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Monte-Carlo approach to multiparticle production in a quark-parton model. II. Transverse momenta, energy dependence of average multiplicities, and inclusive spectra

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We extend our Monte Carlo quark-parton model by introducting explicitly the transverse momenta of partons in the compound state formed by two colliding hadrons. Calculated energy dependence of the average multiplicities of stable hadrons produced in pp collisions and the p_T and y inclusive spectra are in good qualitative agreement with the data. Our results on resonance production at $\sqrt{s} = 53$ GeV coincide with recent CERN ISR data.

I. INTRODUCTION

This paper continues our study[1] of the multiparticle production from the quark-parton–model point of view. The purpose of this paper is to describe the extension of the model of Ref. 1 by introducing explicitly the transverse momenta of quarks and antiquarks and to study the transverse-momentum distributions of hadrons in multiple production, the energy dependence of average multiplicities, and the inclusive spectra of hadrons at different energies.

The philosophy behind this approach is based on the following simple arguments:

(1) The quark-parton model[2] provides a natural framework for the explanation of basic features of deep-inelastic lepton-nucleon scattering. If a coherent and consistent understanding of the structure of hadrons and their interactions is to be obtained, the multiparticle production has to be understood from the quark-parton–model point of view.[3-7]

(2) Multiparticle production is a rather complicated process and at the first sight it is not clear which of numerous pieces of data available at present contains the most relevant information. In this situation it is desirable to have a model which gives a rough description of the general features of the process (rather than a fit to a restricted subset of the data). This leads directly to Monte Carlo models of multiple production.

In a previous paper[1] (hereafter referred to as I) of this series we constructed a simple Monte Carlo quark-parton model of multiple production. The model was based on the following simple-minded picture of the hadron-hadron collision:

The two colliding hadrons are supposed to be[2] coherent superpositions of valence quarks, quark-antiquark pairs (the "sea") and gluons. The collision starts by the interaction of wee partons. This destroys the original coherence of partons within hadrons and leads to the formation of a compound state. During this process gluons are converted to $Q\bar{Q}$ pairs.[8] Hadrons are formed by the recombination of quarks and antiquarks from the compound state. The recombination is supposed to be of short range in rapidity and to give mesons from the 35-plet and baryons and antibaryons from the 56-plets of the SU(6) scheme. In the next step, unstable hadrons decay producing the particles which are observed in experiment.

In our approach we make no attempt to construct a dynamical model for the first stage of the collision but we start directly with generating the exclusive configurations of Q's and $\bar{Q}$'s in the stage following the conversion of gluons to $Q\bar{Q}$ pairs and preceding the recombination of Q's and $\bar{Q}$'s to hadrons. Every such configuration is assigned a weight, given by cylindrical phase space multiplied by the appropriate power of a "coupling constant," by a factor for identical particles (quarks), and by factors which push valence quarks to higher values of momentum fractions.

In I we neglected the transverse momenta of quarks and antiquarks. The effective quark masses which were introduced there represented in fact the average transverse masses. The present work extends the model of I by building in the transverse momenta of partons. The weight of a configuration is modified by introducing an exponential cutoff in p_T.

The masses of confined quarks were fixed at rather low values $m_u = m_d = 0.01$ GeV/c^2, $m_s = 0.16$ GeV/c^2 in accordance with earlier suggestions[9] and with our results[10] on the description of the dimuon production in hadron collisions.

This separation of effects due to transverse momenta and to quark masses enables us to study the energy dependence of average multiplicities and the rapidity spectra at different energies.

The article is organized as follows: In Sec. II we give some details about the generation of exclusive configurations of Q's and $\bar{Q}$'s and we show how the free parameters of the model are fixed.

In Sec. III we discuss the energy dependence of average multiplicities and Sec. IV gives a few examples of the inclusive spectra of hadrons produced in pp collisions at different energies. Comments and conclusions are deferred to Sec. V.

This paper is a continuation of our study begun in I and is not meant to be self-contained; details can be found in I.

II. DISTRIBUTION OF QUARKS AND ANTIQUARKS IN THE SYSTEM OF TWO COLLIDING HADRONS

In the same manner as in I, we start with generating exclusive configurations of Q's and $\overline{Q}$'s in the compound system formed by the two colliding hadrons. These configurations are supposed to reflect the situation just before the recombination of Q's and $\overline{Q}$'s to hadrons.

The probability of finding six valence quarks (in the case of a pp collision) with rapidities and transverse momenta $y_1, \vec{p}_{T1}, y_2, \vec{p}_{T2}, \ldots, y_6, \vec{p}_{T6}$, n quarks with $y_i, \vec{p}_{Ti}$ ($i=7,\ldots,n+6$), and n antiquarks with $y_j, \vec{p}_{Tj}$ ($j=n+7,\ldots,2n+6$) is given by the expression ($N=2n+6$)

$$dP_N(y_1, \vec{p}_{T1}, \ldots, y_N, \vec{p}_{TN}) = KW_{\mathrm{id}}G^n\left(\prod_1^6 |x_i|^{1/2}\right)\exp\left(-\sum_1^N p_{Ti}^2/R^2\right)\delta\left(\sum_1^N \vec{p}_{Ti}\right) \times \delta\left(\sum_1^N p_{Li}\right)\delta\left(E-\sum_1^N E_i\right)\prod_1^N (dy_i d^2p_{Ti}). \qquad (1)$$

Here K is an overall normalization constant, supposed to be independent of energy. In fact K is related to the probability of the hadron-hadron collision. This in turn is given by the interaction of wee partons. Within a simple quark-parton model K should be taken as an energy-independent constant. W_{id} is a factor for identical particles (taken here in the same form as in I); G is a "coupling constant" regulating the mean number of $Q\overline{Q}$ pairs being produced. The factors $|x_i|^{1/2}$ give larger probabilities to those configurations in which valence quarks ($i=1,\ldots,6$) keep larger momentum fractions $|x_i|$. The explicit form of this factor is that used by Kuti and Weisskopf[11] in their study of structure functions in deep-inelastic lepton-nucleon scattering. This explicit form is debatable, but the short-range (in rapidity) character of strong interactions in the parton model requires a mechanism of this type. The factor $\exp(-\sum p_{Ti}^2/R^2)$ expresses the cutoff on transverse momenta of Q's and $\overline{Q}$'s and the rest of Eq. (1) is Lorentz-invariant phase space with energy-momentum conservation.

Equation (1) is perhaps the simplest way to incorporate elementary notions about the nature of the intermediate state in hadron-hadron collisions into probabilities for distributions of Q's and $\overline{Q}$'s.

In I we used an expression similar to Eq. (1), only the term $\exp(-\sum p_{Ti}^2/R^2)$ was omitted and the transverse momenta of Q's and $\overline{Q}$'s were set equal to zero. The parameter R^2 regulates the mean transverse momentum of quarks and antiquarks and is obtained from a comparison of the results with transverse-momentum spectra of hadrons.

In I we used effective quark masses $m_u = m_d = 300$ MeV/c^2, $m_s = 450$ MeV/c^2. Here we have $m_u = m_d = 10$ MeV/c^2 and $m_s = 160$ MeV/c^2 (note that transverse masses in the present version do not differ drastically from those in I). The introduction of transverse momenta leads also to the reevaluation of the probabilities P_u, P_d, and P_s for a given $Q\overline{Q}$ pair to be $u\overline{u}$, $d\overline{d}$, or $s\overline{s}$, respectively. In the present paper we use $P_u = P_d = 0.45$ and $P_s = 0.10$.

The rules for recombinations of $Q\overline{Q}$ pairs to mesons and QQQ and $\overline{QQQ}$ triplets to baryons and antibaryons were taken without change from I. The momentum of a hadron originated by the recombination of particular partons is taken as the vector sum of the momenta of these partons. The decays of unstable hadrons were also treated in the way described in I.

The three parameters G, (P_s/P_u), and R have been fixed by the data at a single energy (150 GeV/c). The coupling constant G has been determined from the averaged charged multiplicity and P_s/P_u by the overall suppression of kaons at this energy ($G=1.15$, $P_s/P_u = \frac{2}{9}$). Technically G is a free parameter, but in fact arguments based on

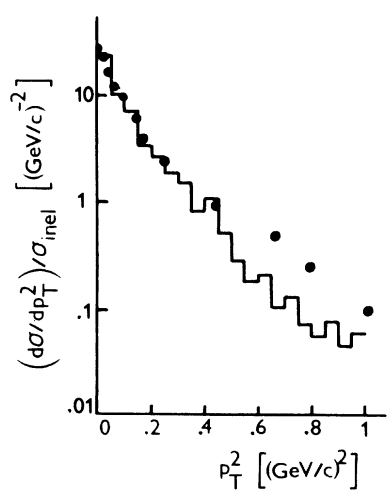

FIG. 1. $d\sigma/dp_T^2$ for negative particles in pp collisions at 150 GeV/c integrated over the backward hemisphere as calculated in our model (the histogram). The points represent the data (Ref. 12) at 205 GeV/c.

estimates of the number of $Q\bar{Q}$ pairs coming from the gluon conversion[8] indicate that this version of the quark-parton model naturally leads to an approximately correct $\langle n_{ch}\rangle$. In order to determine the parameter R, we have calculated the p_T distribution of negative particles in 150 GeV/c pp collisions and compared the results with the data.[12] In Fig. 1 it is shown that $R^2 = 0.20$ GeV$^2/c^2$ gives a good description of the data.

Thus all the parameters of the model are fixed. In Sec. III we present the results on the energy dependence of average multiplicities and on the rapidity spectra.

III. THE ENERGY DEPENDENCE OF AVERAGE MULTIPLICITIES

The energy dependence of average charge multiplicity is essentially given by cylindrical phase space [see Eq. (1)]. Multiplicities of individual particles depend also on other factors, e.g., W_{id}, the distribution of valence quarks within the allowed rapidity region, and on the suppression of strange quarks.

Results on the average multiplicities are presented in Fig. 2. The data shown there were obtained both in electronic experiments (Antinucci *et al.*[13]) and in bubble chambers (Whitmore[14]). These data are somewhat different since in bubble chambers some particles like K_s^0 and Λ are identified, whereas in an electronic experiment one usually registers only their decay products. For

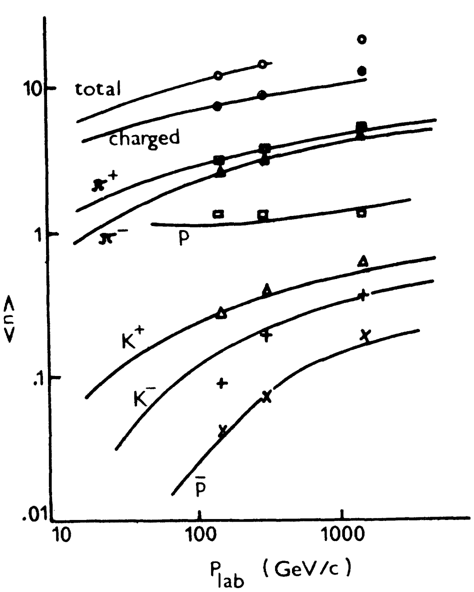

FIG. 2. Energy dependence of average multiplicities. Curves represent the data from Refs. 13 and 14. The points show our results.

the present purposes we shall not elaborate in more detail on these differences.

Figure 2 shows that our model reproduces rather well the overall energy dependence of both the averaged charged multiplicity and the multiplicities of individual particles. In this connection we should note that our calculations of average multiplicities of particles which are rarely produced are rather inaccurate. In generating an event we use numerous random numbers and, as a consequence, results contain rather large fluctuations. We have tried to perform a realistic estimate of the errors of average multiplicities calculated here for CERN ISR energies by comparing the results of several computer runs (at the same energy). In this way we have found

$$\langle n_{\pi^+}\rangle = 5.4 \pm 0.5, \quad \langle n_{\bar{p}}\rangle = 0.19 \pm 0.10 .$$

This indicates that average multiplicities of pions (and, to a lesser extent, of kaons) are determined in our calculations accurately, whereas the situation is worse for antiprotons and similarly worse with Λ's and other particles produced at a rate of about 0.1 per inelastic event. The accuracy can be improved simply by increasing considerably the statistics. The results quoted above correspond to about 12 000 events generated at each energy (this takes roughly 1 hour on a Siemens 4004 computer, which is comparable with a CDC 3300). Note also that—concerning fluctuations—12 000 events generated on a computer are much less than 12 000 events in an experiment. In the latter case nature performs a perfect importance sampling, while in a complicated Monte Carlo program this is almost impossible.

A typical feature of the present model, as well as of earlier ones[4,5] based on the recombination of partons to hadrons is the copious production of resonances. The experimental situation is not quite clear because the extraction of resonance production from the data is a difficult and delicate problem. In Table I we present average multiplicities of particles *directly* produced by recombinations of partons in pp collisions at 150, 300, and 1500 GeV/c as calculated in our model. We have omitted only some resonances which are rarely produced. Average multiplicities of particles predicted for a particular experiment by the present model can be obtained from Table I by using known decay rates of resonances.

In general it seem that the present model predicts higher resonance production that that given by the data. A notable exception is the recent analysis of the multiparticle production at the Split Field Magnet at the CERN ISR. Their analysis[15] indicates that meson resonances are frequently produced and that about 60% of the pions

TABLE I. Calculated average multiplicities of particles directly produced by recombination of partons in pp collisions at 150, 300, and 1500 GeV/c.

	150 GeV/c	300 GeV/c	1500 GeV/c
π^+	0.22	0.22	0.31
π^-	0.17	0.20	0.24
π^0	0.20	0.22	0.34
K^+	0.08	0.15	0.20
K^0	0.03	0.08	0.13
K^-	0.02	0.04	0.03
$\bar{K}^0$	0.03	0.05	0.08
p	0.34	0.35	0.21
n	0.15	0.17	0.25
$\bar{p}$	0.006	0.008	0.010
$\bar{n}$	0.01	0.01	0.02
Δ^{++}	0.21	0.29	0.22
Δ^+	0.75	0.59	0.60
Δ^0	0.29	0.30	0.33
Δ^-	0.03	0.04	0.07
Y^{*+}	0.04	0.08	0.14
Y^{*0}	0.09	0.15	0.18
Y^{*-}	0.02	0.04	0.02
Σ^+	0.04	0.03	0.07
Σ^0	0.04	0.05	0.10
Σ^-	0.01	0.02	0.04
Λ	0.06	0.06	0.08
ρ^+	0.68	0.80	1.01
ρ^0	0.67	0.83	1.15
ρ^-	0.43	0.51	0.72
K^{*+}	0.21	0.27	0.45
K^{*0}	0.14	0.17	0.30
K^{*-}	0.09	0.12	0.30
$\bar{K}^{*0}$	0.06	0.08	0.20
ω	0.57	0.77	1.20
ϕ	0.04	0.05	0.06
η	0.07	0.09	0.18
η'	0.14	0.15	0.31

observed at ISR energies is due to vector-meson decays. Their results are compared with our calculations in Table II. The agreement is surprisingly good.

As the question of resonance production is particularly relevant for quark-parton models we hope to be able to discuss it in more detail later on when the experimental information becomes more complete.

TABLE II. The average multiplicities of meson resonances in pp collision at $\sqrt{s}=53$ GeV. The possibility of f production is not taken into account in our model.

	Calculation	Experiment (Ref. 15)
ρ^0	1.15	1.19 ± 0.25
ω	1.20	1.43 ± 0.26
$K^{*0}+\bar{K}^{*0}$	0.50	0.76 ± 0.23
η	0.18	0.22 ± 0.14
f		0.24 ± 0.13

IV. SINGLE-PARTICLE RAPIDITY SPECTRA AT FERMILAB AND ISR ENERGIES

The data in the Fermilab energy contain relatively detailed information about rapidity distributions of charged pions and the neutral short-living particles K_S^0 and Λ.

The data on single-particle production at ISR energies are available usually at discrete values of p_T (narrow bins) not covering $p_T\approx 0$. Limiting oneself in a Monte Carlo calculation to a narrow p_T bin would lead to results with large fluctuations. Sensible results could be obtained only at the price of increasing considerably the computer time. Because of that we have made no attempts at a detailed comparison of ISR data on p_T and y inclusive spectra with the results of our model.

We perform a comparison with the Fermilab data at 300 GeV/c where we can see also the differences between the present model and its previous version[1] where transverse momenta of the quarks were set equal to zero. Our results are compared in Figs. 3(a) and 3(b) with the data from

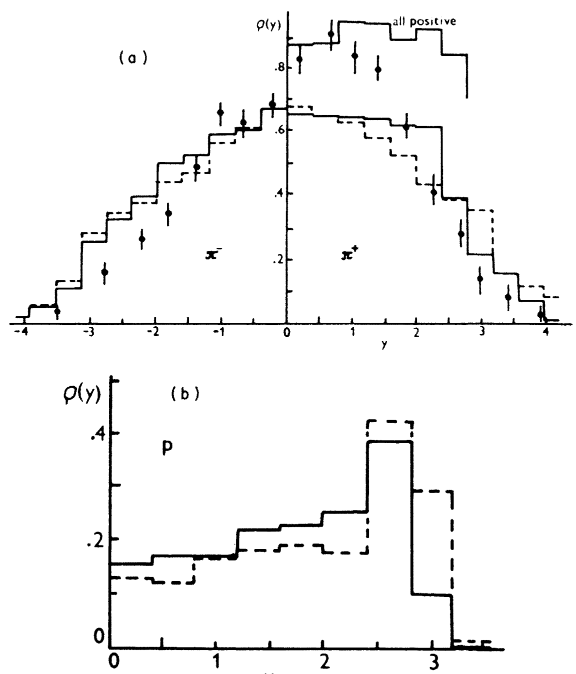

FIG. 3. Inclusive rapidity spectra in pp collisions at 300 GeV/c. Results of the present calculation are shown as solid histograms. For comparison we show also results (dashed histograms) from the previous version of our model (Ref. 1), where transverse momenta of partons were neglected. On the vertical axis we plot the average density of particles per rapadity unit. (a) Positive and negative pions. Data at 200 GeV/c are taken from Ref. 14 (we are not aware of any data of this type at 300 GeV/c). (b) Proton distribution.

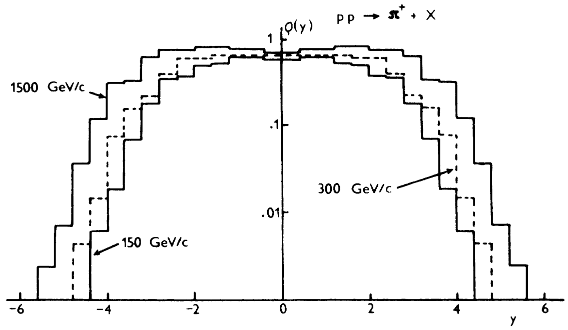

FIG. 4. Positive-pion inclusive rapidity distribution at 150, 300, and 1500 GeV/c as calculated in our model.

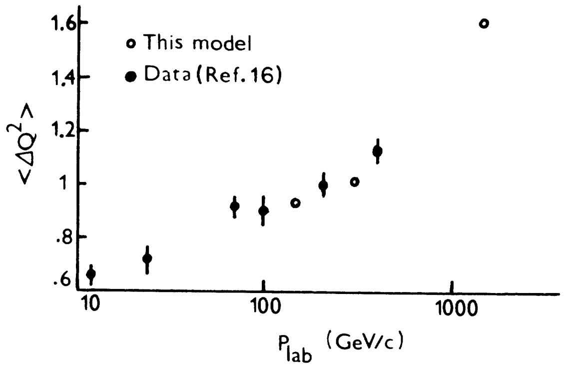

FIG. 5. The energy dependence of the average charge fluctuation $\langle(\Delta Q)^2\rangle$ across $y_{\text{c.m.}}=0$ in pp collisions. Data from Ref. 16 (full circles), our results shown as open circles.

pp collisions at 300 GeV/c and with our previous results.[1] In Fig. 4 we present our results for the inclusive spectra of positive pions in pp collisions at 150, 300, and 1500 GeV/c. Finally in Fig. 5 we compare the calculated energy dependence of charge fluctuation between c.m. rapidity hemispheres with the data.[16] The discussion of this energy dependence in models based on the recombination of quarks and antiquarks to final-state hadrons in given in Ref. 17.

V. COMMENTS AND CONCLUSIONS

The overall qualitative agreement of our model with the limited amount of data studied here indicates that the global and rough features of the multiple production can be understood in a quark-parton model which (1) uses cylindrical phase space (with cutoff transverse momenta), (2) manages to keep valence quarks at the ends of rapidity space, and (3) suppresses in a phenomenological way the production of strange quarks. Without performing further studies of more specific and more refined features of the data it is difficult to judge how much such a model can be pushed farther.

As pointed out in the last section of I the present model is far from being complete. It does not contain *diffractive dissociation* (understood as a process in which the coherence of partons within incoming hadrons is not completely destroyed[18]) and it treats only in a very rough way the *recombination* of quarks and antiquarks to hadrons.

Recently we have used this model to study[10] dimuon production in hadron collisions, in particular in estimating the yield of dimuons originated by annihilations of quarks and antiquarks created during the collision. Working on that problem we have realized that the spacetime evolution[19] of the collision is of primary importance (at least for the dimuon production). In multiparticle production the aspects of the spacetime evolution do not enter in such a clearly visible way since the recombination process is short range in rapidity. Still, we believe that implications of the spacetime evolution of the collision should be studied in more detail. In our calculation this could influence directly the expression for W_{id} (the factor for identical particles, for details see I) and, as far as direct phenomenological aspects are concerned, this should influence the fluctuations of quantum numbers.[20]

In our opinion it is also quite possible that elementary features of the spacetime evolution—and in particular the fact that at any given time only a rather short region of rapidity is excited—can provide a bridge between quark-parton and cluster models of multiple production. This point, in our opinion, deserves attention.

ACKNOWLEDGMENT

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[20]Constraints imposed by the space-time evolution (Ref. 19) would lead to effectively smaller number of quarks contributing to the charge fluctuations (Ref. 17) and therefore lower charge fluctuations.

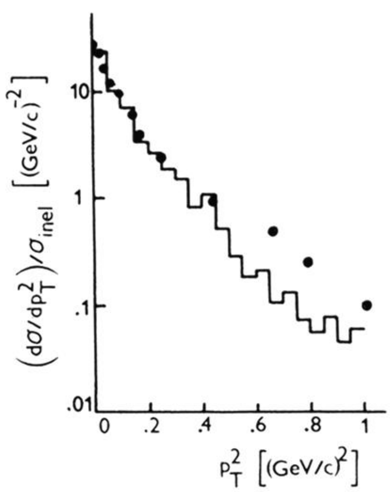
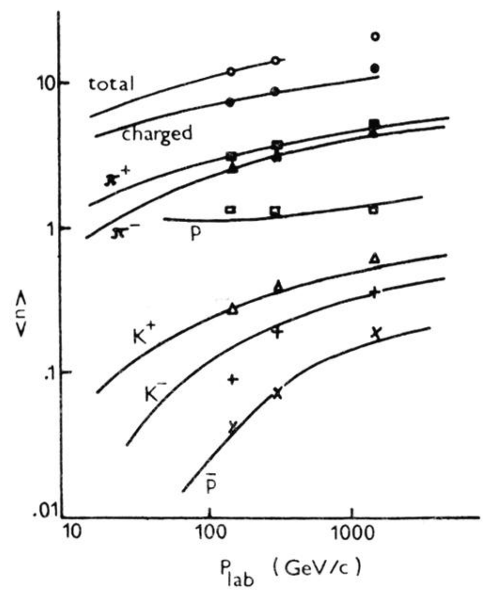


FIG. 1. $d\sigma/dp_T^2$ for negative particles in pp collisions at 150 GeV/c integrated over the backward hemisphere as calculated in our model (the histogram). The points represent the data (Ref. 12) at 205 GeV/c.



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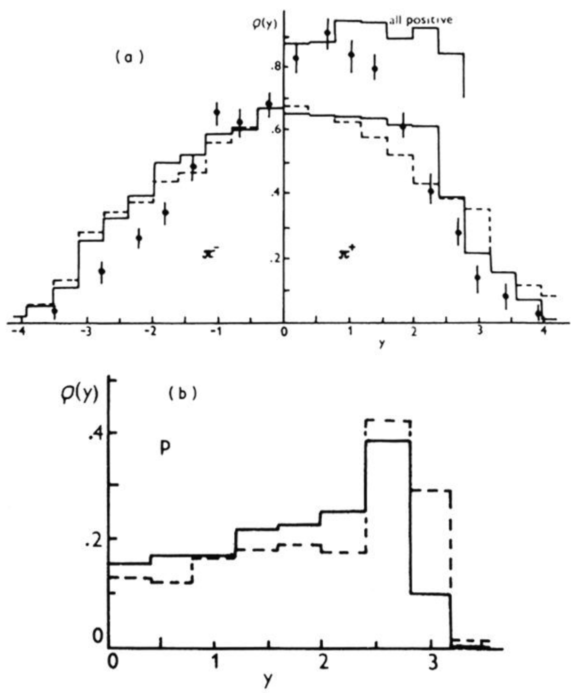
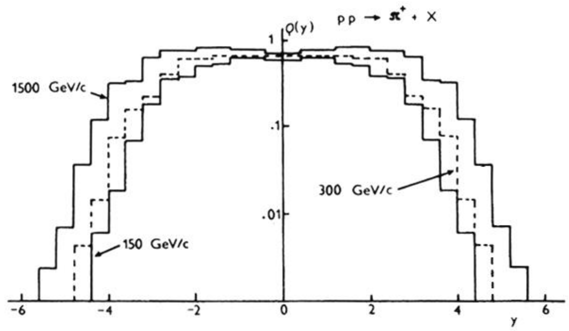


FIG. 3. Inclusive rapidity spectra in pp collisions at 300 GeV/c. Results of the present calculation are shown as solid histograms. For comparison we show also results (dashed histograms) from the previous version of our model (Ref. 1), where transverse momenta of partons were neglected. On the vertical axis we plot the average density of particles per rapadity unit. (a) Positive and negative pions. Data at 200 GeV/c are taken from Ref. 14 (we are not aware of any data of this type at 300 GeV/c). (b) Proton distribution.



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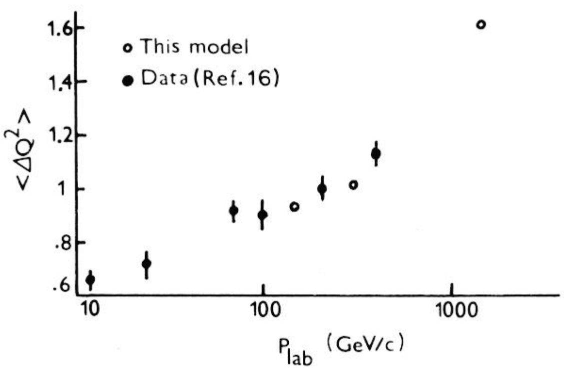

FIG. 5. The energy dependence of the average charge fluctuation $\langle(\Delta Q)^2\rangle$ across $y_{\mathrm{c.m.}}=0$ in pp collisions. Data from Ref. 16 (full circles), our results shown as open circles.