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Soft-pion emission in K^-p interactions

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Through use of PCAC (partially conserved axial-vector current) we calculate the amplitude for the emission of a single soft pion in the process $K^-p \to K^-p$. The ratio of cross sections $\sigma(K^-p \to K^-p\pi^0)/\sigma(K^-p \to K^-p)$ is computed for various incoming kaon momenta. The agreement between theoretical and experimental results is reasonably good.

I. INTRODUCTION

In 1965, Adler[1] developed a formalism for calculating the matrix element of any hadronic process of the type

$$i \to f + \pi , \tag{1.1}$$

in terms of the process $i \to f$, where i and f are the initial and final hadronic states and π is an emitted soft pion. In recent years, this formalism has been applied by the present authors to proton-antiproton annihilation processes.[2,3] In these applications, as in most of the low-energy theorems, current algebra is employed along with the PCAC (partially conserved axial-vector current) hypothesis. PCAC has its origin in weak interactions, and its main usefulness here is in extrapolating the matrix elements of $\partial_\mu A_\mu$ off the pion mass shell over a distance of m_π^2. The error involved in extrapolating from zero to m_π^2 is approximately 10%.

In the following, we apply Adler's formalism to the interaction

$$K^-p \to K^-p\pi^0 . \tag{1.2}$$

We relate the amplitude for this process to that of

$$K^-p \to K^-p \tag{1.3}$$

and find the ratio of the cross sections,

$$\frac{\sigma(K^-p \to K^-p\pi^0)}{\sigma(K^-p \to K^-p)} . \tag{1.4}$$

Since the emission of only one soft pion is involved in the process, only PCAC comes into the picture. In Sec. II we arrive at the matrix element for the process (1.2) and compute its absolute square, averaged over initial and summed over final spin states. The cross section for the single soft-pion emission process (1.2) is expressed as a differential in four kinematic variables. This cross section is normalized to the $K^-p \to K^-p$ cross section, also expressed as a differential. In Sec. III the ratio (1.4), obtained after numerical integration over the kinematic variables, is compared with experimental results.

II. THE $K^-p \to K^-p\pi^0$ AMPLITUDE AND DIFFERENTIAL CROSS SECTION

The matrix element for process (1.2) appears in the well-known reduction formula

$$ik_\mu \langle K^-p | A_\mu^i | K^-p\rangle = \langle K^-p | \partial_\mu A_\mu^i | K^-p\rangle , \tag{2.1}$$

where $i = 1, 2, 3$ are the isospin indices of the axial-vector current operator. According to PCAC,

$$\partial_\mu A_\mu^i = (1/\sqrt{2}) C_\pi \phi_\pi^i , \tag{2.2}$$

with

$$C_\pi = \sqrt{2} G_A M_N m_\pi^2 / g_r(0) ; \tag{2.3}$$

$G_A \approx 1.18$, g_r is the rationalized, renormalized pion-nucleon coupling constant ($g_r^2/4\pi \approx 14.6$), ϕ_π^i is the renormalized pion field, and M_N, m_π are the nucleon and pion masses.

If we introduce the Klein-Gordon operator in Eq. (2.1), there results

$$ik_\mu \langle K^-p | A_\mu^i | K^-p\rangle = \frac{M_N G_A m_\pi^2}{g_r(m_\pi^2 - k^2)} \times \langle K^-p | (m^2 - \Box)\phi_\pi^i | K^-p\rangle . \tag{2.4}$$

One has to investigate this equation in the limit $k \to 0$. As $k \to 0$, the right-hand side approaches $M_N G_A / g_r$ times the matrix element for emission of a zero four-momentum pion and the left-hand side vanishes unless it has pole terms. Pole terms that go as k^{-1} arise when the axial-vector current is attached to the external line that does not terminate. Insertion of A_μ^i into a pseudoscalar meson line is forbidden by parity. Therefore for the reaction $K^-p \to K^-p$ one has to consider only the two diagrams shown in Fig. 1. The contribution of the insertion of A_μ^3 into the initial baryon line [Fig.

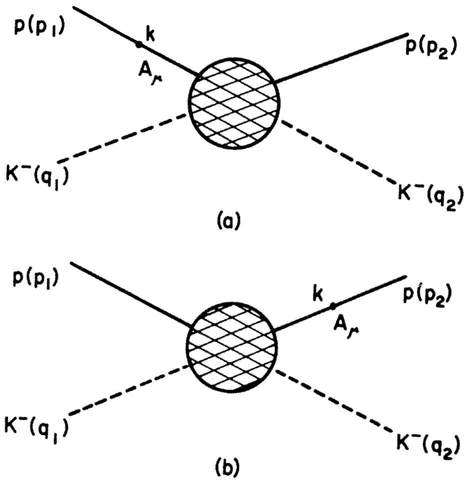

FIG. 1. Diagrams of order k^{-1} in the axial-vector current matrix elements.

1(a)] is

$$\bar{u}_s(p_2)\mathfrak{M}\frac{1}{i\gamma\cdot(p_1-k)+M_N}G_A(\gamma\cdot k)\gamma_5\tau^3 u_r(p_1)\,, \quad (2.5)$$

where r and s indicate the spin states and $\mathfrak{M}$ is the operator that takes the initial state i into the final state f. In terms of relativistic invariants,

$$\mathfrak{M}=A+B\gamma\cdot Q\,, \quad (2.6)$$

where $Q=q_1+q_2$. The contribution of the insertion of A_μ^3 into the final baryon line [Fig. 1(b)] is

$$\bar{u}_s(p_2)G_A(\gamma\cdot k)\gamma_5\tau^3\frac{1}{i\gamma\cdot(p_2+k)+M_N}\mathfrak{M}u_r(p_1)\,. \quad (2.7)$$

Taking a sum of these contributions and retaining only the zeroth-order terms in pion momenta, we have

$$\frac{M_N G_A}{q_r}M^\pi=G_A\bar{u}_s(p_2)(AF_A+BF_B)u_r(p_1)\,, \quad (2.8)$$

where

$$F_A=\gamma_5 M_N(\gamma\cdot k)\left(\frac{1}{p_1\cdot k}-\frac{1}{p_2\cdot k}\right)+2i\gamma_5 \quad (2.9)$$

and

$$F_B=-M_N\gamma_5\left(\frac{\gamma\cdot Q\gamma\cdot k}{p_1\cdot k}+\frac{\gamma\cdot k\gamma\cdot Q}{p_2\cdot k}\right). \quad (2.10)$$

The absolute square of the matrix element summed over final and averaged over initial spin states is given by the usual expression,

$$\langle|M^\pi|^2\rangle\propto\tfrac{1}{2}\mathrm{Tr}\left[(AF_A+BF_B)\frac{M_N-i\gamma\cdot p_1}{2M_N}\gamma_4(AF_A+BF_B)^\dagger\gamma_4\frac{M_N-i\gamma\cdot p_2}{2M_N}\right]. \quad (2.11)$$

In Eq. (2.8) A and B (scalar functions of Q^2, K^2, and $Q\cdot K$, where $K=p_1+p_2$) are unknown constants. To facilitate computation, we neglect terms proportional to A^2, assuming that they are small compared to the B^2 terms.[4] This leads to

$$\begin{aligned}\langle|M^\pi|^2\rangle=-\frac{2g_r^2B^2}{(2M_N)^2}\Bigg[&\frac{M_N^2Q\cdot Qk\cdot k}{(p_1\cdot k)^2}+\frac{4M_N^2Q\cdot kQ\cdot k}{(p_2\cdot k)(p_1\cdot k)}-\frac{2M_N^2Q\cdot Qk\cdot k}{(p_1\cdot k)(p_2\cdot k)}+\frac{M_N^2Q\cdot Qk\cdot k}{(p_2\cdot k)^2}\\&+\frac{1}{(p_1\cdot k)^2}(4Q\cdot kp_1\cdot kp_2\cdot Q+Q\cdot Qk\cdot kp_1\cdot p_2-p_1\cdot Qp_2\cdot Qk\cdot k-2p_1\cdot kp_2\cdot kQ\cdot Q)\\&+\frac{1}{(p_1\cdot k)(p_2\cdot k)}(4p_1\cdot kp_2\cdot kQ\cdot Q+4k\cdot kp_1\cdot Qp_2\cdot Q-2k\cdot kQ\cdot Qp_1\cdot p_2\\&\qquad-4Q\cdot kp_2\cdot kp_1\cdot Q+4Q\cdot kQ\cdot kp_1\cdot p_2-4Q\cdot kp_1\cdot kp_2\cdot Q)\\&+\frac{1}{(p_2\cdot k)^2}(4Q\cdot kp_2\cdot kp_1\cdot Q-2p_1\cdot kp_2\cdot kQ\cdot Q-2p_1\cdot Qp_2\cdot Qk\cdot k+Q\cdot Qk\cdot kp_1\cdot p_2)\Bigg]. \quad (2.12)\end{aligned}$$

The differential cross section is given by the expression

$$d\sigma^\pi=\frac{(2\pi)^4p_1^0q_1^0}{[(p_1\cdot q_1)^2-m_K^2M_N^2]^{1/2}}\frac{M_N}{2p_1^0q_1^0}\langle|M^\pi|^2\rangle\frac{1}{(2\pi)^9}\frac{d\vec{p}_2d\vec{q}_2d\vec{k}}{4q_2^0p_2^0k^0}\delta(p_1+q_1-p_2-q_2-k)\,. \quad (2.13)$$

Out of the nine variables, five can be integrated trivially. The remaining four variables are chosen to be the following:

$$m_{Kp}^2,\ \cos\theta_\pi,\ \cos\theta_{p_2},\ \phi\,, \quad (2.14)$$

where $m_{Kp}^2=-R_1^2$ is the invariant mass of the $(K^-p)_{\mathrm{final}}$ system ($R_1=q_2+p_2$); θ_π is the angle between the pion and the incoming proton in the R $(=p_1+p_2)$ rest frame; θ_{p_2} is the angle between the final proton and the pion in the R_1 rest frame; ϕ is the relative azimuthal angle between the p_1q_1 plane and the R_1 decay plane. The variables are restricted as follows:

$$m_K^2+M_N^2 \leq m_{Kp}^2 \leq M_{Kp}^2\,,\quad 0<\theta_\pi<\pi\,,\quad 0<\theta_{p_2}<\pi\,,\quad 0<\phi<2\pi\,, \tag{2.15}$$

where $R^2=-M_{Kp}^2$. The Appendix tabulates the conversion formulas to the desired kinematic variables. The expression for the differential cross section is then

$$d\sigma^\pi = \frac{M_N^2}{[(p_1\cdot q_1)^2-M_N^2m_K^2]^{1/2}}\frac{\langle |M^\pi|^2\rangle}{8}\frac{1}{(2\pi)^5}\,dm_{Kp}^2 d(\cos\theta_{p_2})d(\cos\theta_\pi)d\phi$$
$$\times\frac{1}{2m_{Kp}^2}[(m_{Kp}^2+m_K^2-M_N^2)^2-4m_{Kp}^2m_K^2]^{1/2}\frac{\pi}{2M_{Kp}^2}[(M_{Kp}^2+m_{Kp}^2-m_\pi^2)^2-4m_{Kp}^2M_{Kp}^2]^{1/2}\,. \tag{2.16}$$

The matrix element for the reaction $K^-p\to K^-p$ is

$$M=\bar{u}_s(p_2)B\gamma\cdot Qu_r(p_1)\,. \tag{2.17}$$

This leads to

$$|M|^2=-\frac{B^2}{2M_N^2}(M_N^2Q\cdot Q+Q\cdot Qp_1\cdot p_2-2Q\cdot p_1Q\cdot p_2)\,. \tag{2.18}$$

The corresponding expression for the differential cross section (see Appendix) is

$$d\sigma=\frac{M_N^2\pi}{[(p_1q_1)^2-m_K^2M_N^2]^{1/2}}\frac{|M|^2}{4(2\pi)^2}d(\cos\theta_{p_2})\frac{1}{m_{Kp}^2}[(m_{Kp}^2+M_N^2-m_K^2)^2-4M_N^2m_{Kp}^2]^{1/2}\,. \tag{2.19}$$

The ratio of Eq. (2.16) to Eq. (2.19) gives the differential cross section for the process $K^-p\to K^-p\pi^0$, normalized to the differential cross section for the same process without the soft pion.

III. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

Expressions (2.16) and (2.19) are integrated[5] to obtain cross sections. The incoming kaon momentum in the laboratory system is varied from 0.55 GeV/c (the threshold momentum for the soft-pion emission process being 0.51 GeV/c) to 1.2 GeV/c. Although the expression (2.16) is valid only for soft pions, in the process of integration no restriction is applied to the pion momenta. The calculated ratios of cross sections are presented in Table I. These are compared with the experimental results of Conforto *et al.*[6] The agreement between experimental and theoretical results is reasonably good, as indicated by the plot in Fig. 2.

TABLE I. Calculated ratios $\sigma(K^-p\to K^-p\pi^0)/\sigma(K^-p\to K^-p)$ for various incoming kaon momenta.

K^- momentum in lab (MeV/c)	$\sigma(K^-p\to K^-p\pi^0)/\sigma(K^-p\to K^-p)$
550	0.0003
600	0.0010
700	0.0070
800	0.0170
900	0.0330
950	0.0430
1000	0.0540
1050	0.0670
1100	0.0820
1150	0.0980
1200	0.1160

Since only PCAC is used in the derivation of our theoretical expressions, the results appear to be a corroboration of the validity of PCAC. The results also seem to indicate that the terms proportional to A^2 are indeed small compared with the B^2 terms as was assumed initially. However, one must be cautious about asserting these conclusions as matters of fact. Indeed the spirit of the calculation has rather been one that has avoided such commitments.

It is well known, for example, that the presence of resonance pole diagrams can complicate soft-pion calculations. Their effect on pion production in πN scattering was noted some time ago by Chang,[7] in NN scattering by Schillaci and Silbar,[8] and in $\bar{p}p\to K\bar{K}$ by Greenhut and Intemann.[9] In the present reaction there are possibilities for resonance poles also (K^*,Δ). As was the case in our earlier calculations of $\bar{p}p$ annihilation,[2,3] our approach here also is not to take into account resonant intermediate states explicitly.

A similar point of view extends to the matter of the neglect of A with respect to B terms. The threshold argument[4] for the plausibility of this can well be supported by information on elastic K^-p phase shifts. Even so, the principal outcome rather is that the initial assumptions, in both areas, lead to results that appear to justify these assumptions.[10] The results then, in a strictly

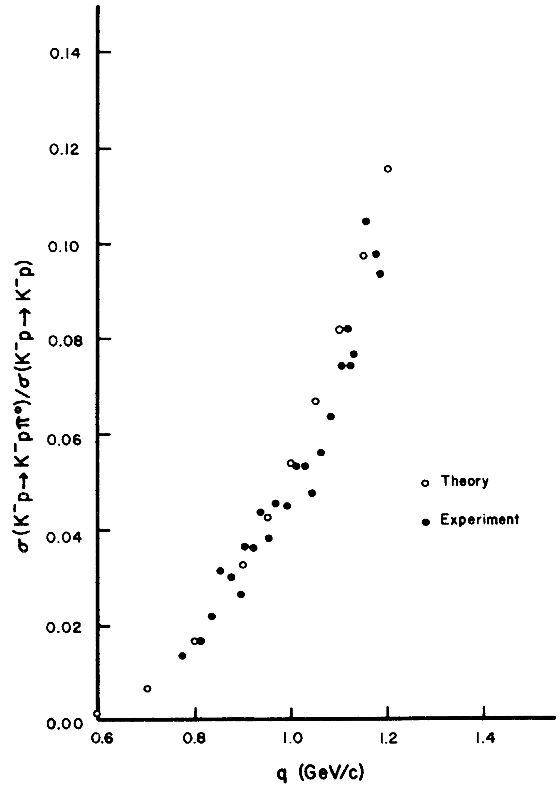

FIG. 2. Experimental and theoretical values of the ratio $\sigma(K^-p \to K^-p\pi^0)/\sigma(K^-p \to K^-p)$ as a function of kaon laboratory momentum q. The experimental values have a typical error of about 10%; see Ref. 6.

logical sense, are less a verification of any fundamental facts, and more of a phenomenological presentation of a successful, consistent calculational scheme[11] for obtaining the ratio of cross sections of $K^-p \to K^-p\pi^0$ and $K^-p \to K^-p$.

A similar procedure involving both PCAC and current algebra is being applied to K^-p interactions with the emission of two soft pions.[12]

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APPENDIX

The terms appearing in Eq. (2.12) must be expressed in terms of the kinematic variables $m_{Kp}{}^2$, $\cos\theta_\pi$, $\cos\theta_{p_2}$, and ϕ. For this purpose we first define

$$R = p_1 + q_1\,, \quad R_1 = p_2 + q_2\,,$$
$$R_2 = k\,, \quad \Delta = p_1 - q_1\,, \qquad \text{(A1)}$$

and

$$S = p_2 - q_2\,.$$

From these variables one can form the following invariants:

$$R_2,\ R_1{}^2,\ R_2{}^2,\ \Delta^2,\ S^2,\ R\cdot R_1,\ R\cdot R_2,\ R\cdot\Delta,\ R\cdot S,$$
$$R_1\cdot R_2,\ R_1\cdot\Delta,\ R_1\cdot S,\ R_2\cdot\Delta,\ R_2\cdot S, \qquad \text{(A2)}$$

and

$$\Delta\cdot S.$$

In terms of these invariants the variables in Eq. (2.12) are given as follows:

$$p_1\cdot p_2 = \tfrac{1}{4}(R\cdot R_1 + \Delta\cdot R_1 + S\cdot R + \Delta\cdot S)\,, \qquad \text{(A3)}$$

$$q_1\cdot q_2 = \tfrac{1}{4}(R\cdot R_1 - \Delta\cdot R_1 - S\cdot R + \Delta\cdot S)\,, \qquad \text{(A4)}$$

$$q_1\cdot p_2 = \tfrac{1}{4}(R\cdot R_1 + S\cdot R - \Delta\cdot R_1 - \Delta\cdot S)\,, \qquad \text{(A5)}$$

$$p_1\cdot q_2 = \tfrac{1}{4}(R\cdot R_1 + \Delta\cdot R_1 - S\cdot R - \Delta\cdot S)\,, \qquad \text{(A6)}$$

$$p_1\cdot q_1 = \tfrac{1}{2}(R^2 + M_N{}^2 + m_K{}^2)\,, \qquad \text{(A7)}$$

$$p_2\cdot q_2 = \tfrac{1}{2}(R_1{}^2 + M_N{}^2 + m_K{}^2)\,, \qquad \text{(A8)}$$

$$p_1\cdot k = \tfrac{1}{2}(R^2 + \Delta\cdot R - R_1 + \Delta\cdot R_1)\,, \qquad \text{(A9)}$$

$$p_2\cdot k = \tfrac{1}{2}(R\cdot R_1 + S\cdot R - R_1{}^2 - R_1\cdot S)\,, \qquad \text{(A10)}$$

$$q_1\cdot k = \tfrac{1}{2}(R^2 - \Delta\cdot R - R\cdot R_1 + \Delta\cdot R_1)\,, \qquad \text{(A11)}$$

$$q_2\cdot k = \tfrac{1}{2}(R\cdot R_1 - S\cdot R - R_1{}^2 + R_1\cdot S)\,, \qquad \text{(A12)}$$

where

$$R^2 = -M_{Kp}{}^2\,, \quad R_1{}^2 = -m_{Kp}{}^2\,, \quad R\cdot R_1 = \tfrac{1}{2}(m_\pi{}^2 - M_{Kp}{}^2 - m_{Kp}{}^2)\,, \quad \Delta\cdot R = -M_N{}^2 + m_K{}^2 = R_1\cdot S\,, \qquad \text{(A13)}$$

$$\Delta^2 = -M_N{}^2 - m_K{}^2 - 2p_1\cdot q_1\,, \quad S^2 = -M_N{}^2 - m_K{}^2 - 2p_2\cdot q_2\,, \qquad \text{(A14)}$$

$$\Delta\cdot R_1 = -[(\Delta^2 - R^2 - 1.8533)/2]^{1/2}\{[R^2R_1{}^2 - (R\cdot R_1)^2]/R^2\}^{1/2}\cos\theta_\pi + (\Delta\cdot R R\cdot R_1)/R^2\,, \qquad \text{(A15)}$$

$$S\cdot R = -[(S^2 - R_1{}^2 - 1.8533)/2]^{1/2}\{[R^2R_1{}^2 - (R\cdot R_1)]/R_1{}^2\}^{1/2}\cos\theta_{p_2} + (R_1\cdot S R\cdot R_1)/R_1{}^2\,, \qquad \text{(A16)}$$

$$\Delta\cdot S = \frac{(D_1)^{1/2}(D_2)^{1/2}\cos\phi + R_2{}^2\Delta\cdot R_1 R\cdot S + R_2\cdot S\Delta\cdot R_2 R\cdot R_1 - R_2\cdot S\Delta\cdot R_1 R\cdot R_2 - R_2\cdot R_1\Delta\cdot R_2 S\cdot R}{R_2{}^2 R\cdot R_1 - R_1\cdot R_2 R\cdot R_2}\,. \qquad \text{(A17)}$$

Also,

$$R_1\cdot R_2=R\cdot R_1-R_1^2\,,\quad R\cdot R_2=R^2-R\cdot R_1\,,\quad \Delta\cdot R_2=\Delta\cdot R-\Delta\cdot R_1\,,\quad R_2\cdot S=R\cdot S-R_1\cdot S\,, \tag{A18}$$

$$D_1=R_2^2\Delta^2R^2-R_2^2\Delta\cdot R\Delta\cdot R-\Delta\cdot R_2\Delta\cdot R_2R^2+\Delta\cdot R_2\Delta\cdot RR\cdot R_2+R\cdot R_2\Delta\cdot R_2\Delta\cdot R-R\cdot R_2\Delta^2R\cdot R_2\,, \tag{A19}$$

$$D_2=R_2^2S^2R_1^2-R_2^2S\cdot R_1S\cdot R_1+R_2\cdot SS\cdot R_1R_1\cdot R_2-R_2\cdot SR_2\cdot SR_1^2+R_2\cdot R_1S\cdot R_2S\cdot R_1-R_2\cdot R_1S^2R_1\cdot R_2\,. \tag{A20}$$

Similarly the terms in Eq. (2.18) must be related to the variables in Eq. (2.19). With R_1, R, Δ, and S defined as before,

$$p_1\cdot q_1=\tfrac{1}{2}(-M_{Kp}^2+M_N^2+m_K^2)=p_2\cdot q_2\,, \tag{A21}$$

$$p_1\cdot p_2=\tfrac{1}{4}(-M_{Kp}^2+\Delta\cdot R+S\cdot R+\Delta\cdot S)\,, \tag{A22}$$

$$p_2\cdot q_1=-M_N^2+p_2\cdot q_2-p_1\cdot p_2=p_1\cdot q_2\,, \tag{A23}$$

$$q_1\cdot q_2=q_1\cdot p_1-m_K^2-q_1\cdot p_2\,, \tag{A24}$$

where

$$\Delta\cdot R=m_K^2-M_N^2=S\cdot R=S\cdot R_1\,, \tag{A25}$$

$$\Delta\cdot S=[(\Delta^2-R^2-1.8533)/2]\cos\theta_{p_2}+(\Delta\cdot R)(S\cdot R)/R^2\,. \tag{A26}$$

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[1]S. L. Adler, Phys. Rev. 139, B1638 (1965).

[2]R. A. Uritam, Phys. Rev. D 6, 3233 (1972).

[3]P. Nuthakki and R. A. Uritam, Phys. Rev. D 8, 3196 (1973).

[4]If we go to the Breit frame of the proton and consider the pion production at threshold, we have

$$\langle |M^{\pi}|^2\rangle \sim -\frac{2}{M_N^2}\left(\frac{p^{0^2}A^2+B^2M_N^2Q^{0^2}}{p^{0^2}}\right)\times(M_N^2+p_1\cdot p_2),$$

where $p^0=p_1^0=p_2^0$. The term A^2 is therefore 10^5 times smaller than the B^2 term and can be neglected.

[5]Integration was performed numerically, using the IBM 1130 computer at Andhra University.

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[8]M. E. Schillaci and R. R. Silbar, Phys. Rev. D 2, 1220 (1970).

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[10]In this sense it is vaguely reminiscent of Weinberg's well known early work on $\pi\pi$ scattering, where he assumed weak $\pi\pi$ scattering in order to be able to apply the soft-pion method successfully. See S. Weinberg, Phys. Rev. Lett. 17, 616 (1966).

[11]This point of view has been adhered to consistently in our past calculations, Refs. 2 and 3; see also J. St. Amand and R. A. Uritam, Phys. Rev. D 9, 3058 (1974); 14, 1883 (1976).

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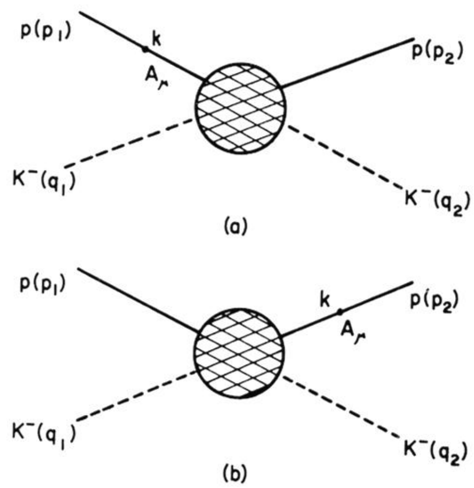
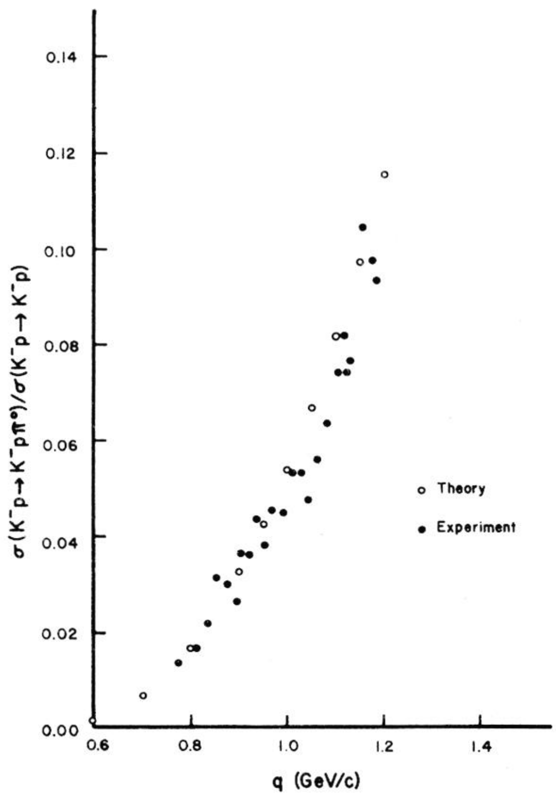


FIG. 1. Diagrams of order k^{-1} in the axial-vector current matrix elements.



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