Thermal properties of Green's functions in Rindler, de Sitter, and Schwarzschild spaces

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The conventional massless scalar Green's functions in the Minkowski, de Sitter, and two-dimensional Schwarzschild spaces are reinterpreted as finite-temperature Green's functions and the corresponding averages of the stress-energy operator are calculated. The renormalization adopted consists of subtracting the zero-temperature quantities. In all cases the averages give the stress tensor of a purely Planck-type perfect gas.

I. INTRODUCTION

This paper is essentially a continuation of an earlier one[1] where I considered the Green's functions on manifolds that possessed a periodic coordinate ranging from 0 to β. This coordinate could be a real angle, as in a physical wedge, or it could be an imaginary time, as in Rindler space. In the latter case the periodic Green's function can be interpreted as a finite-temperature Green's function, and the present paper contains some further elaboration of this fact. In particular I wish to illustrate the theory with some exact and explicit expressions. These will be associated with the Rindler manifold, de Sitter space, and a two-dimensional exterior black hole.

II. GREEN'S FUNCTION FOR MASSLESS PARTICLES: RINDLER SPACE

For convenience I recapitulate a little of the formalism of Ref. 1. Space-time is represented either by standard Cartesian coordinates

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$$

or, in a certain region, by static Rindler coordinates

$$ds^2 = Z^2 dv^2 - dZ^2 - d\vec{R}^2 = g_{\mu\nu} dx^\mu dx^\nu , \tag{1}$$

where $\vec{R}$ stands for a two-dimensional Euclidean vector $\vec{R} = (X, Y)$. (It is possible to work in more than four dimensions. $\vec{R}$ could be extended to an n-dimensional vector allowing the use of a dimensional regularization, if needed.)

As discussed in detail in Ref. 1 the Green's function, which is periodic in the imaginary Rindler time difference $i(v - v')$, is given by the contour integral

$$G_\beta(x, x') = \int_{A'} G_{2\pi}(\bar{x}, \bar{x}') \times \frac{\beta^{-1} e^{2\pi i\alpha'/\beta}}{e^{2\pi i\alpha'/\beta} - e^{-2\pi(v-v')/\beta}} d\alpha' , \tag{2}$$

where the point x is labeled, equivalently, by (t, x, y, z) or $(v, z, \vec{R})$, and similarly for x'. The (complex) points $\bar{x}$ and $\bar{x}'$ have the same spatial $(Z, \vec{R})$ coordinates as x and x', respectively, but have Rindler times which differ by $i\alpha'$. The contour A' has two branches (these could be combined): One is in the upper α' plane starting from and returning to imaginary infinity but passing beneath the point $\alpha' = i\alpha_1$, where

$$\cosh\alpha_1 = (Z^2 + Z'^2 + |\vec{R} - \vec{R}'|^2)/2ZZ' . \tag{3}$$

The other part of A' is obtained from the upper one by reflection in the origin $\alpha' = 0$.

$G_{2\pi}(\bar{x}, \bar{x}')$ is the ordinary Minkowski Green's function and is a function of only the (complex) space-time separation $\bar{\sigma}^2$ between $\bar{x}$ and $\bar{x}'$:

$$\bar{\sigma}^2 = 2ZZ' \cos\alpha' - Z^2 - Z'^2 - |\vec{R} - \vec{R}'|^2 .$$

All the G_β satisfy the same differential equation

$$(\nabla_\mu \nabla^\mu + \kappa^2) G_\beta(x, x') = \delta(x, x')$$

and are distinguished by the boundary conditions (periodicity requirements) they obey. Note that only for $\beta = 2\pi$ is G_β a function of the space-time separation of x and x'. In other words the boundary conditions violate Poincaré invariance, in general.

The singular points $\alpha' = \pm i\alpha_1$ of $G_{2\pi}(\bar{x}, \bar{x}')$ correspond to the light cone $\bar{\sigma}^2 = 0$, and for massive particles are branch points (corresponding to the propagation of fields inside the light cone), but for massless particles they are just simple poles and the integral can be evaluated very easily in terms of the residues of $G_{2\pi}$. For massless fields

$$G_{2\pi}(\bar{x}, \bar{x}') = -\frac{i}{4\pi^2} \frac{1}{\bar{\sigma}^2 - i\epsilon} \tag{4}$$

and a straightforward evaluation yields

$$G_\beta(x, x') = \frac{i}{4\pi\beta ZZ' \sinh\alpha_1} \times \frac{\sinh(2\pi\alpha_1/\beta)}{\cosh(2\pi\alpha_1/\beta) - \cosh[2\pi(v - v')/\beta]} , \tag{5}$$

which has the characteristic form of a thermal Green's function of temperature β^{-1}. An alternative way of expressing G_β is as the infinite periodicity sum[1,2]

$$G_\beta(x,x')=\sum_{m=-\infty}^{\infty} G_\infty(\ldots, v-v'-im\beta)\,, \tag{6}$$

where G_∞ is the zero-temperature Green's function

$$G_\infty(x,x')=-\frac{i}{4\pi^2}\frac{1}{ZZ'}\frac{\alpha_1}{\sinh\alpha_1}\frac{1}{(v-v')^2-\alpha_1{}^2} \tag{7}$$

(*not* a function of σ^2).

III. ENERGY-MOMENTUM TENSOR

G_β can now be used to evaluate the vacuum average of the improved energy-momentum operator $\hat{T}_{\mu\nu}$ as the coincidence limit

$$\langle\hat{T}_{\mu\nu}\rangle_\beta=-\frac{i}{6}\lim_{x'\to x}\,(4\nabla_\mu\nabla_{\nu'}-g_{\mu\nu'}\nabla^\sigma\nabla_{\sigma'} -\nabla_{\mu'}\nabla_{\nu'}-\nabla_\mu\nabla_\nu)G_\beta(x,x')\,, \tag{8}$$

e.g.,

$$\langle\hat{T}_{00}\rangle_\beta=\frac{i}{6}\lim_{x'\to x}\,(3\partial_0\partial_0+\nabla_{0'}\nabla_{0'}+\nabla_0\nabla_0 -Z^2\nabla_i\nabla_{i'})G_\beta(x,x')\,, \tag{9}$$

where $\partial_0=\partial/\partial v$.

I have not been too careful to say exactly how the coincidence limit is to be taken. This is because G_β is to be thought of as having been regularized, in some way, so as to make the coincidence limit finite. It should not then be necessary to specify how x' approaches x. Afterwards, in principle, the regularization is relaxed and the infinities then exhibited are renormalized away. In practice I am simply going to subtract a term from G_β to render the limit finite. For the moment consider this a pure ansatz, the significance of which should appear later.

The interpretation of G_β as a finite-temperature Green's function suggests that the same procedure as in conventional finite-temperature theory is adopted. That is, the infinite part of the zero-temperature expression, coming from the $m=0$ G_∞ in (6), is to be removed. Examples of this procedure can be found in Dowker and Critchley[2] and Brown and Maclay.[3] In general one does not subtract the entire zero-temperature expression; otherwise one would obtain a zero Casimir term, for example. Rather one has the problem of extracting the finite part of $G_\infty(x,x)$. However, in the present instance it seems most natural to delete the whole of the $m=0$ term in (6). The justification for this is, basically, the same as that for removing the corresponding direct term when dealing with the Casimir effect by the image method.[3,4] Equation (6) expresses G_β as an infinite series of images in imaginary time. Both acts stand or fall by their operational significance, that is to say by comparison with experiment.

The subtracted $\langle T_{\mu\nu}\rangle_\beta$ is thus defined by

$$\langle\hat{T}_{\mu\nu}\rangle^\beta_{\text{sub}}\equiv\langle\hat{T}_{\mu\nu}\rangle_\beta-\langle\hat{T}_{\mu\nu}\rangle_\infty\,, \tag{10}$$

and an immediate consequence is that for $\beta=2\pi$ Poincaré invariance has been violated. This allows a nonzero traceless average, which can be interpreted as a thermal average of $\hat{T}_{\mu\nu}$.

When $\beta=2\pi$, $\langle T_{\mu\nu}\rangle_{2\pi}$ is the $T^{(v)}_{\mu\nu}$ of Christensen and Fulling[5] while $\langle\hat{T}_{\mu\nu}\rangle_\infty$ is their $T^{(\eta)}_{\mu\nu}$. An extensive discussion of the various vacuum states has been given by Fulling.[6]

The calculation of $\langle\hat{T}_{\mu\nu}\rangle^\beta_{\text{sub}}$ is straightforward but I give some of the details as they may be of some interest. It is sufficient to consider $\langle\hat{T}_{00}\rangle_\beta$.

The procedure is as follows. Firstly a "partial" coincidence limit of the right-hand side of (9) is evaluated. That is, (X',Y',Z') are set equal to (X,Y,Z) but v' is not equated to v until after $\langle\hat{T}_{00}\rangle_\infty$ has been subtracted. Also, since $v'\neq v$ in the intermediate quantities, any or all of X', Y', and Z' can be set equal to X, Y, and Z, respectively, in order to simplify the algebra. Note that $\alpha_1=0$ if and only if $X'=X$, $Y'=Y$, and $Z'=Z$.

Direct differentiation then yields the following partial coincidence limits

$$\left.\frac{\partial}{\partial Y'}\frac{\partial}{\partial Y}G_\beta\right|_{\substack{X'=X\\Y'=Y\\Z'=Z}}=-(Z^2\alpha_1)^{-1}\left.\frac{\partial G_\beta}{\partial\alpha_1}\right|_{\alpha_1=0}, \tag{11a}$$

$$\left.\frac{\partial}{\partial X'}\frac{\partial}{\partial X}G_\beta\right|_{\substack{X'=X\\Y'=Y\\Z'=Z}}=-(Z^2\alpha_1)^{-1}\left.\frac{\partial G_\beta}{\partial\alpha_1}\right|_{\alpha_1=0}. \tag{11b}$$

To obtain (11a) I set $Y'=Y$ after $X'=X$ and $Z'=Z$, and similarly for (11b), interchanging X and Y.

The calculation of $(\partial/\partial Z')(\partial/\partial Z)G_\beta$ is slightly more complicated. There are three contributions: one purely from the ZZ' in the denominator, another purely from the α_1 dependence, and a third from both these dependences (a cross term). Since it is only the Z's that are of interest here it is convenient to set $X'=X$ and $Y'=Y$ immediately. From (3) it is easily found that

$$\left.\frac{\partial\alpha_1}{\partial Z}\right|_{\substack{X'=X\\Y'=Y}}=Z^{-1}\quad\text{and}\quad\left.\frac{\partial\alpha_1}{\partial Z'}\right|_{\substack{X'=X\\Y'=Y}}=-Z'^{-1}\,, \tag{12}$$

and the quantity we require is given by

$$\left.\frac{\partial}{\partial Z'}\frac{\partial}{\partial Z}G_\beta\right|_{\substack{X'=X\\Y'=Y\\Z'=Z}}=Z^{-2}G_\beta\Big|_{\alpha_1=0}-Z^{-2}\left.\frac{\partial^2 G_\beta}{\partial\alpha_1{}^2}\right|_{\alpha_1=0}+0\,, \tag{13}$$

where the first term comes from the ZZ' and the second from the α_1 dependence. The third (cross) term vanishes.

In order to evaluate (11a), (11b), and (13) it will be sufficient, and convenient, to give the expansion of G_β in powers of α_1 up to α_1^2. This is, from (5),

$$G_\beta = \frac{if^{-1}}{2\beta^2 ZZ'}\left[1-\frac{\alpha_1^2}{6}\left(1+\frac{4\pi^2(3-f)f^{-1}}{\beta^2}\right)\right] + O(\alpha_1^4) \tag{14}$$

with

$$f \equiv 1-\cosh[2\pi(v-v')/\beta]\,.$$

The final partial limit that is needed is

$$\frac{\partial}{\partial v}\frac{\partial}{\partial v'}G_\beta\Bigg|_{\substack{X'=X\\ Y'=Y\\ Z'=Z}} = \frac{2\pi^2 i(3-f)f^{-2}}{\beta^4 Z^2}\,, \tag{15}$$

and $\langle\hat{T}_{00}\rangle_\beta$ can now be found by substituting into (9), Eqs. (11a), (11b), (13), and (15) where (14) gives G_β as far as is necessary. It is important not to forget Christoffel symbols in the ∇_0 and $\nabla_{0'}$ and the result is

$$\langle\hat{T}_{00}\rangle_\beta = -\frac{i}{6}\lim_{v'\to v}\left[\frac{10i\pi^2(3-f)f^{-2}}{\beta^4 Z^2} - 3\,\frac{\partial^2 G_\beta}{\partial\alpha_1^2}\bigg|_0 - G_\beta\big|_0\right]$$
$$= \lim_{v'\to v}\,[2\pi^2(3-f)f^{-2}/\beta^4 Z^2]\,. \tag{16}$$

It can be seen that terms of order β^{-2} cancel. The two contributions to this order, one from G_β and the other from a part of $3(\partial^2 G_\beta/\partial\alpha_1^2)$ are of equal magnitude but opposite in sign.

In order to effect the final limit $v'\to v$, we must first subtract the value of $\langle\hat{T}_{00}\rangle_\beta$ for $\beta=\infty$. This is most easily accomplished by expanding Eq. (16) using

$$(2+\cosh x)(1-\cosh x)^{-2} = 12x^{-4}+\tfrac{1}{60}+O(x^2)\,,$$
$$(1-\cosh x)^{-1} = -2x^{-2}+\tfrac{1}{6}+O(x^2)\,.$$

It is found that

$$\langle T_{00}\rangle_\beta = \lim_{v'\to v}\,\{[6\pi^2 Z^2(v-v')^4]^{-1}+\pi^2/30Z^2\beta^4\}\,.$$

The only divergent terms are those independent of β so that they cancel on subtraction of the $\beta=\infty$ term. This last act produces the final result

$$\langle\hat{T}^0_0\rangle^\beta_{\text{sub}} = \pi^2 T^4/30\,, \tag{17}$$

where I have put $\beta^{-1}=T_0$, the constant temperature, and have introduced the local temperature T by $T=T_0/g_{00}^{1/2}=T_0/Z$.

Equation (17) is the correct form for the Planck energy density for a gas of scalar "photons" and, if $\beta=2\pi$, is precisely the four-dimensional analog of the result obtained by Davies[7] in two dimensions using Bogoliubov transformations and a Hawking-type mode analysis.

The Rindler manifold is flat. For a curved space-time it might be imagined that non-Planck-type terms would arise in $\langle T_{00}\rangle^\beta_{\text{sub}}$. To investigate this possibility I turn now to the case of de Sitter space.

IV. de SITTER SPACE GREEN'S FUNCTION

Again I consider the massless conformally invariant scalar field and also write the metric in the static form

$$ds^2 = [4a^2/(1+Z^2)^2]\,[Z^2 d(t/a)^2 - dZ^2] - a^2[(1-Z^2)/(1+Z^2)]^2\,(d\theta^2+\sin^2\theta\,d\varphi^2)$$

which corresponds to the Rindler metric (1).

The massless Green's function is[8]

$$G_{2\pi}(x,x') = -i(8\pi^2 a^2)^{-1}[p(x,x')-1-i\epsilon]^{-1}\,, \tag{18}$$

where $p(x,x')$ is given by

$$a^2 p(x,x') = [(a^2-r^2)(a^2-r'^2)]^{1/2}\cosh[(t-t')/a] - [(\vec{R}-\vec{R}')^2-r^2-r'^2]/2\,.$$

The variable r is the usual static radial coordinate related to Z by $Z^2=(a-r)/(a+r)$, a being the radius of the de Sitter sphere. $\vec{R}$ and $\vec{R}'$ stand for the spatial points (r,θ,φ) and (r',θ',φ'), respectively, and $(\vec{R}-\vec{R}')^2$ is the usual Euclidean distance between them.

Evaluation of $G_\beta(x,x')$ from an equation exactly like (2), with $v-v'$ replaced by $(t-t')/a$, proceeds as before since $G_{2\pi}(\bar{x},\bar{x}')$ has simple poles at $p(\bar{x},\bar{x}')=1$, i.e. at $\alpha'=\pm i\alpha_1$ where

$$\cosh\alpha_1 = [(a^2-r^2)(a^2-r'^2)]^{-1/2}\times\{a^2-rr'[\cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\varphi-\varphi')]\}\,.$$

Then

$$G_\beta(x,x') = i\,\frac{[(a^2-r^2)(a^2-r'^2)]^{-1/2}}{4\pi\beta\sinh\alpha_1}\times\frac{\sinh(2\pi\alpha_1/\beta)}{\cosh(2\pi\alpha_1/\beta)-\cosh[2\pi(t-t')/a\beta]}\,, \tag{19}$$

which we can again interpret as a finite-temperature Green's function.

The evaluation of $\langle\hat{T}_{\mu\nu}\rangle^\beta_{\text{sub}}$ follows the same pattern as for Rindler space. I omit intermediate details except to note that in this case it is necessary to include the R terms in the improved stress tensor. These cancel against the Christoffel symbol terms and the final result is again completely Planck-type:

$$\langle \hat{T}^0_0 \rangle^\beta_{\rm sub} = \pi^2 T^4/30 \ , \tag{20}$$

with T again the local temperature equal to $(a\beta)^{-1}(g_{00})^{-1/2}$, the usual value.[9]

V. TWO-DIMENSIONAL SPACE-TIME

Unfortunately the most interesting case, the Schwarzschild black hole, is intractable in explicit terms in four dimensions. However, a great deal can be learned from the two-dimensional analog[6,10] to which I now turn.

First consider the simpler Rindler case again, the metric for which is given by (1) after excising the $d\vec{R}^2$. The β-periodic Green's function is still given by (2) with $G_{2\pi}$ the usual two-dimensional Feynman Green's function

$$G_{2\pi}(x,x') = \tfrac{1}{4} H_0^{(2)}(\kappa(\sigma^2 - i\epsilon)^{1/2}) \ ,$$

where κ is the mass of the scalar field. We want $\kappa = 0$ and then there arise the usual problems associated with potentials in two dimensions. These are not serious. They simply mean at most that one has to carry around an infinite constant in the Green's function for convergence. This constant is of no importance when calculating $\langle \hat{T}_{\mu\nu} \rangle$ since it goes out on differentiation.

Then we have

$$G_{2\pi}(x,x') = -\frac{i}{4\pi}\ln(\sigma^2 - i\epsilon) + C_{2\pi}$$

and

$$G_\beta(x,x') = -\frac{i}{4\pi}\ln\left(ZZ'\{\cosh[2\pi(v-v')/\beta] - \cosh(2\pi\alpha_1/\beta)\}\right) + C_\beta \ , \tag{21}$$

which can be expanded in G_∞, if desired [see (6)].

For $\langle \hat{T}_{\mu\nu} \rangle_\beta$ we have

$$\langle \hat{T}_{\mu\nu} \rangle_\beta = -i \lim_{x' \to x} (\partial_\mu \partial_{\nu'} - \tfrac{1}{2} g_{\mu\nu'} \partial^{\sigma'} \partial_\sigma) G_\beta(x,x') \ ,$$

e.g.,

$$\langle \hat{T}_{00} \rangle_\beta = -\frac{i}{2} \lim (\partial_v \partial_{v'} + Z^2 \partial_Z \partial_{Z'}) G_\beta \tag{22}$$

and I quickly find

$$\langle \hat{T}_{00} \rangle_\beta = \pi\beta^{-1} \lim_{v' \to v} f^{-1}$$

If the $\beta = \infty$ term is subtracted there results

$$\langle \hat{T}^0_0 \rangle^\beta_{\rm sub} = (\pi/6\beta^2) g^{00} = (\pi/6) T^2 \ , \tag{23}$$

which is purely Planck-type, as expected.

When $\beta = 2\pi$, (23) is effectively just the result of Davies[7] referred to before. It is also equivalent, mathematically, to a calculation of Candelas and Raine.[11] They actually evaluate $\langle \hat{T}_{\mu\nu} \rangle_\infty - \langle \hat{T}_{\mu\nu} \rangle_{2\pi}$ in our notation. That is, $\langle \hat{T}_{\mu\nu} \rangle_\infty$ is the Rindler average (G_∞ is the Rindler Green's function) and Candelas and Raine renormalize this by subtracting the Minkowski expression ($\beta = 2\pi$). My procedure is exactly the reverse. I renormalize the Minkowski average by subtracting the Rindler one. Hence the finite expression is minus that on page 2104 of Ref. 11.

The Schwarzschild model is virtually identical to the Rindler theory, *if* the metric is written as conformally Rindler:

$$ds^2 = 32M^3 r^{-1} e^{-r/2M}[Z^2 d(t/4M)^2 - dZ^2] \ , \tag{24}$$

where $Z = \exp(r^*/4M)$. This is basically the Kruskal coordinate system. Then the Feynman Green's function is precisely that of the Rindler theory (using the conformal invariance of the equations) and G_β will be identical to (21) if $(v - v')$ is interpreted as $(t - t')/4M$.

The evaluation of $\langle T^0_0 \rangle^\beta_{\rm sub}$ yields no surprises. The answer is just (23) with T defined now by $T^{-1} = 4M\beta(1 - 2M/r)$ and the equivalent scalar gas is again one with a thermal-equilibrium Planck-type spectrum.

VI. COMMENTS AND CONCLUSION

The results (17), (20), and (23) have been obtained on the basis of Eq. (10) as the definition of the subtracted stress-energy tensor. That is the *entire* zero-temperature quantity has been subtracted. Such a process is pure assumption. It is not valid in the cases discussed in Refs. 2 and 3 which are conventional field theories at finite temperature. There, only a part, the infinite part, of the zero-temperature expression was removed. This corresponds to the conventional renormalization of the zero-temperature theory, for which it is the $G_{2\pi}$ of (4) or (18) that is the Green's function. The situation in the present paper is different. The conventional Green's functions $G_{2\pi}$ are reinterpreted as thermal ones and one really ought to consider the standard renormalization of the corresponding zero-temperature theory, for which G_∞ is the Green's function.

For this reason it is probably incorrect to call $\langle T_{\mu\nu} \rangle^\beta_{\rm sub}$ a *renormalized* energy-momentum *tensor* and I have tried to avoid this particular term, although I have referred, somewhat loosely, to the subtraction (10) as a "renormalization."

Possibly the more conventional view, mentioned previously, and used by myself on occasion, is best illustrated in the Rindler case where one obviously (?) renormalizes by subtracting the Minkowski ($\beta=2\pi$) expression to make the renormalized $\langle T_{\mu\nu}\rangle_{\text{ren}}^{\beta=2\pi}$ zero. The $\langle T_{\mu\nu}\rangle_{\text{ren}}^{\beta}$ could then be considered as *the* (observer-independent) renormalized energy-momentum *tensor*. Such is the attitude taken by Candelas and Raine[11] and by Candelas and Deutsch[12] in a four-dimensional flat calculation of the stress-energy above an accelerating plane conductor. I do not wish to quarrel with this procedure. However, it is not yet clear to me that the subtraction (10) does not have an operational significance.

This is clearly an important issue and will be considered elsewhere. In any case what has been calculated in (10), for $\beta=2\pi$, is the difference[5] of the averages of $\hat{T}_{\mu\nu}$ taken in the η and υ vacuums[6] and (20) confirms that this difference is purely Planck-type. (To obtain the remaining components of $\langle T_{\mu\nu}\rangle_{\text{sub}}^{\beta}$ it is only necessary to remark that $\langle T_{\mu}^{\nu}\rangle_{\text{sub}}^{\beta}$ is traceless, due to its construction, and that the subtraction of $\langle T_{\mu}^{\nu}\rangle_{\infty}$ does not destroy any spatial symmetry so that $\langle T_{ij}\rangle_{\text{sub}}^{\beta}$ is proportional to g_{ij}.)

This result is due to the conformal invariance of the field equations and the conformal flatness of the space-times considered. For four-dimensional Schwarzschild space the difference is *not* purely Planck-type. This can be attributed to the more complicated analytic structure of the Schwarzschild Green's function $G_{2\pi}$, the Hartle-Hawking Green's function.[13]

The calculations of the present paper can be extended in several ways. It would be only a technical problem to consider higher-spin fields, for example. Also a more determined attack on more interesting manifolds such as Schwarzschild, Kerr, or Taub-NUT (Newman-Unti-Tamburino) should yield valuable results. In fact for a self-dual Taub-NUT space the wave equation is again exactly soluble being virtually a repulsive Coulomb equation.

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