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Comments and Addenda

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Some comments on the Brayshaw mechanism for generating peaks in the hadron system

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We point out here that the recent proposals of one of us (D.D.B.) enable us (at last) to understand the striking empirical successes of the Peierls mechanism for generating hadron peaks noted many years back. Trivial predictions can now be made for identifying the octet partners of the $N^*(1470)$ Roper resonance. Further applications of the Brayshaw mechanism to systems involving the $\phi(1.020)$ and charm particles are also summarized here. Finally, we comment on the empirically unexplored aspects of the Peierls mechanism, in particular, for low-lying excitations of three-particle systems involving a baryon isobar.

Almost a generation back, Peierls[1] proposed that the mechanism of pion-isobar scattering (with nucleon exchange in the crossed u channel) can generate a sharply energy-dependent effect and behaves like a resonance. Nauenberg and Pais[2] extended Peierls' treatment to the meson system, where in place of $(\Delta(1238), \pi, N)$ for generation of a possible higher pion-nucleon resonance $N^*(1512)$, one considers energy peaks in the meson system generated by (ρ, π, π), (K^*, π, K), and $(K^*, \bar{K}, \pi)$. Here we have employed the notation (X, A, B) to represent the process illustrated in Fig. 1. Peaks are predicted at c.m. energy for $(\rho\pi)$, $(K^*\pi)$, and $(K^*\bar{K})$ at 1090, 1170, and 1410 MeV, respectively. The first and third predictions are in striking agreement with experiment[3] on the $A_1(1100)$ and $E(1420)$, while the second prediction is in satisfactory accord with the lower structure Q_1 at 1200 MeV—considering the experimental uncertainties and the fact that there are *no free parameters in the Peierls formula.* Nevertheless, Salam[4] first questioned whether the particular Peierls singularity (corresponding to $s_+^{1/2}$ c.m. energy in notations to be developed below) discussed then[1] will actually show up when joined with the initial pion-nucleon system via $\pi + N \to \pi + \Delta(1238)$—since all physically observed strong interactions are initiated with stable particles (and isobar-meson scattering is not directly observed). A naive calculation[5] suggests that the singularity becomes only logarithmic in $\pi N \to \pi\Delta$, and introducing a finite width for the $\Delta(1238)$ would largely eliminate the peak effect in the neighborhood of the second pion-nucleon resonance $N^*(1512)$. Later, a more systematic analysis was carried out by Goebel, Chew and Low, and others[6] with the more definitive conclusion that the Peierls (s_+) singularity is on the wrong sheet and unlikely to generate peaks in physically observed strong-interaction processes. Nevertheless, as a *mnemonic rule*, the Peierls s_+ singularity formula is remarkable[7] and suggests that ultimately a sound theoretical basis will be found for the empirical successes (of which the Nauenberg-Pais examples quoted above are just the most illustrative sample). With the above historical introduction, we turn now to the proposals of one of us (D. D. B.).[8]

The proposed mechanism arises from a singularity in the diagram of Fig. 1. This singularity

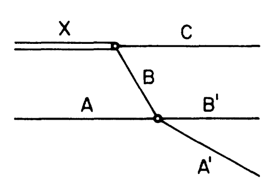

FIG. 1. Rescattering diagram which generates the Brayshaw singularity. The vertex blobs correspond to off-shell scattering amplitudes, and A', B' are at threshold. The situation in which the subenergy $s_{A'B'}$ coincides with a resonant energy of $A' + B'$ gives rise to the Peierls singularity.

corresponds to the diagram being realized as an *on-shell sequential rescattering*, i.e., to A, B, and C describing a physical intermediate state of invariant (three-body) energy $\sqrt{s}$. For our purposes, we choose X to be a resonance. The situation in which the subenergy $s_{A'B'}$ coincides with a resonant energy of $A'+B'$ gives rise to the Peierls' singularity.[1] The present mechanism is different in that we take $s_{A'B'}$ at the subenergy threshold, and a singularity for the $[C, A', B']$ system is generated for $s=(k_A+k_B+k_C)^2$, with $m_X^2=(k_B+k_C)^2$, $s_{A'B'}=(k_A+k_B)^2$, and $\cos\theta_{AC}=-1$. Thus,

$$s=s_{A'B'}+m_C^2+2(s_{A'B'})^{1/2}\epsilon_C,$$
$$\epsilon_C=[\alpha(m_B^2+\bar{k}_B^2)^{1/2}+\bar{k}_B(\alpha^2-m_B^2m_C^2)^{1/2}]/m_B^2,$$
$$\alpha=\tfrac{1}{2}(m_X^2-m_B^2-m_C^2). \tag{1}$$

Here $\bar{k}_B$ is the magnitude of the three-momentum $\vec{k}_B$ in the AB (and $A'B'$) c.m.; i.e.,

$$(m_A^2+\bar{k}_B^2)^{1/2}+(m_B^2+\bar{k}_B^2)^{1/2}=(s_{A'B'})^{1/2}=m_{A'}+m_{B'}. \tag{2}$$

In particular, for the (elastic) case $m_A=m_{A'}$ and $m_B=m_{B'}$, we have $\bar{k}_B=0$, and Eq. (1) reduces to $s=s_c$, with

$$s_c=(m_A+m_B)^2+m_C^2+(m_A+m_B)(m_X^2-m_B^2-m_C^2)/m_B. \tag{3}$$

A detailed analysis shows that this singularity is on the *correct sheet*,[9] and occurs irregardless of whether the initial (XA) state is on shell (e.g., the diagram of Fig. 1 could be preceded by an arbitrary production process). It is thus reasonable to anticipate a resonance near $s=s_c$ in the three-hadron system $A'B'C$ (*and in coupled inelastic channels*). However, the s_c singularity, though near the physical region, is only logarithmic in a given partial wave (a point apparently recognized by several earlier authors[10] in an analogous context). Introduction of input two-body resonances having typical widths of the order of 100 MeV causes s_c to be removed rather far from the real axis (note that m_X is *complex*), and relatively sharp peaks are not predicted. The key aspect of Ref. 8 is to stress that in circumstances where $m_A+m_B>m_{A'}+m_{B'}$, e.g., in

$$p\pi^-\to n\pi^\circ,\quad n\pi^+\to p\pi^\circ,$$
$$\pi^+\pi^-\to\pi^\circ\pi^\circ,$$
$$\bar{K}^\circ K^\circ\to K^+K^-, \tag{4}$$
$$D^+D^-\to D^\circ\bar{D}^\circ,$$

a final "charge exchange" at this vertex can shift the singularity into, or very near, the physical region while the particles themselves remain close to subenergy threshold thus leading to much enhanced effects.

To see this we note that $\bar{k}_B$ as defined by Eq. (2) becomes (positive) imaginary; $\bar{k}_B=i\kappa_B$. Hence

$$\mathrm{Im}s=\frac{2(m_{A'}+m_{B'})}{m_B^2}\times[(m_B^2-\kappa_B^2)^{1/2}\mathrm{Im}\alpha+\kappa_B\mathrm{Re}(\alpha^2-m_B^2m_C^2)^{1/2}], \tag{5}$$

with $\mathrm{Im}\alpha=-\bar{m}_X\Gamma_X$ for $m_X=\bar{m}_X-\frac{1}{2}i\Gamma_X$. For increasing values of $\kappa_B>0$, the second term in the square brackets can lead to a substantial cancellation in certain three-body systems and a corresponding reduction in Ims. In practice, this effect is crucial and leads to conclusions which appear contradictory from the standpoint of the approximate formula of Eq. (3). For example, if X is a typical baryon resonance and A is a pion, Ims_c is almost twice as large for pion exchange $(m_B=m_\pi)$ as for baryon exchange $(m_C=m_\pi)$. Nevertheless, when the exact expression of Eq. (5) is employed, one finds that Ims is considerably smaller for pion exchange (generally speaking, the coefficient of Imα must be small, and hence m_B should be the lesser of the two decay masses). This suggests that the nucleon-exchange diagram is relatively unimportant, a fact which is confirmed by the more rigorous treatment of Brayshaw[8] (it should be noted that the singularity is not a simple pole, and Ims cannot be identified with the width). The importance of the charge-exchange mechanism in generating a significant effect is discussed in some detail in a recent article.[9]

We now consider the Peierls singularity corresponding to Fig. 2, i.e., to (X, A, C). This singularity[11] represents a cut in the s plane (s_+, s_-) for X-A scattering, where

$$s_+=2(m_X^2+m_A^2)-m_C^2,\quad s_-=(m_X^2-m_A^2)^2/m_C^2. \tag{6}$$

For the meson system $(X, A, C)=(\rho, \pi, \pi)$, (K^*, π, K), and $(K^*, \bar{K}, \pi)$ [to which we can now add also the entry[8] $(\delta(970), \pi, \eta)$, where $s_+^{1/2}=1.285$ MeV in close proximity to the D meson], Nauenberg and Pais[2] argued that the energy peaks

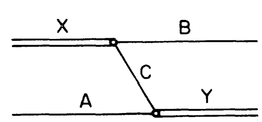

FIG. 2. Rescattering diagram which generates the Peierls singularity. The vertex blobs correspond to off-shell scattering amplitudes, where Y is a resonance.

are due to the s_+ singularity. Here we present a *general kinematic argument for both meson and baryon systems that if $m_A = m_B$ and $m_X = m_Y$, then $s_+ = s_c$.*

The argument proceeds as follows. In Fig. 2, the Peierls singularities correspond to the following conditions for four-momentum conservation:

$$m_X^2 = (k_B + k_C)^2 , \tag{7a}$$

$$m_Y^2 = (k_A + k_C)^2 , \tag{7b}$$

$$s = (k_A + k_B + k_C)^2 . \tag{7c}$$

Solving (7a) and (7b) for the *mass-shell* four-vectors k_A, k_B, k_C (e.g., in the ABC c.m. system), and substituting into (7c) leads to the values $s = s_\pm$ when $\cos\theta_{AB} = \pm 1$. As noted above, the Brayshaw singularity[8] $s = s_c$ comes from

$$m_X^2 = (k_B + k_C)^2 , \tag{8a}$$

$$m_{Y'}^2 = (k_A + k_B)^2 , \tag{8b}$$

$$s = (k_A + k_B + k_C)^2 , \tag{8c}$$

with $\cos\theta_{AB} = +1 = -\cos\theta_{AC}$ and $m_{Y'} = m_A + m_B$; note that only (8b) is different from the Peierls case. Now evaluate in the AB c.m. system, so that $k_A = (m_A, \vec{0})$ and $k_B = (m_B, \vec{0})$. Therefore, if (8a) is satisfied, we also have

$$\begin{aligned} m_X^2 &= (k_B + k_C)^2 = m_C^2 + m_B^2 + 2m_B E_C , \\ (k_A + k_C)^2 &= m_A^2 + m_C^2 + 2m_A E_C \equiv s_{AC} , \end{aligned} \tag{9}$$

so

$$\begin{aligned} E_C &= (m_X^2 - m_B^2 - m_C^2)/2m_B , \\ s_{AC} &= m_A^2 + m_C^2 + m_A(m_X^2 - m_B^2 - m_C^2)/m_B . \end{aligned} \tag{10}$$

Thus, *in the special case*

$$m_A = m_B, \quad m_X = m_Y, \tag{11}$$

this reduces to $s_{AC} = m_X^2 = m_Y^2$; i.e., condition (7b) of the Peierls case is satisfied, *and hence $s_c = s_+$*. Needless to say, the cases[12] considered by Nauenberg and Pais[2] *do satisfy condition (11)*. Hence, although the sheet difficulties[6] prevent direct use of the Peierls s_+ singularity, the *empirical successes recorded at that time can now be understood in the Brayshaw framework*.

Note that as a practical rule, $m_X = m_Y$ usually forces $m_A = m_B$, so the Peierls s_+, with only one type of resonance involved, equals s_c. A counterexample would be a diagram (cf. Fig. 2), where $m_A = m_C = m_\pi$, $m_B = m_N$, $m_X = m_\Delta$, and we take AC to be at resonance $m_Y = m_\rho$. Here s_+ and s_c are *quite different*. For the baryon system, we shall see below that the new outlook presented by Brayshaw (with no sheet problems) enables us now to proceed and identify possible octet partners of the $I = \frac{1}{2}$, $J^P = \frac{1}{2}^+$ Roper resonance $N^*(1470)$.

The Roper $J^P = \frac{1}{2}^+$ octet? Brayshaw[8] had already pointed out that taking $X = \Delta(1236)$, $A = B = \pi$, $C = N$ in Fig. 1, the mechanism for $(N\pi\pi)$ yielded a mass of 1488 MeV, which may correspond to the Roper $N^*(1470)$—especially as the $\pi^+\pi^- \to \pi^0\pi^0$ scattering of Eq. (4) is in $I = 0$, S wave near threshold. Choosing $X = \Sigma_1^*(1385)$, $C = \Lambda(\Sigma)$, and $X = \Xi^*(1530)$, $C = \Xi(1320)$ of the decuplet with $A = B = \pi$ would suggest the following entries for completion of the Roper octet[13] with $J^P = \frac{1}{2}^+$:

$$\begin{aligned} \Lambda^*:&\ \sqrt{s_c} = 1.62 \text{ GeV} , \\ \Sigma^*:&\ \sqrt{s_c} = 1.57 \text{ GeV} , \\ \Xi^*:&\ \sqrt{s_c} = 1.74 \text{ GeV} . \end{aligned} \tag{12}$$

The baryon states of Eq. (12) are expected to share the peculiar properties of $N^*(1470)$ (and A_1), namely, absence of a simple Breit-Wigner signal in hadron-initiated strong processes, different width properties in diffractive and nondiffractive experiments, etc.[14] From the Dalitz quark-model viewpoint,[15] candidates for the Roper octet are $\Lambda^*(1600)$ and $\Sigma^*(1660)$ which may belong to an overall $(56, 0^+)_{N=2}$, where N denotes the harmonic-excitation bands in a shell-model picture. Hence there is nothing inconsistent with the quark-model prediction for Λ^*, and it would be of substantial interest to search for the $\Xi^*(1.74)$ with $J^P = \frac{1}{2}^+$ (we shall comment further on the situation with respect to $\Sigma^*(1660)$ of the quark model below). Again for the above baryon cases, $s_c = s_+ = 2(m_{b^*}^2 + m_\pi^2) - m_b^2$ (where b and b^* are the appropriate baryon and baryon resonance, respectively). However, the assignment of spin-parity and, sometimes, isospin is different. For instance, the Peierls s_+ mechanism sought explanations[1,5,11] of $J^P = \frac{3}{2}^-$ $N^*(1512)$ and a Σ_1^* in the vicinity of 1645 MeV; the dynamics of these $J^P = \frac{3}{2}^-$ states needs to be understood differently, perhaps in terms of a Ball-Frazer mechanism.[16] We have restricted our application here to X identified with well-known low-lying resonances. Removing this restriction would enable us to predict the positions of a host of other states with either parity. However, in these cases where sufficient phase space is available (for three-body systems with X a high-lying resonance), the Brayshaw mechanism might well produce closely degenerate resonances in two or more J^P states and hence become difficult to sort out experimentally. We conclude with an example of X identified with a "higher" resonance [belonging to $(70, 1^-)_{N=1}$], where phase space is not "forbidding," namely, $X = \Lambda(1405)$. With $A = B = \pi$, and $C = \Sigma$, Eq. (3) yields $\sqrt{s_c} = 1.65$ GeV. The predicted $[(\pi\pi)_{I=0}\Sigma]$

state has $I=1$, and given the phase space available to the $\epsilon\Sigma$ configuration [where $\epsilon=(\pi\pi)_{I=0}$], a $J^P=\frac{1}{2}^+$ assignment is most likely. Such a state may well be related to the $\Sigma^*(1660)$ with $J^P=\frac{1}{2}^+$ of the quark shell model mentioned above.

Application to $\phi(1.020)$ *and the charm particles.* The $\phi(1.020)$ is a good testing case for the mechanism[8] since ϕ has relatively narrow strong decay width into $K^\circ\bar{K}^\circ$, and the S-wave scattering $K^\circ\bar{K}^\circ\to K^+K^-$ [Eq. (4)] can be enhanced by threshold-type resonances in final states like the $I=1$ $\delta(970)$ and the $I=0$ $S^*(993)$. Setting $X=\phi$, $A=\bar{K}^\circ$, $B=K^\circ$, $C=\bar{K}^\circ$, and $(A', B')=(K^+, K^-)$, we obtain $\sqrt{s_c}=1.52$ GeV. The final three-body ($K\bar{K}K$) system has $I=\frac{1}{2}$, and because of the small phase space available, the three K mesons are likely to be in mutual S-wave pairs leading to a suggested $J^P=0^-$ for the predicted 1.52-GeV state. Application to charmed particles leads to the results found in Table I. In Table I, the following points are to be noted: (a) Experiments[17] suggest that $D^{*\circ}\to D^+\pi^-$ is forbidden from phase-space considerations though $D^{*+}\to D^\circ\pi^+$ is allowed; hence given the constraint embodied by Eq. (4), it is possible that the predicted state at 2.15 GeV appears in $(D^\circ\pi\pi)^\circ$ but not in $(D^+\pi\pi)^+$. (b) The charmed baryons $\Lambda_c(2.260)$, $\Sigma_c^*(2.426)$, and $\Sigma_c^*(2.500)$ are assumed to have $J^P=\frac{1}{2}^+$, $\frac{1}{2}^+$, and $\frac{3}{2}^+$, respectively, as appear to be consistent with current data and mass systematics.[18,19] (c) The J^P values given in Table I are mere suggestions based on available phase space rather than firm predictions.

TABLE I. Results from application to charmed particles.

X	$A\,(A')$	$B\,(B')$	C	$\sqrt{s_c}$ (GeV)	(I, J^P)
$\psi''(3.772)$	D	$\bar{D}$	D	5.65	$(\frac{1}{2}, 0^-)$
$D^*(2.007)$	π	π	D	2.15	$(\frac{1}{2}, 0^-)$
$\Sigma_c^*(2.426)$	π	π	$\Lambda_c(2.260)$	2.59	$(0, \frac{1}{2}^+)$
$\Sigma_c^*(2.500)$	π	π	$\Lambda_c(2.260)$	2.73	$(0, \frac{1}{2}^+)$

Finally, we comment on the empirically unexplored aspects of the Peierls mechanism,[1] in particular for low-lying excitations of three-particle systems involving a baryon isobar. Brayshaw and Peierls[20] have argued convincingly that though the s_+ singularity is unphysical for the cases we treat,[21] the singularity at s_- [cf. Eq. (6)] is on the physical sheet, and a three-particle resonance might be expected to develop near s_-. The actual occurrence of such resonances is, of course, still a quantitative question (it has been applied to π-d scattering by one of us[22]). However, it is clear that low-lying excitations of three-particle systems involving a baryon isobar would be good candidates to examine. The $\sqrt{s_-}$ values have been summarized[11] for the low-lying baryon isobars (with appropriate u-channel baryon exchange) as follows:

	$(\Delta(1238), \pi, N)$	$(\Sigma^*(1385), \pi, \Lambda)$	$(\Xi^*(1530), \pi, \Xi)$
$\sqrt{s_-}$ (MeV)	$1605-i130$	$1700-i60$	$1760-i7$

(13)

If one assumes that the c.m. momentum of pion-isobar scattering is small, then the lowest partial wave (S wave) is likely to be dominant, and a well-defined angular momentum structure may result. Three particle resonances with $J^P=\frac{3}{2}^-$ and masses near $R\ \sqrt{s_-}$ [given by Eq. (13)] are thus suggested, namely, $N^*(1605)$, $\Sigma^*(1700)$, and $\Xi^*(1760)$. Crudely we may regard $\sqrt{s_-}=E_r-\frac{1}{2}i\Gamma$, where E_r and Γ are, respectively, the resonance mass and width.

The data[3] do not support the existence of an $N^*(1605)$; nor for the meson system does there appear to be a 1^+ $K^*(1550)$ which might evolve from $(K^*(888), \pi, K)$, where $\sqrt{s_-}(\text{MeV})=1550-i90$. So a qualitative conclusion is that for $\sqrt{s_-}$ to be not too far from the real axis (thus not yielding a difficult-to-detect very broad resonance), the half "width" $\frac{1}{2}\Gamma$ has to be smaller than 90 MeV, unless some peculiar inelastic property is associated with the predicted state. The situation with respect to a predicted $\Sigma^*(1700)$ from s_- is far more promising. The data[3,23] refer to a $\Sigma^*(1695)$ with a large branching ratio into the $(\pi\Lambda)$ channel seen in production experiments.[24] There is no well-established bump as a candidate for $\Xi^*(1760)$ to date[23]; such a state can only be searched for in production experiments, e.g., via $K^-p\to K^+\Xi$. In view of the narrow "width" $\frac{1}{2}\Gamma=7$ MeV associated with $\sqrt{s_-}$ (and hence its proximity to the real axis), we urge *concentrated effort for an experimental search of the associated nearby* $\Xi^*(1760)$ *resonance of relatively narrow width.*

We can also explore s_- effects involving $\phi(1020)$, psions, and charm particles. For instance, $\sqrt{s_-}(\text{MeV})=2071.2-i8.3$ for $(\phi(1020), K, \bar{K})$, might generate an $(I, J^P)=(\frac{1}{2}, 1^+)$ state near 2071.2 MeV; $\sqrt{s_-}(\text{MeV})=7627.1-i56.7$ for $(\psi''(3772), D, \bar{D})$ might generate an $(I, J^P)=(\frac{1}{2}, 1^+)$ state near 7627.1

MeV assuming that $\psi''(3772)$ is an $I=0$ state of width 28 MeV; and $\sqrt{s_-}$(MeV) = 2151.8 − i2.2 for ($D^*(2007)$, π, D) might generate a state with $J^P=1^+$ near 2151.8 MeV assuming that D^* has its upper-limit width[25] of 2 MeV and D mass is taken to be 1863 MeV throughout.

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[13]We have identified here the position of the predicted state with $\sqrt{s_c}$. This is an approximation since there are width corrections, etc., to be taken into account (cf. Ref. 8). However, a detailed check by one of us (D.D.B.) supports the approximate version here outlined.

[14]A critical comparison with the experimental situation would be entirely analogous to the detailed analysis for the ($K\pi\pi$) system [$Q_1(1200)$] recently performed by one of us [D. D. Brayshaw, Report No. SLAC-PUB-2130 (unpublished)].

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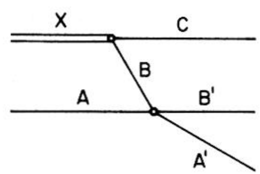

FIG. 1. Rescattering diagram which generates the Brayshaw singularity. The vertex blobs correspond to off-shell scattering amplitudes, and A', B' are at threshold. The situation in which the subenergy $s_{A'B'}$ coincides with a resonant energy of $A' + B'$ gives rise to the Peierls singularity.

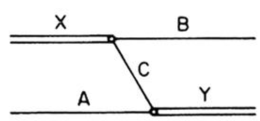

FIG. 2. Rescattering diagram which generates the Peierls singularity. The vertex blobs correspond to off-shell scattering amplitudes, where Y is a resonance.