Scalar-meson model and radiative decays of the K meson

Gerald W. Intemann and Gary K. Greenhut
Department of Physics, Seton Hall University, South Orange, New Jersey 07079
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We investigate the role which scalar mesons play in the radiative decays of the K meson. A scalar-meson model involving the $\delta(980)$ is proposed to describe the three- and four-body K decays $K_L^0 \to \pi^0\gamma\gamma$, $K^\pm \to \pi^\pm\gamma\gamma$, $K \to \pi\pi\pi\gamma$, and $K \to \pi\pi\gamma\gamma$. The calculation of the decay rates is based on the most recent experimental data on $\delta(980)$ and on the Moshe-Singer version of the weak nonleptonic Hamiltonian. For the decays $K \to \pi\gamma\gamma$, the results of the calculation are compared with the predictions of various other models as well as with the present experimental data.

I. INTRODUCTION

In a recent paper[1] we investigated the role which scalar mesons play in the radiative decays of the η meson. We restricted our investigation to contributions made by the $\delta(980)$ resonance, the only well-established scalar meson, which presumably has the quantum numbers $I^G, J^P = 1^-, 0^+$, and which decays predominantly into $\eta\pi$ and $K\bar{K}$. We found that this scalar meson plays a major role in the dynamics of the radiative decays $\eta \to \pi^0\gamma\gamma$, $\eta \to \pi^+\pi^-\pi^0\gamma$, and $\eta \to \pi^+\pi^-\gamma\gamma$.

In the work reported here we perform a similar study of the contributions which the δ scalar meson makes in the nonleptonic radiative decays of the K meson. In particular, we consider the three- and four-body decays $K_L^0 \to \pi^0\gamma\gamma$, $K^\pm \to \pi^\pm\gamma\gamma$, $K \to \pi\pi\pi\gamma$, and $K \to \pi\pi\gamma\gamma$, the amplitudes of which have contributions from a δ-meson intermediate state.

In this study we choose as the framework for analyzing the nonleptonic weak processes a current-current Hamiltonian, with the currents dominated respectively by vector and pseudoscalar and axial-vector particles, as originally suggested by Sakurai.[2] We adopt the $\Delta S = 1$, CP-conserving Hamiltonian proposed earlier by Moshe and Singer[3] which has the form

$$\mathcal{H}_w = \sqrt{2} G_{\mathrm{NL}} [J_\mu^1(x) J_\mu^4(x) + J_\mu^2(x) J_\mu^5(x) - J_\mu^3(x) J_\mu^6(x)], \tag{1.1}$$

where $J_\mu^\alpha = j_\mu^{Va} + j_\mu^{Aa}$, $a = 1, 2, \ldots, 8$ is an SU(3) index and we set[4] $G_{\mathrm{NL}} = 1.1 \times 10^{-5}/m_p^2$. As a result of its form, $\mathcal{H}_w$ accommodates the $\Delta I = \frac{1}{2}$ rule with a minimum of neutral currents and transforms as members of an octet and a 27 representation of SU(3). It should be mentioned that there are alternative choices for $\mathcal{H}_w$. In particular, a Hamiltonian which behaves like a λ_6 vector of an SU(3) octet has frequently been employed in the past.[2,5] However, as pointed out by Moshe and Singer,[3] such a pure octet form leads to difficulties in describing $K_L^0 \to \gamma\gamma$ decay and in calculating the $K_S^0 - K_L^0$ mass difference. On the other hand, the Hamiltonian (1.1) is successful[6] in accounting for $K_L^0 \to \gamma\gamma$.

An interesting feature of the Hamiltonian (1.1) is the absence of a $J^6 J^8$ term, thus forbidding to lowest order a $K_L^0 \to \eta$ weak transition. Albright and Oakes[7] have analyzed various experimental data and have concluded that a $J^6 J^8$ term is not required by the present data.

This paper is arranged as follows: In Sec. II we provide a brief review of the experimental situation with regard to the $\delta(980)$ scalar resonance. In Sec. III we apply the scalar-meson model to the CP-conserving process $K_L^0 \to \pi^0\gamma\gamma$ and calculate its decay rate. In Sec. IV we describe the decay $K^\pm \to \pi^\pm\gamma\gamma$ with the same model, calculating the decay rate and the charged-pion center-of-mass kinetic-energy spectrum. In Secs. V and VI we study the rare four-body decays $K \to \pi\pi\pi\gamma$ and $K \to \pi\pi\gamma\gamma$ in the framework of the δ model. Section VII is devoted to a summary of our results.

II. EXPERIMENTAL STATUS OF THE $\delta(980)$ SCALAR MESON

Before applying the δ-pole model to radiative K decays, it would be instructive to briefly review the experimental situation with regard to the $\delta(980)$ resonance. There were early reports from missing-mass experiments[8-11] on the possible existence of a relatively narrow $I = 1$ meson resonance near 970 MeV. However, the observation of a narrow $\eta\pi$ peak was first announced[12-14] at the 1968 Vienna Conference based on the results of bubble-chamber experiments dealing with the reactions $K^- p \to \Lambda\pi^+\pi^-\eta$ and $\bar{p}p \to D^0\pi^+\pi^- \to \pi^+\pi^-\eta\pi^+\pi^-$. The width of this peak was measured to be in the neighborhood of 50 MeV and the peak was generally interpreted as the decay of the δ scalar meson.

The existence of an $\eta\pi$ enhancement was further confirmed by other bubble-chamber experiments.[15-17]

However, the parameters of the $\eta\pi$ peak had been poorly determined in these experiments mainly because of poor statistics and the use of the neutral decay modes of the η to identify its presence.

Recent experiments involving K^-p interactions[18] and photoproduction[19] have provided particularly clear evidence for the existence of the $\delta(980)$. In one case[18] a complete analysis of the δ was reported from a high-statistics experiment involving the reaction $K^-p \to \Sigma^+(1385)\eta\pi^-$ with the η being detected from its charged decay mode. From this experiment the mass and partial width of the δ (980) meson is quoted to be

$$m_\delta = 981 \pm 6 \text{ MeV}, \quad \Gamma_{\eta\pi} = 55 \pm 5 \text{ MeV}. \tag{2.1}$$

It has also been suggested that the δ is connected with the threshold effect seen in the charged $K\bar{K}$ system, as observed in various low-energy antiproton experiments.[20] Gay *et al.*,[18] and Irving[21] have investigated the possibility that the δ is strongly coupled to $K\bar{K}$ by refitting the $\eta\pi^-$ mass spectrum using a two-channel resonance parametrization.[22] Their analyses indicate that the $\delta K\bar{K}$ coupling is consistent with the SU(3) prediction

$$g_{\delta K\bar{K}}{}^2 = \tfrac{3}{2} g_{\delta\eta\pi}{}^2 . \tag{2.2}$$

In applying the δ-meson model to the radiative K-meson decays, we shall use, for the parameters of the $\delta(980)$, the values given by (2.1) and (2.2).

III. THE δ-MESON MODEL AND $K_L^0 \to \pi^0\gamma\gamma$

We employ the δ-meson pole model to describe the CP-conserving decay $K_L^0 \to \pi^0\gamma\gamma$. The current experimental upper limit quoted[23] for this process is

$$\Gamma(K_L^0 \to \pi^0\gamma\gamma) < 4.6 \times 10^3 \text{ sec}^{-1}. \tag{3.1}$$

In our model we assume that this decay is dominated by a δ intermediate state which virtually decays into two photons. In Fig. 1 we exhibit the two Feynman diagrams contributing in our model. The possibility of a diagram featuring a weak $K_L^0 \to \eta$ transition is ruled out since we are adopting (1.1) as our weak nonleptonic Hamiltonian which forbids a 6–8 transition.

The diagrams of Fig. 1 can be easily evaluated and one obtains, using Eq. (2.2), for the polarized average squared matrix element

$$\langle \mathfrak{M}^2 \rangle = \frac{2 g_{\delta\gamma\gamma}{}^2 g_{\delta\eta\pi}{}^2 G_{K\pi}^0{}^2 (k_1 \cdot k_2)^2}{[(k_1+k_2)^2 - m_\delta^2]^2 (m_K^2 - m_\pi^2)^2} \times \left[\left(\frac{3}{2}\right)^{1/2} + \frac{g_{\eta\pi}}{m_\eta^2 - m_\pi^2}\right]^2, \tag{3.2}$$

where k_1, k_2 are photon momenta, $m_P (P = \pi, K, \eta)$

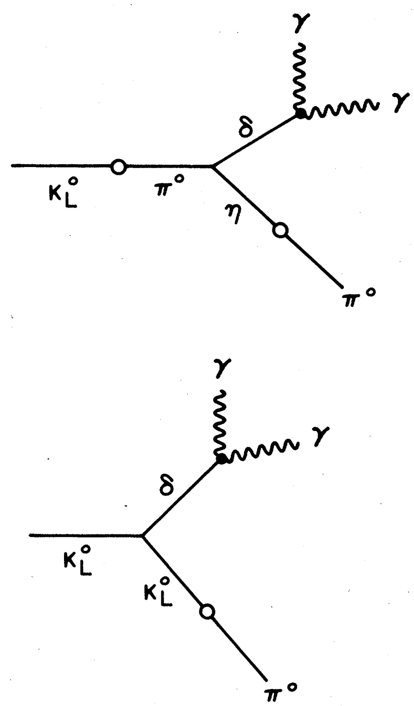

FIG. 1. Diagrams for the decay $K_L^0 \to \pi^0\gamma\gamma$ in the δ model.

are pseudoscalar-meson masses, $g_{\delta\eta\pi}$ denotes the strong coupling of δ to $\eta\pi$, $g_{\delta\gamma\gamma}$ is the effective electromagnetic coupling of δ to two photons, $G_{K\pi}^0$ measures the $K_L^0 - \pi^0$ weak transition, and $g_{\eta\pi}$ is the strength of the η-π^0 electromagnetic transition. In terms of these coupling constants the decay rate for $K_L^0 \to \pi^0\gamma\gamma$ is calculated from Eq. (3.2) to be

$$\Gamma(K_L^0 \to \pi^0\gamma\gamma) = \frac{g_{\delta\gamma\gamma}{}^2 g_{\delta\eta\pi}{}^2 G_{K\pi}^0{}^2 \lambda^2}{256\pi^3 m_K (m_K^2 - m_\pi^2)^2} \times \int_{m_\pi}^{\omega_{\max}} \frac{(\omega^2 - m_\pi^2)^{1/2} (m_K^2 + m_\pi^2 - 2m_K\omega)^2}{(2m_K\omega + m_\delta^2 - m_K^2 - m_\pi^2)^2} d\omega , \tag{3.3}$$

where

$$\omega_{\max} = (m_K^2 + m_\pi^2)/2m_K \tag{3.4}$$

and

$$\lambda = \left(\frac{3}{2}\right)^{1/2} + \frac{g_{\eta\pi}}{m_\eta^2 - m_\pi^2} . \tag{3.5}$$

Numerically, the value of the integral in Eq. (3.3) is $5.14 \times 10^{-3} m_\pi^2$. The remaining task is the evaluation of the various coupling constants appearing in Eq. (3.3).

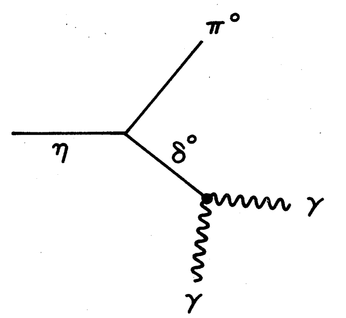

FIG. 2. Diagram for the decay $\eta \to \pi^0\gamma\gamma$ in the δ model.

The value of $g_{\delta\eta\pi}$ can be obtained by calculating the decay width $\Gamma(\delta \to \eta\pi)$ and fitting it to the measured value given by Eq. (2.1). The calculated expression is

$$\Gamma(\delta \to \eta\pi) = \frac{g_{\delta\eta\pi}^2}{16\pi m_\delta^3}(m_\delta^4 + m_\eta^4 + m_\pi^4 - 2m_\delta^2 m_\eta^2 - 2m_\delta^2 m_\pi^2 - 2m_\eta^2 m_\pi^2)^{1/2}, \tag{3.6}$$

which gives

$$g_{\delta\eta\pi} = 2.06 \pm 0.11 \text{ GeV}. \tag{3.7}$$

The coupling $g_{\delta\gamma\gamma}$ can be determined[1] by applying the δ model to the radiative decay $\eta \to \pi^0\gamma\gamma$. The δ contribution to this decay is shown by the diagram in Fig. 2. Evaluating this diagram, we obtain, using (3.7),

$$\Gamma(\eta \to \pi^0\gamma\gamma) = (2.20 \pm 0.24) \times 10^{-7} g_{\delta\gamma\gamma}^2 \text{ GeV}. \tag{3.8}$$

From the experimental[24] decay rate for $\eta \to \pi^0\gamma\gamma$ of 26 ± 9 eV, we find

$$g_{\delta\gamma\gamma} = 0.34 \pm 0.08 \text{ GeV}^{-1}. \tag{3.9}$$

An estimate of the coupling constant $G_{K\pi}^0$ describing the $K_L^0 \to \pi^0$ weak transition can be made using the Hamiltonian (1.1). We have from Eq. (1.1)

$$\mathcal{H}_w(K_L^0 \to \pi^0) = -\sqrt{2}\, G_{\text{NL}} J_\mu^3(x) J_\mu^6(x). \tag{3.10}$$

Using the field-current identities[2]

$$J_\mu^3(x) = -f_{\pi^0} \partial_\mu \phi_{\pi^0}, \tag{3.11a}$$

$$J_\mu^6(x) = -f_{K^0} \partial_\mu \phi_{K_L^0}, \tag{3.11b}$$

where f_{π^0} and f_{K^0} are pseudoscalar-meson decay constants, we then obtain

$$\mathcal{H}_w(K_L^0 \to \pi^0) = -\sqrt{2}\, G_{\text{NL}} f_{\pi^0} f_{K^0} \partial_\mu \phi_{\pi^0} \partial_\mu \phi_{K_L^0}. \tag{3.12}$$

It then follows from Eq. (3.12) that

$$G_{K\pi}^0 = -\sqrt{2}\, G_{\text{NL}} f_{\pi^0} f_{K^0} p^2, \tag{3.13}$$

where p represents the common four-momentum of the K and π meson. Thus we observe that $G_{K\pi}^0$ is momentum dependent and that its numerical value depends on which meson (K_L^0 or π^0) is on the mass shell. Using $G_{\text{NL}} = 1.1 \times 10^{-5}/m_p^2$, $f_\pi = 97$ MeV from experiment,[24] and the Callan-Treiman relation[25] $f_K = 1.28 f_\pi$, we find

$$G_{K\pi}^0(p^2) = \begin{cases} -5.27 \times 10^{-8} \text{ GeV}^2, & p^2 = m_K^2 \\ -3.88 \times 10^{-9} \text{ GeV}^2, & p^2 = m_\pi^2. \end{cases} \tag{3.14}$$

It should be pointed out that the value obtained here for $G_{K\pi}^0(m_K^2)$ is in good agreement with the value obtained[26] by fitting $K_L^0 \to \gamma\gamma$ to a π^0 pole using the Hamiltonian (1.1).

Finally, the coupling $g_{\eta\pi}$ for the $\eta \to \pi^0$ electromagnetic transition is evaluated by considering the decay $\eta \to 3\pi$ in the δ model. In this model $\eta \to 3\pi$ is described by the type of diagram displayed in Fig. 3. The evaluation of this contribution yields, using (3.7),

$$\Gamma(\eta \to 3\pi) = (6.6 \pm 1.3) \times 10^{-3} g_{\eta\pi}^2 \text{ GeV}. \tag{3.15}$$

From the experimental width[24] $\Gamma(\eta \to 3\pi) = 201$ eV, Eq. (3.15) predicts

$$g_{\eta\pi} = (6.0 \pm 0.6) \times 10^{-3} \text{ GeV}^2. \tag{3.16}$$

This value for $g_{\eta\pi}$ is in reasonable agreement with values determined by other methods.[27]

With the various coupling constants determined, we can now calculate the decay rate for $K_L^0 \to \pi^0\gamma\gamma$. Substituting (3.7), (3.9), (3.14), and (3.16) into Eq. (3.3) we obtain

$$\Gamma(K_L^0 \to \pi^0\gamma\gamma) = 10.8 \pm 6.3 \text{ sec}^{-1}. \tag{3.17}$$

This predicted value based on the δ-meson model is to be compared with the experimental upper limit stated in (3.1). We see that the value predicted by the δ model is roughly two orders of magnitude below the present experimental limit. As a basis for theoretical comparison, we have

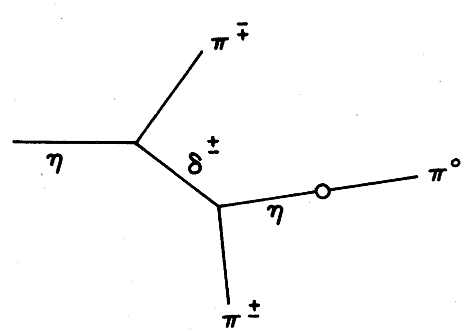

FIG. 3. The δ-meson contribution to $\eta \to 3\pi$.

TABLE I. Comparison of various theoretical predictions for the decay $K_L^0 \to \pi^0\gamma\gamma$; ϵ_4 is a parameter in one of the models and s is the invariant mass squared of the photon pair. Key: (a) Ref. 28; (b) Ref. 29; (c) Ref. 30; (d) Ref. 29; (e) Ref. 31.

Model	Predicted $\Gamma(K_L^0 \to \pi^0\gamma\gamma)$ (sec^{-1})
η pole	(a) 6.3×10^3
Dispersion relations	(b) 1.3×10^1
Current algebra	(c) $<1.5 \times 10^3$
Unitarity	(d) >0.28 ($s > m_K^2 - 7m_\pi^2$)
Moshe and Singer	(e) 2.7 ($\epsilon_4 =$ 1.53) 0.7 ($\epsilon_4 = -3.12$)
δ pole	10.8 ± 6.3

also listed in Table I the predictions for $K_L^0 \to \pi^0\gamma\gamma$ based on various other models.[28-31]

IV. $K^\pm \to \pi^\pm\gamma\gamma$ DECAYS

We now describe the decays $K^\pm \to \pi^\pm\gamma\gamma$ in the δ-meson model. In this case there is only one diagram contributing from a δ intermediate state. This diagram is shown in Fig. 4. Evaluating this diagram gives the polarized average squared matrix element

$$\langle \mathfrak{M}^2 \rangle = \frac{3g_{\delta\gamma\gamma}^{\;2} g_{\delta\eta\pi}^{\;2} G_{K\pi}^{\pm\,2} (k_1 \cdot k_2)^2}{[(k_1 + k_2)^2 - m_\delta^{\;2}]^2 (m_K^{\;2} - m_\pi^{\;2})^2}\,, \tag{4.1}$$

where we have again used the SU(3) relation (2.2), and $G_{K\pi}^{\pm}$ denotes the strength of the $K^\pm \to \pi^\pm$ weak transition.

From Eq. (4.1) the decay rate can be calculated

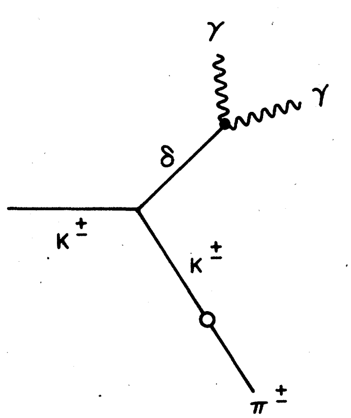

FIG. 4. Diagram for the decay $K^\pm \to \pi^\pm\gamma\gamma$ in the δ model.

TABLE II. Comparison of various theoretical predictions for the decays $K^\pm \to \pi^\pm\gamma\gamma$. Key: (a) Ref. 28; (b) Ref. 34; (c) Ref. 35; (d) Ref. 36; (e) Ref. 37; (f) Ref. 35; (g) Ref. 38; (h) Ref. 3; (i) Ref. 39; (j) Ref. 40.

Model	Predicted $\Gamma(K^\pm \to \pi^\pm\gamma\gamma)$ (sec^{-1})
η pole	(a) 1.6×10^2 (b) 2.4×10^4
π^0 pole	(c) 5.6×10^3 (d) 1.6×10^3
σ pole	(e) 3.2×10^5
Vector dominance	(f) 3.7×10^{-2}
Axial-vector dominance	(g) $1.2 \times 10^3 - 1.6 \times 10^4$
Moshe-Singer	(h) $(1.9 \pm 0.5) \times 10^2$
Fermion loop	(i) 5.2×10^1
Unitarity	(j) >0.11
δ pole	5.6 ± 3.2

in a straightforward manner and we obtain

$$\Gamma(K^\pm \to \pi^\pm\gamma\gamma) = \frac{3g_{\delta\gamma\gamma}^{\;2} g_{\delta\eta\pi}^{\;2} G_{K\pi}^{\pm\,2}}{512\pi^3 m_K (m_K^{\;2} - m_\pi^{\;2})^2} \times \int_{m_\pi}^{\omega_{\max}} \frac{(\omega^2 - m_\pi^{\;2})^{1/2} (m_K^{\;2} + m_\pi^2 - 2m_K\omega)^2}{(2m_K\omega + m_\delta^{\;2} - m_K^{\;2} - m_\pi^{\;2})^2}\, d\omega\,, \tag{4.2}$$

where $\omega_{\max}$ is given by Eq. (3.4). The integral can be numerically calculated and is found to be equal to $4.12 \times 10^{-3} m_\pi^{\;2}$. The value for $G_{K\pi}^{\pm}$ can be determined from the Hamiltonian (1.1). In fact, a consequence of the form of (1.1) is that $G_{K\pi}^{\pm}(p^2) = -G_{K\pi}^{0}(p^2)$. Thus, from (3.14) we deduce that $G_{K\pi}^{\pm}(m_\pi^{\;2}) = 3.88 \times 10^{-9}$ GeV^2 and this is the value which we will use in Eq. (4.2). It should be noted that the value $G_{K\pi}^{\pm}(m_K^2) = 5.27 \times 10^{-8}$ GeV^2, predicted by (1.1), is in reasonable agreement with the value obtained from current algebra.[5]

Evaluating Eq. (4.2), using (3.7) and (3.9) and our value for $G_K^{\pm}(m_\pi^{\;2})$, gives

$$\Gamma(K^\pm \to \pi^\pm\gamma\gamma) = 5.6 \pm 3.2 \text{ sec}^{-1}. \tag{4.3}$$

This value is to be compared with the present experimental upper limit[32] which gives[33]

$$\Gamma(K^\pm \to \pi^\pm\gamma\gamma) < 5.6 \times 10^2 \text{ sec}^{-1}. \tag{4.4}$$

Thus, our predicted value based on the δ model is exactly a factor of 100 below the experimental limit. Once again, we compare our prediction for $K^\pm \to \pi^\pm\gamma\gamma$ to the predictions based on various

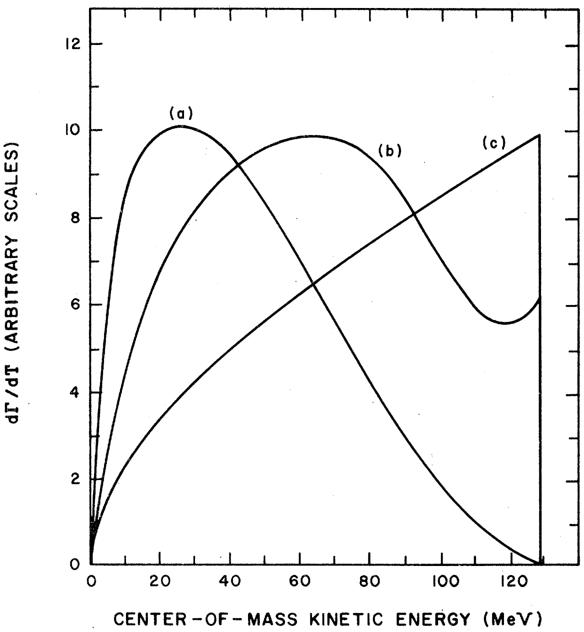

FIG. 5. Center-of-mass kinetic-energy spectrum of the charged pion in $K^\pm \to \pi^\pm\gamma\gamma$ decay predicted by (a) the δ-meson model, (b) the model of Moshe and Singer (Ref. 3), and (c) phase space.

other models.[3,28,34–40] This comparison is shown in Table II.

We have also calculated the charged-pion center-of-mass kinetic-energy spectrum predicted by the δ-meson model. The spectrum is shown in Fig. 5 along with the distributions expected from phase space and from the Moshe-Singer model.

V. THE δ MESON CONTRIBUTION TO $K \to \pi\pi\pi\gamma$ DECAYS

In this section we apply the δ-meson model to the rare four-body decays

$$K^\pm \to \pi^\pm\pi^+\pi^-\gamma\,, \tag{5.1}$$

$$K^\pm \to \pi^\pm\pi^0\pi^0\gamma\,, \tag{5.2}$$

$$K_L^0 \to \pi^+\pi^-\pi^0\gamma\,, \tag{5.3}$$

$$K_L^0 \to \pi^0\pi^0\pi^0\gamma\,. \tag{5.4}$$

The basic diagram describing these decays to lowest order in the δ model is shown in Fig. 6. The δ coupling to $\pi\pi\gamma$ is assumed to occur via vector mesons. The ω-γ and ϕ-γ couplings are given by $e^2 m_\omega^2/2\gamma_\omega$ and $e^2 m_\phi^2/2\gamma_\phi$, respectively,

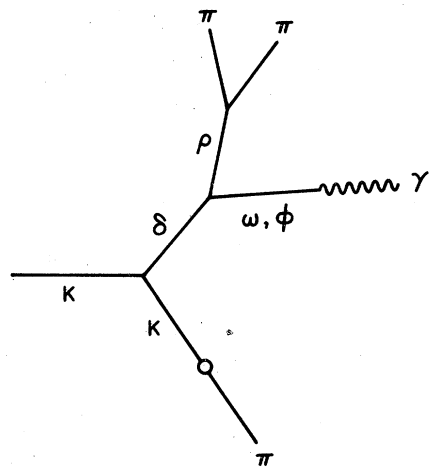

FIG. 6. Lowest-order diagram for $K\to\pi\pi\pi\gamma$ decays in the δ model.

and we use the experimental values[41]

$$\gamma_\omega^2/4\pi = 7.6\,, \quad \gamma_\phi^2/4\pi = 5.9\,. \tag{5.5}$$

The ρ-π-π coupling, obtained from the experimental $\rho\to\pi\pi$ width, is given by $g_{\rho\pi\pi}^2/4\pi = 2.86$.

The $\rho\omega\delta$ and $\rho\phi\delta$ couplings are obtained[1] by assuming that the decay $\delta\to\gamma\gamma$ is mediated by vector mesons. Vector dominance requires

$$g_{\delta\gamma\gamma} = \frac{\alpha}{2} g_{\rho\omega\delta}\left(\frac{\gamma_\rho^2}{4\pi}\right)^{-1/2}\left[\left(\frac{\gamma_\omega^2}{4\pi}\right)^{-1/2} + \beta\left(\frac{\gamma_\phi^2}{4\pi}\right)^{-1/2}\right], \tag{5.6}$$

where $\beta = g_{\rho\phi\delta}/g_{\rho\omega\delta}$ and experimentally[41] $\gamma_\rho^2/4\pi = 0.61$. If the δ meson is an isotriplet of an SU(3) octet, then we expect the ratio β to be equal to the same ratio of vector-meson couplings to the pion. In the vector-dominance picture

$$\frac{\Gamma(\rho\to\pi\gamma)}{\Gamma(\phi\to\pi\gamma)} = \left(\frac{g_{\rho\omega\pi}}{g_{\rho\phi\pi}}\right)^2\left(\frac{\gamma_\rho}{\gamma_\omega}\right)^2\left(\frac{m_\phi}{m_\rho}\right)^3\left(\frac{m_\rho^2-m_\pi^2}{m_\phi^2-m_\pi^2}\right)^3, \tag{5.7}$$

and, using the recent experimental data on these,[24] we find[42] $g_{\rho\phi\pi}/g_{\rho\omega\pi} = 0.073 \approx \beta$. Using this value for β we obtain

$$g_{\rho\omega\delta} = 180\pm40\ \text{GeV}^{-1}. \tag{5.8}$$

The transition rates for the decays (5.1)–(5.4) are obtained from diagrams of the type shown in Fig. 6 and are given by

$$\Gamma(K\to3\pi\gamma) = \frac{\epsilon g_{\delta\omega\rho}^2 g_{\rho\pi\pi}^2 e^2 g_{\delta K\bar K}^2 G_{K\pi}^2}{128(2\pi)^8 m_K \gamma^2 (m_K^2-m_\pi^2)^2}\int \frac{d^3q_1}{\omega_1}\frac{d^3q_2}{\omega_2}\frac{d^3q_3}{\omega_3}\frac{d^3k}{k}\,\frac{[k\cdot(q_1+q_2)]^2(q_2-q_1)^2+[k\cdot(q_2-q_1)]^2(q_1+q_2)^2}{[(p-q_3)^2-m_\delta^2]^2[(q_1+q_2)^2-m_\rho^2]^2}$$
$$\times\,\delta^4(p-q_1-q_2-q_3-k)\,, \tag{5.9}$$

where q_1, q_2, q_3 are pion momenta, p and k are kaon and photon momenta, and we have defined

$$\frac{1}{\gamma}=\frac{1}{\gamma_\omega}+\frac{\beta}{\gamma_\phi}, \tag{5.10}$$

and $\epsilon=2$ for reactions (5.1) and (5.2) and $\epsilon=9, 0$ for processes (5.3) and (5.4), respectively.[43] The integrations over the pion momenta are carried out in the center-of-mass system of the three pions using standard techniques.[1,44] The remaining integrals can be evaluated numerically. Using (5.5), (5.8), and (5.10) we find for the various decay rates

$$\Gamma(K^\pm\to\pi^\pm\pi^+\pi^-\gamma)=1.8\times10^{-3}\ \text{sec}^{-1}, \tag{5.11}$$

$$\Gamma(K^\pm\to\pi^\pm\pi^0\pi^0\gamma)=8.4\times10^{-3}\ \text{sec}^{-1}, \tag{5.12}$$

$$\Gamma(K_L^0\to\pi^+\pi^-\pi^0\gamma)=1.8\times10^{-2}\ \text{sec}^{-1}, \tag{5.13}$$

$$\Gamma(K_L^0\to\pi^0\pi^0\pi^0\gamma)=0 \tag{5.14}$$

These predictions are very small and it is unlikely that such low rates will be experimentally detectable in the foreseeable future.

VI. THE δ-MESON CONTRIBUTION TO $K\to\pi\pi\gamma\gamma$ DECAYS

As a final application of the δ-meson model we consider the radiative decays

$$K^\pm\to\pi^\pm\pi^0\gamma\gamma, \tag{6.1}$$

$$K_L^0\to\pi^+\pi^-\gamma\gamma, \tag{6.2}$$

$$K_L^0\to\pi^0\pi^0\gamma\gamma. \tag{6.3}$$

There are three basic types of diagrams for these decays which are shown in Fig. 7. The evaluation of these diagrams can be done and the decay rates calculated. However, the resulting integrations are quite lengthy so the reader is referred to Ref. 1 where a similar calculation for the decay $\eta\to\pi\pi\gamma\gamma$ has been carried out in detail.

We obtain the following decay rates:

$$\Gamma(K^\pm\to\pi^\pm\pi^0\gamma\gamma)=1.2\times10^{-3}\ \text{sec}^{-1}, \tag{6.4}$$

$$\Gamma(K_L^0\to\pi^+\pi^-\gamma\gamma)=5.9\times10^{-6}\ \text{sec}^{-1}, \tag{6.5}$$

$$\Gamma(K_L^0\to\pi^0\pi^0\gamma\gamma)=2.4\times10^{-3}\ \text{sec}^{-1}. \tag{6.6}$$

Thus, we obtain very small values for these four-body decay rates.

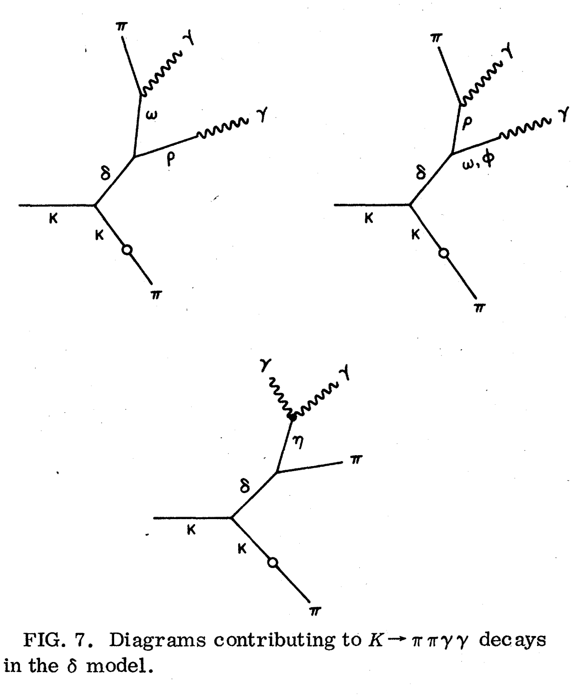

FIG. 7. Diagrams contributing to $K\to\pi\pi\gamma\gamma$ decays in the δ model.

VII. SUMMARY AND CONCLUSIONS

We have investigated the role which the δ(980) scalar meson plays in the nonleptonic radiative decays of the K meson. Using a Hamiltonian with a minimum of neutral currents but consistent with $\Delta I=\frac{1}{2}$ to describe the weak interaction, we have calculated the decay rates for $K_L^0\to\pi^0\gamma\gamma$, $K^\pm\to\pi^\pm\gamma\gamma$, $K\to3\pi\gamma$, and $K\to\pi\pi\gamma\gamma$.

For the processes $K_L^0\to\pi^0\gamma\gamma$ and $K^\pm\to\pi^\pm\gamma\gamma$ we have found that the δ model predicts decay rates which are about two orders of magnitude below the current experimental upper limits for these decays. It will be interesting to see whether or not future experiments find any significant enhancement over these low values for the decay rates.

From inspecting Table I we observe that our predicted rate for $K_L^0\to\pi^0\gamma\gamma$ is comparable to the rates predicted for this process by a dispersion relation calculation[28] and by the Moshe-Singer model.[3] At the same time our calculated decay rate for $K_L^0\to\pi^0\gamma\gamma$ lies between the theoretical upper and lower limits set by current algebra and unitarity calculations.[29,30]

On the other hand, we observe from Table II that a number of theoretical models for $K^\pm\to\pi^\pm\gamma\gamma$ seem to be ruled out by the current experimental upper limit. In addition, the vector-dominance model seems to be in disagreement with the lower bound set by unitarity. Clearly, the Moshe-Singer model predicts the largest decay rate for $K^\pm\to\pi^\pm\gamma\gamma$ which is still below the limit set by experiment. However, any further significant reduction in the experimental upper limit would tend to rule out the Moshe-Singer model.

In order to better understand the exact mechanism for $K^\pm\to\pi^\pm\gamma\gamma$ decay, it will be necessary to also determine the $\pi^\pm$ center-of-mass kinetic-energy spectrum. From Fig. 5 we observe that the spectrum is noticeably different for the δ-meson model and the Moshe-Singer model. The phase-space distribution is also included for comparison. Thus, a careful experimental study of the $\pi^{\pm}$ energy spectrum should reveal a great deal of information as to the exact mechanism for the decay and should provide a definitive test of the various models which have been proposed.

For completeness, the decay rates for the rare four-body processes $K\to 3\pi\gamma$ and $K\to\pi\pi\gamma\gamma$ have also been calculated in the framework of the δ-meson model. These predictions are unfortunately too small to be confronted by experiment in the foreseeable future.

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