Contradiction between deep-inelastic data and the neutron charge radius in naive three-point-quarks models

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(Received 14 July 1977)

Generalizing a previous result, we derive a theorem proving a contradiction between deep-inelastic data and the neutron charge radius for a large class of three-point-quarks nulceon wave functions (including mixing with an arbitrary $\underline{70}$). We briefly discuss the possible ways out. In particular we point out that a nonisoscalar component of the sea, including a pion cloud, might solve the problem.

In the context of parton models, a connection has been established by Sehgal[1] between the sign of the neutron charge radius and the distribution of parton transverse momenta. Moreover, a contradiction between the observed negative sign and the trend of deep-inelastic data was noted by Niegawa and Kiang.[2] Finally, Close, Halzen, and Scott[3] have proposed a solution to this contradiction based on a different interpretation of the data (decrease of $\langle \vec{p}_T^{\,2}\rangle$ at large x).

In a naive quark-model approach with a simple three-point-quarks wave function, but allowing for SU(6) configuration mixing we found[4] also a contradiction between the sign of the mixing angle needed to explain the neutron charge radius and the one needed to explain the trend of F_2^{en}/F_2^{ep}. A qualitative argument using the relation between the spread of a wave function and that of its Fourier transform indicated that this contradiction may not be limited to strict harmonic-oscillator wave functions, as used in Ref. 4. It is to be noted that, in this frame, in contrast to the relation of Sehgal, only the ratio $F_2^{en}(x)/F_2^{ep}(x)$ is involved and the transverse-momentum distribution does not appear separately. Indeed, a wave-function description of the nucleon in terms of three quarks naturally leads to correlations between the transverse quark momenta and the fractions of longitudinal momentum, simply because of rotational invariance of the wave function at rest. The aim of this comment is to make this naive quark-model argument quantitative. We derive the contradiction for a rather large class of wave functions.

If we assign the neutron to a pure $\underline{56}$ representation, in addition to the necessary vanishing of the total charge, the spatial neutron charge distribution

$$\rho_n(\vec{r}) = \left\langle \Psi_n \left| \sum_i e_i \delta(\vec{r} - \vec{r}_i) \right| \Psi_n \right\rangle \tag{1}$$

identically vanishes, as is well known

$$\rho_n(\vec{r}) = 0 \tag{2}$$

due to the factorization of the SU(6) and spatial dependence. The same argument works in momentum space, using the operator

$$\tilde{\rho}(\vec{p}) = \sum_i e_i \delta(\vec{p} - \vec{p}_i) \tag{3}$$

and leads to the independent consequence

$$\tilde{\rho}_n(\vec{p}) = \left\langle \Psi_n \left| \sum_i e_i \delta(\vec{p} - \vec{p}_i) \right| \Psi_n \right\rangle , \tag{4}$$

$$\tilde{\rho}_n(\vec{p}) = 0 . \tag{5}$$

The quantity $\rho_n(\vec{r})$ is usually considered experimentally through its Fourier transform, the electric form factor $G_E^n(q^2)$, and therefore one has from (2):

$$G_E^n(q^2) \simeq -\tfrac{1}{6}|q^2|\langle r^2\rangle_n , \tag{6}$$

$$G_E^n(q^2) = 0 . \tag{7}$$

The quantity $\tilde{\rho}_n(\vec{p})$ may be measured by the so-called "structure functions" of the nucleon. In fact, the quark distribution functions are directly related to the matrix elements of $\langle\Psi|\delta(\vec{p}-\vec{p}_i)|\Psi\rangle$ by the Lorentz boost connecting the $\vec{P}=0$ and $\vec{P}=\infty$ frames. With the simple boost prescription proposed in our previous papers,[5] we get

$$q(x, \vec{p}_\perp) = m_N \langle \Psi_N|\delta(\vec{p}-\vec{p}_i)|\Psi_N\rangle_{p_z = m_N(x-1/3)} . \tag{8}$$

The local vanishing of the charge distribution in the *momentum* space inside the neutron, as expressed in Eq. (5), may then be translated into

$$u(x) - 2d(x) = 0 , \tag{9}$$

where integration has been performed on the transverse momenta. Equation (9) may itself be translated in terms of the directly measured structure functions of the nucleon:

$$F_2^{ep} = x(4u + d)/9 , \tag{10}$$

$$F_2^{en} = x(u + 4d)/9 \quad (11)$$

by

$$f(x) \equiv \frac{1}{x}[F_2^{en}(x) - \tfrac{2}{3}F_2^{ep}(x)]$$

$$= \tfrac{5}{9} m_N [\int d_2\vec{p}\,\tilde{\rho}_n(\vec{p})]_{p_z = m_N(x-1/3)}\,, \quad (12)$$

$$f(x) = 0\,. \quad (13)$$

Note that the relation (8) implies that the structure functions are interpreted in terms of a pure *valence-quark* parton model.

Both (7) and (13) are known to fail, which expresses a failure of (2) and (5). A simple way out of this situation is to give up the pure $\underline{56}$ assignment of the nucleon and introduce a mixing between two configurations with different SU(6) and spatial behavior.[5] Then both $\rho_n(\vec{r})$ and $\tilde{\rho}_n(\vec{p})$ deviate from zero.

We want to establish a connection between the departures from zero of the distributions $\rho_n(\vec{r})$ and $\tilde{\rho}_n(\vec{p})$, or, in terms of directly measured quantities, between $\langle r^2\rangle_n$ and $f(x)$. We consider the class of wave functions

$$\Psi_N = \cos\varphi|56\rangle\Psi^s(\vec{r}_i) + \sin\varphi[|70'\rangle\Psi^{M'}(\vec{r}_i) + |70''\rangle\Psi^{M''}(r_i)]/\sqrt{2}\,, \quad (14)$$

where $\Psi^s(\vec{r}_i)$ is the usual oscillator ground state. It is expected to be a very good approximation for spin-independent potentials. On the contrary, Ψ' and Ψ'' are quite arbitrary save for their symmetry properties—mixed symmetry—to combine with the $\underline{70}$. We must allow for such a freedom of Ψ' and Ψ'' because interference terms could be rather sensitive to the position of the nodes which are expected in this configuration. The $\underline{70}$, on the contrary, has been shown[6] to be the only SU(6) configuration able to describe the behavior of the deep-inelastic structure functions (with φ *small*).

We get, by neglecting the φ^2 terms,

$$\rho_n(\vec{r}) = -\sqrt{2}\sin\varphi\cos\varphi(\tfrac{3}{2})^{3/2} \times \int d_3\vec{\rho}\,\Psi''(\vec{\lambda} = -\sqrt{3/2}\,\vec{r}, \vec{\rho})\Psi^s(\vec{\lambda} = -\sqrt{3/2}\,\vec{r}, \vec{\rho})\,, \quad (15)$$

$$\tilde{\rho}_n(\vec{p}) = -\sqrt{2}\sin\varphi\cos\varphi(\tfrac{3}{2})^{3/2} \times \int d_3\vec{p}_\rho \tilde{\Psi}''(\vec{p}_\lambda = -\sqrt{3/2}\,\vec{p}, \vec{p}_\rho)\tilde{\Psi}^s \times(\vec{p}_\lambda = -\sqrt{3/2}\,\vec{p}, \vec{p}_\rho)\,, \quad (16)$$

where $\vec{\rho}, \vec{\lambda}$ $(\vec{p}_\rho, \vec{p}_\lambda)$ are, respectively, $(\vec{r}_1 - \vec{r}_2)/\sqrt{2}$, $(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{6}$ (their canonical conjugates) and $\tilde{\Psi}$'s are the Fourier transform of Ψ's:

$$\Psi^s = N\exp[-(\vec{\lambda}^2 + \vec{\rho}^2)/2R^2]\,. \quad (17)$$

We then derive

$$\rho_n(\vec{r}) = (3/4\pi)^{3/2}\frac{1}{R^3}\exp(-3\vec{r}^2/4R^2) \times \int d_3\vec{p}\exp(3i\vec{p}\cdot\vec{r}/2)\exp(3\vec{p}^2R^2/4)\tilde{\rho}_n(\vec{p})\,. \quad (18)$$

Note that there is not a Fourier transformation, in general, between $\rho_n(\vec{r})$ and $\tilde{\rho}_n(\vec{p})$. It is true only because of the factorization of Ψ^s in ρ and λ.

We directly get from (18)

$$\langle r^2\rangle_n = -R^4\int d_3\vec{p}\,\vec{p}^2\tilde{\rho}_n(\vec{p}) \quad (19)$$

or, because of (12),

$$\langle r^2\rangle_n = -54/5\, m_N^2 R^4\int_{1/3}^1 dx\,(x - \tfrac{1}{3})^2 f(x)\,. \quad (20)$$

Another useful relation is

$$\rho_n(0) = (3/4\pi)^{3/2}\frac{1}{R^3}\int d_3\vec{p}\exp(\tfrac{3}{4}\vec{p}^2R^2)\tilde{\rho}_n(\vec{p}) \quad (21)$$

$$= \tfrac{9}{5}(3/4\pi)^{3/2}\frac{2}{R^3}\int_{1/3}^1 dx[1 + (3R^2/2)m_N^2(x - \tfrac{1}{3}^2] \times\exp[\tfrac{3}{4}R^2m_N^2(x - \tfrac{1}{3})^2]f(x)\,. \quad (22)$$

We have moreover

$$\int_{1/3}^1 f(x)dx = 0 \quad (23)$$

by orthogonality of Ψ'' and Ψ^s which belong to different representations of S_3.

Having established some general relations, we now demonstrate the announced paradox. Experimentally $f(x)$ has the following qualitative form (Fig. 1). Since we have (23) and since in (20) and (22) $f(x)$ is weighted by increasing functions of x, we deduce[6]

$$\int_{1/3}^1 dx\,f(x)(x - \tfrac{1}{3})^2 < 0\,,$$

$$\int_{1/3}^1 dx\,f(x)[1 + \tfrac{3}{2}R^2m_N^2(x - 1/3)^2] \times\exp[\tfrac{3}{4}R^2m_N^2(x - 1/3)^2] < 0\,,$$

whence

$$\langle r^2\rangle_n > 0\,, \quad (24)$$

$$\rho_n(0) < 0\,. \quad (25)$$

Equation (24) is in clear contradiction with experiment. It is not so clear for (25), which depends on the integral

$$\int d_3\vec{q}\,G_E^n(\vec{q}^2)$$

and therefore also on larger-q^2 data.

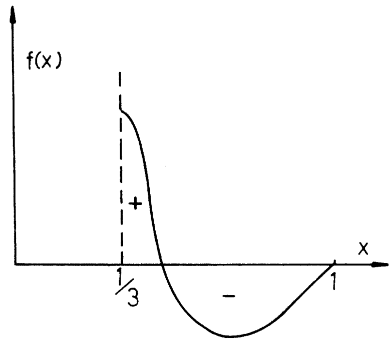

FIG. 1. Experimental qualitative shape of the $f(x)$ function.

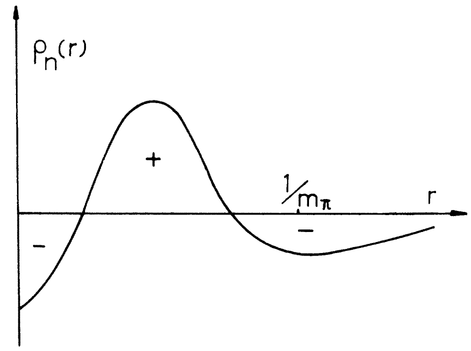

FIG. 2. Qualitative shape of the expected charge density in the quark model with a pion cloud.

We now discuss some of the above assumptions to find a way out of the paradox:

(a) Is not the assumed form of the wave function (14) too restrictive? We feel that our conclusion is not very sensitive to the $\underline{56}$ spatial wave function, and that SU(6) configurations other than the $\underline{56}$ and $\underline{70}$ would contribute only weakly if the mixing parameters are small, for reasons explained in Ref. 5. Only drastic changes in the wave function could change our conclusion.

(b) The connection expressed in (8) between $\vec{P}=0$ wave functions and structure functions is not present in standard parton-model works, although it has deep significance to relate $\vec{P}=0$ wave functions and deep-inelastic data. It is certainly very crude, and we think it fails for x very near 1 because we have not the kinematical bound $q(x)=0$ for $x=1$. Moreover, gluons and a $q\bar{q}$ sea are known to be present in addition to valence quarks. But they can be included in a natural manner by the method of Altarelli *et al.*,[7] as a dressing of each point quark by a cloud of gluons and $q\bar{q}$ pairs. It then appears that the theoretical $f(x)$, defined as

$$f(x)=\tfrac{5}{9}m_N\Big[\int d_2\vec{p}_\perp\tilde{\rho}_n(\vec{p})\Big]_{p_z=m_N(x-1/3)}\,, \qquad (26)$$

should still have qualitatively the same behavior as the experimental one:

$$f(x)=\frac{1}{x}\big[F_2^{en}(x)-\tfrac{2}{3}F_2^{ep}(x)\big] \qquad (27)$$

and the contradiction remains. The wave function Ψ now describes the motion of dressed quarks. Note that the conclusion relies on the usual assumption that the sea is isoscalar, as is suggested by the equality $F_2^{en}(x)\simeq F_2^{ep}(x)$ as $x\to 0$.

(c) The most interesting way out seems to us to question the description of $G_E^n(\vec{q}^{\,2})$ by a simple three-point-quarks model at very small q^2. The dressing of a bare neutron by pions is known[8] to generate a charge radius of the right sign, although too large. This is due to the virtual process:

$$n\to p+\pi^-,\quad n+\pi^0$$

which endows the neutron with a pion cloud, negatively charged and extending at rather large distances $\sim 1/(2m_\pi)$, balanced by a positive core at shorter distances. This mechanism could combine with the deformation of the wave function which acts in the *opposite* direction, but which is mainly localized inside the core range (i.e., inside the usual nucleon radius) due to the rapidly falling Gaussian wave function. The combination of the two effects could explain why the resultant charge radius squared is smaller than expected from the pion cloud.

An interesting conjecture is that the result for the central charge density $\rho_n(0)<0$ (25) still holds. One then expects the behavior indicated in Fig. 2. One expects also that $G_E^n(q^2)$ becomes negative and has therefore a zero, since $\int d_3\vec{q}\, G_E^n(q^2)<0$. The zero should lie in the range $|q^2|\gtrsim 4m_\pi^2$. Indeed there is some evidence[9] for a negative $G_E^n(q^2)$ in the region $2.5\ \mathrm{fm}^{-2}\leqslant|q^2|$ and $|q^2|\leqslant 5\ \mathrm{fm}^{-2}$, i.e. for $|q^2|>5m_\pi^2$.

The structure functions would hopefully not be significantly modified by the pion cloud, except perhaps in the small-x region (cf. Kleinert[10]), and therefore the above description of structure functions in the valence-quark region would remain unchanged.

Note added in proof. We have just received a report by A. C. Davis and E. J. Squires, University of Durham report, 1977 (unpublished). They draw an opposite conclusion to ours. We shall comment on this discrepancy later.

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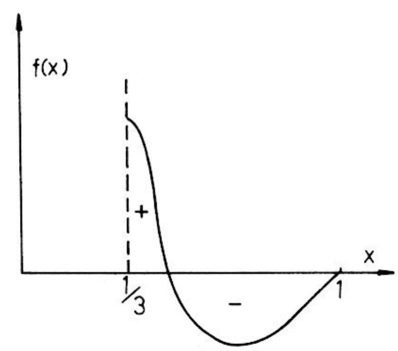

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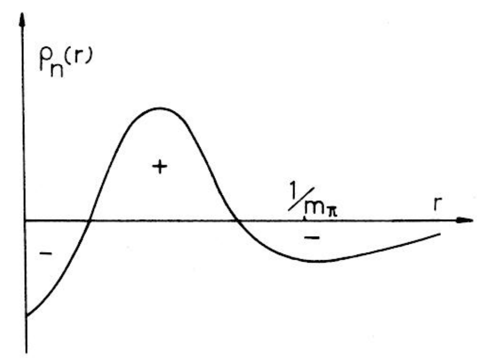

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