PHYSICAL REVIEW D VOLUME 18, NUMBER 6 15 SEPTEMBER 1978

Melosh transformation in interacting quark theories

Anjan S. Joshipura
Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India
(Received 7 September 1977)

We make a general study of the existence of a Melosh-type transformation in interacting quark field theories quantized on the null plane. Following Carlitz and Tung this transformation is required to decompose the total angular momentum operators of these theories into mutually commuting orbital angular momentum and spin parts. This allows the identification of a U(6) × U(6) × O(3) classification group in these theories. It is shown that the requirement of commutativity between the generators of the U(6) × U(6) subgroup and the three momentum operators P^+, $\vec{P}^\perp$ generating translations on the null plane, when combined with the spin properties of the former, automatically assures the exact conservation of the entire U(6) × U(6) algebra in a Poincaré-invariant theory. A part of this transformation which is bilinear in the quark field is shown to exist in these theories. This part commutes with the operators P^+ and $\vec{P}^\perp$ showing that the breaking of U(6) × U(6) symmetry can take place only through the quark pair terms appearing in the transformation.

I. INTRODUCTION

The evidence in favor of a broken $SU_W(6) \times O(3)$ symmetry among the physical hadrons is impressive.[1] However the construction of such an algebra in realistic field-theoretical models is not straightforward and was not carried out until recently. The first example of this algebra was given by Melosh[2] in the free quark-field theory. He, in fact, demonstrated in this theory the existence of a bigger group, namely, U(6) × U(6) × O(3), which can provide the basis for the quark-model classification scheme for the hadrons.[3] Recently, Carlitz and Tung[4] have used a semirelativistic approximation scheme towards constructing a Melosh-type transformation in the presence of interactions among quarks. They have explicitly considered a Yukawa-type interaction and given a transformation up to the second order in the inverse power of the quark mass m. The purpose of the present paper is to generalize their considerations to higher orders in m^{-1} as well as to other interactions.

Theories quantized on the null plane[5] form a natural framework for the description of hadronic symmetries and have been extensively used in recent years because of the numerous theoretical and phenomenological advantages they offer.[6,7] Certain components of the physical weak and electromagnetic currents in these theories have simple forms in terms of the canonical field $q_+(x)$—known as the current quark field—of the theory. The charges obtained by intergrating the local currents formed out of the fields $q_+(x)$ on the null plane generate the lightlike analog of the $SU_W(6)$ algebra of Lipkin and Meshkov.[8] Unfortunately the complicated form of the angular momentum generators in these theories makes this $SU_W(6)$-currents group unsuitable as a classification group for hadrons. In fact in these theories the total angular momentum generators $J^{1,2}$, apart from having an involved dependence on dynamics, fail to possess the separation into mutually commuting orbital and spin angular momenta. These properties of the angular momentum generators, arising from the choice of the restricted Dirac field $q_+(x)$ as a canonical field of the theory, are responsible for the bad spin properties of the $SU_W(6)$ generators. However, the angular momentum operators of the free quark theory can be decomposed[4,9] into the orbital and spin parts by expressing them in terms of a nonlocal field $\varphi_+(x) = V_{\text{free}}\, q_+(x) V^{-1}_{\text{free}}$, where V_{free} is the transformation constructed by Melosh.[2] Consequently the generators of the $SU_W(6)$ group, defined in terms of the constituent quark field $\varphi_+(x)$, have the right spin properties and so can be used to classify the physical hadronic states.

Attempts to generalize this transformation to theories with interacting quarks have been made both in first-[10] as well as second-quantized theoreis.[4,11] In general the complex forms of the angular momentum generators $J^{1,2}$ and of the Hamiltonian in the null-plane–quantized theories have made it almost impossible to construct the transformation exactly. However, in the limiting case of the quark mass tending to infinity, these operators develop simple forms and the transformation becomes unnecessary.[4] This is quite analogous to the situation encountered in the construction of the Foldy-Wouthuysen transformation in the presence of interactions.[13] This analogy led Bell and Ruegg,[10] and Carlitz and Tung[4] to construct an approximate transformation V by expanding it in the inverse powers of the quark mass[12] m. The former authors have concentrated on bringing out symmetries of the Hamiltonian while the latter have used this approximation to "diagonalize" the angular momentum generators. Carlitz and Tung chose a Yukawa-type interaction between quarks and a scalar field for the actual construction of the transformation. A similar thing can be done for the other interactions as well, since the basic philosopy behind the approach—namely, the appearance of an exact $SU_W(6)$ symmetry in the limit $m\to\infty$—is a model-independent aspect.[4] However, it is not *a priori* obvious that the transformation V will exist for other interactions; nor is it clear that the transformation which decomposes $J^{1,2}$ into the desired form will possess other properties required to assure the existence of a well-defined U(6) × U(6) algebra.[6,9,14] These properties include the commutativity of the generators of the U(6) × U(6) group with the three momentum generators P^+ and $\vec{P}^\perp$ that generate translations in the null plane,[6] and with the third component J^3 of the angular momentum operators $\vec{J}$. In what follows we investigate the question of the existence of a transformation which solves the angular conditions[2] in interacting quark theories without scrificing any of these properties.

Section II contains a discussion of the consequences that follow from the imposition of the above-mentioned kinematical constraints on the generators of the U(6) × U(6) symmetry group. More specifically we show there that the exact conservation of the generators of the U(6) × U(6) group follows in a Poincaré-invariant theory once they satisfy the angular conditions and commute with the three momentum operators. Possible consequences of this result are also discussed in the same section. In Secs. III and IV we investigate the nature of the transformation $V\equiv e^{iY}$ in interacting quark field theories using the semi-relativistic approximation scheme of Bell, Ruegg, Carlitz, and Tung. In general the operator Y is found to contain[4] terms bilinear in the quark fields as well as the multilinear terms which are responsible for producing quark pairs inside the hadrons. In Sec. III we formulate the sufficiency condition for the existence of a $P^+, \vec{P}^\perp$ conserving bilinear part of Y to all orders in m^{-1}. These conditions are highly nontrivial but, as we show in Sec. IV, they can always be satisfied in any rotationally invariant theory. This section also contains a discussion on the expected solution for the multilinear part of Y. The existence of this part of Y to the lowest order is demonstrared in the Appendix. Rotational invariance plays an instrumental role in assuring the existence of V. As illustrated in the concluding section, V fails to exist in a model containing second-quantized quarks interacting with the external vector-gluon field because of the lack of rotational invariance in this particular theory. Expressions are also given in Sec. V for the transformation to order $O(m^{-2})$ in fully second-quantized vector-gluon theory and a comparison with the scalar-gluon-model transformation of Carlitz and Tung is made.

II. CONSTRAINTS ON THE U(6) × U(6) ALGEBRA

The generators of the group U(6) × U(6) × O(3) × $O(3)_{\text{currents}}$ [to be distinguished[2] from the classification group U(6) × U(6) × $O(3)_{\text{strong}}$] have the following form in terms of the current quark fields $q_+(x)$,

$$F^a_\alpha=\frac{1}{\sqrt{2}}\int d^4x\,\delta(x^+)q^\dagger_+(x)\Gamma^a_\alpha q_+(x)\,, \tag{2.1a}$$

$$\hat{F}^a_\alpha=\frac{1}{\sqrt{2}}\int d^4x\,\delta(x^+)q^\dagger_+(x)\Gamma^a_\alpha\epsilon(\eta)q_+(x)$$
$$(\alpha=0,\ldots,8;a=0,\ldots,3)\,, \tag{2.1b}$$

$$L^i=-\tfrac{1}{2}\epsilon^{ij}\int d^4x\,\delta(x^+)q^\dagger_+(x)\times\left[x^j\left(\frac{m^2-\partial_\perp{}^2}{2\eta}\right)-x^j\eta-x^-i\partial^j\right]q_+(x)+\text{H.c.}+J^i_B$$
$$\equiv L^i_q+J^i_B\quad(i,j=1,2;\epsilon^{12}=-\epsilon^{21}=1)\,, \tag{2.1c}$$

$$L^3=\sqrt{2}\int d^4x\,\delta(x^+)q^\dagger_+(x)(\epsilon^{ij}x^i i\partial^j+\tfrac{1}{2}\sigma^3)q_+(x)+J^3_B$$
$$\equiv L^3_q+J^3_B\,, \tag{2.1d}$$

where

$$\Gamma^a_\alpha=\lambda_\alpha\quad\text{for } a=0$$
$$=\beta\sigma^a\frac{\lambda_\alpha}{2}\quad\text{for } a=1,2$$
$$=\sigma^3\frac{\lambda_\alpha}{2}\quad\text{for } a=3$$

and $\lambda_0=2$. Moreover, $\epsilon(\eta)=\eta/|\eta|$ is an integral operator like η^{-1},[4,9] while η and ∂^j are derivatives with respect to the variables x^- [$x^\pm\equiv(x^0\pm x^3)/\sqrt{2}$] and x^j; $\vec{J}_B$ is the contribution of the fields other than quark fields to the total angular momentum.[15] These fields are assumed to be singlets under the group U(6) × $U(6)_{\text{currents}}$. The operator $\vec{L}_q$ represents the quark orbital angular momentum.

The operator L^3 has a simple relationship with the third component of the total angular momentum

$$L^3=J^3-F^3_0\,, \tag{2.2}$$

while $L^{1,2}$ are not related simply to $J^{1,2}$ owing to the dependence of the latter on the bad component

$q_-(x)$ of the Dirac field $q(x)$ $(q_\pm(x) \equiv \frac{1}{2}(1 \pm \alpha^3) q(x))$. In the free quark model $L^{1,2}$ can be related to $J^{1,2}$ by rotating the spin basis of the quarks[2,14]:

$$V_{\rm free}^{-1} J^i V_{\rm free} = L_q^i + S^i \quad (i = 1, 2), \tag{2.3}$$

where $S^i \equiv \hat{F}_0^i$ describes the transverse components of the quark spin operator.[4] In general the interactions between the quarks modify the form of the angular momentum generators[4,5] and a mere rotationof the quark spin does not bring $J^{1,2}$ into a form similar to that in Eq. (2.3). Carlitz and Tung[4] have proposed an interaction-dependent transformation V which, apart from rotating the quark spin, brings $J^{1,2}$ into the desired form. Such a V enables one to define in any theory the $\mathrm{U}(6) \times \mathrm{U}(6) \times \mathrm{O}(3)_{\rm strong}$ algebra through the relations

$$(W_\alpha^a,\ \hat{W}_\alpha^a) \equiv V(F_\alpha^a, \hat{F}_\alpha^a) V^{-1}, \tag{2.4a}$$

$$\vec{\mathrm{L}}(W) \equiv V \vec{\mathrm{L}}\, V^{-1}, \tag{2.4b}$$

where V is a solution of the equation

$$V^{-1} J^{1,2} V = L^{1,2} + S^{1,2}. \tag{2.4c}$$

These generators W_α^a, $\hat{W}_\alpha^a$ (to be called W charges) satisfy the angular conditions if V commutes with J^3. In addition, if they commute with the null-plane three-momenta $P^+, \vec{\mathrm{P}}^\perp$, then every irreducible basis of the $\mathrm{U}(6) \times \mathrm{U}(6)$ algebra has a definite spin and three-momentum so that it can be taken as a meaningful classification algebra.

The transformation $V_{\rm free}$ of Eq. (2.3) satisfies all these requirements. Furthermore the $\mathrm{U}(6) \times \mathrm{U}(6)_{\rm strong}$ algebra is exactly conserved in this theory. The latter property may seem to be a peculiar feature of the free quark theory where the Hamiltonian is free of γ matrices and even the algebra $\mathrm{U}(6) \times \mathrm{U}(6)_{\rm currents}$ is conserved.[6] These simplifications not being present in general, the difference in the forms of the Hamiltonian and $J^{1,2}$ would lead one to expect the breakdown of $\mathrm{U}(6) \times \mathrm{U}(6)_{\rm strong}$ symmetry at some stage.[4] However, it turns out that, irrespective of the form of the operators $J^{1,2}$ and P^-, the assumed commutativity of the W charges with the three momentum operators P^+ and $\vec{\mathrm{P}}^\perp$ implies the conservation of the former in a Poincaré-invariant theory.

Crucial for proving the last statement is the fact that the operators P^+ and $\vec{\mathrm{P}}^\perp$ keep the null plane $x^+ = 0$ invariant and thus have interaction-independent forms

$$P^c = \sqrt{2} \int d^4x\, \delta(x^+)\, q_+^\dagger(x) i\, \partial^c q_+(x) + P_B^c, \tag{2.5}$$

where P_B^c denotes the three momentum operators of the other fields and $c = +, 1, 2$. The Poincaré algebra, together with the Eq. (2.1c), implies the following relations when use is made of the canonical anticommutation relations of the quark fields $q_+(x)$[4]:

$$[P^j, L^i] = \frac{1}{\sqrt{2}}\, \epsilon^{ij} (P^-_{\rm free} - P^+), \tag{2.6a}$$

$$[P^j, S^i] = 0 \quad (i, j = 1, 2). \tag{2.6b}$$

Here

$$P^-_{\rm free} = \sqrt{2} \int d^4x\, \delta(x^+)\, q_+^\dagger(x) \left\{ \left(\frac{m^2 - \partial_\perp{}^2}{2\eta} \right) \right\} q_+(x) + P_B^-,$$

where P_B^- denotes the contribution of the fields other than the quark fields.

The W charges, unlike the F charges of Eqs. (2.1a), (2.1b) do not commute with the operators P^c un less $V^{-1} P^c V$ is invariant under $\mathrm{U}(6) \times \mathrm{U}(6) \times \mathrm{U}(6)_{\rm currents}$. Thus if

$$V^{-1} P^c V \equiv P^c + \hat{P}^c, \tag{2.7}$$

then the necessary and sufficient condition for the commutativity of the W charges with the operators P^c is

$$[\hat{P}^c, F_\alpha^a] = [\hat{P}^c, \hat{F}_\alpha^a] = 0 \tag{2.8}$$

$$(c = +, 1, 2;\ a = 0, \ldots, 3;\ \alpha = 0, \ldots, 8).$$

The commutator of Eq. (2.4c) with the momentum generators P^j leads to the following equation when use is made of the Poincaré algebra and Eqs. (2.6) and (2.8):

$$V^{-1} P^- V = P^-_{\rm free} + \hat{P}^+ + \frac{i}{\sqrt{2}}\, \epsilon^{ij} [L^i, \hat{P}^j] + \frac{i}{\sqrt{2}}\, \epsilon^{ij} [S^i, \hat{P}^j]. \tag{2.9}$$

It follows by virtue of the invariance of the right-hand side of Eq. (2.9) under $\mathrm{U}(6) \times \mathrm{U}(6)_{\rm currents}$ that the generators of the $\mathrm{U}(6) \times \mathrm{U}(6)_{\rm strong}$ group commute with the full Hamiltonian P^- provided they also commute with the three momentum generators P^+ and $\vec{\mathrm{P}}^\perp$.[16] This peculiar feature, which is a consequence, among other things, of the nonlocality of the W charges, is absent in the case of local charges since the latter automatically commute with the kinematical components of the four momentum operators[5,6] irrespective of their conservation properties. If $\mathrm{U}(6) \times \mathrm{U}(6)_{\rm strong}$ symmetry is to break down at some stage[4] then, as implied by this result, its generators will fail to commute with the operators P^+ or/and $\vec{\mathrm{P}}^\perp$. In particular, the commutativity of W charges with the operator P^+ is essential to assure their vacuum annihilation properties.[5,6] Consequently the simultaneous breakdown of the commutativity of the W charges with the operator P^+ and the Hamiltonian would result in the vacuum-breaking effects of the same order of magnitude as the explicit violation of $U(6)\times U(6)$ symmetry. This can make the W charges as bad as their timelike counterparts. In fact, it is in order to avoid vacuum-breaking effects associated with the timelike charges[17] that one uses lightlike charges,[6] or equivalently goes to the infinite-momentum frame.[18] It is interesting to note that the generators of the $SU_W(6)_{\text{strong}}$ subgroup of the $U(6)\times U(6)_{\text{strong}}$ group may still commute with P^+ and $\vec{P}^\perp$ even when $SU_W(6)_{\text{strong}}$ symmetry is broken. As evident from Eq. (2.9), this can happen only if the bilocal charges $\hat{F}^a_\alpha$ of Eq. (2.1) fail to commute with the three momentum operators. In practice, this commutativity will depend upon the nature[19] of the solution V of Eq. (2.4c). In view of these results it becomes necessary to investigate the detailed nature of the transformation V to which we now turn.

III. INTERACTING QUARK FIELD THEORIES

We consider in this section interacting quark field theories quantized on the null plane $x^+=0$. The existence of such theories and their equivalence to field theories quantized on a spacelike surface have been demonstrated by Kogut and Soper[20] and by Chang *et al.*[21] Specifically they considered nonderivative interactions between fermions and scalar or vector bosons. Tomboulis carried out the quantization of non-Abelian gauge theories[22] which were later shown by Casher[23] to be equivalent to corresponding theories quantized on a spacelike surface. We do not restrict ourselves to any of these theories but consider instead a general form for the rotational generators valid in all the above-mentioned theories[24]:

$$J^i=\int d^4x\,\delta(x^+)q_+^\dagger(x)\mathcal{J}^i(x)q_+(x)+J^i_{\text{Coulomb}}+J^i_B\,, \tag{3.1a}$$

where

$$\begin{aligned}\mathcal{J}^i(x^-,\vec{x}^\perp,\eta^{-1},B(x))\equiv\mathcal{J}^i(x)\\ =-\epsilon^{ij}(m^2x^jp_0^-+mx^jp_1^-+x^jp_2^-\\ -x^j\eta+m\Sigma_1^j+\Sigma_2^j)\\ (i,j=1,2)\,.\end{aligned} \tag{3.1b}$$

Here the operators $p^-_{0,1,2}$ are functions of the derivatives $\eta,\vec{\partial}_\perp$ of the good Dirac matrices[6] of the integral operators like η^{-1} and of the mediating fields [which we collectively denote as the gluon field $B(x)$]. They are related to the Hamiltonian P^- through the equation

$$P^-=\sqrt{2}\int d^4x\,\delta(x^+)\,q_+^\dagger(x)(m^2p_0^-+mp_1^-+p_2^-)q_+(x)+P_B^-+P^-_{\text{Coulomb}}\,. \tag{3.1c}$$

The operators $\Sigma^j_{1,2}$ receive contribution from the spin parts of the generators $J^{1,2}$. They have the same type of functional dependence as $p^-_{0,1,2}$. The terms J^i_{Coulomb} and P^-_{Coulomb} are quartic in the quark fields and appear only if the gluon field is vector in character.[20,21] The operators J^i_B and P^-_B denote the angular momentum generators and the Hamiltonian of the gluon field, respectively. The former has a general form

$$J_B^i=-\epsilon^{ij}\int d^4x\,\delta(x^+)\{x^j[\tilde{P}_B^-(x)-\tilde{P}_B^+(x)]+x^-\tilde{P}_B^j(x)+\tilde{\Sigma}_B^j(x)\}\,, \tag{3.1d}$$

where $\tilde{P}^\mu(x)$, $\tilde{\Sigma}^j_B(x)$, respectively, denote the four-meomentum density and the "spin density" expressed in terms of the canonically independent components of the gouon field.

The operators $J^{1,2}$ depend upon the time derivatives as well as upon the bad (noncanonical) fields of the theory. In the above-mentioned theories it is possible to express the bad field at any value of x^+ in terms of the good fields at the same x^+.[19,20] The use of these constraint equations leads one to the form (3.1) for $J^{1,2}$. The most important feature of this form is the model independence of the terms p_0^- and Σ_1^j. They have the same form as in the free quark model, namely,

$$p_0^-=\tfrac{1}{2}\eta^{-1}\quad\Sigma_1^j=\tfrac{1}{2}i\gamma^j\eta^{-1}\,. \tag{3.2}$$

The operators $J^{1,2}$ are to be brought into the form of Eq. (2.4c) by a unitary transformation V, for which we write an asymptotic expansion[4]

$$V\equiv e^{iY}\,,\quad Y=\sum_{n=1}^{\infty}m^{-n}Y_n\,. \tag{3.3}$$

The operator Y can be written as

$$Y=\int d^4x\,\delta(x^+)\,\mathfrak{z}(x)\,. \tag{3.4}$$

The form of $\mathfrak{z}(x)$ is restricted by various constraints on V. In particular its dependence on $(x^-,\vec{x}^\perp)$ can come only through its dependence on the canonical fields occurring in it if Y is to commute with the operators P^+ and $\vec{P}^\perp$.[6] This property of Y is sufficient to guarantee the commutativity of the W charges with P^+ and $\vec{P}^\perp$. In what follows we impose these conditions on the bilinear part of Y from the outset and investigate its existence.

As suggested by Carlitz and Tung,[4] the solution to Eq. (2.4c)—if it exists—can be determined recursively by making use of the expansion (3.3). To any particular order n, one encounters the commutator of the unknown operator Y_n with

$$\mathcal{J}_0^i = -\tfrac{1}{2}\epsilon^{ij}\int d^4x\,\delta(x^+)\,q_+^\dagger(x)x^j\eta^{-1}q_+(x)\qquad (i,j=1,2)\,,$$

experssed in terms of the known quantities containing, in general, terms bilinear as well as multilinear in the quark field $q_+(x)$. Owing to the interaction independence of $\mathcal{J}_0^i$, the commutator $[iY_n, \mathcal{J}_0^i]$ contains the same number of quark fields as contained in Y_n itself; thus solutions containing different numbers of quark pairs can be treated separately. For the moment let us concentrate on the bilinear terms for which we write

$$\mathfrak{z}_{\rm bl}(x) \equiv \sqrt{2}\,q_+^\dagger(x)Oq_+(x)\,. \tag{3.5a}$$

The operator O can be separated into a free part and an interaction-dependent part, each of which can be expanded in powers of m^{-1}:

$$O \equiv O_{\rm free}(\eta, \eta^{-1}, |\partial_\perp|) + O_{\rm int}(x)$$
$$\equiv \sum_{n=1}^{\infty} m^{-n}(O_{n\,\rm free} + O_{n\,\rm int})\,. \tag{3.5b}$$

The bilinear part of $V^{-1}J^{1,2}V$ may be written as

$$(V^{-1}J^i V)_{\rm bl} = \int d^4x\,\delta(x^+)\,q_+^\dagger(x)[e^{-iO}\mathcal{J}^i(x)e^{iO} + \sqrt{2}\,e^{-iO}J_B^i e^{iO} - \sqrt{2}\,J_B^i]\,q_+(x) \tag{3.6a}$$

$$\equiv -\epsilon^{ij}\int d^4x\,\delta(x^+)q_+^\dagger(x)\left(\frac{1}{2}m^2x^j\eta^{-1} + \hat{l}^j + s^j + \sum_{n=-1}^{\infty} m^{-n}C_n^j\right)q_+(x)\quad (i,j=1,2)\,. \tag{3.6b}$$

Here

$$\hat{l}^j \equiv -\left(x^j\frac{\partial_\perp^{\,2}}{2\eta} + x^j\eta + ix^-\partial^j - \frac{\partial^j}{2\eta}\right), \tag{3.6c}$$

$$s^j \equiv \frac{i}{\sqrt{2}}\,\gamma^j\epsilon(\eta)\,, \tag{3.6d}$$

$$C_n^j \equiv x^jA_{n+2} + B_{n+2}^j\eta^{-1} + \left[\frac{-iO_{n+2}}{2}\,, x^j\eta^{-1}\right]. \tag{3.6e}$$

In obtaining Eq. (3.6a) we have made use of the equal-time anticommutations relations of the quark fields.[4] Equation (3.6b) is obtained by expanding O in powers of m^{-1} and using Eq. (3.1b). The terms involving $J_B^{1,2}$ arise in second-quantized theories because of their noncommutativity with $O_{\rm int}(x)$. In general each A_n $(n\geq 2)$[25] involves the quantities $p^-_{0,1,2}$, η, and/or the commutator of $O_1, O_2, \ldots, O_{n-1}$ with $p^-_{0,1,2}, \eta$ and the gluon momentum desnsities $\bar{P}_B^\pm(x)$. Similarly each B_n^j $(n\geq 2)$ contains $\Sigma^j_{1,2}$ and/or the commutator of $O_1, O_2, \ldots, O_{n-1}$ with x^j, $\Sigma^j_{1,2}$, $x^-\bar{P}_B^j(x)$, and $\bar{\Sigma}_B^j(x)$. A_2, B_2^j get additional contributions from the term $-(\hat{l}^j + s^j)$ which serve to cancel the $(\hat{l}^j + s^j)$ term present in Eq. (3.6b). Moreover,

$$A_1 \equiv p_1^-\,,\; B_1^j \equiv \Sigma_1^j\eta\,.$$

It now follows that if the bilinear parts of $Y_1, \ldots, Y_{n-1}$ exist and commute with the operators $P^+, \vec{P}^\perp$, then A_n and B_n^j do not have any explicit dependence on the coordinates $x^-, \vec{x}^\perp$. This fact can be exploited in formulating the condition for the existence of O_n for any arbitrary n.

Equation (3.6b) develops the required form of Eq. (2.4c) provided the equation

$$\left[\frac{iO_n}{2}, x^j\eta^{-1}\right] = x^jA_n(x) + B_n^j\eta^{-1} \tag{3.7}$$

is satisfied for every n. It is not difficult to see that the solution to Eq. (3.7), without any explicit dependence on the coordinates $\vec{x}^\perp, x^-$, can exist if and only if there exist operators $O_1, \ldots, O_{n-1}$ with such properties and the following condition is satisfied[26]:

$$[A_n(x), x^j] = [B_n^j, \eta^{-1}]\,. \tag{3.8}$$

The solution in this case is given by[27]

$$\frac{iO_n}{2} = \eta(\eta^{-1}A_n(x))\eta + \frac{iO_{n\,\rm free}}{2}\,. \tag{3.9}$$

This does not involve the coordinates $\vec{x}^\perp, x^-$ explicitly. Moreover the commutativity of the translation generators $P^\pm$ with the third component J^3 of the angular momentum $\vec{J}$ can be used to show that Y_n commutes with J^3 if $Y_1, Y_2, \ldots, Y_{n-1}$ do the same. Thus if Eq. (3.8) is shown to hold for all n then the bilinear part of Y must exist and commute with the generators P^+, $\vec{P}^\perp$, and J^3. However, conditions (3.8) are nontrivial and imply stringent restrictions on various quantities appearing in $J^{1,2}$ of Eq. (3.1). For example, for $n=2$,

$$A_2 = p_2^- + \tfrac{1}{2}[-iO_1, p_1^-] + \frac{\partial_\perp^{\,2}}{2\eta}\,, \tag{3.10a}$$

$$B_2^j = \{\Sigma_2^j - \Sigma_1^j\eta p_1^- + \tfrac{1}{2}[-iO_1, \Sigma_1^j] + ix^-\partial^j - \tfrac{1}{2}\partial^j\eta^{-1} - s^j\}\eta\,. \tag{3.10b}$$

These equations, when used with (3.9), lead to the condition

$$[p_2^-, x^j] - [\Sigma_2^j, \eta^{-1}]\eta + 2[p_1^-, \Sigma_1^j]\eta = 0\,. \tag{3.11}$$

It is not *a priori* clear that these and similar equations for higher n would be satisfied in an arbitrary interacting theory. However, as shown in Sec. IV, rotational invariance is sufficient to assure the existence of Eq. (3.8) for all n.

IV. EXISTENCE OF Y_{b1}

The Lorentz invariance of the theories[21] considered in Sec. III implies in particular the commutation relations

$$[J^i, J^k] = i\epsilon^{ik} J^3 , \tag{4.1a}$$

$$[J^i, q_+(x)] = -\frac{1}{\sqrt{2}} \mathcal{J}^i(x) q_+(x) + \ldots , \tag{4.1b}$$

where $i,k = 1,2$. The second equation follows when use is made of the canonical anticommutation relations of the quark fields[4] and Eq. (3.1a). The dots in Eq. (4.1b) represent the terms carrying more than one quark field. By commuting Eq. (4.1b) with J^k and noting that $\mathcal{J}^i(x)$ does not involve any time derivative as well as any quark field [see Eq. (3.1b)], one obtains

$$[\mathcal{J}^i(x), \mathcal{J}^k(x)] + \sqrt{2}\,[J_B^i, \mathcal{J}^k(x)] - \sqrt{2}\,[J_B^k, \mathcal{J}^i(x)] = \sqrt{2} i\,\epsilon^{ik} \mathcal{J}^3 , \tag{4.2a}$$

where $\mathcal{J}_3$ is defined by

$$J^3 = \int d^4x\, \delta(x^+) q_+^\dagger(x) \mathcal{J}^3 q_+(x) + J_B^3 . \tag{4.2b}$$

Equation (4.2a) is a consequence of rotational invariance of the theory and its validity can be checked, for instance, in scalar or vector field theory by a lengthy calculation. This equation implies in particular the relation

$$e^{-iO}[\mathcal{J}^i(x) + \sqrt{2}\, J_B^i, \mathcal{J}^l(x) + \sqrt{2}\, J_B^l] e^{iO} = \sqrt{2}\, i\epsilon^{il} e^{-iO}(\mathcal{J}^3 + \sqrt{2}\, J_B^3) e^{iO} , \tag{4.3}$$

where the operator O is defined by Eq. (3.5b). When the left hand side of Eq. (4.3) is expressed in terms of the quantities C_n^j of Eq. (3.6e), one finds that

$$\epsilon^{ij}\epsilon^{lk}\left\{\left[\tfrac{1}{2}m^2 x^j \eta^{-1} + \hat{l}^j + s^j + \sum_{n=-1}^{\infty} m^{-n} C_n^j ,\ \tfrac{1}{2} m^2 x^k \eta^{-1} + \hat{l}^k + s^k + \sum_{n=-1}^{\infty} m^{-n} C_n^k\right]\right\}$$
$$+ \sqrt{2}\left\{-\epsilon^{ij}\left[\sum_{n=-1}^{\infty} m^{-n} C_n^j , J_B^i\right] + \epsilon^{ij}\left[\sum_{n=-1}^{\infty} m^{-n} C_n^j , J_B^l\right]\right\} = \sqrt{2}\, i\epsilon^{il}(e^{-iO} g^3 e^{iO} + \sqrt{2}\, e^{-iO} J_B{}^3 e^{iO} - \sqrt{2}\, J_B{}^3) . \tag{4.4}$$

The right-hand side of Eq. (4.4) is the term appearing in the bilinear part of $V^{-1} J_B^3 V$ (modulo $\sqrt{2}$) and thus it reduces to $\sqrt{2}\, g^3$ if V commutes with J^3.

Equation (4.4) is sufficient to prove the existence of the bilinear part of Y to all orders in m^{-1}. Let us assume that the conditions of Eq. (3.9) are satisfied for $n = 1, \ldots, N$. Then, as discussed in Sec. III, $Y_{1\,b1}, Y_{2\,b1}, \ldots, Y_{N\,b1}$ exist and commute with J^3, P^+, and $\vec{P}^\perp$. This means several things: (1) All C_n^j for $n = 1, \ldots, N-2$ are zero [see Eqs. (3.6) and (3.7)]. (2) A_{N+1} and B_{N+1}^j do not have any explicit dependence on $\vec{x}^\perp$. (3) The right-hand side of Eq. (4.4) is of the order $O(m^{-(N+1)})$, apart from one $O(m^0)$ term, namely $\sqrt{2}\, g^3$. This last statement implies that the coefficient of $m^{-(N-1)}$ in the left-hand side of Eq. (4.4) is zero, yielding[28] the result

$$\epsilon^{ij}\epsilon^{lk}\left\{\left[x^j\eta^{-1}, x^k A_{N+1} + B_{N+1}^k \eta^{-1} + \left[-\frac{iO_{N+1}}{2}, x^k\eta^{-1}\right]\right] - (j \to k)\right\} = 0$$

or

$$\{x^k([x^j, A_{N+1}] - [\eta^{-1}, B_{N+1}^j])\eta^{-1} + [x^j, B_{N+1}^k]\eta^{-2}\} - \{j \to k\} = 0 \tag{4.5}$$

for $(j,k = 1,2)$.

As A_{N+1}, B_{N+1}^j are free of any explicit dependence on $\vec{x}^\perp$, the above equation implies the condition

$$[A_{N+1}, x^j] = [B_{N+1}^j, \eta^{-1}] . \tag{4.6}$$

Thus the existence of $O_1, \ldots, O_N$ implies the existence of O_{N+1} also. The existence of O_1 can be trivially demonstrated by the same argument, completing the proof of the existence of Y_{b1} commuting with P^+, $\vec{P}^\perp$, and J^3 to all orders in m^{-1}. From the arguments of Sec. II it now follows that this solution not only diagonalizes $J^{1,2}$ (up to terms multilinear in quark fields) but also removes γ-dependent bilinear terms from $V^{-1}P^- V$. Multilinear terms in $V^{-1}J^{1,2}V$ and $V^{-1}P^- V$ arise[4] in second-quantized theories because of the dependence of Y_{b1} on the gluon field. The cancellation of these multilinear terms requires the introduction of similar terms in Y. It is not *a priori* obvious that this cancellation can always be achieved. Even if this can be done, the commutativity or noncommutativity of this additional piece in Y with P^+ and $\vec{P}^\perp$ is not obvious. One can hopefully try to answer these questions along the same line as was adopted in proving the existence of Y_{b1}. The formulation of the existence condition analogous to Eq. (3.8) and the exploitation of the consequences of the rotational invariance is mathematically more complicated in this case. However, as shown in the Appendix, the lowest-order transformation can be constructed in all interacting theories under consideration in the same way as in the scalar-gluon model of Carlitz and Tung.[4]

Finally, it is gratifying to note that even if the multilinear terms in Y fail to ensure the conservation of the W charges to some order in m^{-1}, the commutativity of V with the operator P^+ is not destroyed to that order and so the vacuum-breaking effects are automatically suppressed.[29] This is ensured by the very structure of the angular momentum operators. The noncommutativity of V with the operator P^+ can arise only through the explicit dependence of $\mathfrak{d}(x)$ [see Eq. (3.4)] on x^-. Since the explicit dependence of the angular momentum densities corresponding to $J^{1,2}$ on x^- is contained in the terms $x^- P^j(x)$ [$P^j(x)$ being components of transverse-momentum density of the fields], it follows that Y_n for any arbitrary n cannot depend explicitly on x^- as long as $Y_1, Y_2, \ldots, Y_{n-1}$ commute with the transverse-momentum operator $\vec{P}^\perp$. Thus the commutativity of the W charges with $\vec{P}^\perp$ (and thereby with the Hamiltonian) will be violated prior to their commutativity with the operator P^+. This would suppress the vacuum-breaking effects.

V. AN ILLUSTRATIVE EXAMPLE

In this section we consider an ordinary SU(3) triplet of quarks interacting with a vector-gluon field assumed to be a singlet under the color as well as flavor SU(3) groups. The extension to the case in which gluons carry color degrees is straightforward. The Lagrangian density is

$$\mathcal{L}=\bar{q}(i\vec{\gamma}_\perp\cdot\vec{D}_\perp - m)q - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}\,,$$
$$D_\mu \equiv \partial_\mu - igB_\mu(x)\,, \tag{5.1}$$
$$F_{\mu\nu}\equiv[\partial_\mu B_\nu(x) - \partial_\nu B_\mu(x)]\,.$$

This theory can be quantized[20] on the null plane by choosing the special gauge $B^+(x)=0$, where $B^\pm(x) = [B^0(x)\pm B^3(x)]/\sqrt{2}$. The component $B^-(x)$ of the gluon field in this gauge can be expressed on the null plane in terms of the transverse components $B^j(x)$, which play the role of canonical fields:

$$B^-(x)=i[\eta^{-1}\vec{\partial}_\perp\cdot\vec{B}_\perp(x)] - [\eta^{-2}J^+(x)]\,, \tag{5.2}$$

where $J^+(x)=\sqrt{2}\,gq_+^\dagger(x)q_+(x)$, and η^{-2} is an integral operator similar to η^{-1}.[8] The dependent component $q_-(x)$ of the quark field likewise satisfied the following constraint equation:

$$q_-(x)=\tfrac{1}{2}\eta^{-1}(-i\vec{\gamma}_\perp\cdot\vec{D}_\perp + m)\gamma^+ q_+(x)\,. \tag{5.3}$$

The angular momentum generators involve the bad fields $q_-(x), B^-(x)$, and the time derivatives of the good fields. On eliminating them, one arrives[30] at the $J^{1,2}$ of the form of Eq. (3.1a), (3.1b) with

$$p_2^- = \tfrac{1}{2}(\vec{\gamma}_\perp\cdot\vec{D}_\perp)\eta^{-1}(\vec{\gamma}_\perp\cdot\vec{D}_\perp) - ig[\eta^{-1}\vec{\partial}_\perp\cdot\vec{B}_\perp(x)]\,, \tag{5.4a}$$

$$p_1^- = \tfrac{1}{2}g[\eta^{-1},\vec{\gamma}_\perp\cdot\vec{B}_\perp(x)]\,, \tag{5.4b}$$

$$\Sigma_2^j = -\tfrac{1}{2}\gamma^j\eta^{-1}(\vec{\gamma}_\perp\cdot\vec{D}_\perp) - ix^-\partial^j - ig(\eta^{-1}B^j)\,, \tag{5.4c}$$

$$J^i_{\text{Coulomb}} = -\frac{1}{2\sqrt{2}}\,\epsilon^{ij}\int d^4x\,\delta(x^+)x^j J^+(x)\eta^{-2}J^+(x)\,, \tag{5.4d}$$

$$J^i_B = \frac{1}{\sqrt{2}}\,\epsilon^{ij}\int d^4x\,\delta(x^+)\left[(-i\partial^+B^k(x))\left(x^j\eta + x^- i\partial^j + \frac{x^j}{2\eta}\,\partial_\perp{}^2 - \frac{\partial^j}{2\eta}\right)B^k(x) + 2(-i\partial^+B^k(x))\,\partial^k\eta^{-1}B^j(x)\right]$$
$$(i,j,k=1,2)\,. \tag{5.4e}$$

With these explicit forms of the various quantities at hand, one can construct the lower-order Y's by making use of Eq. (3.9) and the result of the Appendix. The expressions for Y_1 and Y_2 are given by[31]

$$Y_1 = -\sqrt{2}\int d^4x\,\delta(x^+)q_+^\dagger(x)\vec{\gamma}_\perp\cdot\vec{D}_\perp q_+(x)\,, \tag{5.5a}$$

$$\begin{aligned}Y_2 = \sqrt{2}\int d^4x\,\delta(x^+)q_+^\dagger(x)\{&\sqrt{2}\,|\eta|\vec{\gamma}_\perp\cdot\vec{\partial}_\perp - \tfrac{1}{2}g[\eta,[\vec{\gamma}_\perp\cdot\vec{\partial}_\perp,(\eta^{-1}\vec{\gamma}_\perp\cdot\vec{B}_\perp(x))]_+ + ig(\eta^{-1}B_\perp{}^2(x))]_+\\ &-2g\eta(\eta^{-1}\vec{\partial}_\perp\cdot\vec{B}_\perp(x))\eta\}q_+(x)\\ +\tfrac{1}{2}ig^2\int d^4x\,\delta(x^+)\Big[\sum_{a=1}^{\infty}[&(q_+^\dagger(x)(\vec{\eta}\vec{\eta})^{1-a}\gamma^l q_+(x))\,\eta^{-1}(q_+^\dagger(x)(\vec{\eta}\vec{\eta})^{a-1}\gamma^l q_+(x))\\ &-4(q_+^\dagger(x)(\vec{\eta}\vec{\eta})^{1-a}q_+(x))\,\eta^{-3}(q_+^\dagger(x)(\vec{\eta}\vec{\eta})^{a}q_+(x))]\Big]\,.\end{aligned} \tag{5.5b}$$

The algebraic properties of the operators $Y_{1,2}$ can be easily read off from the above equations. To the lowest order in m^{-1}, Y transforms under $SU_W(6)_{\text{currents}}$ in the same way as in the free quark model. However to order m^{-2} it contains a term transforming as a member of the $\underline{35}\times\underline{35}$ representation of $SU(6)_{\text{currents}}$. This latter property is to be contrasted with the scalar-gluon model[4] where the quartic term present in Y_2 transforms as an $SU_W(6)_{\text{currents}}$ singlet. More interesting are the properties of Y under the O(2) generator L^3 [Eq. (2.1d)]. Both $Y_{1\,\text{int}}$ and $Y_{2\,\text{int}}$ fail to possess a definite value, unlike their free counterparts which change L^3 by ±1. This is the consequence of the spin of the gluon field and the definition (2.1d) of the O(2) generators. The latter induces only spatial rotation on the quark fields and both spin and spatial rotation on the gluon fields $B^i(x)$. This gives rise to the complicated O(2) properties for $Y_{1\,\text{int}}$ and $Y_{2\,\text{int}}$. This can be overcome by including only the orbital part of the gluon angular momentum in the definition of the generators of O(3). Just as in the free quark theory, the operators $J_B^{1,2}$ appearing in Eq. (5.4e) fail to possess clear-cut separation into the orbital angular momentum and the gluon spin part.[30] This can probably be brought about by making a unitary transformation V_B similar to the Melosh transformation[2] on Eq. (2.4c). This transformation, being dependent on the gluon field only, will not alter the structure of the operators L_q^i and S^i appearing in the right-hand side of that equation; thus angular conditions will still be satisfied. No such transformation is needed in the spin-zero case since Y_1, Y_2 already have the definite value of L^3.[4]

We conclude this section with a brief consideration of the theory with an unquantized gluon field $B^\mu(x)$, having the component $B^+(x)=0$. The angular momentum generators still retain the form of Eq. (3.1) with

$$p_1^- = \tfrac{1}{2}g[\eta^{-1},\vec{\gamma}_\perp\cdot\vec{B}_\perp(x)]\,, \tag{5.6a}$$

$$p_2^- = \tfrac{1}{2}(\vec{\gamma}_\perp\cdot\vec{D}_\perp)\eta^{-1}(\vec{\gamma}_\perp\cdot\vec{D}_\perp) - gB^-\,, \tag{5.6b}$$

$$\Sigma_2^j = -ix^-\partial^j - \tfrac{1}{2}\gamma^j\eta^{-1}(\vec{\gamma}_\perp\cdot\vec{B}_\perp)\,. \tag{5.6c}$$

In this theory no component of the gluon field depends upon the fermion current J^+ and thus neither the term $-ig(\eta^{-1}B^j)$ of Eq. (5.4c) nor the Coulomb term appears in the definition of the operators $J^{1,2}$. As a consequence of this, Eq. (3.8) is not satisfied and a transformation, free of any explicit dependence on $\vec{x}^\perp$, cannot exist from order $O(m^{-2})$ onwards. In fact it is easy to see that even if the requirement of the commutativity with the translation generators $P^+,\vec{P}^\perp$ is given up, the operator Y_2 does not exist in general, since the equation determining[31] O_2,

$$\left[\frac{iO_2}{2},x^j\eta^{-1}\right] = x^j\left\{-gB^- - \frac{ig}{4}\left[(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp)(\vec{\gamma}_\perp\cdot\vec{B}_\perp(x)) + (\vec{\gamma}_\perp\cdot\vec{B}_\perp(x))(\vec{\gamma}_\perp\cdot\vec{\partial}_\perp) + igB_\perp{}^2(x),\eta^{-1}\right]_+\right\} - \frac{ig}{2}B^j(x)\eta^{-1} - \frac{i}{\sqrt{2}}\,\epsilon(\eta)\gamma^j$$

does not possess any solution for arbitrary nonzero $\vec{B}^\perp(x)$.

VI. DISCUSSIONS

We have attempted the construction of a U(6) × U(6) × O(3) algebra which can be used to classify the physical hadronic states. This work is a generalization of the work of Carlitz and Tung in two directions. First, we have generalized their results obtained for Yukawa-type interactions among quarks to other types of interactions.[24] Second, we have demonstrated the existence of the bilinear part of the transformation V to all orders in m^{-1} in these theories. We have not been able to give a similar proof for the existence of the terms multilinear in the quark fields. A further study of such terms is desirable in view of their importance in determining the commutativity properties of U(6) × U(6) charges with the four momentum generators. Like Melosh,[2] and Carlitz and Tung,[4] we have concentrated on the diagonalization of the total angular momentum operator $\vec{J}$. However, this makes the consequent algebra suitable for classifying states at rest only. The same algebra can be used to classify the collinear states only if V commutes with the Lorentz boost along the z axis, which, however, is not the case even in the free quark theories. This inadequacy of V can have appreciable effects on the structure of the matrix elements of various currents between unequal-mass states.[4,9,10]

Quite remarkably, the solution obtained for the bilinear part of the operator Y removes all the interaction-dependent bilinear terms from the Hamiltonian P^- of Eq. (3.1c). This rather unexpected result[4] is the consequence of the commutativity of the operator Y_{bl} with the operators P^+, $\vec{P}^\perp$ as already discussed in Sec. II. This result in particular implies that any mass difference between U(6)× U(6) multiplets in this scheme can arise only through the terms multilinear in the quark fields. The presence of these symmetry-breaking terms as well as their structure can be inferred from the general study of the multilinear part of Y.

The pratical utility of the transformation relating the group $SU_W(6)_{\text{currents}}$ to the classification group $SU_W(6)_{\text{strong}}$ has been in predicting the algebraic structure of the physical hadrons under the group $SU(6)\times O(2)_{\text{currents}}$. Most of the work along this line has been carried out using the free-quark-model transformation of Melosh. The transformations considered in this paper can be used to find the effect of interactions on the algebraic structure predicted by the free-quark–model transformation. The latter transformation beautifully describes the algebraic structure of the matrix elements of the local axial and electromagnetic currents.[1] However, it is inadequate in accounting for the algebraic properties of the bilocal currents as revealed by the studies of the deep-inelastic lepton-hadron scattering.[32] The approximation schemes[12] of Carlitz and Weyers and of Fuchs can be used together with the algebraic properties of the transformation in the vector- or scalar-gluon theory to study the effects of interactions on the structure of local and bilocal currents. Fuchs[12] has in fact made such studies by exploiting the relationship of the Melosh transformation with the Moller wave matrices connecting the Heisenberg in and out states with the Dirac representation states.[11] His results are encouraging enough for undertaking more detailed phenomenological studies within this scheme.

ACKNOWLEDGMENTS

I would like to thank Dr. Probir Roy for his constant help and encouragement during this work and for suggesting some changes in the manuscript. I have also benefited from discussions with Dr. P. P. Divakaran and Dr. A. K. Kapoor.

APPENDIX

In this Appendix we construct explicitly the quartic part of the operator Y_2 of Eq. (3.3) for the various types of interactions between quarks. Specifically, we consider quarks interacting with scalar, pseudoscalar, or vector gluons. They can be massive or massless and can have internal degrees such as isospin or color. The last, not being relevant, shall be suppressed in the following.

The operator $Y_{2\,\text{pair}}$ is the solution of[4]

$$\int d^4x\,\delta(x^+)[iY_{2\,\text{pair}}, q_+^\dagger(x)x^j\eta^{-1}q_+(x)] = \int d^4x\,\delta(x^+)[-iY_{1\,\text{int}}\,q_+^\dagger(x)x^j p_1^- q_+(x)]_{\text{pair}} + \epsilon^{ji}J^i_{\text{Coulomb}}\,, \tag{A1}$$

where the suffix "pair" refers to the terms quartic in the quark fields. The term J^i_{Coulomb} appears only for the vector-exchange part and has the form given by Eq. (5.4d):

$$Y_{1\,\text{int}} = -2i\sqrt{2}\int d^4x\,\delta(x^+)q_+^\dagger(x)\eta(\eta^{-1}p_1^-)\eta q_+(x)\,. \tag{A2}$$

In general, p_1^- is linear in the gluon field and has the following structure[31]:

$$p_1^- = \tfrac{1}{2}g[\eta^{-1}, V_1]_\pm \tag{A.3}$$

where $V_1 \equiv B(x)$, $\gamma_5 B(x)$, and $\vec{\gamma}_\perp\cdot\vec{B}_\perp(x)$ when the gluon field is scalar, pseudoscalar, and vector, respectively. The upper (lower) sign applies for the scalar (vector and pseudoscalar) case. g is the coupling constant. By making use of Eqs. (A1), (A2), (A3), and the equal-time commutation between gluon fields, one arrives at the solution[4]

$$Y_{2\,\text{pair}} = \pm\frac{ig^2}{2}\int d^4x\,\delta(x^+)\left\{\sum_{a=1}^{\infty}\left[(q_+^\dagger(x)\Gamma^l(\overleftarrow{\eta}\mp\overrightarrow{\eta})(\overleftarrow{\eta}\overrightarrow{\eta})^{1-a}q_+(x))\overrightarrow{\eta}^{-3}(q_+^\dagger(\overleftarrow{\eta}^{-1}\mp\overrightarrow{\eta}^{-1})\Gamma^l(\overleftarrow{\eta}\overrightarrow{\eta})^a q_+)\right.\right.$$
$$\left.\left. -2ig^2(q_+^\dagger(x)(\overrightarrow{\eta}\,\overleftarrow{\eta})^{1-a}q_+)\overrightarrow{\eta}^{-3}(q_+^\dagger(\overleftarrow{\eta}\,\overrightarrow{\eta})^a q_+)\right]\right\}\,. \tag{A4}$$

Here η and η^{-1} act inside the parentheses in the direction indicated by the arrows. Γ^l collectively represents 1, γ_5, or γ^l in the case of scalar, pseudoscalar, and vector gluons, respectively. The last term is present only in the case of the vector-gluon model.

[1]For reviews see J. Weyers, in *Particle Interactions at Very High Energies, Part B*, edited by D. Speiser, F. Halzen, and J. Weyers (Plenum, New York, 1974); F. J. Gilman, in *Strong Interactions, SLAC Summer Institute on Particle Physics, 1974*, edited by M. C. Zipf (National Technical Information Service, U. S. Dept. of Commerce, Springfield, Va., 1974), Vol. 1, p. 307; A. J. G. Hey, Southampton University Lecture Notes and references therein (unpublished).

[2]H. J. Melosh, Phys. Rev. D 9, 1095 (1974). The construction of a conserved SU(6) algebra within the free quark model in the spacelike formulation was achieved long ago by Riazuddin and L. K. Pandit, Phys. Rev. Lett. 14, 462 (1965).

[3]J. L. Rosner, Phys. Rep. 11C, 190 (1974).

[4]R. Carlitz and Wu-ki Tung, Phys. Rev. D 13, 3446 (1976).

[5]H. Leutwyler, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, New York, 1969), Vol. 50, p. 29; J. Jersak and J. Stern, Nucl. Phys. B7, 413 (1968); K. Bardakci and M. B. Halpern, Phys. Rev. 176, 1686 (1968).

[6]S. P. De Alwis and J. Stern, Nucl. Phys. B77, 509 (1974); R. Jackiw, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, Berlin, 1972), Vol. 62, p. 1.

[7]R. Carlitz, D. Heckathorn, J. Kaur, and W.-k. Tung, Phys. Rev. D 11, 1234 (1975).

[8]H. J. Lipkin and S. Meshkov, Phys. Rev. Lett. 14, 670 (1965).

[9]H. Osborn, Nucl. Phys. B80, 90 (1974).

[10]J. S. Bell and H. Ruegg, Nucl. Phys. B93, 12 (1975); G. Eyre and H. Osborn, Nucl. Phys. B116, 281 (1976).

[11]H. Fuchs, Phys. Rev. D 11, 1569 (1975); 15, 3717 (1977).

[12]Purely within quark models, an expansion in m^{-1} is equivalent to the expansion in the parameter $(\vec{P}_q^{\perp}/m)$, $\vec{P}_q^{\perp}$ denoting the transverse momentum of the quarks inside hadrons. Physically, $(\vec{P}_q^{\perp}/m)$ corresponds to the ratio of the quark size to the hadronic dimension and can be shown to be small on phenomenological grounds. See R. Carlitz and J. Weyers, Phys. Lett. 56B, 154 (1975). However, in the presence of the gluon fields, an additional parameter characterizing the ratio of the hadronic size to the gluon size enters the expansion. This also can be shown to be small. See N. Fuchs, Phys. Rev. D 13, 1309 (1976). Moreover an expansion in m^{-1} may not be incompatible with an approximate chiral SU(3) ×SU(3) symmetry since the quarks may acquire large masses through a dynamically induced pseudoparticle–generated spontaneous breakdown of the chiral symmetry. See, for example, D. G. Caldi, Phys. Rev. Lett. 39, 121 (1977).

[13]L. Foldy and S. Wouthuysen, Phys. Rev. 78, 29 (1950); J. D. Bjorken and S. Drell, in *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964).

[14]E. Eichten, F. Feinberg, and J. Willemsen, Phys. Rev. D 8, 1204 (1973).

[15]Note that $\vec{J}_B$ is expressed in terms of the canonically independent components of the gluon field and thus commutes with the operator $\vec{L}_q$.

[16]The commutativity of O(3) generators between forward states found in the free quarks theory does not follow in general unless V itself commutes with the operators P^+, $\vec{P}^{\perp}$. Note that the last term of Eq. (2.9) is zero as long as Eq. (2.8) holds.

[17]S. Coleman, Phys. Lett. 19, 144 (1965).

[18]V. De Alfaro, S. Fubini, G. Furlan, and C. Rossetti, *Currents in Hadron Physics* (North-Holland, Amsterdam, 1973).

[19]This will happen if, for instance, Y contains at some stage a γ-independent term which, in addition to depending upon the quark and gluon fields, also depends upon x^- and/or $\vec{x}^{\perp}$.

[20]J. B. Kogut and D. Soper, Phys. Rev. D 1, 2901 (1970); D. Soper, Ph.D. Thesis, SLAC Report No. 137, 1971 (unpublished).

[21]S.-J. Chang, R. G. Root, and T.-M. Yan, Phys. Rev. D 7, 1133 (1973); T.-M. Yan, *ibid.* 7, 1760 (1973).

[22]E. Tomboulis, Phys. Rev. D 8, 2736 (1973).

[23]R. Casher, Phys. Rev. D 14, 452 (1976).

[24]The theory of quarks coupled to a massive vector boson can be quantized after making a field-dependent phase transformation on the quark fields. (See Ref. 21.) In the particular case of the axial-vector gluon this results in a complicated form for p_1^- and Σ_1^j and the considerations of the Appendix need modification. However, since this theory is not of much physical interest, we shall exclude it from our considerations. No such problem arises for the rest of the theories considered in Refs. 20 and 21.

[25]Due to the γ independence of the free quark Hamiltonian, all A_n's are zero in the free quark theory. Thus the A_n's always come only from the gluon fields. We shall write for brevity $A_n(\eta, \vec{\partial}_{\perp}, \eta^{-1}, B(x)) \equiv A_n(x)$.

[26]This equation uniquely fixes the interaction-dependent part of B_n^j in terms of A_n through the relation $B^j_{n\ \mathrm{int}} = [\eta(\eta^{-1}A_n)\eta, x^j]$.

[27]This solution is unique up to a part commuting with η^{-1} and satisfying other requirements discussed in Ref. 14. However, such a nonuniqueness is present is the free quark theory also. (See Ref. 2.)

[28]For $N=1$ there will be an additional contribution coming from the commutator of $\hat{l}^j + s^j$ with $\hat{l}^k + s^k$. However, this will cancel $\sqrt{2}\,g^3$ in the right-hand side of Eq. (4.4), and thus Eq. (4.5) will still remain valid.

[29]This, of course, assumes that the operator Y turns out to be a good operator in the sense defined in Ref. 6.

[30]G. Bart and S. Fenster, Phys. Rev. D 16, 3554 (1977).

[31]The suffix + in $[\ ]_+$ means the anticommutator.

[32]S. P. De Alwis, Nucl. Phys. B55, 427 (1973); A. S. Joshipura and P. Roy, Ann. Phys. (N.Y.) 104, 460 (1977).