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Approach to scaling of lepton pair production in hadronic collisions

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The effect of hadron and parton masses and parton transverse momentum on the cross section $d\sigma/dmdy$ for lepton pair production is investigated using a simple parton model. The fragmentation of heavy quarks, and parton transverse momentum in e^+e^- annihilation, are briefly discussed.

I. INTRODUCTION

With the advent of accurate data on the cross section for high-energy hadronic production of high-mass muon pairs,[1] there is renewed interest[2,3] in testing the Drell-Yan formula.[4] To do this one needs an accurate determination of the quark distributions. Various scaling possibilities have been proposed, for example, Ref. 5, but it is now well known that there are scaling violations in deep-inelastic leptoproduction.[6] The usual explanation[7] is that these are due to asymptotic-freedom effects, but the production of charmed particles may be very important.[8]

The parton model[9] for deep-inelastic leptoproduction predicts scaling asymptotically, but there is, of course, a source of effective scaling violation in the approach to scaling. The way hadron and parton masses control the approach to scaling in the parton model has been extensively studied (see Refs. 10–12, and references within these). In Ref. 12 a method was presented for finding the next-to-leading order terms in a simple model for deep-inelastic leptoproduction. This method could be used easily for the hadronic production of lepton pairs, in the parton model. The approach to scaling of the cross section $d\sigma/dm$ was studied, where m is the invariant mass of the lepton pair.[13] While in leptoproduction what matters is the mass of the parton after it has absorbed the virtual photon, what matters in lepton pair production is how far the partons are off shell before they annihilate. This is related to parton transverse momentum.[12] Here we wish to investigate the approach to scaling of the cross section $d\sigma/dmdy$. The results unfortunately depend on parameters which cannot be fixed elsewhere by experimental data, but we make what we hope are representative choices[12] to give some idea of the order of magnitude involved.

We find that scaling is approached from below, and the next-to-leading order term has a large coefficient. This is similar to results[12,14] for $d\sigma/dm$. Over the Fermilab kinematic range,[1] the effective scaling violations are ~10–30 % in the model. This is the same order of magnitude, and in the opposite direction to a guess we make for asymptotic-freedom scaling violations.

Using related models[12] we briefly discuss, in the scaling limit, the fragmentation of heavy quarks, and parton-transverse-momentum effects in e^+e^- annihilation.

II. THE DRELL-YAN MECHANISM

The Drell-Yan quark-antiquark annihilation mechanism for $AB \to \mu^+\mu^- X$, where A and B are hadrons, is shown in Fig. 1. The asymptotic cross section is[4]

$$\frac{2E}{\sqrt{s}}\frac{d\sigma}{dmdx_F}=\frac{d\sigma}{dmdy}=\frac{8\pi\alpha^2}{9m^3}\sum_a e_a{}^2 x_1 q_a^A(x_1)x_2 q_{\bar{a}}^B(x_2)\,, \qquad (1)$$

where m and y are the invariant mass and rapidity of the pair, the sum is over quarks and antiquarks with a factor $(\frac{1}{3})$ for color, $q_a^A(x)$ is the fractional longitudinal momentum distribution of quark a in hadron A (and similarly for B), e_a is quark a's charge, $x_{1,2}=(m/\sqrt{s})\exp(\pm y)$, and $s=(p_1+p_2)^2$.

Here we wish to estimate the effects of hadron and parton masses, and parton transverse momentum. We calculate the next-to-leading order corrections to Eq. (1) in a simple model retaining masses and transverse momentum. This calculation is simply an extension of that for $d\sigma/dm$ described in Ref. 12.

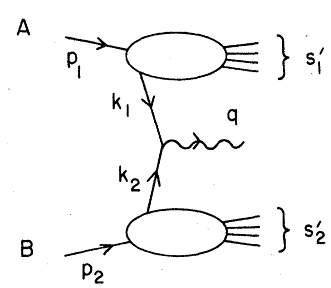

FIG. 1. Drell-Yan mechanism for lepton pair production.

The cross section of Eq. (1) is, generally,

$$\frac{d\sigma}{dm\,dy}=\frac{8\pi\alpha^2}{9m^3}(x_1+x_2)W(\tau,x_F,\nu)\,, \tag{2}$$

where $\nu=p_1\cdot p_2$, and $\tau=m^2/2\nu$. This defines the structure function W, which we will write as

$$W(\tau,x_F,\nu)=W_0(\tau,x_F)+\frac{1}{2\nu}W_1(\tau,x_F)+O\left(\frac{1}{\nu^2}\right). \tag{3}$$

At fixed τ and x_F we will find that scaling is approached from below, which means that $W_1<0$. We consider two points regarding this. First, at fixed s, τ, and x_F, increasing the masses s_1' and s_2' (see Fig. 1) reduces the available energy, and so also, presumably, the cross section. As one increases s at fixed τ and x_F the effects of s_1' and s_2' will go away, suggesting that the cross section rises to its scaling form. Second, it was shown in Ref. 12 that covariant kinematics with k_1^2 and k_2^2 variable and spacelike implies that nonzero parton transverse momentum can only hinder the formation of large m^2. This again suggests that introducing parton transverse momentum will lead to scaling being approached from below.

The question now is to the order of magnitude, and so we calculate Fig. 1. A parton (momentum k_1) and an antiparton (momentum k_2) annihilate forming a virtual photon (momentum q, $q^2=m^2$) such that $\tau=m^2/2\nu$ is finite. The diagram is only gauge invariant in leading order, and the gauge invariance in lower order is a detailed dynamical question. We avoid this problem by taking the partons and the current to have zero spin, and so the calculation can at best give only an indication of the magnitude and the sign of the subasymptotic corrections.

From Eq. (2) we have (for 2π's, see Ref. 15)

$$W(\tau,x_F,\nu)=\frac{\nu}{2\pi^2}\int d^4k_1d^4k_2f(k_1^2,s_1')\bar f(k_2^2,s_2')\times\delta((k_1+k_2)^2-q^2)\delta\left(\frac{k_{1z}+k_{2z}}{\sqrt{s}/2}-x_F\right), \tag{4}$$

$$s_1'=(p_1-k_1)^2,\quad s_2'=(p_2-k_2)^2.$$

Here f ($\bar f$) is the imaginary part of a forward (antiparton) parton-hadron amplitude.

Following Ref. 12, we write

$$\begin{aligned}k_1&=\left(\xi_1+\frac{b-1}{2M_1^2b}\eta_1\right)p_1+\frac{\eta_1}{2\nu b}p_2+k_{1T},\\ k_2&=\frac{\eta_2}{2\nu b}p_1+\left(\xi_2+\frac{b-1}{2M_2^2b}\eta_2\right)p_2+k_{2T},\\ b&=\left(1-\frac{M_1^2M_2^2}{\nu^2}\right)^{1/2}=1+O\left(\frac{1}{\nu^2}\right).\end{aligned} \tag{5}$$

The two-dimensional vectors k_{1T} and k_{2T} are orthogonal to both p_1 and p_2. In terms of these variables, and the angle θ between k_{1T} and k_{2T},

$$\begin{aligned}W(\tau,x_F,\nu)=\frac{1}{32\pi}\int&\left\{\prod_{i=1}^{2}dk_i^2ds_i'd\xi_id\eta_idk_{iT}^2\,\delta((\xi_i-1)\eta_i+(\xi_i-1)^2M_i^2-k_{iT}^2-s_i')\delta(s_i'+\eta_i+(2\xi_i-1)M_i^2-k_i^2)\right\}\\ &\times\int_0^{2\pi}d\theta f(k_1^2,s_1')\bar f(k_2^2,s_2')\delta\left(\xi_1\xi_2-\tau+\frac{1}{2\nu}(k_1^2+k_2^2-2k_{1T}k_{2T}\cos\theta)+O\left(\frac{1}{\nu^2}\right)\right)\\ &\times\delta\left(\xi_1-\xi_2-x_F+\frac{1}{2\nu}(k_2^2-k_1^2+(\xi_1-\xi_2-1)(M_1^2-M_2^2)+s_1'-s_2')+O\left(\frac{1}{\nu^2}\right)\right).\end{aligned} \tag{6}$$

This expression displays the purpose of the change of variables in Eq. (5)—the only energy dependence appears in the last two δ functions. We now specialize to equal masses, $M_1=M_2=M_{\rm nucleon}$, and expanding the δ functions in Taylor series, we find

$$\begin{aligned}W_0&=F_2(\xi_1)\bar F_2(\xi_2)/(\xi_1+\xi_2),\\ F_2(\xi)&=\tfrac14\int dk^2ds'f(k^2,s')\theta(k_T^2),\\ k_T^2&=-(1-\xi)k^2-\xi s'+\xi(1-\xi)M^2.\end{aligned} \tag{7}$$

By F_2 we mean the quark distribution, not a complete structure function. The correction coefficient is

$$W_1(\tau,x_F)=-\frac{\partial}{\partial x_F}W_A(\tau,x_F)-\frac{\partial}{\partial\tau}W_B(\tau,x_F)\,, \tag{8}$$

where, with k_{iT}^2 given by Eq. (7),

$$W_{A,B}(\tau, x_F) = \frac{1}{16}\int \left(\prod_{i=1}^{2} dk_i^2 ds_i' \theta(k_{iT}^2)\right) f(k_1^2, s_1') \bar{f}(k_2^2, s_2') \times \delta(\xi_1\xi_2 - \tau)\delta(\xi_1 - \xi_2 - x_F)[(k_2^2 - k_1^2 + s_1' - s_2'), (k_1^2 + k_2^2)]. \tag{9}$$

This also defines ξ_1 and ξ_2 in terms of x_F and τ.

We will evaluate Eq. (8) numerically, but first we define

$$\frac{1}{4}\int dk^2 ds' f(k^2, s')\theta(k_T^2)s' = s_0 F_2(\xi),$$

$$\frac{1}{4}\int dk^2 ds' f(k^2, s')\theta(k_T^2)k^2 = -\left(\frac{\langle k_T^2\rangle + \xi s_0}{1-\xi} - \xi M^2\right)F_2(\xi). \tag{10}$$

There are similar definitions for the antiquark distributions.

In principle, and most likely in practice, $\langle k_T^2\rangle$ and s_0 are functions of ξ, but for our present purpose we choose them to be constants. In what follows we choose[12] $\langle k_T^2\rangle = 0.5$ GeV^2, and $s_0 = 1$ or 4 GeV^2 for both quarks and antiquarks. It is hoped that this will give some idea of the real answer. We need also a choice of quark distributions, and we take those of Ref. 3, which have been specifically adjusted to fit the high-mass Fermilab data[1]

$$xu(x) = 2.99x^{1/2}(1-x)^4(1+5.99x-2.63x^{1/2}),$$
$$xd(x) = 1.02x^{1/2}(1-x)^5(1+5.75x), \tag{11}$$
$$xf_{\text{sea}}(x) = 0.145(1-x)^{11}(1+10x).$$

The results of all this are shown for $-W_1(\tau, x_F = 0)/W_0(\tau, x_F = 0)$ in Fig. 2. Note that the correction is negative, and can be quite large. However, at 400 GeV over the currently measured high-mass range, relevant to Drell-Yan, $5 \lesssim m \lesssim 14$ GeV this correction is only ~10–30%. At 200 GeV for these masses, this, of course, means ~20–60%.

Now we consider the changes of shape of the distributions in x_F. These are shown in Fig. 3 for a variety of values of p_L and m. In each figure, the solid line is the "asymptotic" cross section, with no correction. The dashed line has $s_0 = 1$ GeV^2, and the dotted line has $s_0 = 4$ GeV^2.

We turn now to the effects of asymptotic freedom on lepton pair production. There has been considerable interest in this problem,[16,17] but there is as yet no definite conclusion. It has been argued[17] that the parton distributions in Eq. (1) should be replaced by those obtained from leptoproduction data at photon mass squared $q^2 = -m^2$, and in what follows we will use this ansatz. For this m^2 dependence we take the empirical parametrization of D.H. Perkins, P. Schreiner, and W.G. Scott,[18]

$$xq(x, m^2) = xq(x)(m^2)^{0.25-x}. \tag{12}$$

We use this prescription for both valence and sea quarks, whereas it has been deduced from data which depends largely on the valence distributions.

What we will plot, in Fig. 4, are the ratios

$$\frac{\left(m^3 \dfrac{2E}{\sqrt{s}} \dfrac{d\sigma}{dm\,dx_F}\right)_{p_L,\; m/\sqrt{s}=x}}{\left(m^3 \dfrac{2E}{\sqrt{s}} \dfrac{d\sigma}{dm\,dx_F}\right)_{p_L=400\text{ GeV},\; m/\sqrt{s}=x}} \tag{13}$$

at $x_F = 0$, of the cross section at $p_L = 200$ GeV, 300 GeV to the cross section at 400 GeV, at fixed $x = m/\sqrt{s}$. These ratios are unity for exact scaling. Figure 4(a) shows the result from the approach to scaling, Fig. 4(b) shows the result from asymptotic freedom, Eq. (12), and Fig. 4(c) shows the result from putting both together. We show the results for $s_0 = 4$ only. This gives the larger effect.

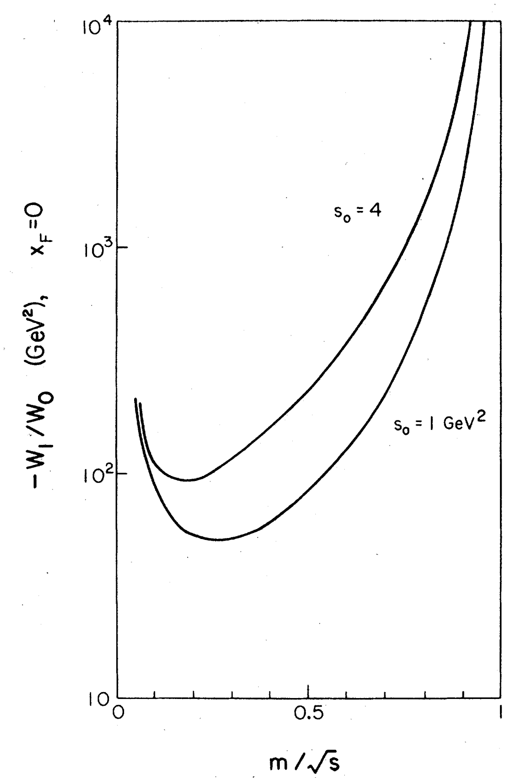

FIG. 2. $-W_1/W_0$ for $x_F = 0$, from Eqs. (8) and (9).

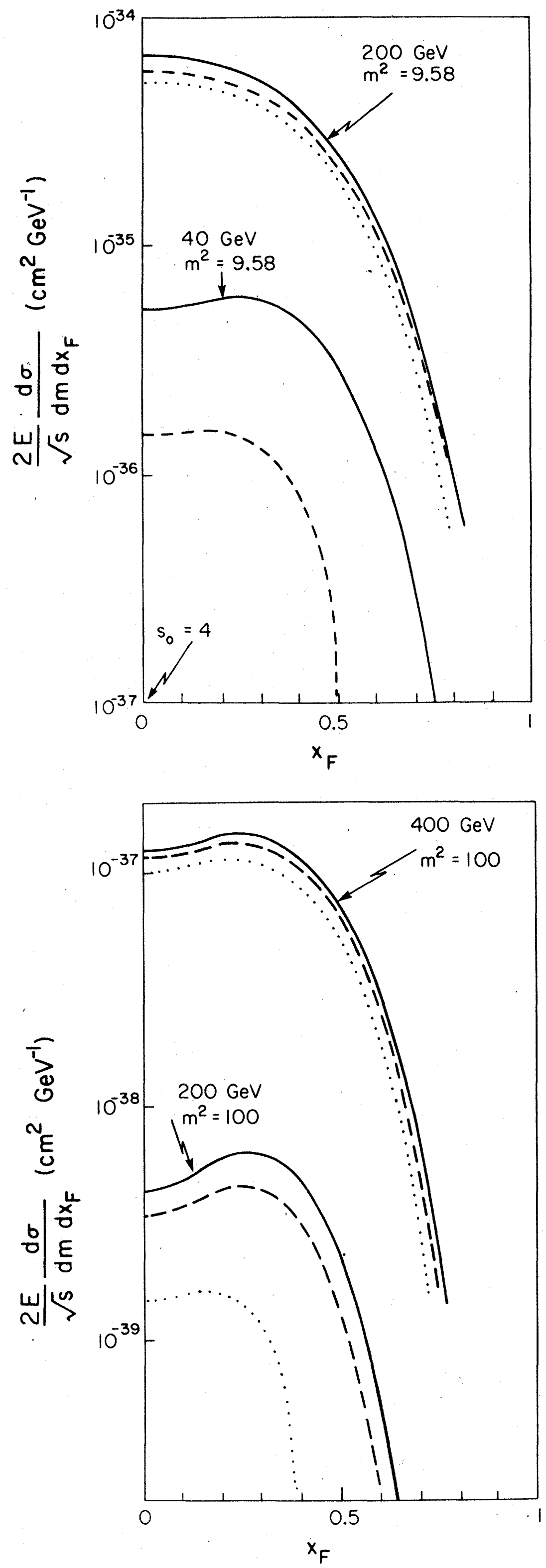

FIG. 3. $(2E/\sqrt{s})d\sigma/dm dx_F \simeq d\sigma/dm dy$ for four choices of p_L, m^2. The solid lines are the asymptotic curve, the dashed line has $s_0 = 1$, and the dotted line has $s_0 = 4$.

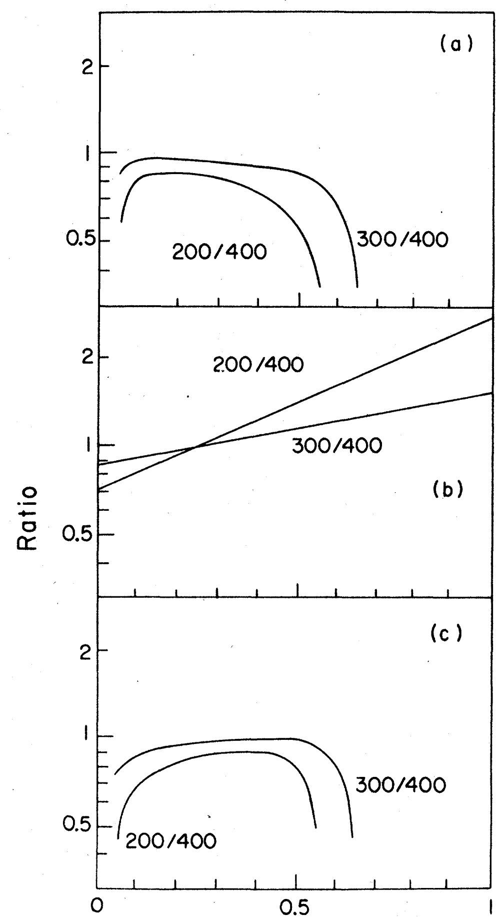

FIG. 4. The ratio defined in (13) which is unity for exact Drell-Yan scaling. (a) The approach to scaling, with $s_0 = 4$. (b) The asymptotic freedom guess with $xq(x, m^2) = xq(x)(m^2)^{0.25-x}$. (c) The result of combining (a) and (b).

The conclusion is clear—it may well be hard to disentangle asymptotic-freedom effects from the approach to scaling in the Fermilab-SPS kinematic range.

III. e^+e^- ANNIHILATION

A simple model, related to the one described above, was presented in Ref. 12, to investigate the effect of masses on the shape of deep-inelastic structure functions, and the x dependence of parton transverse momentum. This was a simplification of earlier models. The model for e^+e^- annihilation is indicated in Fig. 5. The parton propagator carrying momentum k is softened, and one finds, for a fragmentation function appropriate for a meson which is chosen to behave as $z \to 1$ as $D(z) \sim (1-z)$, that[12]

$$D(z) \propto \frac{(1-z)z^2}{z s_0 + (1-z)M^2 - z(1-z)m_q^2}, \tag{14}$$

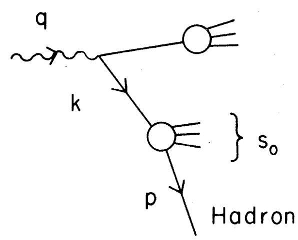

FIG. 5. A model for $e^+e^- \to$ hadron $+X$.

where M is the mass of the produced hadron, and the constant s_0 is defined in Fig. 5. For $z \to 0$, $D(z) \sim z^2$ which is not realistic for light quarks. However, for heavy quarks Eq. (14) may provide a relevant contribution. So we investigate this as there has been recent interest[19] in the fragmentation of heavy quarks.

Let $M = m_q +$ finite become large, and impose, as is usual for charm,

$$\int_0^1 D(z)dz = 1 . \tag{15}$$

It is now straightforward to show from Eq. (14) that $D(z)$ peaks at

$$z \simeq 1 - \left(\frac{s_0}{2M^2 - m_q^2} \right)^{1/2} . \tag{16}$$

The Bjorken x (and Q^2) dependence of the transverse momentum of partons in a hadron has been the subject of some attention.[20-22] The matter still has not been resolved. We wish to comment on the results for e^+e^- annihilation.

Define $\langle p_T^2(z) \rangle$ to be the average squared momentum transverse to the production jet (quark) axis of a hadron carrying fraction $z = x^{-1}$ of the jet's momentum. A common assumption has been that $\langle p_T^2(z) \rangle$ is independent of z. The model considered here[12] has for the z dependence,[23]

$$\langle p_T^2(z) \rangle \propto z s_0 + (1-z)M^2 - (1-z)m_q^2 . \tag{17}$$

For suitable choices of s_0 and m_q^2, $\langle p_T^2(z) \rangle$ is an increasing function of z. It will be interesting to find out which of these possibilities, if either, is favored experimentally.

It should be possible to directly measure $\langle p_T^2(z) \rangle$ in leptoproduction and e^+e^- annihilation. There is some evidence[6] already for a "seagull" effect, as has been predicted for the transverse momentum of quarks in a nucleon.[21] Another place where $\langle p_T^2(z) \rangle$ may be relevant is in the angular dependence in e^+e^- annihilation. For the inclusive production of a hadron,

$$\frac{d\sigma}{d\cos\theta} \propto 1 + \frac{1-R}{1+R} \cos^2\theta , \tag{18}$$

where $R = \sigma_L/\sigma_T$, which in the parton model[9] is

$$R = 4 \frac{[\langle p_T^2(z) \rangle / z^2] + m_q^2}{q^2} . \tag{19}$$

In leptoproduction σ_L/σ_T is experimentally large and theoretically ill understood.[6] $\langle p_T^2(z) \rangle \sim q^2$ in annihilation may lead to large values of R. Experimental information on σ_L/σ_T in this new context of e^+e^- annihilation may help to throw light on the dynamics involved.

IV. CONCLUSIONS

In what we emphasize are very simple models, we have investigated the approach to scaling in lepton pair production, the fragmentation of heavy quarks, and parton transverse momentum in e^+e^- annihilation. The conclusions are as follows:

(i) For $d\sigma/dmdy$, as well as for $d\sigma/dm$,[11,22] scaling is approached from below.

(ii) For $d\sigma/dmdy$ the approach to scaling may be of the same order of magnitude, over the Fermilab kinematic range, and in the opposite direction to asymptotic-freedom effects. At higher energies the effects we have described here disappear, of course.

(iii) The fragmentation function for a heavy quark fragmenting into a heavy hadron peaks at $z = 1 - O(s_0^{1/2}/m_q)$.

(iv) Hadron transverse momentum in e^+e^- annihilation is still theoretically uncertain.

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