rev. D 13, 2161 (1976).

[15]H. Georgi and H. D. Politzer, Phys. Rev. D 14, 1829 (1976).

[16]G. 't Hooft, Nucl. Phys. B61, 455 (1973); J.-L. Gervais and B. W. Lee, *ibid.* B12, 627 (1969); S. Weinberg, Phys. Rev. D 8, 3497 (1973); C. Callan, report, 1973 (unpublished).

[17]At this point we should remark that the masses undergo changes too, e.g., $m_t \to \hat{m}_t$. This is a higher-order calculable effect that we shall ignore here.

[18]We normalize the light-quark operators with external momenta squared to $-\mu^2$.

[19]D. Gross, in *Methods in Field Theory*, edited by R. Balian and J. Zinn-Justin (North-Holland, Amsterdam, 1976).