# Scaling and the inclusive semileptonic decays of charmed particles

X. Y. Pham and R. P. Nabavi

*Division de Physique Théorique,* Institut de Physique Nucléaire, Paris* †
*and Laboratoire de Physique Théorique des Particules Elémentaires, Paris*†

(Received 29 March 1977)

A general formula describing the semileptonic decays of charmed particles is written in terms of structure functions. A procedure is proposed to estimate semileptonic decay widths by supposing that these structure functions may be averaged by their scaling limits.

When a hadron carrying a new flavor is produced, its semileptonic decay modes are particularly interesting since they can provide a distinctive signal for a new quantum number. Thus, before the recent discovery of charmed mesons at SLAC,[1] the dilepton events in deep-inelastic neutrino scattering[2] and the inclusive lepton spectrum in $e^+e^-$ annihilation[3] already constituted indirect evidence for new-flavored hadrons. Furthermore, leptonic decays provide a powerful tool for investigating the space-time structure of the weak currents associated with the new flavors.

Because of the high mass of charmed particles, it is clear that, *a priori*, semileptonic decays into multihadronic final states may be important. However, we have no reliable ways of calculating the partial rates of such modes, although estimates have been made.[4] The large number of accessible decay channels prompts us to consider here a new approach. By analogy with deep-inelastic neutrino scattering, we describe inclusive semileptonic decay in terms of five structure functions.[5] In view of the large momentum transfer and high missing mass in charmed-particle decay, we then extend the appealing idea of Bloom and Gilman[6] that inclusive and exclusive processes merge smoothly, i.e., that scaling occurs, in some average sense, in the resonance region.

We start by writing the differential decay width of a heavy particle $H\to l\nu+$anything:

$$\frac{d\Gamma(H\to l^{\pm}\nu+\cdots)}{dt\,dM_X^2\,d(\cos\theta)}=\frac{G_F^2(t-\mu^2)^2P}{32\pi^3M^2t}\left[W_1(t,M_X^2)+\frac{P^2}{2t}\sin^2\theta\,W_2(t,M_X^2)\mp\frac{P}{2M}\cos\theta\,W_3(t,M_X^2)+O(\mu^2/t)\right],\tag{1}$$

where

$$O(\mu^2/t)=\frac{\mu^2}{t}\left\{\tfrac{1}{2}W_1(t,M_X^2)+\left[\frac{(M^2-M_X^2)^2-t^2}{4M^2t}+\frac{EP\cos\theta}{t}+\frac{P^2\cos^2\theta}{2t}\right]W_2(t,M_X^2)\right.$$
$$\left.+\frac{t}{2M^2}W_4(t,M_X^2)+\left(\frac{E+P\cos\theta}{M}\right)W_5(t,M_X^2)\right\}.\tag{2}$$

Here, $t=q^2=(p_l+p_\nu)^2$ is the square of the invariant mass of the lepton-neutrino pair, $M_X^2=(p_H-q)^2$ is the square of the invariant mass of the hadronic system, $\theta$ is the angle between the charged-lepton momentum and the recoil momentum of the hadronic system in the lepton-pair rest frame, and $\mu$ and $M$ are the masses of the lepton $l$ and the decaying hadron $H$, respectively. Also,

$$E=\frac{M^2+t-M_X^2}{2M},\quad P^2=E^2-M_X^2,$$

and $G_F$ is the Fermi constant ($1.03\times10^{-5}\,M_p^2$). Equation (1) tells us that the three structure functions $W_1$, $W_2$, and $W_3$ can be separated experimentally by looking at the lepton-hadron angular distribution in the lepton-pair rest frame at fixed $t$ and $M_x^2$.

The dimensionless structure functions $W_1,\ldots,W_5$ are defined formally in the same way as those of deep-inelastic neutrino scattering.[7] They are related to the helicity structure functions $W_+$, $W_-$, and $W_0$ defined in Ref. 8 and advocated there as the most convenient way to analyze inclusive semileptonic decay processes:

$$W_\pm=W_1\mp(P/2M)W_3,$$
$$W_0=W_1+(P^2/t)W_2.\tag{3}$$

Neglecting the lepton mass, and integrating Eq. (1) over the angle $\theta$, we have

$$\frac{d\Gamma}{dt\,dM_X^2}=\frac{G_F^2tP}{16\pi^3M^2}\left[W_1(t,M_X^2)+\frac{P^2}{3t}W_2(t,M_X^2)\right].\tag{4}$$

The formulas (1) and (4) are quite general, and are valid for both mesons and baryons. For illustration, we consider the decay of the $D(1.87)$ meson, for which the domain of integration $\mathfrak{D}_1$ is shown in Fig. 1.

Now, in general, the energy released in a typical charmed-particle decay is large compared with the rest masses of the particles in the final state. This suggests that it may be reasonable to assume that the resonances and the background (formed by the many-body modes) may average out to a smooth scaling function. Since the structure functions $W_1$ and $(\nu/M)W_2$ are known to tend towards well-defined scaling limits in the kinematic regions appropriate to deep-inelastic neutrino scattering and high-energy $e^+e^-$ annihilation (large values of $t$ and $M_X^2$), we make the hypothesis that, on average over the range of integration in the decay region, they may be approximated by these same scaling limits using an appropriate scaling variable. In spirit, this assumption is similar to that employed by Bloom and Gilman[6] in their application of scaling ideas to electroproduction in the resonance region. Furthermore, the Bloom-Gilman variable $x'$ is not positive-definite in the case of particle decay, and thus cannot be used here. For simplicity, therefore, we employ the standard Bjorken scaling variable $x$ (without necessarily claiming that this is the best possible choice), and we make the replacements

$$W_1(t,M_X^2) \to F_1(x)\,,$$

$$\frac{P^2}{t} W_2(t,M_X^2) = \left(\frac{\nu}{2xM} - 1\right) W_2(t,M_X^2) \to \frac{1}{2x} F_2(x)\,, \tag{5}$$

where

$$x = \frac{1}{\omega} = \frac{t}{2M\nu} = \frac{t}{t+M^2-M_X^2}\,. \tag{6}$$

We hope that this assumption will be justified *a posteriori* by the fact that it gives a reasonable numerical result [see Eq. (15)]. Hence, we put

$$\int_{\mathfrak{D}_1} dt\, dM_X^2 Pt[W_1(t,M_X^2) + \frac{P^2}{3t} W_2(t,M_X^2)]$$

$$= \int_{\mathfrak{D}_2} dt\, dx \frac{Pt^2}{x^2}[F_1(x) + \frac{1}{6x} F_2(x)]\,. \tag{7}$$

The new domain $\mathfrak{D}_2$ of integration of this expression is shown in Fig. 2, for the case of the $D$ meson.

In the quark-parton model, and with the Glashow-Iliopoulos-Maiani current,[9]

$$2xF_1(x) = F_2(x) = 2xC_D(x)\,, \tag{8}$$

where $C_D(x)$ is the valence charmed-quark distribution in the $D$ meson, normalized to 1. Here, $x$ can be interpreted as the fraction of the $D$-meson momentum carried by the charmed quark, in the infinite-momentum frame. Although of course this interpretation makes little sense in the case of a decaying particle, we suppose that, having defined the structure functions in the appropriate kinematic region, we may make an analytic extrapolation to the decay region using the $x$ variable. Integrating Eq. (7) over the variable $t$, we find

$$\Gamma(D \to l\nu + \cdots) = \frac{G_F^2 M_D^5}{12\pi^3} \int_0^{1/2} dx \frac{C_D(x) I(x)}{x^2}\,, \tag{9}$$

where

$$I(x) = M_D^{-7} \int_{4x^2 M_D^2}^{xM_D^2/(1-x)} dt\, Pt^2 \tag{10}$$

$$= x^7\{(z^2-1)^{1/2}(2z^3 + \tfrac{16}{3} z^2 + 3z - \tfrac{16}{3}) - 5\ln[z + (z^2-1)^{1/2}]\}\,, \tag{11}$$

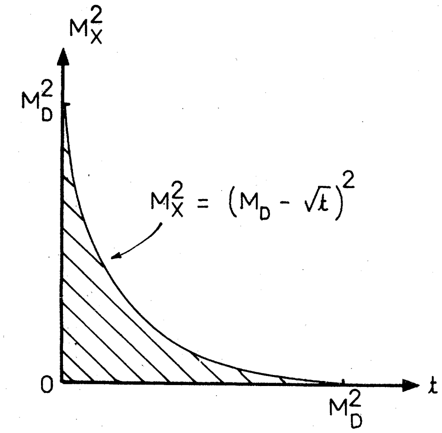


FIG. 1. Domain $\mathfrak{D}_1$ of integration of $d^2\Gamma/dM_X^2\,dt$ for the $D$ meson.

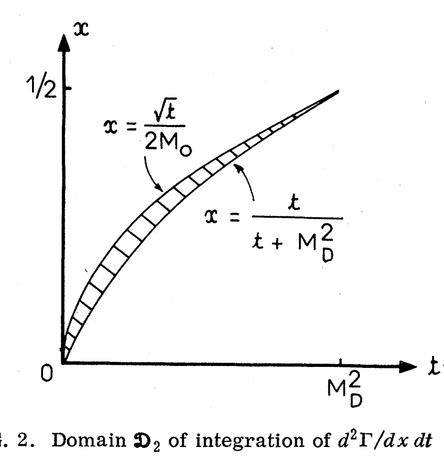


FIG. 2. Domain $\mathfrak{D}_2$ of integration of $d^2\Gamma/dx\,dt$ for the $D$ meson.

with

$$z=[2x(1-x)]^{-1}-1\,. \tag{12}$$

The distribution $C_D(x)$ can be determined by looking at $e^+e^-$ annihilation into $D$ + anything, at high energy. Such data are not yet available, but it seems reasonable to suppose that $C_D(x)$ is the same as the distribution functions of the valence quarks $u$, $d$, and $s$ in the familiar $\pi$ and $K$ mesons, for which parametrizations exist in the literature.[10] Using one such parametrization,

$$C_D(x)=0.94\frac{(1-x)^5}{\sqrt{x}}+\begin{cases}45.1x^{3/2}e^{-7.5x} & (x<0.35)\\ 1.03(1-x) & (x\geq 0.35)\,,\end{cases} \tag{13}$$

and normalizing

$$\int_0^1 C_D(x)\,dx=1\,, \tag{14}$$

Eq. (9) can be integrated numerically to yield, for the total semileptonic decay width of the $D$ meson,

$$\Gamma(D\to l\nu+\cdots)=0.16\,\frac{G_F^2M_D^5}{192\pi^3}$$
$$=1.2\times 10^{11}\ \text{sec}^{-1}\,. \tag{15}$$

We have checked that the use of other plausible parametrizations for $C_D(x)$ does not yield significantly different results (less than 10% variation). Inasmuch as this is true, the coefficient 0.16 appearing in expression (15) is the same for all mesons.

To get an idea of the reliability of this procedure, it is interesting to see what we predict for the semileptonic decay width of the $K$ meson. We consider only $K\to e\nu+\cdots$, since the muon mass is not negligible in comparison with the kaon mass. Equation (15) becomes

$$\Gamma(K\to e\nu+\cdots)=0.16\,\frac{G_F^2\sin^2\theta_C\,M_K^5}{192\pi^3}$$
$$=7.6\times 10^{6}\ \text{sec}^{-1}\,, \tag{16}$$

to be compared with the experimental values $3.9\times 10^6$ sec$^{-1}$ for $K^+$ and $7.5\times 10^6$ sec$^{-1}$ for $K^0$. The agreement is good, given that this is an extreme case; we are trying to average, by a smooth scaling function, a $\delta$ function representing a process which has only one significant mode, $\pi e\nu$. We can reasonably expect that the approximation will be no worse for the decays of heavier particles.

The method considered here may be compared with another calculation, namely that mentioned in Ref. 4 and discussed in more detail in Ref. 8, where the total semileptonic decay width of a charmed particle is assumed to be close to that of a free charmed quark. This gives

$$\Gamma(\text{charm}\to l\nu+\cdots)$$
$$=\frac{G_F^2m_C^5}{192\pi^3}\,(1-8r^2+8r^6-r^8-24r^4\ln r)\,, \tag{17}$$

where $r=m_s/m_c$ is the ratio of the strange- and charmed-quark masses. Note that it is now the quark masses which enter, predicting the same total semileptonic decay widths for all charmed particles. Expression (17) is sensitive to the quark masses taken; for example with $m_c=1.6$ GeV and $m_s=0.3$, 0.4, and 0.5 GeV, it gives respectively 2.7, 2.2, and $1.7\times 10^{11}$ sec$^{-1}$. This is of the same order as our estimate, though rather higher.

The procedure outlined here may also be applied to the semileptonic decay of a charmed baryon such as the $\Lambda_c(2.26)$ recently identified.[11] Only a slight modification of Eqs. (9) and (12) is needed, to take account of the fact that baryon-number conservation demands that there must always be at least a nucleon mass in the final state. Thus, the upper limit in Eq. (9) becomes $\frac{1}{2}(1-M_N/M_{\Lambda_c})$, and the $z$ variable in (12) becomes $z'$, where:

$$z'=\frac{(1-M_N^2/M_{\Lambda_c}^2)}{2x(1-x)}-1\,. \tag{18}$$

One must also use a parametrization $C_\Lambda(x)$ appropriate to a baryon. Putting[10]

$$C_\Lambda(x)=\frac{1.03(1-x)^7}{\sqrt{x}}$$
$$+0.7(1-x)\times\begin{cases}90.2x^{3/2}e^{-7.5x} & (x<0.35)\\ 5(1-x)^3 & (x\geq 0.35)\,,\end{cases} \tag{19}$$

we find

$$\Gamma(\Lambda_c\to l\nu+\cdots)=0.04\,\frac{G_F^2M_{\Lambda_c}^5}{192\pi^3}$$
$$=0.8\times 10^{11}\ \text{sec}^{-1}\,. \tag{20}$$

Finally, we note that this method may be applied to other new-flavored hadrons, provided that a large region of final-state phase space is available in their semileptonic decays.

---

*Laboratory associated with CNRS.
†Postal address: Université Pierre et Marie Curie, Tour 16-E1 4, Place Jussieu, 75230 Paris-Cedex 05, France.

[1] G. Goldhaber *et al.*, Phys. Rev. Lett. 37, 255 (1976); I. Peruzzi *et al.*, *ibid.* 37, 569 (1976).
[2] A. Benvenuti *et al.*, Phys. Rev. Lett. 35, 1199 (1975); 35, 1203 (1975); 35, 1249 (1975); B. C. Barish *et al.*,

*ibid.* 36, 939 (1976); H. Deden *et al.*, Phys. Lett. 58B, 361 (1975); J. Blietschau *et al.*, *ibid.* 60B, 207 (1976); J. Von Krogh *et al.*, Phys. Rev. Lett. 36, 710 (1976).

[3]See, for example, B. H. Wiik, in *Proceedings of the XVII International Conference on High Energy Physics Tbilisig, 1976*, edited by N. N. Bogolubov *et al.* (JINR, Dubna, 1977), Vol. II, p. N75.

[4]M. K. Gaillard, B. W. Lee, and J. Rosner, Rev. Mod. Phys. 47, 277 (1975); J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B100, 313 (1975).

[5]The use of structure functions in analyzing semileptonic decays of charmed particles has previously been considered by V. Barger and R. J. N. Phillips, Phys. Rev. D 14, 80 (1976).

[6]E. D. Bloom and F. Gilman, Phys. Rev. Lett. 25, 1140 (1970). This idea is extensively used in J. D. Bjorken and J. Kogut, Phys. Rev. D 8, 1341 (1973); the spirit of our hypothesis is related to that considered by J. Sakurai, Phys. Lett. 46B, 207 (1973).

[7]C. H. Llewellyn Smith, Phys. Rep. 3C, 281 (1972); H. Pietschmann, *Formulae and Results in Weak Interactions* (Springer, Berlin, 1974), p. 51.

[8]R. P. Nabavi, X. Y. Pham, and W. N. Cottingham, J. Phys. G 3, 1485 (1977). J. Phys. G: Nucl. Phys. London (to be published).

[9]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[10]R. Blankenbecler *et al.*, Report No. SLAC-PUB-1531, 1975 (unpublished); see also V. Barger, T. Weiler, and R. J. N. Phillips, Nucl. Phys. B102, 439 (1976); M. Duong Van, Phys. Lett. 60B, 287 (1976).

[11]B. Knapp *et al.*, Phys. Rev. Lett. 37, 882 (1976).