PHYSICAL REVIEW D VOLUME 18, NUMBER 1 1 JULY 1978

Production of large-mass diffractive states at high energies*

Paul M. Fishbane and John K. Kotsonis
Physics Department, University of Virginia, Charlottesville, Virginia 22901
(Received 13 April 1977)

We investigate the forward production of large-but-fixed-mass fermion diffractive states in the multichannel eikonal model in the high-energy limit. The sum over paths to reach the final diffractive state can be treated and solved as a random-walk problem with constraints, i.e., it is a diffusion problem with various boundary conditions. We find that the sum of paths is not dominated by a "single-hard-multiple-soft" transition path, unlike fixed-angle elastic scattering.

I. INTRODUCTION

A steady flow of experimental data has in the past few years made the properties of hadronic diffractive excitations (DE's) much better known. Production of DE's was first discussed by Good and Walker and since then it has been studied extensively by many authors.[1,2]

For example, nucleon DE's can be studied by the reaction $p+p \to p+X$ where the Feynman scaling variable is near 1, by exclusive reactions of the type $p+p \to (p+n\pi\text{'s})+p$ where the n pions have momentum near to an accompanying p, by similar reactions in nuclear targets, etc. Reactions involving DE's resemble closely high-energy elastic scattering of the hadronic ground states in their (constant) energy dependence and to some extent in the sharply forward-peaked shape of the differential cross section. We now have a systematic picture of the variation of experimentally interesting quantities in the production of DE's: (i) The ratio of the elastic cross section to the production cross section, summed over all DE masses, is of order 1, and not rapidly varying with energy at several hundred GeV and above. (ii) The slope of the DE production differential cross section $d\sigma/dt\,|_{0\to \text{DE}}$ at $t=0$ shows a systematic decrease (flattening) as the mass of the DE increases. (iii) $d\sigma/dt\,|_{0\to\text{DE}}$ has strong dips which disappear as the mass of the DE increases. (iv) In impact-parameter space, the production profiles are peripheral (they peak at some nonzero impact parameter). (v) The elastic scattering cross sections of nucleon DE's on nucleons show a systematic decrease as the mass of the DE increases.

One mode of analysis of these processes which has proven useful is the Regge formalism, with triple-Pomeron contributions.[2,3]

Another mode of analysis employs a multichannel eikonal approach.[4,5,6] This approach pushes the phenomenology down one level, to "elementary" couplings, transition and elastic, of DE's with (absorptive) t-channel exchanges. The model has the advantage of being explicitly s-channel unitary, and has several nontrivial consequences of interest. Moreover, a class of these models with physically reasonable assumptions ("hopping" models) seems to be qualitatively in agreement with the experimental properties listed above.[7] The hopping models are characterized by elementary transitions which are nonzero only between nearest neighbors, i.e., between DE's which differ in mass by one unit, say a pion mass. They also happen to be exactly soluble.

We seek in this paper to uncover certain properties of multichannel eikonal models associated with forward production of large-mass DE's. We are motivated in this by other work, some recent and some not so recent, on large-momentum-transfer processes. We refer in particular to high-energy fixed-angle processes in potential scattering,[8] to the same process in quantum field theory[9,10] (and by extension[11] to large-$p_\perp$ inclusive scattering of hadrons in the quark-scattering picture), and to large-$p_\perp$ inclusive scattering of hadrons in the constituent-exchange model. In each of the above problems, t is a fixed fraction of s; a common dynamical feature which emerges is that, in a multistep picture, the large-t final state is reached by a single hard step in which most of the momentum transfer takes place accompanied by multiple soft steps. Naturally this fact, if it goes beyond the models in which it can be verified, leads to a considerable simplification in treatment of these processes.

However, we do not treat here the case in which the final DE mass is a fixed fraction of $\sqrt{s}$; this case would be the true analog of the processes mentioned above. Rather we report a modest beginning to this end, in which $s\to\infty$ while the final DE mass is fixed but much greater than the ground state. This limit may, of course, be of interest in its own right. In this limit, the structure of any graph contributing to the eikonal is, once the elementary transition couplings have been extracted, identical to the topologically equivalent graph appearing in the elastic one-channel eikonal problem. Since the latter problem has been solved long ago, the problem reduces to that of summing over all possible mass-change paths which lead to the given final DE mass, with weights of these paths determined by their respective elementary-coupling sequences. We have been able to treat this latter problem by using techniques well known from the theory of random walks in one dimension. We have drawn heavily on Chandrasekhar's treatment of Markovian processes.[12]

We show that, generally speaking, the amplitude for the forward production of large fixed DE masses in the high-energy limit is not dominated by the one-hard–multiple-soft terms in the sum over paths. This result holds for a wide range of dependence of the elementary-coupling strength on the mass change, as long as this coupling strength decreases as the mass change increases, an assumption which seems reasonable.

In Sec. II we work out the details of the sum over paths, while in the last section we compare the results with the "one-hard–multiple-soft" approximation.

II. SUM OVER PATHS

We consider the high-energy forward (or fixed transverse momentum transfer) scattering to two-nonidentical fermions. This process is described by multiple exchange in all possible orders of some connected unit. This unit completely characterized the eikonal function $i\chi(s,b)$ through the impact-parameter Fourier transform of the amplitude when one unit is exchanged. Multiple exchange then ensures s-channel unitarity. The unit exchanged may contain multiparticle intermediate states and thus give an absorptive eikonal function. In the multichannel eikonal picture we consider here, DE's are treated explicitly rather than through the absorptivity of the exchange. (We assume only one of the fermions can form DE's for simplicity.) The elementary-transition strengths are taken as a simple function of the DE masses, as shown in Fig. 1; in that figure j indexes the DE state, by mass. Otherwise we assume the DE behaves like the ground state in the multi-unit-exchange amplitude.

Now in the dominant pieces of the multi-unit-exchange, the fermion masses along the sides play no role; recall the n-unit-exchange amplitude has a Fourier transform proportional to the Fourier transform of the single-unit-exchange to the nth power. In this case, the amplitude for production of a given final state m_f factorizes into a dynamical piece characterizing the one-channel problem times a product of coupling strengths defining a path to reach m_f from the initial m_0. Figure 2 shows one path to reach m_5 (say) in a triple-exchange graph. For each graph we must sum over paths. It is this sum over paths with which we are concerned here.

The Nth-order amplitude is then

$$M_N = M_N^{(0)} \sum_{\alpha} \prod_{j=1}^{N} \phi(m_{i_{j-1}}, m_{i_j}), \tag{2.1}$$

where α indexes the path, and where $M_N^{(0)}$ is the Nth-order amplitude in the one-channel problem. There are two physical restrictions in this sum over paths: first that the sum over mass changes is $m_f - m_0$, and second that at no intermediate stage does the mass m_{int} drop below m_0.

We shall consider ϕ to be a function only of the mass change (and, by time-reversal invariance, of the magnitude of the difference):

$$\phi(m_{i_{j-1}}, m_{i_j}) = \phi(|m_{i_j} - m_{i_{j-1}}|) \equiv \phi(|\delta m|). \tag{2.2}$$

The sum of paths in Eq. (2.1) is then symbolically written

$$\mathcal{S} \equiv \sum_{\alpha} \prod_{j=1}^{N} \phi(m_{i_{j-1}}, m_{i_j}) = \sum_{\alpha} [\phi(|\delta m|)]^N. \tag{2.3}$$

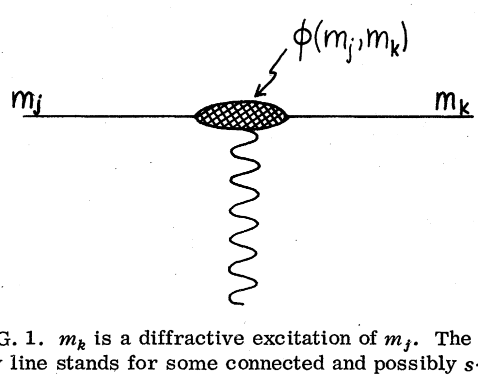

FIG. 1. m_k is a diffractive excitation of m_j. The wavy line stands for some connected and possibly s-channel absorptive unit.

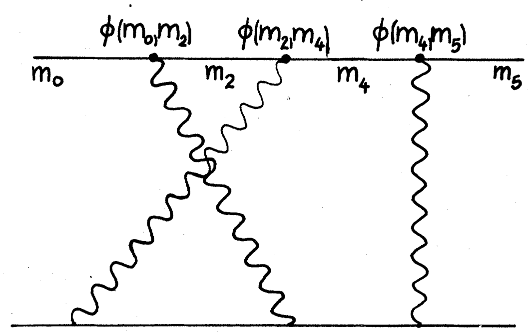

FIG. 2. A sample graph for the production of m_5. Only the top line is allowed to have diffractive excitation.

We now use our large-final-mass m_f condition by approximation of the sum over α by an integral and by working with probability densities. Namely, let us denote the unrestricted sum over mass changes (substitute the label q for δm)

$$\sum_{\delta m}\phi(q)\to\int_{-\infty}^{\infty}\phi(q)dq=K. \qquad (2.4)$$

Then

$$\tau(q)=\frac{1}{K}\phi(q) \qquad (2.5)$$

is a probability density for the (elementary) mass change q.

The calculation of the restricted sum over paths in Eq. (2.3) is then converted to the calculation of the differential probability $W_N(\Delta_F)d\Delta_F$ of coming within $d\Delta_F$ of a net mass change

$$\Delta_F=m_f-m_0 \qquad (2.6)$$

in N steps,

$$\mathcal{S}\to W_N(\Delta_F)d\Delta_F=\int_\Omega[\tau(q)dq]^N, \qquad (2.7)$$

subject to

$$\int_{-\infty}^{\infty}\tau(q)dq=1 \qquad (2.8)$$

and

$$\sum_{j=1}^{L}q_j\geq 0\ \forall L\leq N, \qquad (2.9)$$

where the integration Ω is restricted to the region

$$\Delta_F-\tfrac{1}{2}d\Delta_F\leq\Delta\equiv\sum_{j=1}^{N}\delta m_j\leq\Delta_F+\tfrac{1}{2}d\Delta_F, \qquad (2.10)$$

and by dropping the subscripts on $\delta m_j=q_j$ in Eq. (2.7) we have assumed that the probability $\tau(\delta m_j)$ of a mass change δm_j on the jth step is the same for all j.

In order to solve (2.7) with the restrictions (2.8) and (2.9), we first solve it with the restriction (2.8) alone [case (a)], and then use this result to solve for case (b), when (2.9) is added. The method we use is adapted from the study of random flights in Ref. 12.

We first treat the integration region (2.10) by integrating the q's over all space with a weight D_1 which is unity where (2.10) holds and 0 otherwise:

$$D_1=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin(\frac{1}{2}\rho\,d\Delta_F)}{\rho}\exp[i\rho(\Delta-\Delta_F)]d\rho\,. \qquad (2.11)$$

Additionally, since $d\Delta_F$ is small, we approximate the sine function in (2.11) by its argument. Then, exchanging the ρ and q_j integrations,

$$W_N(\Delta_F)d\Delta_F=\frac{d\Delta_F}{2\pi}\int_{-\infty}^{\infty}d\rho\exp(-i\rho\Delta_F)A_N(\rho)\,, \qquad (2.12)$$

where

$$A_N(\rho)\equiv\left[\int_{-\infty}^{\infty}dq\,\tau(q)\exp(iq\rho)\right]^N. \qquad (2.13)$$

The region $q\approx\rho^{-1}$ dominates in (2.13). Expanding the exponential and keeping only the first few terms,

$$\begin{aligned}A_N(\rho)&\approx\left\{\int_{-\infty}^{\infty}dq\,\tau(q)[1+iq\rho-\tfrac{1}{2}(\rho q)^2]\right\}^N\\&\approx\exp(iN\rho\langle q\rangle-\tfrac{1}{2}\rho^2N\langle q^2\rangle)\\&=\exp(-\tfrac{1}{2}\rho^2N\langle q^2\rangle)\,,\end{aligned} \qquad (2.14)$$

where we have approximated by large N and recalled $\tau(q)=\tau(|q|)$, so that $\langle q\rangle=0$. The expressions above define the average $\langle\ \rangle$. Using (2.14) the integral in (2.12) can be performed.

$$W_N(\Delta_F)\approx(2\pi N\langle q^2\rangle)^{-1/2}\exp\left(-\frac{\Delta_F^{\,2}}{2N\langle q^2\rangle}\right). \qquad (2.15)$$

The probability $W_N(\Delta_F)$ is correctly normalized to unity.

$W_N(\Delta_F)$ has the appearance of a diffusion function. Indeed we shall now form a diffusion equation of which $W_N(\Delta_F)$ is the solution. The equation can then be solved with the remaining restriction (2.9) as a boundary condition, giving us the solution of the original problem.

Since in (2.15), N appears analogous to time in a diffusion function, we form a continuous parameter time t by the relation

$$N=\kappa t\,; \qquad (2.16)$$

κ is the number of collisions (mass changes or eikonal exchanges) per unit time. Then

$$W_{\kappa t}(\Delta)=(4\pi Dt)^{-1/2}\exp\left(-\frac{\Delta^2}{4Dt}\right)\equiv W(\Delta,t)\,, \qquad (2.17)$$

where we have defined the "diffusion constant" D as

$$D\equiv\tfrac{1}{2}\kappa\langle q^2\rangle\,. \qquad (2.18)$$

Finding the equation for W is now a standard procedure, which we repeat only for convenience. Consider a time interval δt large enough for the number of collisions to be large but short enough for $\langle|\delta\Delta|^2\rangle$ to be small on a scale set by D. The probability of an increment $\delta\Delta$ in δt is determined by the distribution $W(\delta\Delta,\delta t)$. Then

$$W(\Delta,t+\delta t)=\int_{-\infty}^{\infty}d(\delta\Delta)W(\Delta-\delta\Delta,t)W(\delta\Delta,\delta t)\,. \qquad (2.19)$$

Since $\langle|\delta\Delta|^2\rangle$ is small, expand $W(\Delta-\delta\Delta,t)$ in a Taylor series about Δ under the integral, and do the same about t on the left-hand side. Then

$$W(\Delta,t)+\frac{\partial W}{\partial t}\delta t+O((\delta t)^2)=W(\Delta,t)+(4\pi D\delta t)^{-1/2}\int d(\delta\Delta)\exp\left(-\frac{(\delta\Delta)^2}{4D\delta t}\right)\left\{\left(-\delta\Delta\frac{\partial W}{\partial\Delta}\right)+\tfrac{1}{2}(\delta\Delta)^2\frac{\partial^2 W}{\partial\Delta^2}\right\}.$$

The first term within the large curly brackets vanishes by symmetry, and we have

$$\frac{\partial W}{\partial t}\delta t=\frac{1}{2}\frac{\partial^2 W}{\partial\Delta^2}(4\pi D\delta t)^{-1/2}\times\int d(\delta\Delta)(\delta\Delta)^2\exp\left[-\frac{(\delta\Delta)^2}{4D\delta t}\right],$$

or

$$\frac{\partial W}{\partial t}=D\frac{\partial^2 W}{\partial\Delta^2},\qquad(2.20)$$

just the standard diffusion equation.

We could then have treated the unrestricted case by solving the diffusion equation (2.20) with boundary conditions

$$W(\Delta,0)=\tau(\Delta).\qquad(2.21a)$$

The restriction (2.9) is then added by a second boundary condition,

$$W=0\ \text{for}\ \Delta=0.\qquad(2.21b)$$

This condition prevents the mass-change sequence from crossing the point $\Delta=0$; we assume the initial ground-state position lies infinitesimally above this.

We solve (2.20) with conditions (2.21) by standard Fourier-transform methods.[13] Before we solve the restricted case [with boundary condition (2.21b)], it is instructive to re-examine the unrestricted case [with boundary condition (2.21a)]. The solution is

$$W(\Delta,t)=(4\pi Dt)^{-1/2}\int_{-\infty}^{\infty}d\Delta'\tau(\Delta')\exp\left[-\frac{(\Delta-\Delta')^2}{4Dt}\right].\qquad(2.22)$$

We find a small-t (small-N) form by expanding around $\Delta\approx\Delta'$:

$$W(\Delta,t)\underset{t\to0}{\simeq}\tau(\Delta)+\tau''(\Delta)Dt,\qquad(2.23a)$$

or

$$W_N(\Delta)\approx\tau(\Delta)+\tfrac{1}{2}\langle\Delta^2\rangle\tau''(\Delta)N.\qquad(2.23b)$$

By expanding (2.22) around $\Delta'\approx0$ [reflecting the dropoff of $\tau(\Delta)$] we find large-N corrections to Eq. (2.15):

$$W(\Delta,t)\underset{t\to0}{\simeq}(4\pi Dt)^{-1/2}\exp\left(-\frac{\Delta^2}{4Dt}\right)\times\left[1-\langle\Delta^2\rangle\left(\frac{1}{4Dt}-\frac{\Delta^2}{8D^2t^2}\right)\right],\qquad(2.24a)$$

or

$$W_N(\Delta)\underset{\text{large N}}{\simeq}(2\pi N\langle\Delta^2\rangle)^{-1/2}\exp\left(-\frac{\Delta^2}{2N\langle\Delta^2\rangle}\right)\times\left[1-\frac{1}{2}\left(\frac{1}{N}-\frac{\Delta^2}{N^2\langle\Delta^2\rangle}\right)\right],\qquad(2.24b)$$

which is more relevant to our problem.

Addition of the boundary condition (2.21b) simply eliminates even Fourier components in the solution of (2.20). We find for this restricted case

$$W(\Delta,t)=(4\pi Dt)^{-1/2}\int_0^{\infty}d\Delta'\tau(\Delta')\left\{\exp\left[-\frac{(\Delta-\Delta')^2}{4Dt}\right]-\exp\left[-\frac{(\Delta+\Delta')^2}{4Dt}\right]\right\}.\qquad(2.25)$$

We expand this about $\Delta'=0$ to find the interesting large-N behavior:

$$W(\Delta,t)\underset{t\to\infty}{\simeq}(4\pi)^{-1/2}(Dt)^{-3/2}\Delta\exp\left(-\frac{\Delta^2}{4Dt}\right)\langle\Delta\rangle_+,\qquad(2.26a)$$

where

$$\langle\Delta^n\rangle_+\equiv\int_0^{\infty}d\Delta\,\Delta^n\tau(\Delta).\qquad(2.27)$$

In terms of N rather than t, (2.26a) reads

$$W_N(\Delta)\underset{\text{large N}}{\simeq}\left(\frac{2}{\pi}\right)^{1/2}(N\langle\Delta^2\rangle)^{-3/2}\langle\Delta\rangle_+\Delta\times\exp\left(-\frac{\Delta^2}{2N\langle\Delta^2\rangle}\right).\qquad(2.26b)$$

The small-N behavior continues to be linear in N, as in Eq. (2.23b).

III. SINGLE-HARD-COLLISION APPROXIMATION

Here we wish to compare the results of Sec. II with an approximation in which the paths containing a single large (hard) jump are dominant. Denoting the weight for this path by P, we have

$$P=\int_0^{\beta}d\Delta_1\int_{\Delta_F-\beta}^{\Delta_F+\beta}d\Delta_2\sum_{k=1}^{N-1}W_k^{(i)}(\Delta_1)\tau(\Delta_F-\Delta_1-\Delta_2)\times W_{N-k-1}^{(f)}(\Delta_2),\qquad(3.1)$$

where β is some small range characterizing the size of the soft collisions, $\beta\ll\Delta_f$; $W_k^{(i)}(\Delta_1)$ and $W_{N-k-1}^{(f)}(\Delta_2)$ must also be calculated under the re-

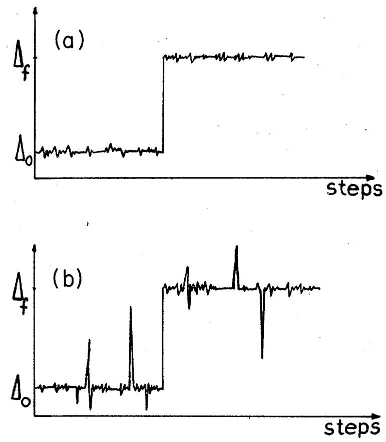

FIG. 3. (a) The one-hard–multiple-soft path configuration for a net mass change Δ_F. (b) These paths are explicitly excluded.

striction that no large excursions are permitted in making the net small mass changes Δ_1 and Δ_2. For convenience we also let β characterize this restriction, namely $W^{(i)}_{(\Delta,t)}$ and $W^{(f)}_{(\Delta,t)}$ satisfy Eq. (2.20) with the boundary conditions $W^{(i)}=0$ for $\Delta=0$ and β and $W^{(f)}=0$ for $\Delta=\Delta_f-\beta$ and $\Delta_f+\beta$. We replace the argument of τ in Eq. (3.1) by Δ_F.

Equation (3.1) can be pictured graphically as in Fig. 3(a), while the paths ruled out by the restrictions in $W^{(i)}$ and $W^{(f)}$ are shown in Fig. 3(b).

We can use large and small numbers of step forms for the W in Eq. (3.1) by dividing the sum over k into three regions,

$$\sum_{k=1}^{N}=\sum_{k=1}^{k_1}+\sum_{k=k_1+1}^{N-k_1}+\sum_{k=N-k_1+1}^{N},$$

where k_1 is a convenient division point. Generally, the N dependence of the first and third sums is the same as the second sum but with a small coefficient (because these sums are basically endpoint contributions). Thus we can write

$$P\approx\tau(\Delta_F)\int_0^\beta d\Delta_1\int_{\Delta_F-\beta}^{\Delta_F+\beta}d\Delta_2\sum_{k=k_1}^{N-k_1}W_k^{(i)}(\Delta_1)\times W_{N-k}^{(f)}(\Delta_2). \qquad (3.2)$$

The solution $W^{(i)}(\Delta,t)$ is

$$W^{(i)}_{(\Delta,t)}=\sum_n c_n(\beta)\exp\left[-Dt\left(\frac{\pi n}{\beta}\right)^2\right]\sin\frac{\pi n\Delta}{\beta}, \qquad (3.3a)$$

$$c_n(\beta)=\frac{2}{\beta}\int_0^\beta\tau(\Delta')\sin\frac{n\pi\Delta'}{\beta}d\Delta'. \qquad (3.3b)$$

We can deal with this expression by noting that, since the large-N (large-t) forms of W are needed, we expect that for finite β

$$Dt\gg\beta^2.$$

Therefore only the first few terms of the sum over n are of interest. In particular, keeping only $n=1$ yields

$$W^{(i)}(\Delta,t)=c_1(\beta)\sin\frac{\pi\Delta}{\beta}\exp\left(-Dt\frac{\pi^2}{\beta^2}\right)$$

or (3.4)

$$W_{k'}^{(i)}(\Delta)=\text{const}\times\sin\frac{\pi\Delta}{\beta}\exp\left(-\frac{\pi^2\langle\Delta^2\rangle k'}{2\beta^2}\right).$$

For $W^{(f)}$ we have

$$W^{(f)}(\Delta,t)=\sum_n d_n(\beta)\exp\left[-Dt\left(\frac{\pi n}{2\beta}\right)^2\right]\sin\frac{\pi n\delta}{2\beta}, \qquad (3.5a)$$

$$d_n(\beta)=\frac{1}{\beta}\int_0^{2\beta}\tau(\delta')\sin\frac{n\pi\delta'}{2\beta}d\delta', \qquad (3.5b)$$

where $\delta=\Delta-\Delta_F+\beta$. As above, this function can be approximated in Eq. (3.2) by

$$W_{k'}(\Delta)=\text{const}\times\sin\frac{\pi\delta}{2\beta}\exp\left(-\frac{\pi^2\langle\Delta^2\rangle k'}{8\beta^2}\right). \qquad (3.6)$$

Inserting (3.4) and (3.6) into (3.2), we have

$$P\approx\text{const}\sum_{k=k_1}^{N-k_1}\exp\left\{-\frac{\pi^2\langle\Delta^2\rangle}{2\beta^2}\left[k+\tfrac{1}{4}(N-k-1)\right]\right\}\tau(\Delta_F)$$

$$=\text{const}\times N\int_0^1 dy\exp\left[-\frac{\pi^2\langle\Delta^2\rangle}{8\beta^2}N(1+3y)\right]\tau(\Delta_F)$$

$$=\text{const}'\times\left[\exp\left(-\frac{\pi^2\langle\Delta^2\rangle}{8\beta^2}N\right)-\exp\left(-\frac{\pi^2\langle\Delta^2\rangle}{2\beta^2}N\right)\right]\tau(\Delta_F). \qquad (3.7)$$

This exponential N dependence is to be compared with the complete result given in Eq. (2.26b). We see that for fixed Δ_F and increasing N, P is a vanishing fraction of the complete sum over paths.

*Work supported in part by NSF under Grant No. MPS-72-05166-A01.

[1]M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960).

[2]H. I. Miettinen, in *High Energy Physics*, proceedings of the European Physical Society International Conference, Palermo, 1975, edited by A. Zichichi (Editrice Compositori, Bologna, 1976), p. 731; and references con-

tained therein.
[3]A. Capella *et al*., Phys. Rev. Lett. 31, 497 (1973).
[4]P. Fishbane *et al*., Phys. Rev. D 10, 3056 (1974).
[5]J. Schaffner *et al*., Phys. Rev. D 12, 244 (1975).
[6]P. Camillo and P. Fishbane, Phys. Rev. D 13, 740 (1976).
[7]P. Camillo and P. Fishbane, Phys. Rev. D 14, 3213 (1976).
[8]L. I. Schiff, Phys. Rev. 103, 443 (1956).
[9]J. L. Cardy, Nucl. Phys. B17, 493 (1970); B28, 477 (1971); B33, 139 (1971).
[10]I. G. Halliday *et al*., Nucl. Phys. B83, 189 (1974).
[11]D. Sivers *et al*., Phys. Rep. 23C, 1 (1976).
[12]S. Chandrasekhar, in *Noise and Stochastic Processes*, edited by N. Wax (Dover, New York, 1954).
[13]See, e.g., J. Mathews and R. Walker, *Mathematical Methods of Physics* (Benjamin, New York, 1965), Sec. 8-4.