PHYSICAL REVIEW D VOLUME 18, NUMBER 7 1 OCTOBER 1978

Disintegration of deuteron by neutrino-deuteron scattering and the photon-neutrino weak-coupling theory

Karabi Sen and Pratul Bandyopadhyay
Indian Statistical Institute, Calcutta-35, India

(Received 31 May 1977; revised manuscript received 4 August 1977)

We study here disintegration of the deuteron by neutrino-deuteron scattering according to the photon-neutrino coupling theory. It is found that the value of σ calculated for the transition $^3S \to {}^1S$ in the energy range of 50 MeV is of the order of 10^{-42} cm^2. A comparison is also made here with the predictions of the Salam-Weinberg theory.

I. INTRODUCTION

Recently there has been considerable interest in the possible existence of a weakly interacting neutral lepton current. The existence of neutral-lepton-current interactions with hadrons has been studied experimentally in both inclusive[1] and exclusive processes. The recent results of the Harvard-Pennsylvania-Wisconsin[2] collaboration for the neutral-current–induced inclusive processes establish the fact that the Lorentz character of a hadronic neutral weak current may not be the familiar $V-A$ of the charged weak currents. This has encouraged a more systematic study of the nature of the neutral-current events. The properties of the neutral currents can be utilized to test the gauge-theory strategy where these currents have definite isospin and Lorentz properties. This is done by measuring the different pieces of hadronic weak currents and their relative contributions in a number of elementary-particle and nuclear transitions. Attempts have been made to interpret these neutral-current events on the basis of (i) Salam-Weinberg gauge theories, (ii) Sakurai's baryon-current model, and (iii) the electromagnetic form factor of neutrinos. But unfortunately none of these approaches are perfectly successful in explaining these neutral-current events. Processes such as $\nu_\mu e^- \to \nu_\mu e^-$, $\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-$, and $\nu_\mu N \to \nu_\mu X$ are not allowed by the conventional $V-A$ theory. An extension of the Salam-Weinberg model has been made by Achiman,[3] but this extended model has the drawback that we have to introduce two heavy neutrinos to have weak universality. Sakurai's baryon-current model is also inconsistent with the data.

A detailed study of the observation of the neutral-current effect in the disintegration of the deuteron up to intermediate neutrino energy has been made by Ali and Dominguez.[4] They calculated the total cross sections for the neutrino-induced transitions $^3S \to {}^1S$, $^3S \to {}^1p$, and $^3S \to {}^3p$ of the neutron-proton system by applying the Salam-Weinberg theory.

In this note, we try to interpret the $\nu D \to \nu np$ process on the basis of photon-neutrino weak coupling and show that we obtain the correct order of magnitude for the total cross section for transition $^3S \to {}^1S$.

II. DERIVATION OF TRANSITION CROSS SECTIONS FOR THE PROCESS $\nu d \to \nu np$

A. Kinematics and approximations

We study here the neutral-lepton-current process on the basis of the photon-neutrino weak-coupling theory. Our process is similar to photodisintegration of the deuteron as shown by McVoy and Van Hove,[5] except that, in place of the electromagnetic coupling constant, we take the weak coupling constant.

McVoy and Van Hove showed that the Hamiltonian operator describing the electron-nucleon interaction for two-component nucleons, correct to order q^2/M^2, can be written as

$$H' = -\frac{4\pi e^2}{q^2}\left\langle u(k_f)\right| F_1 e^{-q\cdot x} - \frac{F_1}{2M}[\vec{P}\cdot\vec{\alpha}e^{-iqx} + e^{-iqx}\vec{P}\cdot\vec{\alpha}] - \frac{F_1+\kappa F_2}{2M} i\vec{\sigma}\cdot(\vec{q}\times\vec{\alpha})e^{-iq\cdot x} + \frac{q^2}{8M^2}(F_1+2\kappa F_2)e^{-iq\cdot x} + \frac{F_1+2\kappa F_2}{8M^2} i\vec{\sigma}\cdot[\vec{P}\times(\omega\vec{\alpha}-\vec{q})e^{-iq\cdot x} - e^{-iq\cdot x}(\omega\vec{\alpha}-\vec{q})\times\vec{P}]\left| u(k_i)\right\rangle . \qquad (1)$$

The following notation is used in Eq. (1): $u(k_f)$ and $u(k_i)$ are the spinors describing the final and initial electrons, respectively; F_1 and F_2 are the nucleon form factors; κ is the anomalous magnetic moment of the nucleon in nuclear magnetons. The angular brackets indicate that a matrix element of the electron spinors is to be taken. Now, in the nucleon space, $\vec{P}$ is the momentum operator and $\vec{\sigma}$ is the (2×2) spin operator for the nucleon, while $\vec{\alpha}$ is the usual (4×4) Dirac operator which acts on the electron spinors. The assumption is made that the nucleon form factors in Eq. (1) are the same as the free-nucleon form factors, i.e., there is no distortion due to binding.

The five terms in the interaction given by Eq. (1) describe, respectively, the static Coulomb interaction, the convection current, the spin current, the Darwin-Foldy interaction,[6] and the spin-orbit interaction. For unpolarized nucleons the spin-orbit term does not contribute to order q^2/M^2 (Ref. 5) and will be neglected. It has also been shown that the contribution of the convection-current term is quite small near the quasi-elastic peak of the inelastic spectrum.[7] We will neglect then the second and last terms in Eq. (1) and take for the transition current

$$J_0=\left\langle\psi_f\left|\left[F_{1p}+\frac{q^2}{8M^2}(F_{1p}+2\kappa_pF_{2p})\right]\exp(-i\vec{q}\cdot\vec{r}_p)+\left[F_{1n}+\frac{q^2}{8M^2}(F_{1n}+2\kappa_nF_{2n})\right]\exp(-i\vec{q}\cdot\vec{r}_n)\right|\psi_i\right\rangle, \tag{2}$$

$$\vec{J}=-i\left\langle\psi_f\left|\frac{F_{1p}+\kappa_pF_{2p}}{2M}(\vec{\sigma}_p\times\vec{q})\exp(-i\vec{q}\cdot\vec{r}_p)+\frac{F_{1n}+\kappa_nF_{2n}}{2M}(\vec{\sigma}_n\times\vec{q})\exp(-i\vec{q}\cdot\vec{r}_n)\right|\psi_i\right\rangle. \tag{3}$$

To obtain Eqs. (2) and (3) we have set the time for the proton equal to the time for the neutron. The integration over the time then gives an energy-conserving δ function which is suppressed in Eqs. (2) and (3).

We have written Eqs. (2) and (3) in terms of the Dirac and Pauli form factors F_1 and F_2 since this is the form used by McVoy and Van Hove. The formulas for the current and cross sections are simplified by the use of G_E and G_M rather than F_1 and F_2. The connection between the two sets of form factors is given by

$$G_E=F_1+\frac{Q^2\kappa}{4M^2}F_2, \tag{4}$$

$$G_M=F_1+\kappa F_2. \tag{5}$$

Using these relations we can rewrite Eqs. (2) and (3) as

$$J_0=\langle\psi_f|G_{Ep}(1-\tfrac{1}{2}\eta)\exp(-i\vec{q}\cdot\vec{r}_p)+G_{En}(1-\tfrac{1}{2}\eta)\exp(-i\vec{q}\cdot\vec{r}_n)|\psi_i\rangle, \tag{6}$$

$$\vec{J}=-\frac{i}{2M}\langle\psi_f|G_{Mp}(\vec{\sigma}_p\times\vec{q})\exp(-i\vec{q}\cdot\vec{r}_p)+G_{Mn}(\vec{\sigma}_n\times\vec{q})\exp(-i\vec{q}\cdot\vec{r}_n)|\psi_i\rangle, \tag{7}$$

where $\eta=-q^2/4M^2$. The calculation of the current J_μ now depends on a choice of the deuteron wave function ψ_i and a wave function to describe the final n-p system ψ_f. If the final-state interactions of the outgoing nucleons are neglected, the final wave function in the c.m. system of the outgoing nucleons is simply a plane wave, i.e.,

$$\psi_f=\sum_{s=0,1}e^{i\vec{p}\cdot\vec{r}}z_s^{m'}(s_n,s_p), \tag{8}$$

where $\vec{r}$ is the relative n-p coordinate and $z_s^{m'}$ is the spin function for the two nucleons.

To a first approximation, the D-state component of the deuteron wave function can be neglected, so that

$$\psi_i=\sum_{s=0,1}\frac{u(r)}{(4\pi)^{1/2}r}z_s^{m'}(s_n,s_p). \tag{9}$$

B. Matrix element for the process $\nu d\to\nu np$

We depict the process $\nu d\to\nu np$ on the basis of photon-neutrino weak coupling in Fig. 1. It is noted that, according to this theory, disintegration of the deuteron by the neutrino and antineutrino will have identical cross sections since the diagrams will be identical in both these cases. The matrix element for the process with the assumption listed in Sec. II A is

$$M=\frac{1}{[32(2\pi)^{15}]^{1/2}}\frac{1}{(E_\nu E_pE_n)^{1/2}}g\,|\bar{u}(k_\nu)\gamma_\mu(1+\gamma_5)u(k_\nu)|\,J_\mu\frac{1}{q^2},$$

where J_μ is the transition current for the d-np system, so that

$$M^2=\frac{1}{32(2\pi)^{15}}\frac{g^2}{E_\nu E_pE_n}4\int_0^\pi(\cos\theta-1)d\theta\times\frac{1}{4E_\nu^4(\cos\theta-1)^2}\times\left[F^2(\vec{p},\vec{q},z)\left(\frac{G_{En}^2}{1+\eta}+\frac{\eta G_{Mn}^2}{1+\eta}\right)+F^2(\vec{p},\vec{q},\pi-z)\left(\frac{G_{Ep}^2}{1+\eta}+\frac{\eta G_{Mp}^2}{1+\eta}\right)\right], \tag{10}$$

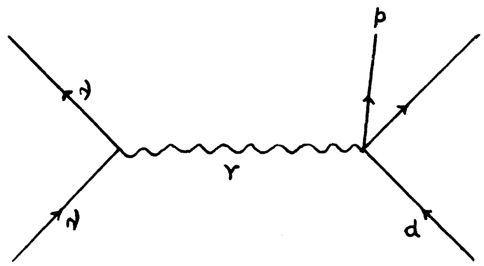

FIG. 1.. Diagram for the process $\nu d \to \nu n p$.

where

$$F(\vec{p},\vec{q},z) \equiv \int_0^\alpha j_0[(|\vec{p}|^2 + \tfrac{1}{4}|\vec{q}|^2 - |\vec{q}||\vec{p}|\cos z)^{1/2} r] \times u(r) r\, dr \qquad (11)$$

and z is the angle between $\vec{q}$ and $\vec{p}$ in the center-of-momentum system of the outgoing nucleons and $j_0(|\vec{p} - \frac{1}{2}\vec{q}|\, r)$ is the spherical Bessel function. In the argument of the spherical Bessel function $|\vec{p} - \frac{1}{2}\vec{q}|$ is a minimum and hence the coincidence cross section is a maximum when $z = 0$, i.e., when $\vec{p}$ and $\vec{q}$ are in the same direction.

C. Total cross section and the effect of the final-state interaction of the neutron and proton

To include the effect of the final-state interaction of the neutron and proton, the plane wave describing the motion of final nucleons must be replaced by a distorted wave. This is done most conveniently by using a partial-wave expansion for the final-state wave function

$$\psi_f = 4\pi \sum_{JLMS} i^L \exp(-i\delta_{JLS}) \frac{F_{JLS}(pr)}{pr} \times Y_L^{m_L *}(\hat{p})(LS m_L m_S | LSJM) \mathcal{Y}_{JLS}^M , \qquad (12)$$

where δ_{JLS} are the n-p scattering phase shifts. The quantity $(LSm_L m_S/LSJM)$ is the usual Clebsch-Gordan coefficient and $\mathcal{Y}_{JLS}^M$ is the spin-angle function defined by

$$\mathcal{Y}_{JLS}^M(\hat{r}) = \sum_{m\mu} (LSm\mu | LSJM) Y_L^m(r) z_s^\mu . \qquad (13)$$

The radial part of the wave function has the asymptotic form

$$F_{JLS}(pr) \underset{r\to\infty}{\longrightarrow} \sin\left[pr - \left(\frac{L\pi}{2}\right) + \delta_{JLS}\right] . \qquad (14)$$

In our calculation we consider only energy in the MeV range, so we calculate the transition cross section for $^3S \to {}^1S$ only. The total cross section in the center-of-momentum frame of outgoing nucleons is given by

$$\sigma = 4\pi^2 \int \frac{d^3k}{E_\nu} \int \frac{d^3p_p}{E_p} \int \frac{d^3p_n}{E_n} \delta^4(P_f - P_i) \frac{1}{(2S_1+1)(2S_2+1)} \sum_{S_i} \sum_{S_j} |m_{fi}|^2 ,$$

where S_1 and S_2 are the spins of the colliding particles in the initial state, so that

$$\begin{aligned}\sigma &= \frac{1}{2(2\pi)^3} \int E_\nu dE_\nu \int_0^\pi 2\pi \sin\theta\, d\theta\, g^2 |\bar{u}(k_\nu)\gamma_\mu(1+\gamma_5)u(k_\nu)|^2 \frac{1}{q^4} J_\mu J_\mu^\dagger \\ &= \frac{g^2}{2(2\pi)^2} \int \frac{dE_\nu}{E_\nu^3} \int_0^\pi \frac{\sin\theta\, d\theta}{\cos\theta - 1} \left\{ F^2(\vec{p},\vec{q},z) \left[\frac{2M^2 G_{En}^2}{2M^2 + E_\nu^2(\cos\theta - 1)} + \frac{E_\nu^2(\cos\theta-1)G_{Mn}^2}{2M^2 + E_\nu^2(\cos\theta-1)} \right] \right. \\ &\qquad \left. + F^2(\vec{p},\vec{q},\pi - z) \left[\frac{2M^2 G_{Ep}^2}{2M^2 + E_\nu^2(\cos\theta-1)} + \frac{E_\nu^2(\cos\theta-1)G_{Mp}^2}{2M^2 + E_\nu^2(\cos\theta-1)} \right] \right\} .\end{aligned}$$

Now,

$$G_{En} = 0, \quad G_{Mn} = -1.91 \frac{e}{2M}, \text{ where } e \text{ is the proton charge}, \quad G_{Ep} = e, \quad G_{Mp} = -2.79 \frac{e}{2M}.$$

In the center-of-momentum frame of the n-p system, the cross section for the transition $^3S \to {}^1p$ is given by

$$\begin{aligned}\sigma &= \frac{g^2}{2(2\pi)^2}(1.91)^2 e^2 \left[\frac{1}{2E_\nu^2} \ln\left(1 - \frac{E_\nu^2}{M^2}\right) - \frac{1}{2M^2} \ln\left(1 - \frac{M^2}{4E_\nu^2}\right) \right] F^2(\vec{p},\vec{q},z) \\ &+ \frac{g^2 e^2}{2(2\pi)^2} \frac{1}{2M^2} \left[\left(1 - \frac{M^2}{E_\nu^2}\right) \ln\left(1 - \frac{M^2}{E_\nu^2}\right) - \left(1 - \frac{M^2}{E_\nu^2}\right) \right] F^2(\vec{p},\vec{q},\pi - z) \\ &+ \frac{g^2 e^2}{2(2\pi)^2} \left[-\frac{1}{2E_\nu^2} \ln\left(\frac{2M}{E_\nu}\right) + \frac{1}{2E_\nu^2} \right] F^2(\vec{p},\vec{q},\pi - z) \\ &+ \frac{g^2 (2.79)^2 e^2}{2(2\pi)^2} \left[\frac{1}{2E_\nu^2} \ln\left(1 - \frac{E_\nu^2}{M^2}\right) - \frac{1}{2M^2} \ln\left(1 - \frac{M^2}{4E_\nu^2}\right) \right] F^2(\vec{p},\vec{q},\pi - z) .\end{aligned}$$

Taking $g^2 = 9\times10^{-21}$,[8] $M = 940$ MeV, and

$$F^2(\vec{p}, \vec{q}, \pi - z) = \int_0^\alpha j_0([|\vec{p}|^2 + \tfrac{1}{4}|\vec{q}|^2 - |\vec{q}||\vec{p}| \times \cos(\pi - z)]^{1/2} r) u(r) r\, dr$$

$$= \int_0^\alpha j_0((\tfrac{1}{4}E_\nu^2 + \tfrac{1}{4}E_\nu^2 + \tfrac{1}{2}E_\nu^2)^{1/2} r) u(r) r\, dr$$

$$= 2.32\times10^4 \text{ for } E_\nu \text{ from 10 to 50 MeV},$$

with $z = 0$, $u(r) = -(\beta^2 - \gamma^2)/(e^{(\beta-\gamma)r} - 1)$, where $\beta = 15.5\times10^{12}$ cm^{-1}, $\gamma = 2.3\times10^{12}$ cm^{-1}. The value of σ calculated for 40-MeV energy is

$$\sigma = 2.46\times10^{-42} \text{ cm}^2 .$$

Experimentally, Gurr *et al.*[10] have presented a new upper limit for the antineutrino disintegration of the deuteron at the reactor energies ($E_{\bar\nu} = 2.2$–5 MeV). This energy range is very close to the binding energy of the deuteron. To have a positive conclusion regarding the nature of the reaction, experiments involving high-energy neutrinos are needed.

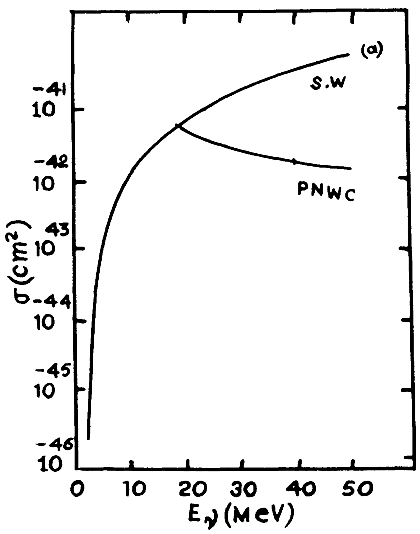

FIG. 2. A comparison of the prediction of the cross sections according to the present theory with that of the Salam-Weinberg theory. S-W represents the predictions of the Salam-Weinberg theory and PNWC represents the photon-neutrino weak-coupling theory. The curve for Salam-Weinberg theory has been drawn for $\sin^2\theta_W = 0.3$ and has been taken from Ref. 4.

III. DISCUSSION

Recent experiments have confirmed the rise of the ratio

$$R = \frac{\sigma(\bar\nu_\mu N \to \mu^+ X)}{\sigma(\nu_\mu N \to \mu^- X)}$$

with energy. This together with the high y anomaly and nonobservation of parity nonconservation in atomic physics, has put the Salam-Weinberg theory in doubt. However, the photon-neutrino weak-coupling theory can well explain the neutral-lepton-current events along with rise of the ratio R and the high-y anomaly.[11] Besides, since the theory allows only the weak neutrino current for neutral-lepton-current interactions, this rules out the parity nonconservation in eN scattering, in conformity with experiments. Also, in a previous paper[12] we have studied the threshold pion production $\nu_\mu N \to \nu_\mu N\pi$ on the basis of this theory and found that it explains the magnitude as well as the behavior of the cross section with respect to the invariant $N\pi$ mass distribution. All these developments indicate that the study of various processes on the basis of the photon-neutrino weak coupling is worthwhile.

Finally, we may add that like the Salam-Weinberg theory, the photon-neutrino weak coupling is also a gauge theory where the weak hadronic current is just the electromagnetic current and, as such, all neutral-current cross sections are related to the corresponding electromagnetic cross section, except for an overall factor. However, unlike the Salam-Weinberg theory, the photon-neutrino weak coupling is of long-range nature, as it occurs via photon exchange. This points to the similarity of the weak-neutral-current interactions with electromagnetic interactions and, unlike the Salam-Weinberg theory, the cross section will decrease as energy increases instead of rising with energy (see Fig. 2). In fact this comprises the crucial test of the theory. In view of this, a precise determination of the cross section at various energies is most welcome.

[1]F. J. Hasert *et al.*, Phys. Lett. 46B, 138 (1973).
[2]A. Rosset, in *Neutrinos—1974*, proceedings of the Fourth International Conference on Neutrino Physics and Astrophysics, Philadelphia, edited by C. Baltay (AIP, New York, 1974), p. 141.
[3]Y. Achiman *et al.* Nuovo Cimento 21A, 45 (1974).
[4]Ahmed Ali and C. A. Dominquez, Phys. Rev. D 12, 3673 (1975).
[5]K. McVoy and L. Van Hove, Phys. Rev. 125, 1034 (1962).
[6]L. L. Foldy, Rev. Mod. Phys. 30, 471 (1958).
[7]L. Durand, Phys. Rev. 115, 1020 (1959).
[8]P. Bandyopadhyay, Phys. Rev. 173, 1481 (1968); Nuovo Cimento 38, 1912 (1965).

[9]H. A. Bethe and C. Longmire, Phys. Rev. 77, 647 (1950).
[10]H. Gurr, F. Reines, and S. W. Sobel, Phys. Rev. Lett. 33, 179 (1974).
[11]P. Bandyopadhyay and S. Bhattacharya, Lett. Nuovo Cimento 18, 259 (1977); report (unpublished).
[12]P. Bandyopadhyay and K. Sen, Nuovo Cimento 39A, 529 (1977).

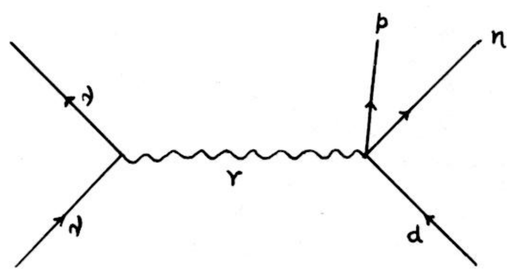

FIG. 1.. Diagram for the process $\nu d \to \nu n p$.

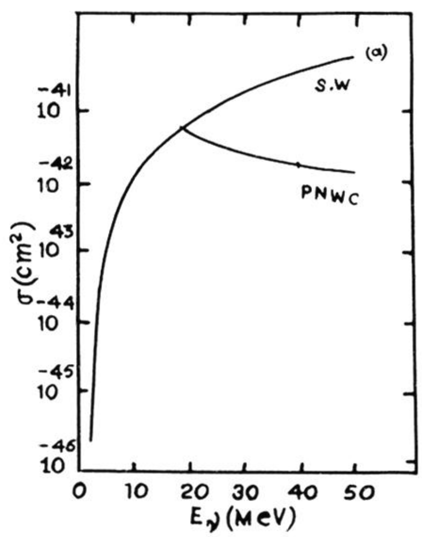

FIG. 2. A comparison of the prediction of the cross sections according to the present theory with that of the Salam-Weinberg theory. S-W represents the predictions of the Salam-Weinberg theory and PNWC represents the photon-neutrino weak-coupling theory. The curve for Salam-Weinberg theory has been drawn for $\sin^2\theta_W = 0.3$ and has been taken from Ref. 4.