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Large transverse momenta from a pre-equilibrium fireball state and fireball spin

Parthajyoti, Datta and D. Bondyopadhyay
Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India
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A phenomenological whole-region formula for inclusive $p_\perp$ distribution at 90°, based on the concept of varying temperature in a pre-equilibrium state, is set up and its predictions are compared with the data on π^0, π^-, and K^+ inclusive distributions at CERN ISR energies. For comparison, prediction of large-$p_\perp$ distributions in the statistical bootstrap model including spin is also considered.

I. INTRODUCTION

Very large transverse momenta of a fraction of the secondaries produced in high-energy hadronic collisions remain a challenging problem for the models of high-energy multiparticle production reactions. In particular, a unified description of large-$p_\perp$ and small-$p_\perp$ phenomena has turned out to be extremely difficult to achieve and usually it is convenient to parametrize two regions of the distribution ($p_\perp < 1$ GeV and $p_\perp > 1$ GeV) with two different functional forms.[1] Nevertheless, models based on the quark-parton hypothesis and the soft- and hard-interaction pictures[2,3] have been used to devise whole-region formulas for $p_\perp$ distributions. The problem has also been considered from the point of view of different statistical models,[4] including the statistical bootstrap model (SBM) modified to include spin.[5,6] Hagedorn[7] has pointed out that the SBM description effectively accounts for 99% of the observed pions in the transverse direction, although serious deviations are observed in the remaining 1%. Several explanations in the framework of statistical bootstrap models have been offered to account for the very high transverse momenta of these small groups of pions; notable among these are the hypothesis of contributory effects from high-spin states and the possible existence of higher-temperature pre-equilibrium situations.

Somewhat detailed theoretical and numerical analyses exists for the effect of the spin of a fireball on $p_\perp$ distribution, but the possibility of the existence of pre-equilibrium states has not been studied so far. The effect of fireball spin will be considered later, but this alone cannot explain the large proportion of heavier particles among the high-$p_\perp$ secondaries. In this paper we have attempted to set up a phenomenological whole-region transverse-momentum distribution formula for pions and kaons from pp collisions at CERN ISR energies, by assuming contributions coming from a pre-equilibrium state that is defined by an initial higher-temperature system which subsequently cools to the universal equilibrium temperature characteristic of the slope of the distribution at low $p_\perp$. The possible existence of a higher-temperature state will be most prominently manifested only through the distributions and correlations of large-$p_\perp$ particles.

That these considerations are not inconsistent with SBM has recently been shown by Fre and Page[8] from a study of such systems. It is shown that the fireballs are characterized by a "correlation volume" which fixes their unique ultimate temperature, and the possibility of the occurrence of a wide variation of temperature inside the fireball matter is established. The model that is being considered here is based on a time evolution of temperature rather than a static temperature distribution. It is plausible that fireballs with varying temperatures would somehow undergo transitions, and a change of temperature would occur during a very short-lived pre-equilibrium condition. However, a master equation for the time evolution of the SBM system that would lead to an appropriate time-dependent pre-equilibrium distribution has not been formulated until now.

There exists the possibility of a time evolution of temperature in the hydrodynamical models[2,9] which, however, usually predict a unique temperature during the particle-emission phase. In a recent paper Gorenstein *et al.*[10] have discussed the possibility of very-high-transverse-momentum production in hydrodynamical models. But for comparision with experiment, an extra assumption of "evaporation" from the initial higher-temperature state becomes necessary. The approach is therefore not essentially different from the usual phenomenological models with two fixed temperatures to account for the high-$p_\perp$ and low-$p_\perp$ particles.

II. INCLUSIVE CROSS SECTIONS

In accordance with our assumptions, the inclusive cross sections averaged over time will be given by[9]

$$E\frac{d^3\sigma}{dp^3}=\frac{\int_0^\infty\left[\exp\left(\frac{(p^2+m^2)^{1/2}}{T(t)}\right)-1\right]^{-1}dt}{\int_0^\infty dt\int\left[\exp\left(\frac{(p^2+m^2)^{1/2}}{T(t)}\right)-1\right]^{-1}d^3p}$$

$$=\text{const}\times\int_0^\infty\left[\exp\left(\frac{(p^2+m^2)^{1/2}}{T(t)}\right)-1\right]^{-1}dt\,. \tag{1}$$

$T(t)$ is a time-dependent function representing temperature which is to be chosen in such a way that a complete description of the transverse-momentum distribution should be contained in formula (1). $T(t)$ is chosen to have the simple form

$$T(t)=T_0+(Ae^{-\lambda t}-T_0)[1-\theta(t-t_0)]\,. \tag{2}$$

It refers to an exponential fall of temperature from a higher value designated by A until it becomes equal to the universal Hagedorn temperature T_0. In terms of the variable $\tau=t/C$ with $\tau_0=t_0/C$, where C is a sufficiently large interval within which pion production from the fireball is complete, the cross section at $\theta=90°$ becomes

$$E\frac{d^3\sigma}{dp^3}=V(s)\left\{\int_0^{\tau_0}[\exp((p_\perp{}^2+m^2)^{1/2}/Ae^{-\ln(A/T_0)\tau/\tau_0})-1]^{-1}d\tau+[\exp((p_\perp{}^2+m^2)^{1/2}/T_0)-1]^{-1}(1-\tau_0)\right\}. \tag{3}$$

τ_0 is the fraction of time for which the high-temperature condition of the fireball exists and is crucial in producing the large-$p_\perp$ tail of the distribution. Formula (3) is only a phenomenological way of interpreting the effect of a possible kind of pre-equilibrium situation since in reality such a situation may not be describable in terms of definite temperatures. A and τ_0 are the energy-dependent parameters of the model; $V(s)$, the normalization parameter, is also energy-dependent. For the π^0 and π^- inclusive cross sections at energies, $V(s)$ may be fitted to the following form:

$$V(s)=Bs^n\,,$$

with $n=\frac{1}{3}$, and $B_{\pi^0}=8.46$, $B_{\pi^-}=14.0$ (ml $\text{GeV}^{-2}c^3$)

III. THE EFFECT OF FIREBALL SPIN

Fireballs possessing high spins will behave like exploding flywheels and may give rise to very large transverse momenta. Attempts to solve the statistical bootstrap model including the effect of spin have been made by Hamer[5] and by Hagedorn and Wambach.[6] Working under the approximation of a single fireball production and integration over a Gaussian volume, Hamer has worked out the following formula for transverse-momentum distribution:

$$E_1\frac{d^3\sigma}{dp^3}\propto E_1\exp\left(\frac{-E_1}{T}\right)\left\{1+\frac{9}{16}\frac{\langle E\rangle^2}{\langle p^2\rangle}\left[\frac{p_\perp{}^2}{\langle p^2\rangle}\left(1+\frac{\cos^2\theta}{4}\right)-\frac{1}{3}\frac{E_1}{\langle E\rangle}\right]\right\}, \tag{4}$$

where E_1 is the single-particle energy and $\langle p^2\rangle$ and $\langle E\rangle$ refer to the average momentum and energy of the pions. Calculation of the averages will involve detailed knowledge of the transverse momentum and rapidity distributions of all kinds of secondaries. As these distributions are not always available experimentally at different energies, it becomes necessary to invoke some models for their calculation. We have approximately calculated the averages by assuming a Bose distribution at Hagedorn temperature T_0. Neglecting the mass

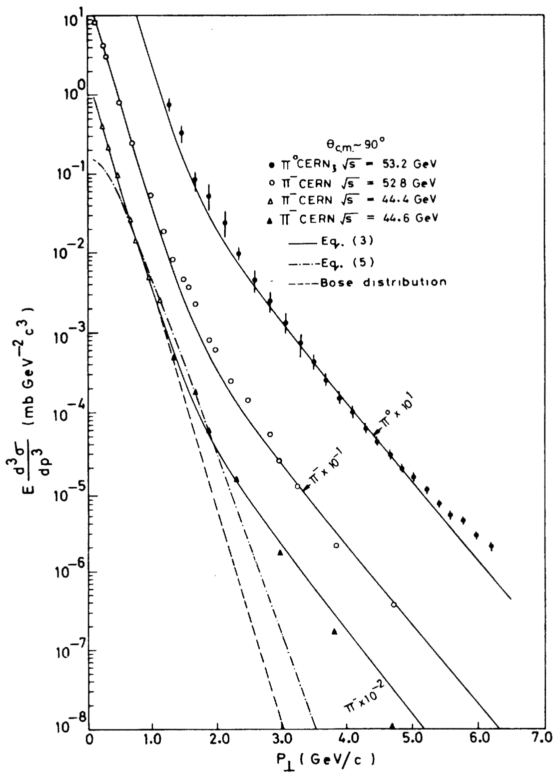


FIG. 1. Inclusive single-particle invariant cross-sections at 90° for π^0 at $\sqrt{s}=53.2$ GeV (Ref. 11) and π^- at $\sqrt{s}=52.8$ and 44.6 GeV (Ref 12) as a function of $p_\perp$. The solid curves refer to Eq. (3)

of the mesons, one obtains

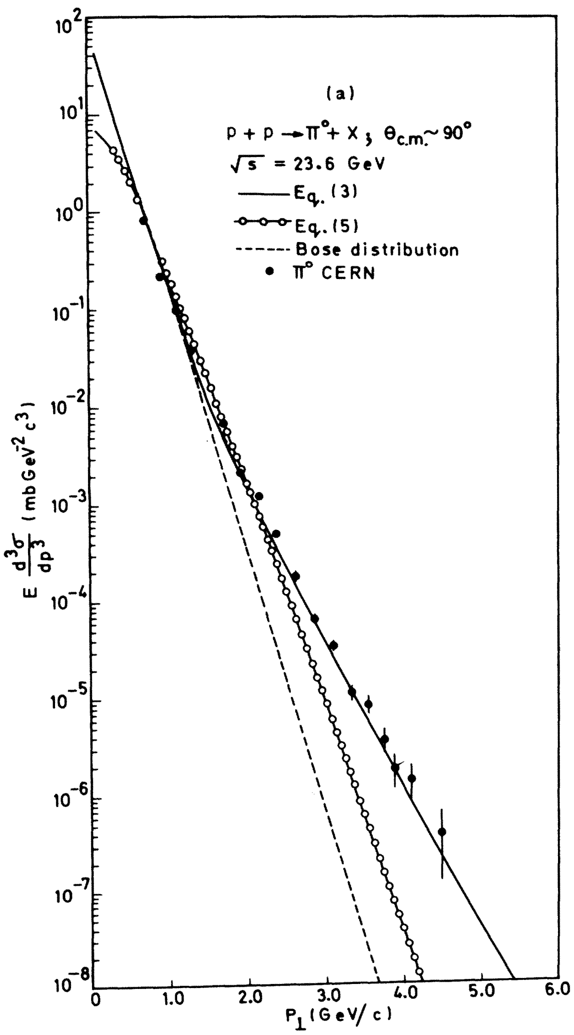
$$\langle p^2 \rangle = \frac{\int_0^\infty (3E^4/2\pi^2)[1/(e^{E/T_0}-1)]\,dE}{\int_0^\infty (3E^2/2\pi^2)[1/(e^{E/T_0}-1)]\,dE} = 10.37T_0^{\,2},$$

$$\langle E \rangle = 2.71T_0.$$

The inclusive distribution now assumes the form

$$E_1\frac{d^3\sigma}{dp^3} = \text{const} \times (p_\perp^{\,2}+m^2)^{1/2}\exp\left[-\frac{(p_\perp^{\,2}+m^2)^{1/2}}{T_0}\right] \times \left\{1+\frac{9}{16}\frac{(2.71T_0)^2}{10.37T_0^{\,2}} \times\left[\frac{p_\perp^{\,2}}{41.48T_0^{\,2}}-\frac{(p_\perp^{\,2}+m^2)^{1/2}}{8.13T_0}\right]\right\}. \qquad (5)$$

Hamer has pointed out that when the statistical-model cross section σ^F is low compared to σ^C, the thermodynamical and coherent production cross sections, then the single-particle distribution will deviate significantly from the statistical model exponential behavior. Single-particle distributions from pp, $\pi^\pm p$, etc., reactions show good exponential behavior for the bulk of the cross sections. This shows that σ^F is overwhelmingly important for these reactions and Hamer's formula should give a good description of the data. This formula is expected to be more applicable in the small-$p_\perp$ region because of the low value of fireball spin that its derivation assumes (J/m small). Plots of formula (5) at various energies can be seen in Figs. 1 and 2(a)–2(d), where the best possible fits are obtained by varying the



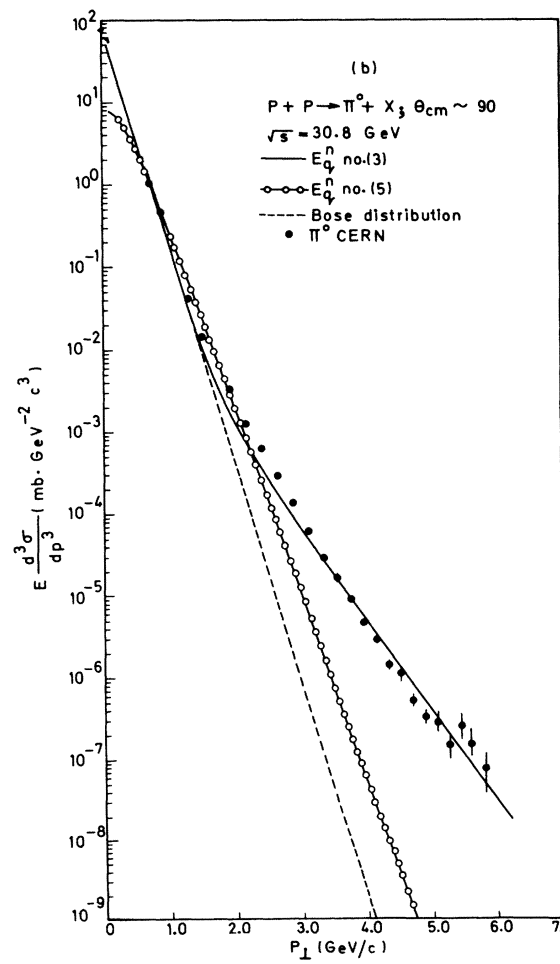


FIG. 2. Data (Ref 11) for π^0 inclusive invariant single-particle cross section at 90° as a function of $p_\perp$; (a) $\sqrt{s}=23.6$, (b) $\sqrt{s}=30.8$, (c) $\sqrt{s}=45.1$, (d) $\sqrt{s}=62.9$ GeV. Curves in each figure are plotted from Eqs. (3), (5), and the Bose distribution.

constant term. The results are not very sensitive to the values of the averages adopted. There are no other adjustable parameters in (5).

IV. DISCUSSIONS

In Figs. 1, 2(a)–2(d), and 3, the results of the calculation of inclusive transverse-momentum distribution using formulas (3), (5), and the Bose distribution are compared with CERN data[11,12,13,14] for π^- at $\sqrt{s}=44.6$ and 52.8 GeV, π^0 at $\sqrt{s}=23.6$, 30.8, 45.1, 53.2, and 62.9 GeV, and K^+ at $\sqrt{s}=23$, 31, 45, and 53 GeV. Formula (5), which fails to account for the observed data even at low $p_\perp$, is perhaps a too simple way of taking into account the effect of fireball spin. The single-fireball approximation and the approximation of the integration over a Gaussian volume may not be very realistic. By pursuing a somewhat different idea and performing detailed phase-space integrations, Hagedorn and Wambach have demonstrated that even these approximations may lead to observed large-$p_\perp$ distributions. This model, however, is not a mere refinement of the Hamer model, and although it may be more realistic it cannot be said to be a pure statistical model in the sense of the Fermi model[15] or the Hagedorn-Frautschi[16] model. It belongs to the class of phenomenological cluster models as it treats single-cluster-production cross sections in a phenomenological manner; rigorous spin-augmented SBM theory is used to describe only the subsequent decay of the cluster. Hamer's treatment, on the other hand, does not consider production and decay separately and is therefore a straightforward extension of the SBM model with no adjustable parameters appearing. The wide difference in the results obtained in these two models is therefore not entirely unexpected.

The $p_\perp$ distribution represented by formula (3) can be made to follow the experimental curves by adjusting parameters A and τ_0. In Table I, A and τ_0 values refer to π^0, but the π^- distributions can

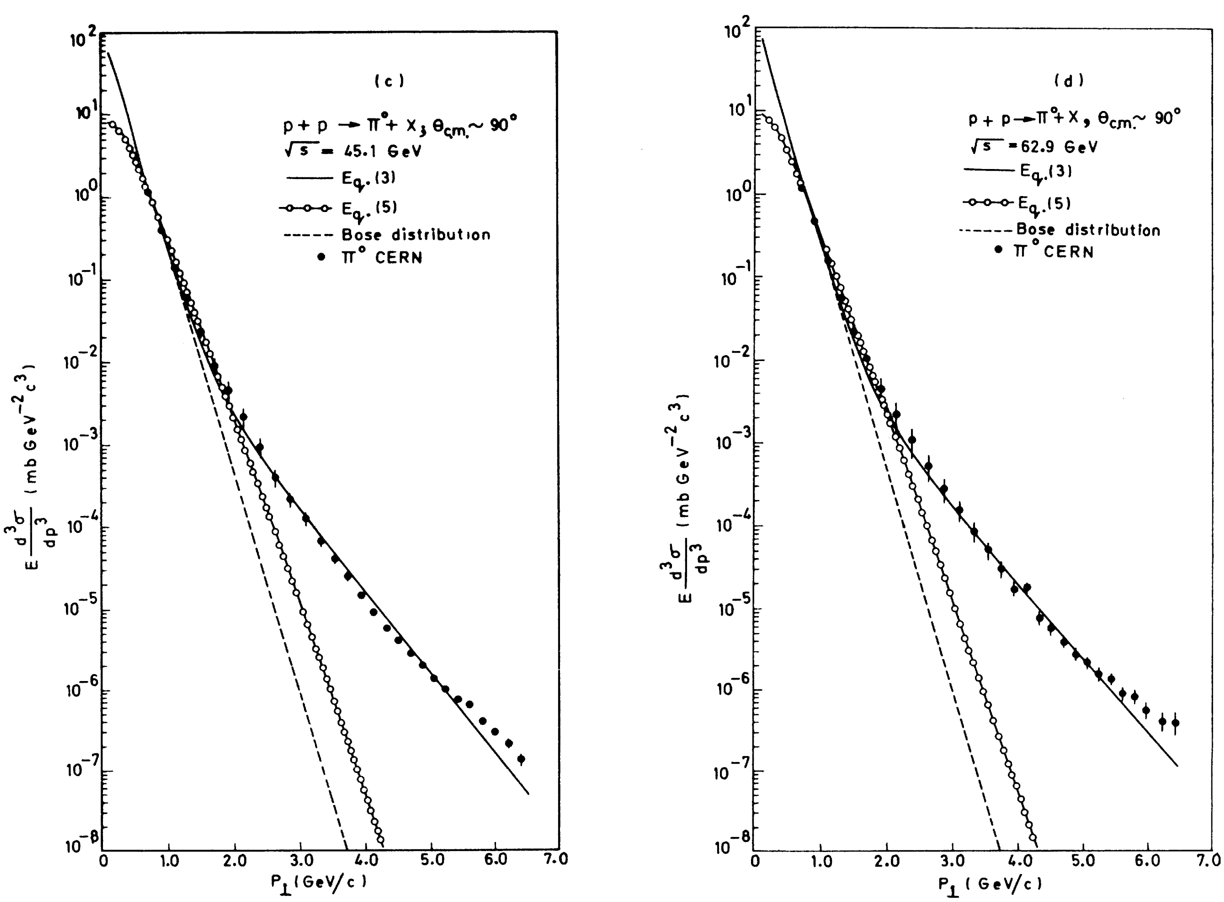


Fig. 2. (*Continued*)

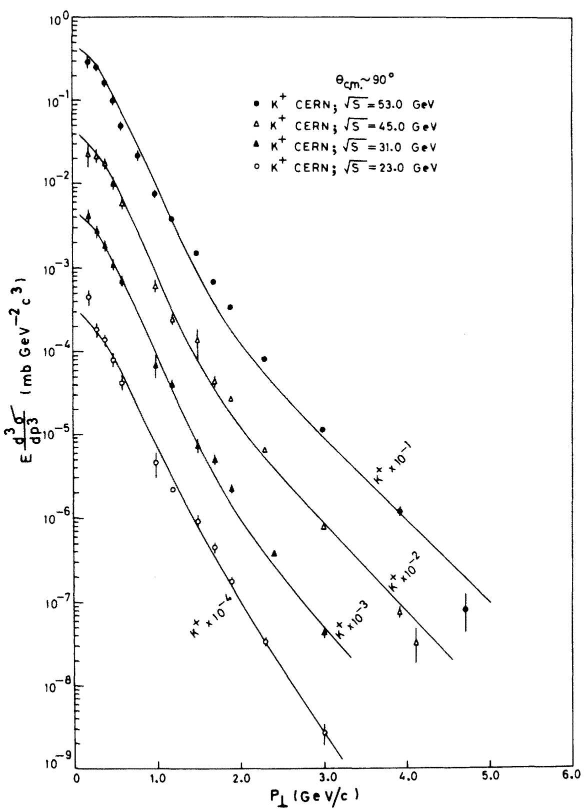


FIG. 3. Inclusive single-particle invariant cross sections at 90° for K^+ as a function of $p_\perp$ at $\sqrt{s}=23$, 31, 45, and 53 GeV (Ref. 14).

also be characterized by almost identical values of these two parameters. Somewhat different values of the parameters are necessary to characterize the K^+ distributions[14] at the corresponding

TABLE I. Fits of the parameters in Eq. (3) for π^0 inclusive distributions.

$\sqrt{s}$ GeV	A GeV	τ_0
23.6	0.32	0.05
30.8	0.435	0.0065
45.1	0.485	0.006
53.2	0.50	0.004
62.9	0.535	0.003

TABLE II. Fits of the parameters in Eq. (3) for K^+ inclusive distributions.

$\sqrt{s}$ GeV	A GeV	τ_0	B (mb GeV)$^{-2}c^3$)
23	0.32	0.05	6.5
31	0.42	0.007	9.0
45	0.47	0.006	6.5
53	0.50	0.004	6.5

energies, as is evident from Table II.

While A slowly increases, τ_0 decreases fairly rapidly with an increase of c.m. energy. τ_0 always remains a small fraction, indicating that a pre-equilibrium state of only small duration is indeed necessary to reproduce the observed high-$p_\perp$ distribution.

An analysis of data at ISR energies and certain observations at cosmic-ray energies lend support to the hypothesis of the existence of high temperatures in fireball matter. In certain observations[17] on large-$p_\perp$ hadron production at cosmic-ray energies (2×10^{15} eV and 10^{17} eV laboratory energy), it has been found that the relative number of heavier particles is very large; $p_\perp$ also becomes quite high and the distribution shows a nearly exponential behavior. These features suggest that large-$p_\perp$ particles are produced from higher-temperature interaction regions within the fireball. It has been suggested[17,18] that the mechanism of the Fermi statistical model is operative for the production of these particles. This will mean an increase of temperature with energy that accounts for the higher multiplicities of the heavier particles seen in this region. In the present phenomenological approach a transient state acts as a high-temperature source and leads to a whole-region description of the $p_\perp$ distribution.

ACKNOWLEDGMENTS

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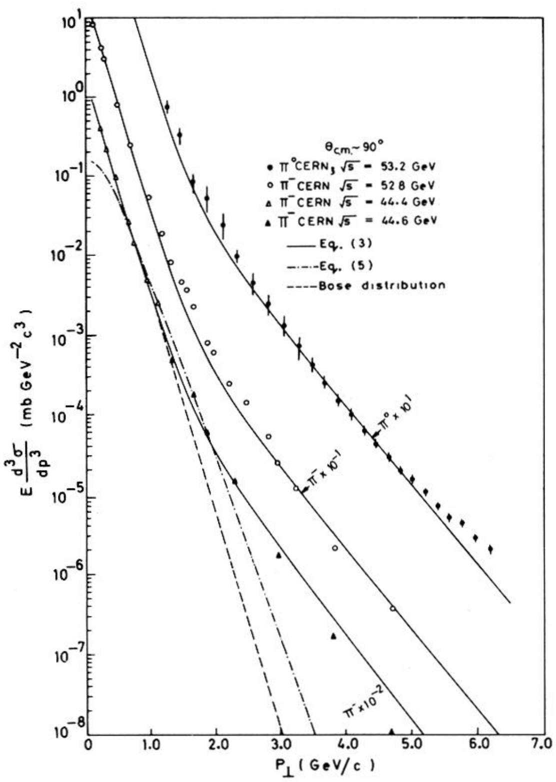


FIG. 1. Inclusive single-particle invariant cross-sections at 90° for π° at $\sqrt{s}$ = 53.2 GeV (Ref. 11) and π^- at $\sqrt{s}$ = 52.8 and 44.6 GeV (Ref 12) as a function of $p_\perp$. The solid curves refer to Eq. (3)

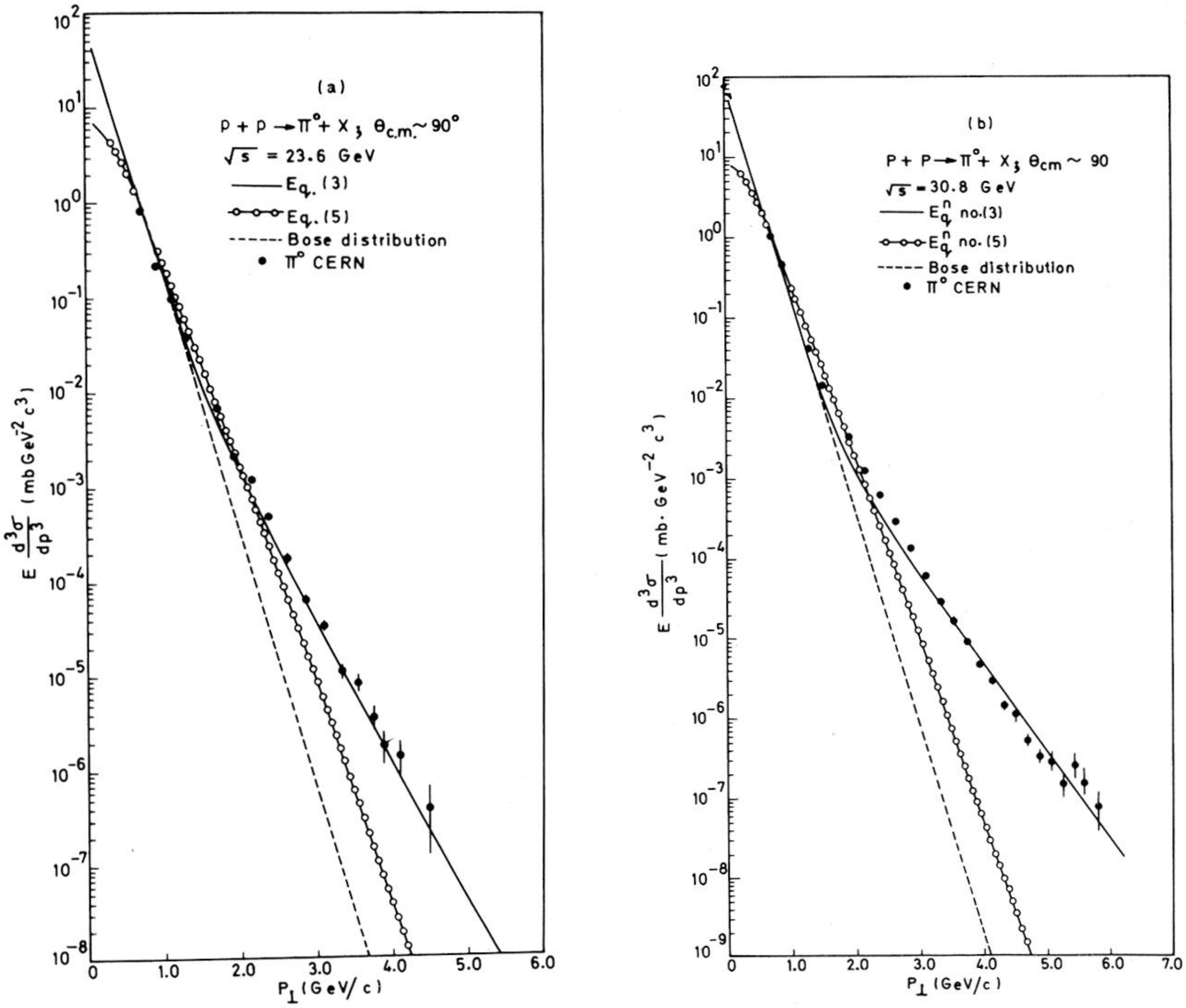
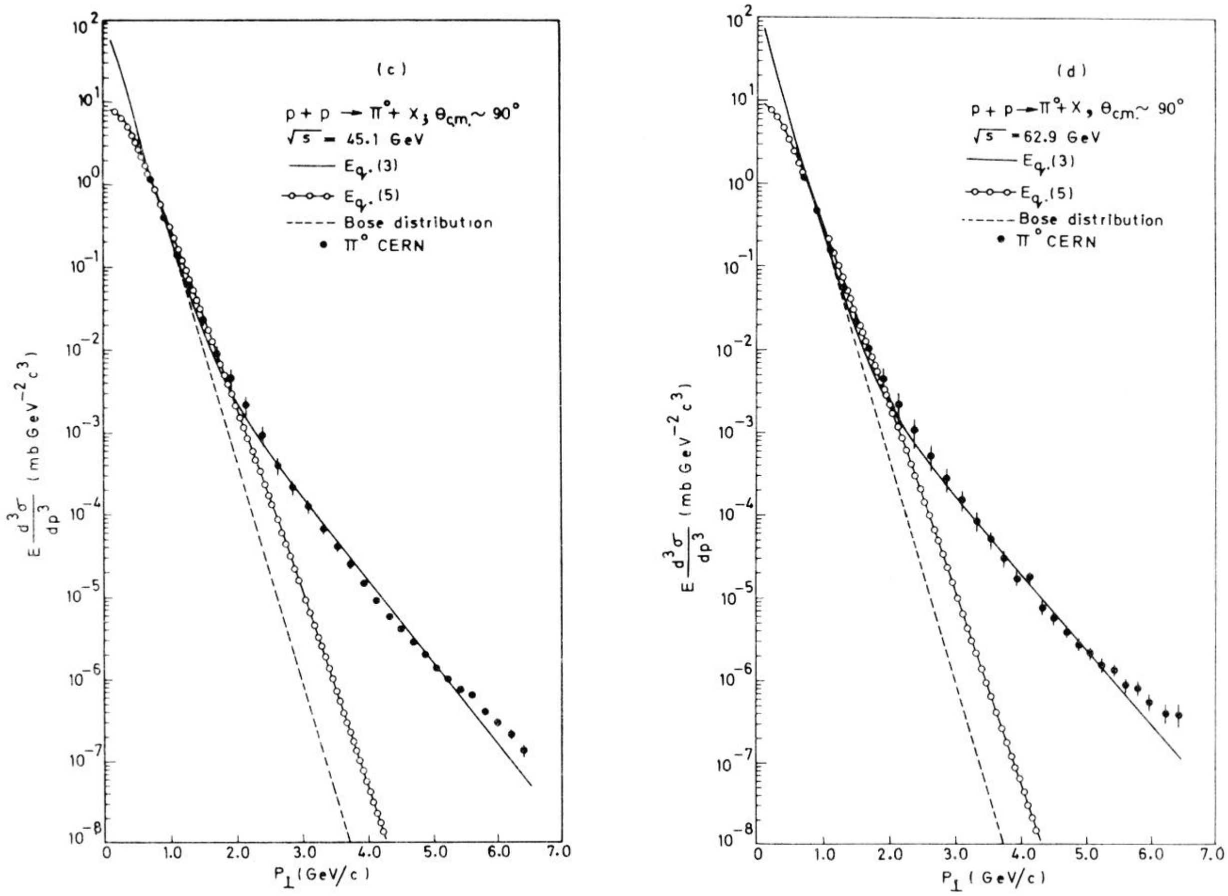


FIG. 2. Data (Ref 11) for π° inclusive invariant single-particle cross section at 90° as a function of $p_\perp$; (a) $\sqrt{s}$ = 23.6, (b) $\sqrt{s}$ = 30.8, (c) $\sqrt{s}$ = 45.1, (d) $\sqrt{s}$ = 62.9 GeV. Curves in each figure are plotted from Eqs. (3), (5), and the Bose distribution.



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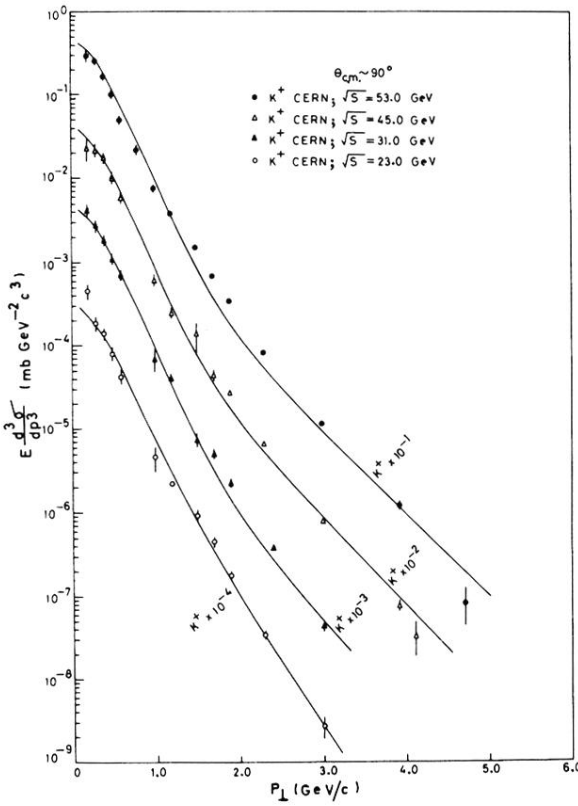

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