Observation of a new regularity in hadronic spectra

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We present evidence using 100-GeV/c $\bar{p}p$ interactions and pp interactions at 102, 205, 303, and 405 GeV/c for the hypothesis that the ratio of an inclusive cross section $ab \to \bar{c} + X$ to $ab \to \bar{c} + X_s$, where X_s is a subset of X, is a function of M^2, the effective mass squared of X (and X_s) only and not of s and t. A summary of the rationale leading up to the hypothesis is also given. The hypothesis is seen to be obeyed in all areas of phase space in the reactions considered.

I. INTRODUCTION

In Ref. 1, it was shown that the subtraction formulas $C_{1,2}=B_{1,2}-A_{1,2}$ (where $B_{1,2}$ denote the inclusive cross sections $\bar{p}p \to \pi^\pm$, $A_{1,2}$ the inclusive cross sections $pp \to \pi^\pm$, and $C_{1,2}$ the inclusive cross sections $\bar{p}p \to \pi^\pm$ in annihilations) violate charge symmetry. Symmetry considerations suggested the alternate formulas

$$C_1 + C_2 = (B_1 + B_2) - (A_1 + A_2)$$

for the part even under inversion and

$$\frac{C_1 - C_2}{C_1 + C_2} = \frac{B_1 - B_2}{B_1 + B_2}$$

for the odd part. The even part is clearly an approximation, since it involves differences between $\bar{p}p$ and pp. The formula for the odd part, however, involves $\bar{p}p$ cross sections only, and the accuracy to which it is obeyed at 12 GeV/c (Ref. 1) leads one to believe that it may be an exact relation. The existence of the odd-part formula is guaranteed if one hypothesizes the three-body scattering $abc \to X$ [Fig. 1(a)] to take place in two steps, first the formation of a fireball of mass M^2 and then its subsequent decay [Fig. 1(b)]. This implies that the cross section for $abc \to X$ can be written as a product of two terms

$$\sigma(abc \to X) = F(M^2, s, t) D_X(Q) \ ,$$

where $F(M^2, s, t)$ describes the formation of the fireball and $D_X(Q)$ describes the decay of the fireball into X. D_X is a function of Q, the set of quantum numbers describing the fireball. Similarly,

$$\sigma(abc \to X_s) = F(M^2, s, t) D_{X_s}(Q) \ ,$$

where X_s is a subset[2] of X. The formation term is clearly the same, since it is the same fireball that is formed. Therefore,

$$\frac{\sigma(abc \to X_s)}{\sigma(abc \to X)} = \frac{D_{X_s}(Q)}{D_X(Q)} = \alpha_{X_s}(Q) \ .$$

The fireball is characterized by its additive quantum numbers which remain fixed for a given channel, and its dynamic quantum numbers, which are M^2 and ρ, its density matrix. $\alpha_{X_s}(Q)$ is the branching ratio of the fireball into subset X_s. It can be shown from general arguments[3] that the branching ratio of an unstable quantum-mechanical system cannot depend on its density matrix. Thus though ρ may depend on s and t, α_{X_s} can only depend on M^2. Therefore,

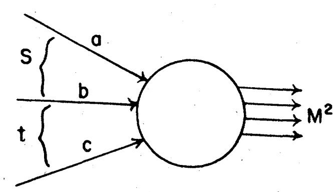
$$\frac{\sigma(abc \to X_s)}{\sigma(abc \to X)} = \alpha_{X_s}(M^2) \ .$$

Following Mueller,[4] one can continue t to the physical region for the scattering $ab \to \bar{c} + X$ to get

$$\frac{f(ab \to \bar{c} + X_s)}{f(ab \to \bar{c} + X)} = \alpha_{X_s}(M^2) \ ,$$

where f denotes the inclusive cross section.

Applying this result to $\bar{p}p$ annihilations, one gets



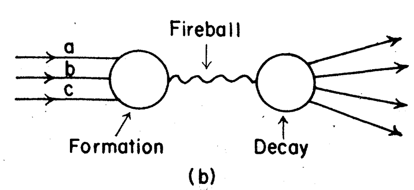


FIG. 1. (a) The diagram for three-body scattering. s and t denote the subenergies of the particle combinations shown. The subenergy of the third combination, u, is determined once the other variables are given. (b) Diagram illustrating formation and decay of the fireball.

$$\frac{C_1}{B_1}=\alpha_1(M^2) \text{ and } \frac{C_2}{B_2}=\alpha_2(M^2) .$$

However,

$$C_1(M^2,s,t)=C_2(M^2,s,u)$$

and

$$B_1(M^2,s,t)=B_2(M^2,s,u)$$

from C invariance in $\bar{p}p$. s,t,u are the usual Mandelstam variables. This implies that $\alpha_1(M^2) \equiv \alpha_2(M^2)$ and leads to

$$\frac{C_1-C_2}{C_1+C_2}=\frac{B_1-B_2}{B_1+B_2} ,$$

the desired result.

II. COMPARISON WITH EXPERIMENT

If the above hypothesis is correct, it should be applicable to any X_s, so long as X_s is a decay product of the fireball in question.[2] We now present evidence for the hypothesis in the channel $\bar{p}p \to \pi^- + X$ at 100 GeV/c.[5] We test the hypothesis for two separate subsets; the first subset (henceforth denoted subset I) comprises all events with a primary multiplicity of 2, 4, or 6 prongs and the second subset (subset II) consists of 12, 14, and 16 prongs. Figure 2(a) shows the M^2 distribution and Fig. 2(b) the t distribution for overall data, subset I and subset II. The M^2 and t distributions for the two subsets differ in shape and magnitude.

Figure 3 is a plot of the ratio of $d\sigma/dM^2$ for subsets I and II to $d\sigma/dM^2$ overall, as functions of M^2. We denote these functions $\alpha_{I}(M^2)$ and $\alpha_{II}(M^2)$. The full curves are fits to the data of a third-order polynomial in M^2. If the hypothesis as outlined in Sec. I is correct, the overall data should mimic subset I (II) in all parts of phase space when weighted[6] by $\alpha_{I}(M^2)$ [$\alpha_{II}(M^2)$]. For each final state π^- (overall data) with missing mass squared M^2, we attach a weight $\alpha_{I}(M^2)$ as given by the polynomial fit. The same procedure is repeated to mimic subset II using $\alpha_{II}(M^2)$.

Figure 4 is a plot of the t distribution for subset I in various M^2 ranges that together cover the whole of phase space. Superimposed is the overall data weighted by $\alpha_{I}(M^2)$. The agreement between the two distributions is seen to be excellent in each M^2 range.

Figure 5 is the corresponding plot for subset II. Once again, the overall data reproduce the subset data when appropriately weighted, even though the shape of the distribution varies considerably with M^2.

Figure 6 (7) is the plot of M^2 distributions for subset I (II) in various t ranges. The M^2 distributions vary greatly in shape as a function of t as well as between subsets. The overall data nevertheless are capable of reproducing these variations in detail when appropriately weighted. Striking, in particular, is the enhancement seen in Fig. 6 (for subset I) in the t range $0.0 < -t < 1.5$ (GeV/c)^2 and in the M^2 region 100–130 (GeV)^2. The overall data weighted by $\alpha_{I}(M^2)$ reproduce the enhancement adequately. The enhancement is suppressed for subset II in the same region

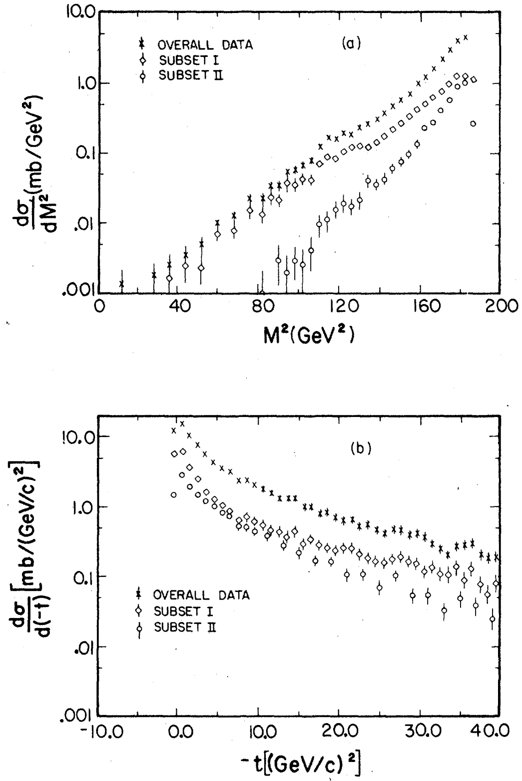


FIG. 2. (a) M^2 distributions for overall data, subset I and subset II. (b) t distributions for overall data, subset I and subset II.

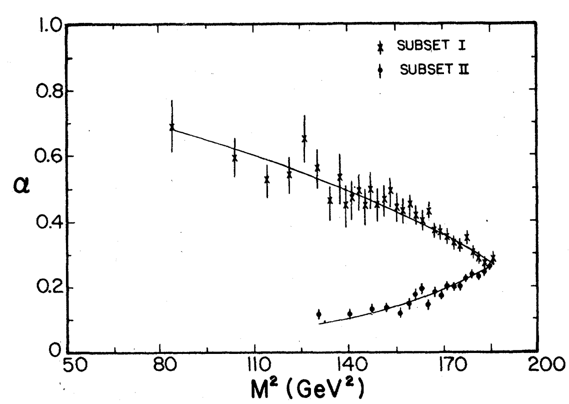


FIG. 3. Ratio of M^2 distributions of subsets I and II to overall data. Curves are fits to a cubic polynomial.

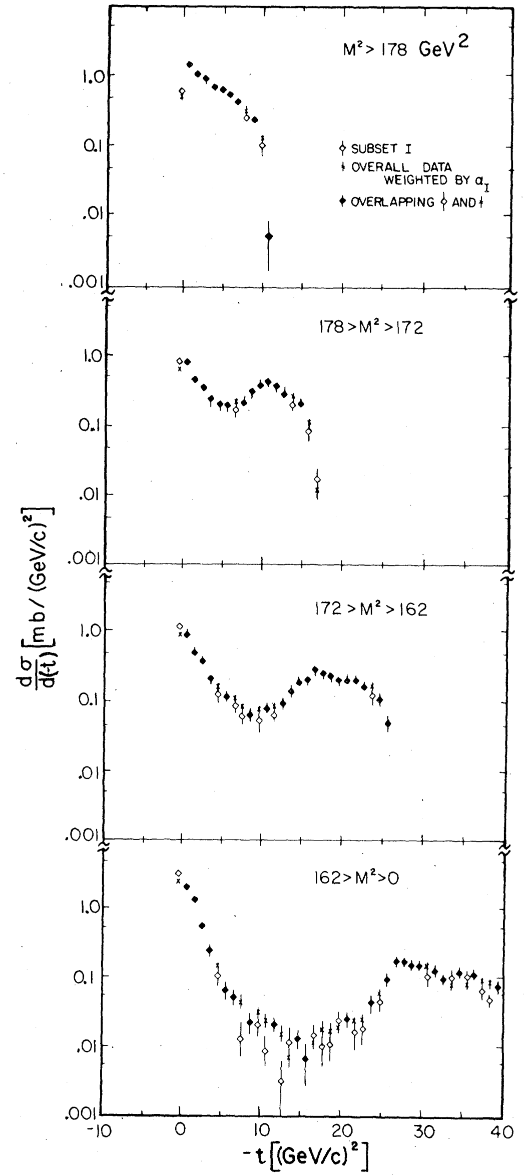


FIG. 4. Comparison of the t distribution for subset I with overall data weighted by $\alpha_I\,(M^2)$ for various M^2 ranges.

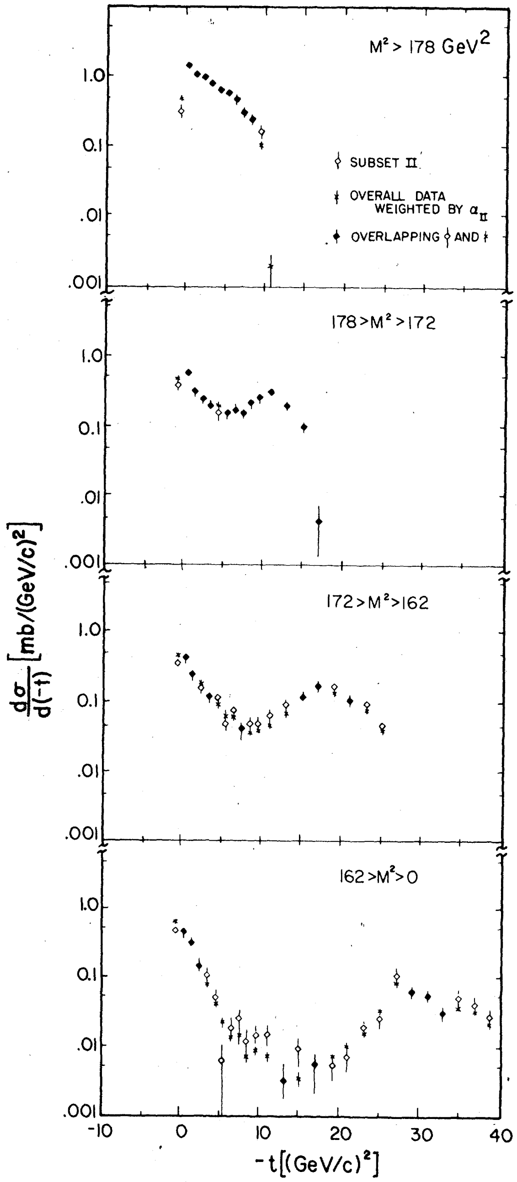


FIG. 5. Comparison of the t distribution for subset II with overall data weighted by $\alpha_{II}\,(M^2)$ for various M^2 ranges.

(Fig. 7) of phase space. Weighting the overall data by $\alpha_{II}(M^2)$ leads to a suppression by just the required amount.

The author has tested the hypothesis under consideration for the subset consisting of events with a primary multiplicity of 8 or 10. The agreement is equally good, but limitations of space prevent their presentation.

We have thus shown that the ratio of the inclusive cross sections $\bar{p}p \to \pi^- + X_s$ to $\bar{p}p \to \pi^- + X$ does not depend to any detectable amount on t.

s independence

Owing to the unavailability of data in suitable form we cannot test the s independence of the ratio for the channel $\bar{p}p \to \pi^- + X$. Instead we shall examine the channel $pp \to$ slow $p + X$ at three different values of beam momentum, 102, 205, and and[7] 405 GeV/c. Figure 8 is a plot of the ratio

$$R_n = \frac{d\sigma_n}{dM^2} \Big/ \frac{d\sigma_{\text{overall}}}{dM^2}$$

(where $n = 2$, 4, and 6 prongs) vs M^2. The data are seen to be consistent with the hypothesis under

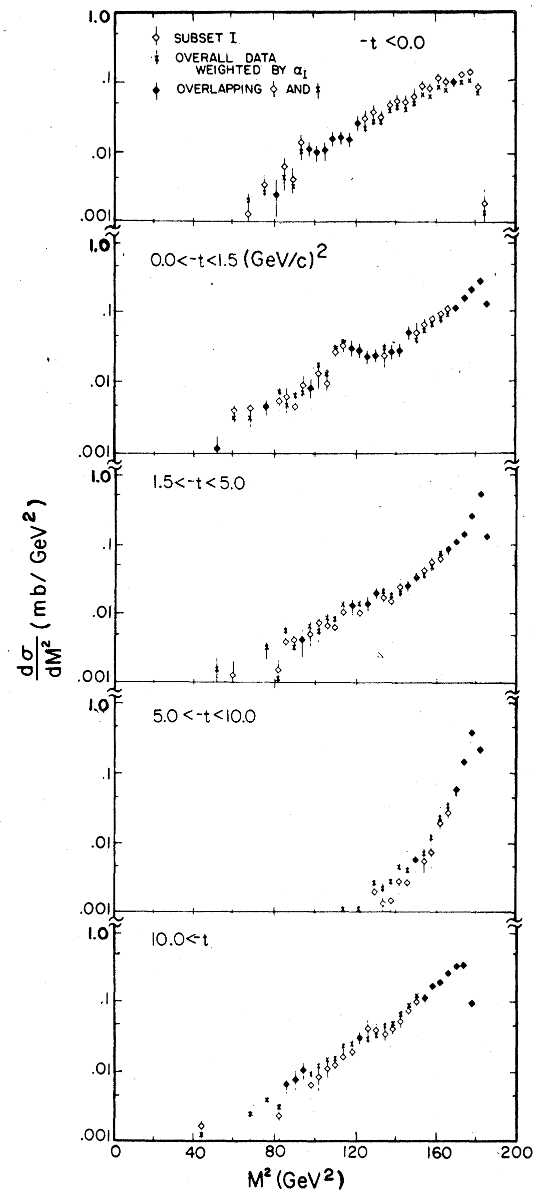


FIG. 6. Comparison of the M^2 distribution for subset I with overall data weighted by α_I (M^2) for various t ranges.

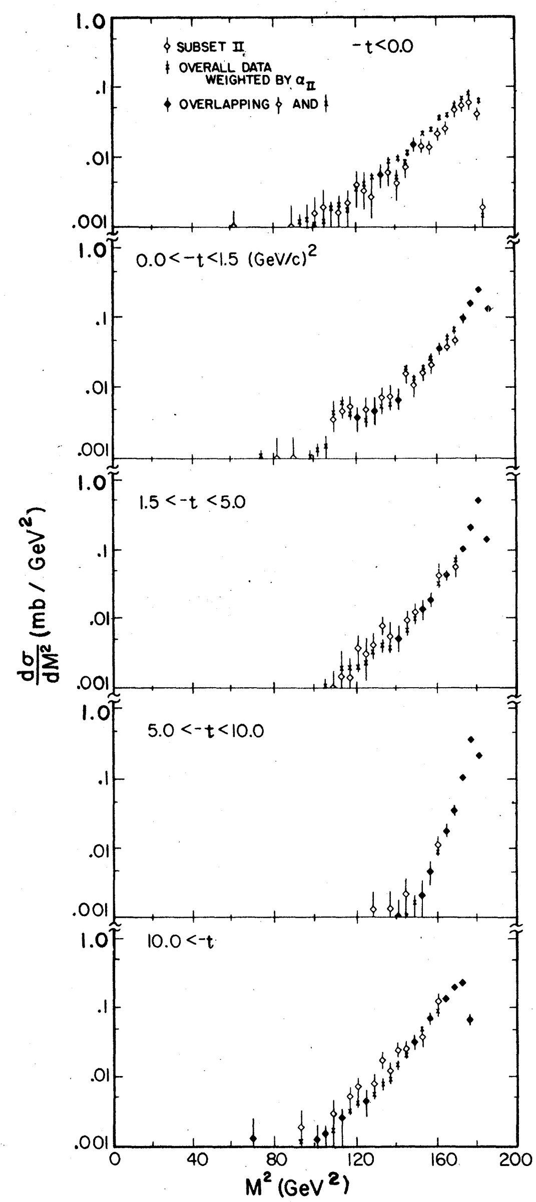


FIG. 7. Comparison of the M^2 distribution for subset II with overall data weighted by α_{II} (M^2) for various t ranges.

consideration here. There is no discernible dependence of R_n on s. One can also show that the mean recoiling multiplicity $\langle n_X\rangle$ for a given channel $ab\to\bar{c}+X$ is independent of s and t, for

$$\langle n_X\rangle=\sum_i \frac{n_i f_i(M^2,s,t)}{f(M^2,s,t)}=\sum_i n_i\alpha_i(M^2)\ ,$$

where f_i = semi-inclusive cross section for X to have multiplicity n_i, f = overall inclusive cross section.

Figure 9 is a plot of $\langle n_X\rangle$ for the reaction $pp\to\text{slow}\,p+X$ at 4 different beam momenta[7] (102, 205, 303, and 405 GeV/c). The data again are consistent with the hypothesis under consideration. As M^2 changes by a factor of 4 (say from 100 to 400 GeV^2), $\langle n_X\rangle$ changes by ~2.5 units. Yet as s changes by a factor of 4, there is no discernible change in $\langle n_X\rangle$. The data thus once again support the hypothesis in question. As shown above, $\langle n_X\rangle$ should be independent of t as well.

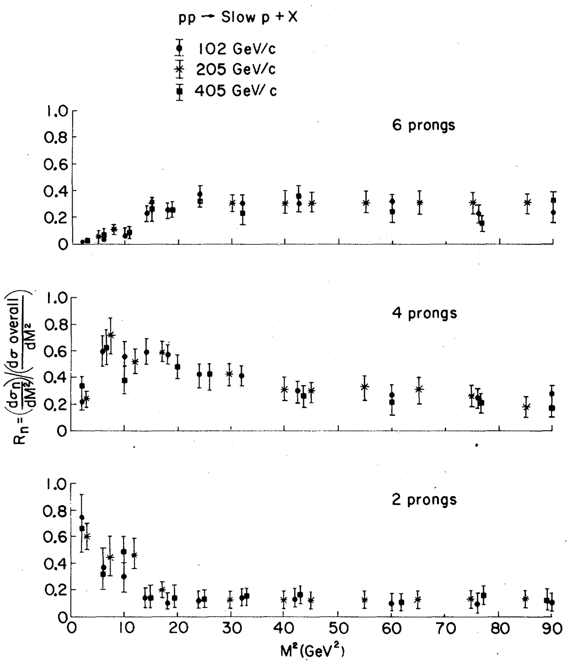


FIG. 8. The ratio of the semi-inclusive cross section to the overall cross section vs M^2 for 2, 4, and 6 prongs at beam momenta 102, 205, and 405 GeV/c.

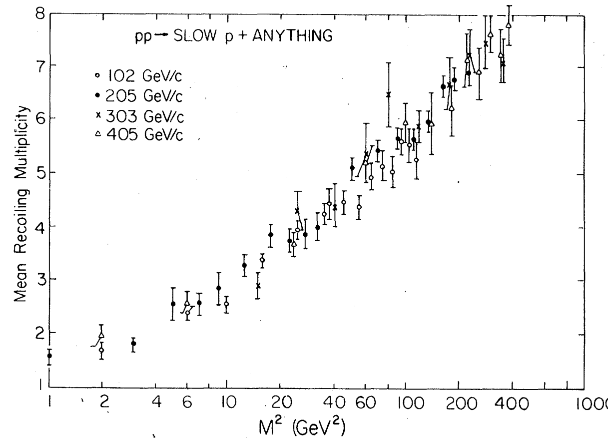


FIG. 9. The mean recoiling multiplicity as a function of M^2 at beam momenta 102, 205, 303, and 405 GeV/c.

Figure 10 is a plot of $\langle n_X \rangle$ vs M^2 for various t ranges for the reaction $pp \to p + X$ at 205 GeV/c. The data are consistent with $\langle n_X \rangle$ being t independent, as required by the hypothesis.

One must add a note of caution here. While testing the t independence of $\langle n_X \rangle$ for reactions that vary rapidly with t (such as central production of pions), it is possible to generate artificial t

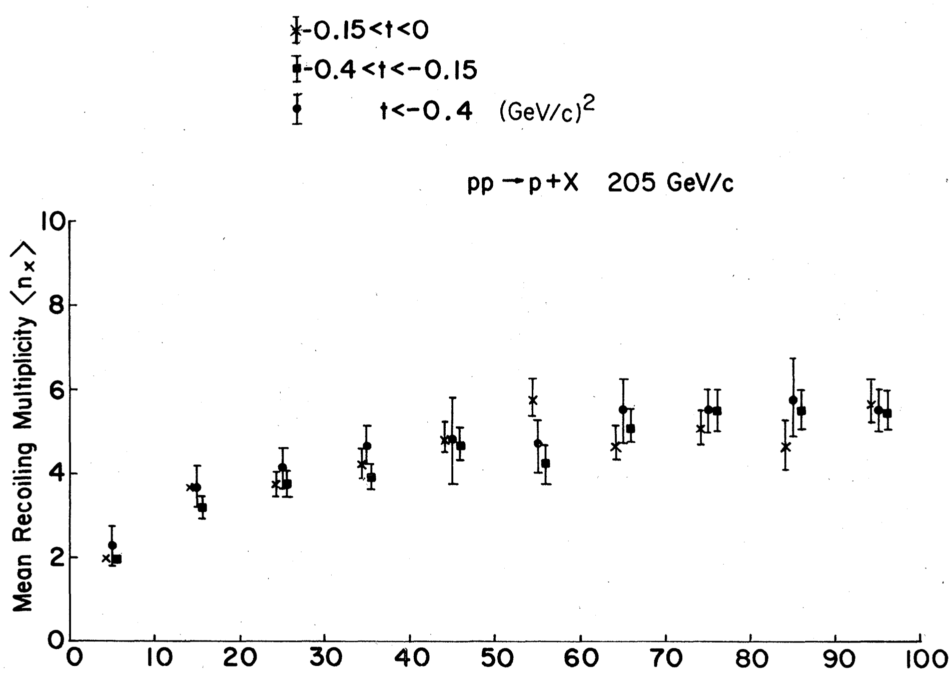


FIG. 10. The mean recoiling multiplicity as a function of M^2 for various t ranges at 205 GeV/c.

dependences by choosing large ranges of M^2 over which the data are averaged.

Connection with Mueller-Regge theory[8]

Since proposing the hypothesis in question, it has come to the author's attention that certain triple-Regge formulas also obey the hypothesis. For example, it can be shown that the inclusive cross section for the process $\pi^- p \to \pi^0 +$ anything has the following form, in the triple-Regge limit:

$$E\frac{d^3\sigma}{dp^3} = G_{\rho\pi\pi}(t)(1-x)^{1-2\alpha_\rho(t)} ,$$

where x is Feynman x of the π^0, $\alpha_\rho(t)$ is the ρ Regge trajectory, $G_{\rho\pi\pi}(t)$ is the $\rho\pi\pi$ coupling constant. For the channel $\pi^- p \to \pi^0 +$ neutrals, using the multiperipheral model in addition, one can show that

$$E\frac{d^3\sigma}{dp^3} = G_{\rho\pi\pi}(t)\left(\frac{1}{M^2}\right)^{1-\tilde{\alpha}_N}(1-x)^{1-2\alpha_\rho(t)} ,$$

where $\tilde{\alpha}_N$ is the intercept of a (pseudo)neutral pole ≈ 0. Since the second reaction is a subset of the first, the ratio of the two should be a function of M^2 only as postulated here. Division of the second by the first yields $(1/M^2)^{1-\tilde{\alpha}_N}$.

Thus although the triple-Regge formulas given above are approximate and have limited validity (they only hold in the triple-Regge limit), they nevertheless exhibit a far more general property, namely, "the ratio of an inclusive cross section $ab \to \bar{c} + X_s$ to $ab \to \bar{c} + X$ is a function of only M^2."

The author realizes that the hypothesis under consideration is provable from unitarity if the system that decays is a bound state or a resonance. It is the purpose of this paper to show that the factorization property holds irrespective of the lifetime of the system under consideration, especially for such short-lived systems as the fireballs considered. It is also worth noting that fireballs of mass M can be though of as poles in the amplitude with infinitesimal residue; there are an infinite number of them along the M^2 axis and the integral over these "little poles" generates the threshold cut.

ACKNOWLEDGMENTS

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[1]R. Raja, Phys. Rev. D 16, 142 (1977).

[2]Since the fireball cannot decay into a single particle, X_s cannot be a resonance or a stable particle. Thus all the channels with X_s as a single particle should be subtracted. For example, in $pp \to p + X$, the elastic channel $pp \to p + p$ should be subtracted.

[3]See for example, H. Pilkuhn, *Interaction of Hadrons* (North-Holland, Amsterdam, 1967), p. 54.

[4]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

[5]Data were obtained in the 30 in. bubble chamber at Fermilab. For a description of how the π^- channel was separated see R. Raja, Fermilab Report No. Fermilab-PUB-77/29-EXP (unpublished).

[6]We prefer this way of demonstrating the t independence of the ratio, rather than explicitly displaying the ratio as a function of M^2 and t following reasons: By displaying the ratio only, all information on the shape of the distribution is lost; but more importantly, the ratio is only independent of t at a fixed M^2. While calculating the ratio, one is obliged to take large ranges of M^2 in one bin especially where the statistics is poor. This can result in artificial t dependences being introduced due to the range of M^2 involved.

[7]The pp data are taken from the following: (102, 405) GeV/c—J. W. Chapman *et al*., Phys. Rev. Lett. 32, 257 (1974); Phys. Lett. 52B, 477 (1974); 205 GeV/c—S. J. Barish *et al*., Phys. Rev. Lett. 31, 1080 (1973); J. Whitmore *et al*., Phys. Rev. D 11, 3324 (1975); 303 GeV/c—F. T. Dao *et al*., Phys. Lett. 45B, 399 (1973); 45B, 402 (1973); see also J. Whitmore, Phys. Rep. 10C, 273 (1974).

[8]See Fermilab Proposal 350 (cospokesmen: G. C. Fox and R. Kenney) (unpublished). The author is grateful to Dr. G. C. Fox for drawing his attention to these formulas.