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Widths of electron-photon cascades

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A review of experimental evidence and theoretical calculations concerning the widths of electron-photon cascades in a range of media from air to lead shows that a universal curve characterizes the lateral cascade distribution for all these materials, provided the widths are measured in Molière units. Discrepancies between previously reported results are resolved in favor of the narrower cascade widths.

I. INTRODUCTION

Recent calculations[1-4] based on Monte Carlo simulations of electromagnetic cascades in air have yielded cascade widths which are narrower, by approximately a factor of 2, than those obtained with the commonly accepted analytic approximation based on the work of Nishimura and Kamata.[5,6] Greisen[7] has put their results into a useful parametric form which is hereafter referred to as the NKG formula. The resolution of this discrepancy is important not only for the interpretation of extensive-air-shower (EAS) measurements, but also for the design and calibration of satellite and balloon-borne spectrometers employed in high-energy cosmic-ray and γ-ray astronomy. For EAS, an understanding of the lateral development of electromagnetic cascades is critical to the determination of the energy of primary cosmic rays from ground-based measurements of the charged-particle distribution. Similarly, for lower-energy cosmic-ray electron and γ-ray measurements, the effective geometrical factor and the energy calibration of scintillation spectrometers employed as total absorption shower counters depend on the widths of the cascades produced by incident primary photons.

It has been suggested by Nishimura[8] that the difference between the analytic results and those obtained with the Monte Carlo simultations may be due to the difference in the energy of the primary photon. In the case of the NKG approximation, the primary energy is effectively infinite, in contrast to the 10 GeV used in the Monte Carlo calculations with which the discrepancy was first discovered.[1,2,9] The Monte Carlo simultations were extended down to 1 GeV and up to 100 GeV primary energy but there was no detectable change in the widths obtained.

In the present work, independent calculations and a reexamination of measurements made with accelerator beams have been employed to resolve the discrepancy. From the theoretical and experimental evidence presented in the following section, we conclude that there exist reliable and self-consistent calculations and measurements which present a clear picture of the lateral development of electromagnetic cascades in materials ranging in atomic number from that characterizing air through that of lead, but which result in a scale width substantially narrower than that obtained with the NKG approximation for air. The significance of these results is discussed in the final section.

II. CRITICAL REVIEW OF CALCULATIONS AND EXPERIMENTAL RESULTS

Observations of EAS lateral structure near the shower axis have long been considered to be in substantial agreement with the NKG approximation, as if the electrons belonged to a purely electromagnetic cascade with an age parameter in the range $0.8 \lesssim S \lesssim 1.6$.[10] It should be noted, however, that the shower age as well as the energy are calculated from the measured lateral distribution, so that agreement with the analytic forms suggested by NKG may be achieved by compensating errors in the calculated and the observed parameters. Furthermore, the development of EAS depends also on hadronic interactions, so that the observed lateral spread may well be significantly wider than the characteristic widths of the component electromagnetic cascades.

Messel and Crawford[11] have carried out extensive and elaborate Monte Carlo calculations for cascades in various materials, but there is serious disagreement in a number of respects between their results and those from the other, much simpler, Monte Carlo calculations and also with NKG.[1-4] Crawford[12] reports that problems with the detailed simulations of electromagnetic cascades in air are due to a computer programming error in which the Compton cross section was effectively set equal to zero.

The lateral and longitudinal development of electromagnetic cascades has been studied experimentally in some detail, using electron beams from accelerators.[13-20] Because most of these measurements have been carried out in media with densities and atomic numbers much higher than those of air, the applicability of the results to the problem of cascades in air must be approached with caution. Moreover, some of these measurements have shown that serious instrumental effects are introduced when inhomogeneities in the cascade medium are large compared to the distance in which the distributions of photons and electrons in the cascade change significantly.[16,18] The most recent experimental studies[20] have shown that distortions of the electron flux within the cascade occur "upstream" as well as "downstream" of the material interface. In particular, gaps for the insertion of a probe can distort both the lateral and longitudinal distributions being measured. Another difficulty can arise from the artifical broadening of the measured lateral distribution of energy deposition when a scintillating salt of higher atomic number than that of the medium is employed as a detector.[17]

The causes of experimental error in the measured cascade lateral distributions using the scintillator technique are evident from an examination of the energy spectra and spatial distributions of the cascade particles and photons obtained from the Monte Carlo simulations.[2] In an earlier publication,[15] one of us made the suggestion that positrons are responsible, through annihilation in flight, for an anomaly that is seen in the lateral distribution but not in the longitudinal distribution. The calculations show that positrons cannot be responsible because they are concentrated closer to the axis than are either the electrons or photons. The key lies in the energy spectrum of the photons, which shows greater softening as one goes to larger radii than it does as one goes to greater depth. The same qualitative behavior is apparent from the analytical calculations of Nishimura and Kamata.[5] Interactions of the low-energy photons in detectors with a higher atomic number than that of the cascade medium must be expected to give an apparent broadening of the lateral distributions, as was indeed suggested previously by Yuda *et al.*[19]

Several measurements have been carried out in ways by which these difficulties are avoided. The lateral and longitudinal development of cascades induced by 950-MeV electrons in water have been measured with an anthracene $[C_6H_4:(CH)_2:C_6H_4]$ detector.[17] Because the medium was liquid and because the atomic numbers of the detector scintillator and its associated light guide were nearly the same as that characterizing water, no significant inhomogeneities in the cascade medium were introduced. In other experiments.[18,19] measurements were made on cascades induced by 600- and 1000-MeV electrons in aluminum, copper, and lead using photosensitive films sufficiently thin to cause negligible distortion in the cascade distribution. The lateral and longitudinal distributions of the energy deposition in water[21,22] and in lead[23] have also been obtained theoretically from Monte Carlo simulations of electron-induced cascades. Good agreement between measurement and calculation was obtained in both cases.

Once instrumental difficulties have been taken into account, the suggestion by Nelson *et al.*[14] that a universal radial distribution may be used to describe cascade structure, independent of the medium, is seen to be in agreement with both theory and experiment. All the reliable accelerator measurements and the corresponding Monte Carlo simulations yield the same radial distribution, which is independent of incident energy and of the atomic number of the cascade medium providing that the radius is measured in Molière units[7] (cf. Fig. 2 of Ref. 17 and Fig. 7 of Ref. 19).

The remaining difficulty is that of relating the measurements and calculations at accelerator energies to the calculations for electromagnetic cascades in air. Experimental considerations restrict measurements made in dense media to radial distances greater than 0.1 Molière unit from the cascade axis, while the Monte Carlo simulations in air have the best statistical accuracy at distances less than 0.5 Molière unit. A suitable parameter describing the cascade width, which lies in this range and which does not depend on the detailed shape of the lateral distribution, is the median width. The median width itself may be defined in a number of different ways. It may be taken, as is generally useful for accelerator measurements, as that radius within which one half of the cascade energy is deposited. Alternatively it may refer to particle numbers, either

TABLE I. Median width of an electromagnetic cascade with respect to energy.

	Cascade maximum (Molière units)	Average
Computed for air		
Allan *et al.*	0.15	0.20
Experimental universal curve	0.17	0.24

TABLE II. Median width of an electromagnetic cascade with respect to particle number.

Calculated for cascade maximum in air with a cutoff energy of 4 MeV	(Molière units)
Allan *et al.*	0.12
Hillas and Lapikens	0.15
NKG	0.27
Calculated for cascade maximum in air with no cutoff energy	**(Molière units)**
NKG	0.32

at cascade maximum or averaged over all depths. The appropriate median radii derived from the results discussed here are presented in Tables I and II. Physical dimensions were converted to Molière units using the values for critical energy and radiation length given by Dovzhenko and Pomanski.[24] It is clear that a reduction of the NKG value for the cascade width in air by a factor of 2 resolves the discrepancy in favor of the Monte Carlo simulations.

III. DISCUSSION

The physical basis for the universal radial distribution curve is not immediately apparent. The Molière unit, in which the cascade widths are expressed, appears naturally only in the high-energy approximation to cascade theory, approximation A.[5,6] It is defined in terms of the radiation length X_R, the critical energy E_c, and the scattering energy E_s as

$$1 \text{ Molière unit} = \frac{E_s}{E_c} X_R . \qquad (1)$$

The radiation length is the scale length for energy losses by electron bremsstrahlung, $dE/dx = -E/X_R$; while the critical energy is that energy at which bremsstrahlung and ionization losses are equal. The scattering energy is a constant in the expression for the mean-square scattering angle in the Rossi-Greisen[25] approximation to multiple scattering, $\langle\theta^2\rangle = (E_s/E_c)^2 x/X_R$, for an electron traversing a distance x. Because of the approximations under which the component parameters were derived, the Molière unit appropriately characterizes the scattering length in a medium only if the scattering angles are small and then only for particle energies greater than E_c.

The requirement of small scattering angles is well satisfied for particles above the critical energy in air, but not in lead. The particle distributions obtained with the Monte Carlo calculations[1-3] indicate another difficulty. While the critical energy for air is ~80 MeV, the median energy for cascade electrons in air is found to be ~30 MeV. Hence only a small fraction of the cascade particles have energies for which even X_R is a significant parameter. It should be noted that the value of 30 MeV for the median energy agrees well with the earlier analytical results of Richards and Nordheim.[26] (The published Nishimura-Kamata[5,6] calculations give 12 MeV, but Nishimura[8] informs us that this figure is incorrect; the error is due to the choice of approximation method for evaluating the relevant integrals.)

Some other parameter, with the same dependence on the atomic number of the medium as the Molière unit, may represent the physical basis for the universal radial distribution curve. The Molière unit is rather insensitive to Z because X_R and E_c are both nearly proportional to $1/Z$, so that the dependence nearly cancels, and the scattering energy has the value 21 MeV, independent of the scattering medium. For lead ($Z = 82$), the Molière unit (in g cm^{-2}) is less than a factor of 2 times larger than it is for air ($Z = 7$). Because the minimum attenuation length for photons is greater than the corresponding radiation length in all media, energy is transported beyond the cascade maximum and off the cascade axis by photons. The minimum attenuation length might, therefore, be expected to characterize the lateral distribution of cascades in the same way as it characterizes the decreasing portion of longitudinal distributions. This scale length also changes relatively little with Z, but the change is in the opposite sense. For lead, the minimum attenuation length is a factor of 3 times smaller than it is for air.

The reason for there being a universal curve may be that the low-energy photons are launched at greater angles to the cascade axis in lead than they are in air. In this context, low energy means less than E_c, and E_c is 10 times smaller in lead than in air. The greater angles are sufficient to more than compensate for the smaller absorption length in lead compared with air. The fact that widths are the same within the computational and experimental accuracy when measured in Molière units is significant and of practical importance, but it cannot be attributed to any simple similarity in the pattern of the cascades.

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