PHYSICAL REVIEW D VOLUME 18, NUMBER 5 1 SEPTEMBER 1978

$\underline{56}$-$\underline{70}$ octet mixing and $|g_A/g_V|$ for stable baryons

S. G. Kamath
Department of Physics, University of Delhi, Delhi-110007, India

(Received 1 July 1977)

The $|g_A/g_V|$ measured recently by Décamp *et al.* from the semileptonic decay $\Sigma^- \to n e^- \bar{\nu}_e$ is calculated in a constituent-quark model with harmonic-oscillator forces in even partial waves only as proposed recently by Mitra. The model provides a natural facility for an ideal mixing between the $\underline{8}_d$ states of the $(\underline{56},0^+)_0$ and $(\underline{70},0^+)_0$ to represent the nucleon octet, with the Roper octet as the orthogonal counterpart. The theoretical prediction obtained is 0.15, to be compared with $0.17^{+0.07}_{-0.09}$ reported by Decamp *et al.* For the $\Lambda \to p e^- \bar{\nu}_e$ decay the corresponding $|g_A/g_V|$ is calculated as 0.778, to be compared with an experimental average value of 0.66 ± 0.05.

In a recent paper Décamp *et al.*[1] report on a measurement of the $|g_A/g_V|$ for semileptonic decay of the Σ^-, quoting a value (0.17) of the order of 3 standard deviations from the previous figure of 0.435 ± 0.035 obtained by the Yale-Fermilab-BNL group.[2] This ratio of the axial-vector to the vector coupling constants for the "stable" baryons[3] has been of prime interest in the study of the symmetries, e.g., SU(6), associated with the constituent-quark model. Thus the SU(6) prediction[4] of $|g_A/g_V| = 5/3$ for the neutron β decay has long been regarded as anomalous (assuming no renormalization of the axial-vector constant at the quark level), and has motivated various corrections to the nonrelativistic quark model.[5] For instance, Bogoliubov[6] has shown that this discrepancy could be due to the basic nonrelativistic character of this model and that a relativistic treatment using Dirac (instead of Pauli) spinors leads to a value of $\frac{5}{3}(1-2\delta)$ for the neutron case, where δ is a positive quantity related to the norm of the small components in the quark Dirac spinors.

Le Yaouanc *et al.*[7] have recently viewed the problem of $|g_A/g_V|$ for the neutron by relaxing the assumption of no renormalization effects on the axial-vector constant at the constituent-quark level, but in the context of the twin effects of internal relativistic quark motion and SU(6) breaking. While thereby suggesting a structure for the quarks (a point of view that is open to question), the renormalized quark axial-vector constant (G_A') appears as a parameter in their theory, the magnitude of which is fixed from the experimental value of $|g_A/g_V|$ for the neutron. More important, however, is the nature of the other assumptions involving other free parameters, e.g., the hypothesis of SU(6) breaking at the nucleon level, that influence the estimation of G_A'. Indeed, the octet mixing introduced between the $(\underline{56},0^+)_0$ and the radially excited $(\underline{70},0^+)_2$ supermultiplets suggested by Le Yaouanc *et al.*[7] to represent the "mixed" nucleon is not easily defendable because the $(\underline{70},0^+)_2$ states are placed a little too high in energy in the harmonic-oscillator approach adopted by these authors.

The basic aim of this note is to keep to the structureless view of the quarks in the constituent-quark model, and to evaluate the $|g_A/g_V|$ for the stable baryons (especially Σ^- and Λ) using an altogether different point of view for the basic interaction forces between the quarks inside the baryon. We have in mind here the recently proposed even-wave harmonic-oscillator (h.o.) theory of baryonic states[8,9] by Mitra, which differs from conventional h.o. theories[10] in that the odd partial waves in a pair-wise h.o. quark-quark interaction are heavily depressed relative to their even counterparts. While referring the reader to the literature[8,9] for details, it would suffice our purpose here to mention one important consequence of this input, namely, the prediction of a ground state of $(\underline{70},0^+)_0$ that lies below the $(\underline{70},1^-)$ and above the $(\underline{56},0^+)_0$ supermultiplets. This naturally allows for an easy mixing[9] between the octet members of the $(\underline{56},0^+)_0$ and $(\underline{70},0^+)_0$, besides providing an interesting alternative[11] to the conventional $(\underline{56},0^+)_2$ for the Roper resonance. Note that in the conventional h.o. approach adopted by Le Yaouanc *et al.*[7] the $(\underline{70},0^+)_2$ lies at a much higher excitation to facilitate mixing with the $(\underline{56},0^+)_0$ state. In the even-wave model[8] on the other hand, this very state is significantly depressed in energy so as to appear as an effective ground state of the $\underline{70}$ series, thus providing a more natural candidate for mixing than in the conventional h.o. model.

The nucleon octet in the even-wave model is thus taken to be[9]

$$|N\rangle = |\underline{56},0^+\rangle \cos\phi + |\underline{70},0^+\rangle \sin\phi \qquad (1)$$

with the Roper octet as the orthogonal state, viz.,

$$|N_R\rangle = -|\underline{56}, 0^+\rangle \sin\phi + |\underline{70}, 0^+\rangle \cos\phi \qquad (2)$$

The magnitude of this mixing angle (ϕ) in Eqs. (1) and (2) is fixed by the $|g_A/g_V|$ input for the neutron, assuming no renormalization effects for the quark axial-vector constant G_A' and it works out to[9]

$$\tan^2\phi = 0.5. \qquad (3)$$

which is strongly reminiscent of an ideal mixing angle. This value of the mixing angle checks very well, as shown in Ref. 9, with the two other important low-energy parameters governing $L^P = 0^+ \to 0^+$ transitions, namely the pion-nucleon coupling constant $G_{NN\pi}$ and the $\Delta \to N\pi$ width. As already observed by Mitra and Sood,[9] the problem of simultaneous consistency among these three experimental parameters has been genuine in most constituent-quark-model theories (whether h.o. or otherwise) that are available, since none of these makes use of a mixed nucleon.

Let us now examine the consequences of Eqs. (1) and (3) for the $|g_A/g_V|$ value obtained for the $\Sigma^- \to ne^-\nu_e$ reported by Décamp *et al.*[1] and the $\Lambda \to pe^-\bar{\nu}_e$, the latter being reported in the Particle Data Group[12] at an average value of 0.66 ± 0.05. Using the well-known totally symmetric spin-unitary-spin functions[13] for the $\underline{8}_d$ components of the $\underline{56}$ and $\underline{70}$, and the quark weak-interaction Hamiltonian in the notation given by[5]

$$\begin{aligned} H_w = \int d^3x [\bar{q}(x)(G_V'\gamma_\mu + G_A'\gamma_\mu\gamma_5) \\ \times (\alpha^+ \cos\theta + \beta^+ \sin\theta) q(x)] \\ \times [\bar{l}(x)\gamma_\mu(1+\gamma_5)\nu(x)] \\ + \text{H.c.}, \end{aligned} \qquad (4)$$

we find that for no renormalization effects on the axial-vector constant G_A', the $\Sigma^- \to ne^-\bar{\nu}_e$ mode is predicted to have

$$|g_A/g_V| = 0.15 \qquad (5)$$

in excellent agreement with the value of $0.17^{+0.07}_{-0.09}$ by Décamp *et al.*[1] Note however, that with a pure $(\underline{56}, 0^+)_0$ assignment for the Σ^- the corresponding $|g_A/g_V|$ works out to 0.44, thus exhibiting a poorer overlap with the results of Ref. 1.

For the $\Lambda \to pe^-\bar{\nu}_e$ decay, the theoretical prediction works out to

$$|g_A/g_V| = 0.778 \qquad (6)$$

to be compared to the averaged experimental value[12] of 0.66 ± 0.05. The fit is again very good in relation to that predicted by a pure $\underline{56}$ assignment, which is calculated to be unity. In both of these semileptonic decays, the hadron momentum in the final state is too small to be of any significant effect at the level of the form factors. A quick estimate with the even-wave h.o. wave functions shows that the correction is only of the order of 7–8%. This again seems to be in excellent favor experimentally with the even-wave h.o. predictions over that of the conventional model.[10]

We would like to conclude this paper with the observation that recently Slaughter and Oneda[14] have used the values of $|g_A/g_V|$ for $\Sigma^- \to ne^-\bar{\nu}_e$ obtained by Tanenbaum *et al.*[2] in support of their conjecture on the existence of a ninth $\frac{1}{2}^+$ baryon at 1700 MeV. Since Décamp *et al.*[1] obtain a value which is 3 standard deviations smaller, it would be interesting to determine the effect of this revised value on Slaughter and Oneda's conjecture.

It is a pleasure to thank Professor A. N. Mitra for his encouragement and a critical reading of the manuscript.

---

[1] D. Décamp *et al.*, Phys. Lett. 66B, 295 (1977).

[2] W. Tanenbaum *et al.*, Phys. Rev. Lett. 33, 175 (1974); Phys. Rev. D 12, 1871 (1975).

[3] The phrase "stable baryon" is used here in the sense employed by the Particle Data Group, Ref. 12.

[4] R. Van Royen and V. Weisskopf, Nuovo Cimento 60A, 517 (1967).

[5] See, e.g., J. J. J. Kokkedee, *The Quark Model* (Benjamin, New York, 1969).

[6] P. N. Bogoliubov, Ann. Inst. Henri Poincaré 8, 163 (1968). See also A. Le Yaouanc, L. Oliver, O. Pène, and J-C. Raynal, Phys. Rev. D 9, 2636 (1974).

[7] A. Le Yaouanc, L. Oliver, O. Pène, and J.-C. Raynal, Phys. Rev. D 15, 844 (1977); 12, 2137 (1975).

[8] A. N. Mitra, Phys. Lett. 51B, 149 (1974); Phys. Rev. D 11, 3270 (1975).

[9] A. N. Mitra and S. Sood, Phys. Rev. D 15, 1991 (1977).

[10] See, e.g., R. P. Feynman, M. Kislinger, and F. Ravndal, Phys. Rev. D 3, 2706 (1971); D. Faiman and A. W. Hendry, Phys. Rev. 173, 1720 (1968).

[11] For comparison of hadronic decay width predictions of all baryon resonances ($L \leq 2$) in the framework of even-wave and conventional h.o. type theories see S. G. Kamath, Ph.D. thesis, Delhi University, 1975 (unpublished); also S. G. Kamath and A. N. Mitra, Phys. Rev. D 17, 390 (1978).

[12] Particle Data Group, Rev. Mod. Phys. 48, S1 (1976).

[13] The notation used follows A. N. Mitra, Ann. Phys. (N.Y.) 43, 126 (1967).

[14] M. D. Slaughter and S. Oneda, Phys. Rev. D 14, 799 (1976).