Possible evidence for neutrino oscillations in the Brookhaven solar-neutrino experiment

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Pomeranchuk has suggested that one might directly observe neutrino oscillations in a solar-neutrino experiment if the oscillation wavelength were comparable to the annual earth-sun–distance variation from perihelion to aphelion, $\Delta r = 5 \times 10^6$ km. We find that data from the Brookhaven solar-neutrino experiment can be interpreted to marginally favor the neutrino-oscillation hypothesis at the 2-standard-deviation level. If the effect is not a statistical fluctuation, the estimated value for $\Delta m^2 = m_{\nu_1}{}^2 - m_{\nu_2}{}^2 \sim 4 \times 10^{-10}$ eV^2 is such that terrestrial tests would seem to be unfeasible.

INTRODUCTION

During the last decade, Davis and his Brookhaven associates have been performing a very careful search deep underground for neutrinos from the sun.[1] With the gradual accumulation of data, their results have become more precise, and disagreement with the prediction of standard solar models has become more acute, thereby spawning a large number of suggested resolutions of the problem of the "missing" solar neutrinos.[1,2] A recently reported value[3] from the Brookhaven experiment is 1.6 ± 0.4 SNU (1 SNU $= 10^{-36}$ captures per atom per second). This value, which results after subtracting a cosmic-ray background of 0.4 ± 0.2 SNU, differs from a recent standard-solar-model value,[4] 4.7 SNU, by a factor of 3. One possible resolution of the missing-solar-neutrino puzzle is based on Gribov and Pontecorvo's suggestion[5] of neutrino oscillations. Neutrino oscillations arise if at least one neutrino type has nonzero mass, and if muon- and electron-number conservation is not absolute, permitting the transformations $\nu_e \rightleftarrows \nu_\mu$. In this case, Gribov and Pontecorvo suggested that some fraction of the solar-electron neutrinos would transform into muon neutrinos before reaching earth, and go undetected in the Brookhaven experiment. Pomeranchuk[5] has noted that because of an annual earth-sun–distance variation of 3%, it might be possible to directly observe such neutrino oscillations if their wavelength is comparable to 5×10^6 km, the range of the earth-sun–distance variation from aphelion to perihelion. The purpose of this paper is to note that recently reported data from the Brookhaven experiment marginally favor the neutrino-oscillation hypothesis according to this author's interpretation of the data.

INTERPRETATION OF DATA FROM THE BROOKHAVEN EXPERIMENT

The results of runs 18–47 from the Brookhaven ^{37}Cl solar-neutrino experiment are shown in Fig. 1.[6] Davis and his associates have already suggested[1] that it is conceivable that there are time variations present in the data, but that most theorists agree that it is extremely unlikely that the solar-neutrino flux would vary. However, as noted above, an *annual* variation could arise from neutrino oscillations rather than a time-varying flux at the source, and hence such a variation is intrinsically more plausible than any other. The ability to make a statistically meaningful statement about the possible presence of time variations in the Brookhaven data is significantly greater if one combines runs from different years on a single annual time scale. In Fig. 2 we have plotted the Brookhaven data on a single annual scale starting at 4 January, the date of perihelion. Plotting the data in this way makes it somewhat easier to visually spot any systematic annual variation. There is, of course, one obvious source of an annual variation, namely the change in solid angle subtended at the detector during the course of the year. However, this effect is small, and the appropriate correction factor ranging from +3% to −3% has been applied to the data of Fig. 2 (and Fig. 5). If there are annual variations in the corrected data arising from neutrino oscillations as a function of earth-sun distance, then the variations must be symmetrical about 4 January and 4 July, the dates of perihelion and aphelion. The data of Fig. 2 do in fact appear to be reasonably consistent with symmetry about the date of aphelion. For example, a fit to the function $N(t) = \beta_1 + \beta_2 \cos(t - t_0)$, yields a best value for

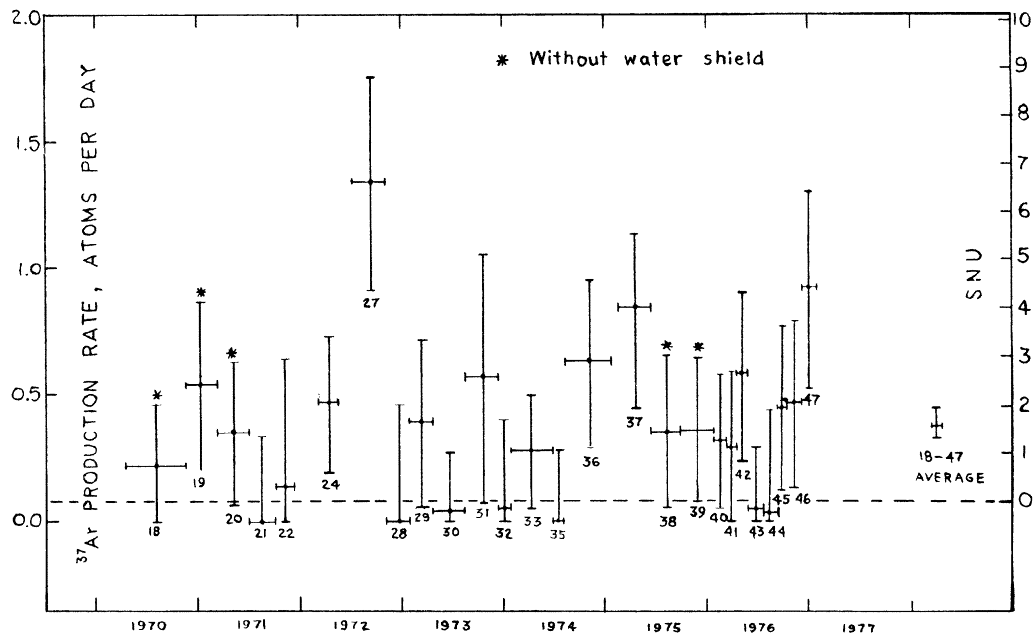

FIG. 1. Solar-neutrino counting rate in ^{37}Ar atoms per day and SNU versus time. Graph is taken from Ref. 3.

$t_0 = 13 \pm 46$ days, in good agreement with the expected value $t_0 = 0$ required by symmetry, where t is measured from the perihelion date $t_0 = 0$.

In testing the hypothesis that there exists an annual variation in counting rate, it is of some significance that seven out of eight runs occurring in the middle of the year fall below the average value. For example, if we compute the average

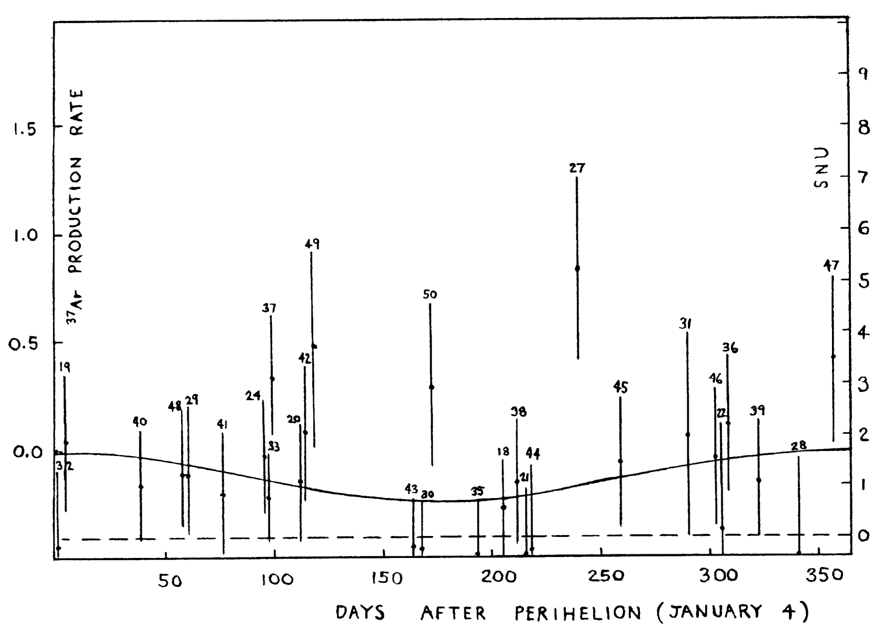

FIG. 2. Brookhaven data corrected for varying solid angle subtended by the detector during the course of a year, and plotted on a single annual scale commencing at the date of perihelion (4 January). The horizontal bars of Fig. 1, showing the length of time each run lasts, have been omitted for clarity.

counting rates in the first, second, and third 4 months of the year (measured from 4 January), we obtain

$$N_1 = 1.9 \pm 0.5 \text{ SNU},$$

$$N_2 = 0.7 \pm 0.5 \text{ SNU},$$

$$N_3 = 1.7 \pm 0.6 \text{ SNU}.$$

This yields for the quantity $\Delta N = \frac{1}{2}(N_1 + N_3) - N_2$ a value $\Delta N = 1.1 \pm 0.6$ SNU that is significantly different from zero, the expected value for a nonvarying counting rate.

The evidence against a constant counting rate, becomes even more compelling when we examine the statistical treatment of runs with low counting rates in the Brookhaven experiment. To quote Davis and his associates:

> "The counts occurring in the energy (FWHM) rise time (90%) window corresponding to the Auger electrons from ^{37}Ar decay were resolved by the statistical treatment into a decaying component with a 35-day half-life and a nondecaying background. The number of counts observed is small, therefore the statistical treatment gives the probability distribution function for the number of ^{37}Ar atoms produced in each experimental run. From this distribution function is obtained the most likely value and the 34% confidence ranges above and below this most likely value. *In the cases where the most likely value is low and the area under the probability distribution function below the most likely value is less than 34%, the upper bound given includes 68% of the area under the probability distribution function.*" (emphasis added)

Thus, the definition that Davis and associates use for the error bars for the low data points means that if all data points lie off a fitted curve by the same multiple of their error bars, then the low data points would decrease the probability of a fit by a significantly greater amount than the others. An alternative approach would be to treat high and low data points in the same manner, showing the 34% confidence ranges above and below the best value of each run. Using this definition, there would be roughly the same contribution to a χ^2 or likelihood function when any given data point lies off a fitted curve by a given multiple of its error bar. It is quite clear that such a reduction in the error bars of the low data points most of which lie in the middle of the year, would have a significant effect on the goodness of fit for the constant-counting-rate hypothesis.

There is, moreover, a second reason why it may be desirable to treat high and low data points in identical fashion, and thereby allow both data points and error bars to go below zero. Suppose that the true neutrino-induced ^{37}Ar production rate were exactly zero. In this case, because of the way the ^{37}Ar rate is extracted, statistical fluctuations in the background may well yield a best value on many runs that is positive. If negative (unphysical) values are not allowed, then a systematic error is introduced when one computes the time-averaged counting rate over all runs.[7] In fact, as the number of runs becomes arbitrarily large, the error in the average value will approach zero, but the average rate will be positive and therefore statistically different from zero.

IMPLICATIONS FOR NEUTRINO-OSCILLATION HYPOTHESIS

We shall now consider the implications of an observed varying counting rate for the neutrino-oscillation hypothesis. Further runs of the Brook-Haven ^{37}Cl experiment should be able to settle the question of whether the effect discussed here is simply a statistical fluctuation.

The neutrino flux observed at a distance R from the place they are produced is given by[8]

$$\phi(R) = \tfrac{1}{2}\,\phi(0)\,(1 + \cos kR)\,, \qquad (1)$$

where

$$\lambda = \frac{2\pi}{k} = \frac{2.5\,E_\nu}{(m_{\nu_1}{}^2 - m_{\nu_2}{}^2)}\ \text{km},$$

$\phi(0)$ is the neutrino flux at a distance R in the absence of neutrino oscillations, E_ν is the neutrino energy in GeV, R is the source-detector distance in km, and m_{ν_1}, m_{ν_2} are the masses in eV of the eigenstates of the neutrino mass matrix defined by Gribov and Pontecorvo.[5] Bahcall[8] has noted that the effect of averaging Eq. (1) over neutrino energy E_ν will smear out any neutrino oscillations for large values of kR, and simply result in an average counting rate which is one half the expected rate in the absence of oscillations. This follows from the fact that the fractional range in energies $\Delta E_\nu/E_\nu$ for solar neutrinos having continuous spectra is much greater than the fractional range in distance $\Delta R/R$ due to the annual variation in earth-sun distance. However, oscillations for monoenergetic solar neutrinos would produce observable annual variations in $\phi(R)$. In the standard solar model[2] about 80% of the expected counting rate in the ^{37}Cl experiment[6] arises from ^{8}B neutrinos having a continuous spectrum extending from 0 to 14 MeV. The monoenergetic neutrinos making the greatest contribution to the expected counting rate are the 0.86-MeV neutrinos from ^{7}Be decay which are expected to contribute 0.99 SNU.[6] Hence, if neutrino oscillations are

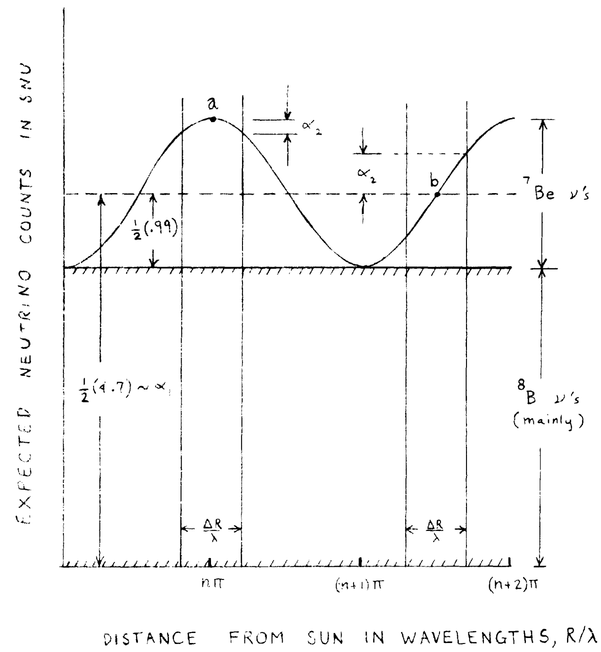

FIG. 3. Expected neutrino counting rate in SNU as a function of distance from the sun in neutrino-oscillation wavelengths R/λ. The continuous-spectrum neutrinos (mainly ^{8}B) give a counting rate independent of R, due to averaging Eq. (1) over neutrino energy. The monochromatic neutrinos (mainly ^{7}Be) show an oscillation of amplitude α_2, which depends on both on the ratio $\Delta R/\lambda = (R_A - R_p)/\lambda$, and also the phase of neutrino oscillations when the earth is midway between perihelion and aphelion, $R_0 = \frac{1}{2}(R_p + R_A)$. For $\Delta R/\lambda < 1$, the maximum value of the oscillation amplitude α_2 occurs for $kR_0 = (n+\frac{1}{2})\pi$ (R_0/λ at point b), and the minimum value of the oscillation amplitude occurs for $kR_0 = n\pi$ (R_0/λ at point a).

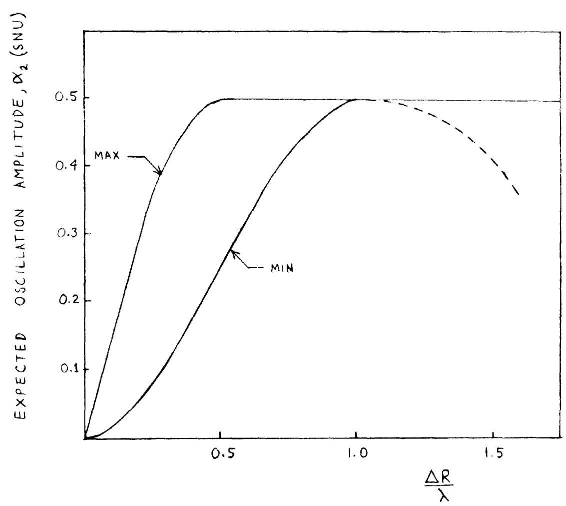

FIG. 4. Minimum and maximum expected amplitude (α_2) for the cosine term in Eq. (2), as a function of the ratio $\Delta r/\lambda$. λ is the neutrino-oscillation wavelength and Δr is the earth-sun-distance variation between perihelion and aphelion. The falloff for $\Delta r/\lambda \gtrsim 1$ (dashed curve) is due to neutrino oscillations being averaged over during some of the long runs in the Brookhaven experiment, and it only indicates a trend not the actual numerical prediction.

occurring one might expect to find a neutrino counting rate that varies with R according to

$$N(R) = \alpha_1 + \alpha_2 \cos(kR)\,, \qquad (2)$$

where $\alpha_1 \approx \frac{1}{2}(4.7) \approx 2.4$ SNU and $|\alpha_2| \lesssim \frac{1}{2}(0.99) \approx 0.50$ SNU (see Fig. 3). The maximum expected value for $|\alpha_2| \approx 0.50$ SNU would arise if the neutrino-oscillation wavelength is less than the seasonal earth-sun-distance variation, i.e., $\Delta R/\lambda > 1$. In general, the expected amplitude of the cosine term α_2 also depends on the phase of the oscillations when the earth is midway between perihelion and aphelion, i.e., $R_0 = \frac{1}{2}(R_A + R_P)$. As indicated in Figs. 3 and 4, the largest expected amplitude occurs if $kR_0 = (n+\frac{1}{2})\pi$, and the smallest expected amplitude α_2 occurs if $kR_0 = n\pi$. For neutrino-oscillation wavelengths less than ΔR, the expected value of α_2 is 0.50 SNU, independent of phase. However, owing to the fact that oscillations are averaged over each run, about half of which last over 100 days, a drop in observable amplitude occurs (indicated by the dotted curve in Fig. 4) for oscillation wavelengths much below ΔR.

In Fig. 5, the Brookhaven data, corrected for varying solid angle subtended at the detector is shown as a function of R, the earth-sun distance in AU (astronomical units). For each data point, a value of R is obtained by averaging over the time interval of the run. The black data points show individual runs, and the circles show averages over all runs occurring in four equal intervals in R. Combining groups of runs gives smaller vertical error bars and moreover conforms to the fact that many runs cover a significant fraction of a year. Three fits to the circled points are shown in Fig. 5. The poorest of these fits, with a χ^2 probability of 10%, is that to a nonvarying counting rate, of 1.5 SNU. While this probability is *not* so low as to reasonably rule out the hypothesis of a constant counting rate, better fits do result to functions having the form of Eq. (2). Two such fits are shown in Fig. 5 for neutrino-oscillation wavelengths equal to ΔR [curve (a)] and $2\Delta R$ [curve (b)]. Clearly, the data are not sufficiently precise to give much information on a value for the neutrino wavelength, when even the presence of neutrino oscillations is open to question. It should, however, again be noted that the statistical evidence becomes considerably stronger if the

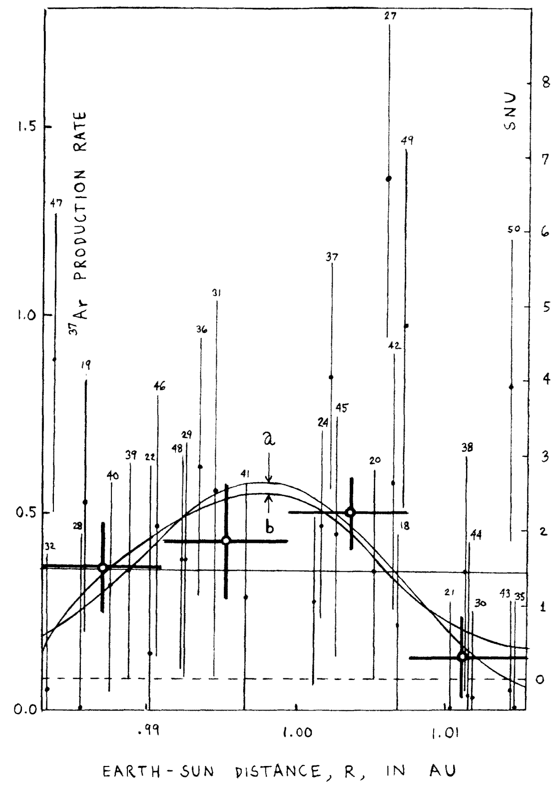

FIG. 5. Brookhaven data corrected for varying solid angle subtended at the detector during the course of a year and plotted on a single annual scale in terms of earth-sun distance, R in AU (astronomical units), instead of time. The R value for each run has been averaged over the time interval for that run. The white (circled) points show the average neutrino counting rate in four equal intervals of R. The horizontal line is a fit to the circled points assuming a constant counting rate. Curves (a) and (b) are fits assuming a variation of the form of Eq. (2), with neutrino-oscillation wavelengths equal to ΔR [curve (a)] and $2\Delta R$ [curve (b)], respectively.

errors on the low data points (and, in particular, the last circled point in Fig. 5) are significantly reduced, on the grounds discussed earlier. Moreover, some additional information on the neutrino-oscillation wavelength results from the requirement that the maximum value for the amplitude should be ~0.50 SNU. This would favor curve (a) ($\alpha_2 = 0.98 \pm 0.55$ SNU) over curve (b) ($\alpha_2 = 2.4 \pm 1.2$ SNU), indicating that the oscillation wavelength is in the neighborhood of ΔR.

The expected value of the constant term in Eq. (2), $\alpha_1 = 2.4$ SNU is somewhat high, when compared to the average of the four circled data points, 1.5 SNU. This difference may be statistical, or it may be narrowed by variations on the standard solar model. Alternatively, the discrepancy can also be eliminated through any mechanism that reduces the expected counting rate of the ^{8}B neutrinos somewhat, while not substantially affecting the expected counting rate for the ^{7}Be neutrinos. Leiter and Glass[9] have, in fact, suggested just such a mechanism based on a postulated nonminimal gravitational coupling (NMGC) for finite-mass fermions. In another recent article, Mann and Primakoff[10] consider the form of neutrino oscillations given the existence of an arbitrary number of neutrino types. Under certain conditions, the time-averaged neutrino flux is reduced by $1/N_\nu$, where N_ν is the number of "communicating" neutrino types. Thus, the expected counting rate can be reduced to any desired value for the ^{8}B neutrinos by postulating a sufficiently large number of neutrino types. Unfortunately, however, any oscillations produced by the ^{7}Be neutrinos would then probably be unobservable in the Brookhaven data. For example, suppose that there exists three types of neutrinos, then, for the expected value of α_1 in Eq. (2), we find α_1 may be reduced to $\frac{1}{3}(4.7) \approx 1.6$ SNU. For the mononergetic ^{7}Be neutrinos, the equation analogous to Eq. (1) now has a linear combination of three cosine terms corresponding to all possible pairs of the three neutrino types.[10] The maximum amplitude of one cosine term is given[10] by $\frac{2}{9}\,\phi(0) = (0.22)\,(0.20)\,(4.7) = 0.21$ SNU, a value too low to produce observable oscillations in the Brookhaven data.

We summarize our results as follows:

1. The data from the Brookhaven ^{37}Cl solar-neutrino experiment is interpreted here as giving marginal evidence favoring the hypothesis of neutrino oscillations.

2. The evidence is not yet statistically strong enough to rule out a constant counting rate, although if the error bars on the low count runs are reduced to include only 34% of the probability distribution above the best values, the evidence against a constant counting rate probably is statistically significant.

3. A test of the significance of the oscillations can be made in a fairly short time with the present ^{37}Cl experiment. If future runs are kept short (close to the ^{37}Ar half life), much more data can be accumulated in a given period of time, since the error bars on long runs are not significantly less than on short runs, due to saturation effects for the ^{37}Ar.

4. Concerns that Davis and associates have previously expressed about uncertainties in backgrounds are relevant to the average rate they report (1.6 ± 0.4 SNU), but any observation of a statistically significant annual oscillation are unlikely to be affected by background uncertainties.

5. If the effect discussed in this paper is not a statistical fluctuation, then the neutrino-oscillation wavelength is comparable to 5×10^6 km for neutrinos having an energy of 0.87 MeV, yielding a value for $\Delta m^2 = m_{\nu_1}{}^2 - m_{\nu_2}{}^2 \sim 4\times10^{-10}$ eV^2.

6. Given the above value for Δm^2, observations of vacuum neutrino oscillations over terrestrial distances would seem to be unfeasible.

7. If the effect is not a statistical fluctuation, it constitutes evidence that electron- and muon-number conservation breaks down at some level.

ACKNOWLEDGMENTS

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[6]Figure 1 was taken from Ref. 3. The numerical values corresponding to these data plus three additional runs 48–50 were kindly supplied by Dr. Raymond Davis, Jr.
[7]The statistical analysis of the data from the Brookhaven experiment has been carried out by Dr. Bruce Cleveland. According to Dr. Davis and Dr. Cleveland the average value they report does not contain the systematic error discussed here since it is obtained not by averaging data points for all runs, but by combining the raw data for all runs and doing a maximum-likelihood fit to a constant counting rate plus an exponential decay (private communication).
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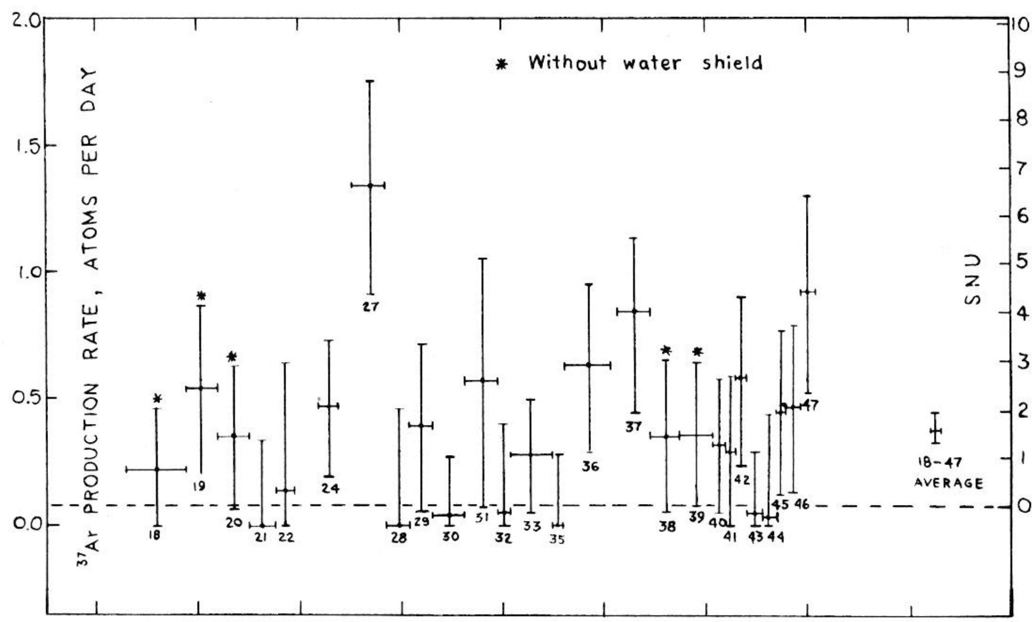
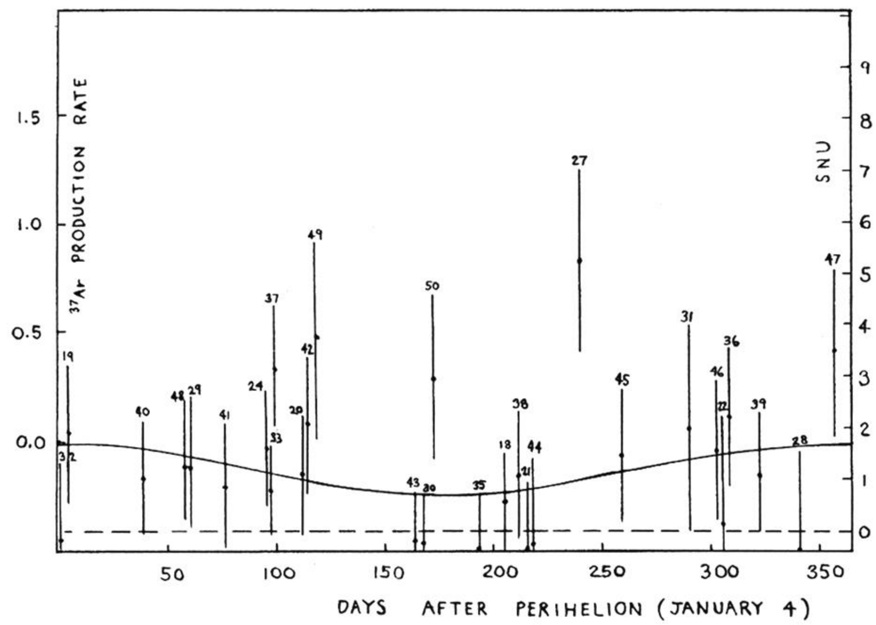


FIG. 1. Solar-neutrino counting rate in ^{37}Ar atoms per day and SNU versus time. Graph is taken from Ref. 3.



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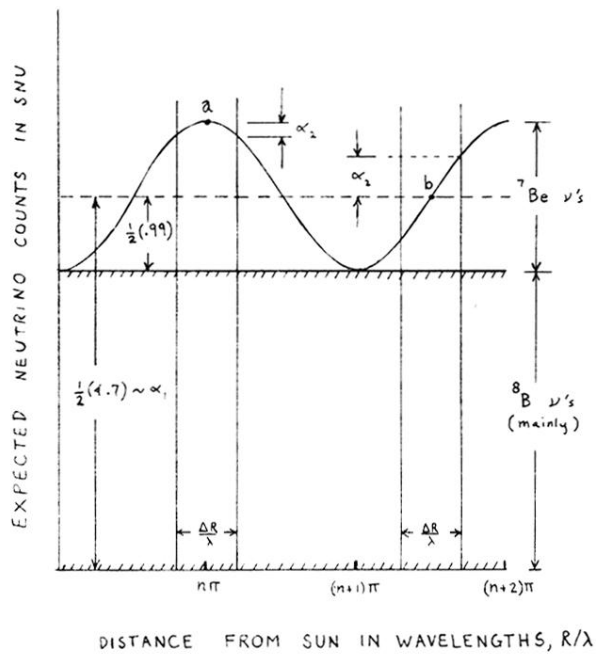

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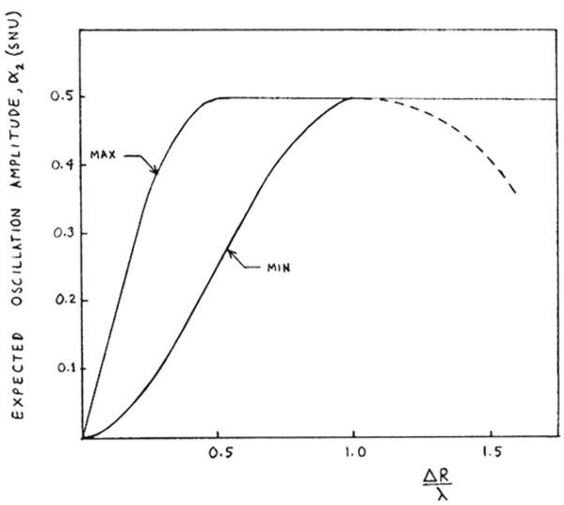

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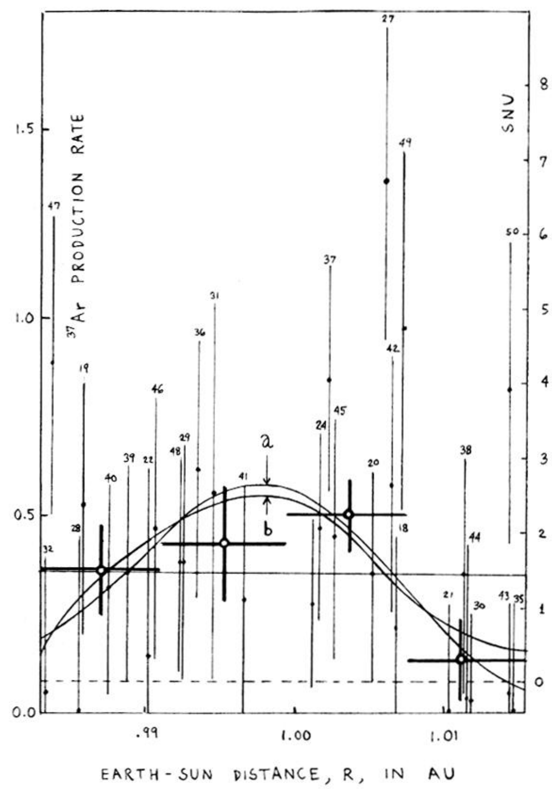

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