

Diffraction scattering and the parton structure of hadrons

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We apply parton-model concepts to the "soft" (small-momentum-transfer) processes which make up the majority of the hadronic total cross section. Diffraction is calculated as the shadow of these soft processes. We obtain an attractive picture of the essential features of total, elastic, and diffractive-inelastic scattering. In particular, the rather large cross section for inelastic diffraction, which is observed experimentally, results from fluctuations in the distribution of the wee partons which initiate interactions. These fluctuations lead to diffractive production by the mechanism of Good and Walker. The observed peripheral character of diffraction as a function of impact parameter, the absence of a forward dip in $d\sigma/dt dm^2$, the correct integrated cross section, and the correct small- t slope of diffraction all follow naturally in our approach.

I. INTRODUCTION

Consider the interaction of a hadronic particle with a hadronic target at high energy. There is strong production of a rich variety of inelastic states. Through s -channel unitarity, this production implies a large imaginary elastic amplitude at impact parameter $b \lesssim 1$ fm, which gives rise to the forward peak in $d\sigma/dt$ for elastic scattering. In the language of optics, the forward peak is thus due to "diffraction," or "shadow scattering."

Hadrons are composite objects. Our incident high-energy particle is therefore a quantum-mechanical superposition of states which contain various numbers, types, and configurations of constituents. The various states in this superposition are absorbed in different amounts by the target, so the superposition of states which arises from shadow scattering is not simply proportional to the incident one. Hence shadow scattering leads not only to elastic scattering, but also to production of inelastic states which have the same internal quantum numbers as the incident particle. This fundamental basis for inelastic diffraction has been known for a long time.¹

Inelastic diffraction arises from the *differences* in absorption probabilities for various components of the hadron's wave function. The fact that these absorption probabilities must lie between 0 and 1 leads to an upper bound, $\sigma_{\text{diff}}(b) + \sigma_{\text{el}}(b) \lesssim \frac{1}{2} \sigma_{\text{tot}}(b)$, which limits the size of diffractive production at each impact parameter.²⁻³ Measurements in inelastic diffraction at the CERN ISR⁴ fall within a factor of 2 of saturating this bound.⁵ This implies a very large spread in the interaction probabilities, as has been shown quantitatively by Fiałkowski and Miettinen.⁶ Thus, for example, in a head-on pp collision ($b=0$), there exist some arrange-

ments of the constituents for which the interaction probability is nearly zero, and other arrangements for which it is nearly unity. The average of these probabilities, which can be deduced from the elastic data, is about 0.75.

The large cross section observed for diffractive production raises two major questions which we attempt to answer in this paper:

1. What are the states which diagonalize the diffractive part of the S matrix, so that their interactions are described simply by absorption coefficients?
2. What causes the large variations in the absorption coefficients at a given impact parameter, which are implied by the large cross section for diffractive production?

Our answer to the first question is that the diagonal states are the states of the parton model.⁷ They are characterized by a definite number N of partons, which have definite impact parameters $\vec{b}_1, \dots, \vec{b}_N$, and definite longitudinal momenta, which we describe in terms of rapidities y_1, \dots, y_N . The partons are structureless "pointlike" constituents. [It is attractive to hypothesize that they are the valence quarks + sea quarks + gluons of quantum chromodynamics (QCD). However, we will not make any use of such hypothesis in this paper. We assume the parton-parton interactions to be of short range in rapidity. This *could* be true for the effective interactions in QCD—in spite of the spin-1 character of the gluons—because of color confinement effects.]

Interactions in the parton model have a short range in rapidity. This fits the observation that two-particle correlations in the central region of rapidity are small for rapidity separations $|y| \gtrsim 2$.⁸ When two particles scatter at very high energy, in the center of mass, only wee partons can interact,

because the parton-parton interaction is of short range and the wave functions do not contain partons which move fast in the "wrong" direction. Since the wee partons are responsible for initiating the soft hadronic collisions, which build up the diffractive cross sections through unitarity, we see that the global properties of diffraction depend on the distribution and interactions of the wee partons alone. Furthermore, since hadronic total cross sections are roughly energy independent, we see that the wee-parton distributions must be roughly independent of the momenta of the parent particles.

In our search for a physical interpretation of the states which diagonalize the diffractive part of the S matrix (the "bare particle" states of Good and Walker) we were led to the parton-model approach through the following considerations. First, let us recall our basic assumption about the dynamical origin of inelastic diffraction, namely, that dissociation processes are *regeneration* processes. They are caused by the fact that hadrons are composite systems, and the different components of the wave functions are absorbed in different amounts by the target. From this assumption it follows directly that, in order to find states which scatter only nondiffractively and—through shadow scattering—elastically, but which do not undergo dissociation, we must find states which have *no internal structure*.

Consider then a state consisting of a fixed number of constituents at fixed impact parameters and fixed rapidities,

$$|\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N\rangle.$$

Assume, furthermore, that the constituents have no internal structure. Consider the collision of this state by an absorbing potential. We see that, since all the variables on which the absorption of the state may depend have fixed values, the state will be absorbed with a well defined absorption strength. Since the state does not regenerate, and since we assumed diffraction dissociation to be due to regeneration, this state is indeed an eigenstate of diffraction. This leads us to identify the parton states as the eigenbasis for diffraction, which is our answer to question 1.

Among the parton states which describe a high-energy hadron, there are some which are rich in wee partons, and are therefore likely to interact, while other states have few or no wee partons, and correspond to the transparent channels of diffraction. This is our answer to question 2. A similar point of view has been advocated by Grassberger.⁹ In more detail, we will show that the fluctuations in interaction probability which generate diffractive production arise in three different ways: from fluctuations in the *number* of wee partons, in their

rapidities, and in their *impact parameters*.

The organization of this paper is as follows. In Sec. II we express the above ideas in mathematical form. In Sec. III we present a simple model which incorporates our ideas, and demonstrate that it agrees with the essential experimental observations. In Sec. IV, we use the model to solve a long-standing problem regarding the t dependence of diffractive production. In Sec. V we restate our conclusions, compare our analysis to some previous work, and suggest some directions for future work.

II. OPTICAL-MODEL FORMULATION

To obtain a framework of analysis which is mathematically and physically simple, we replace the target particle by an average optical potential. This approximation leaves intact the essential physics of the diffraction of the beam particle, while simplifying the analysis.

The beam particle is a linear combination of states which are eigenstates of diffraction:

$$|B\rangle = \sum_k C_k |\psi_k\rangle, \quad (1)$$

$$\text{Im}T |\psi_k\rangle = t_k |\psi_k\rangle, \quad (2)$$

where $\text{Im}T$ is the imaginary part of the scattering amplitude operator, $\text{Im}T = 1 - \text{Re}S$, and the eigenvalue t_k is the probability for the state $|\psi_k\rangle$ to interact with the target. These eigenvalues vary, of course, with impact parameter. We normalize so that

$$\langle B|B\rangle = \sum_k |C_k|^2 = 1. \quad (3)$$

The imaginary part of the elastic amplitude is

$$\langle B|\text{Im}T|B\rangle = \sum_k |C_k|^2 t_k \equiv \langle t \rangle. \quad (4)$$

In other words, it is given by the average over absorption coefficients, which are weighted according to their probability of occurrence in the particle $|B\rangle$. The total cross section and the elastic cross section (ignoring any contribution from the real part) are given by

$$d\sigma_{\text{tot}}/d^2\vec{b} = 2\langle t \rangle, \quad (5)$$

$$d\sigma_{\text{el}}/d^2\vec{b} = \langle t \rangle^2. \quad (6)$$

The cross section for diffractive production, with elastic scattering removed, is

$$\begin{aligned} d\sigma_{\text{diff}}/d^2\vec{b} &= \sum_k |\langle \psi_k | \text{Im}T | B \rangle|^2 - d\sigma_{\text{el}}/d^2\vec{b} \\ &= \sum_k |C_k|^2 t_k^2 - \left(\sum_k |C_k|^2 t_k \right)^2 \\ &= \langle t^2 \rangle - \langle t \rangle^2. \end{aligned} \quad (7)$$

Hence inelastic diffraction is proportional to the dispersion $\langle (t - \langle t \rangle)^2 \rangle$ in cross sections for the diagonal channels. Equations (5)–(7) imply $\frac{1}{2}\sigma_{\text{tot}}(b) - \sigma_{\text{el}}(b) - \sigma_{\text{diff}}(b) = \langle t \rangle - \langle t^2 \rangle$. Hence the requirement $0 \leq t_k \leq 1$ for the absorption probabilities leads to the upper bound $\sigma_{\text{diff}} + \sigma_{\text{el}} \leq \frac{1}{2}\sigma_{\text{tot}}^{2,3,6}$. The bound is saturated if each t_k is either completely transparent or fully absorbed, so that $\langle t \rangle = \langle t^2 \rangle$.

Our basic assumption is that the eigenstates of diffraction are parton states, so Eq. (1) takes the form

$$|B\rangle = \sum_N \prod_{i=1}^N \int d^2\vec{b}_i dy_i C_N(\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N) \times |\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N\rangle. \quad (8)$$

The labels in C_N refer to impact parameters and rapidities of the wee partons. The large-momentum parton labels are not indicated explicitly, since these partons have negligible probability to interact with the target. A sum over those labels is implicit whenever matrix elements are calculated. We assume for simplicity that the parton interactions are independent of spin. The impact-parameter variables \vec{b}_i are defined relative to the impact parameter of the incident particle. Hence a sum over all of the partons would yield $\sum_i \vec{b}_i = 0$; but the contribution of the wee partons to this sum is negligible, so it provides no constraint on them.

For simplicity, we shall consider a model in which the wee partons are not correlated with each other. The total probability associated with N wee partons is then given by a Poisson distribution, with mean number G^2 . We have

$$|C_N(\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N)|^2 = e^{-G^2} (G^{2N}/N!) \prod_{i=1}^N |C_i(\vec{b}_i, y_i)|^2, \quad (9a)$$

where

$$\int d^2\vec{b}_i dy_i |C(\vec{b}_i, y_i)|^2 = 1. \quad (9b)$$

Next we calculate the probability of a given parton state to interact with the target. If the probability for a parton i to interact is τ_i , then the probability for it *not* to interact is $1 - \tau_i$, the probability for N partons *not* to interact is $\prod_{i=1}^N (1 - \tau_i)$, and hence the probability for one or more of the N partons to interact is $1 - \prod_{i=1}^N (1 - \tau_i)$. We have calculated this quantity using the “conservation of prob-

ability” explicitly. Our results therefore depend directly on s -channel unitarity.

Our optical model is now completely specified. To summarize it, we have independent wee-parton states

$$|\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N\rangle,$$

which appear in the incident particle with probability

$$e^{-G^2} G^{2N}/N! \prod_{i=1}^N |C(\vec{b}_i, y_i)|^2 d^2\vec{b}_i dy_i, \quad (10)$$

and which interact with the target with probability

$$t(\vec{b}_1, \dots, \vec{b}_N; y_1, \dots, y_N; \vec{b}) = 1 - \prod_{i=1}^N (1 - \tau(\vec{b}_i + \vec{b}, y_i)), \quad (11)$$

where \vec{b} is the impact parameter of the incident hadron, and $\tau(\vec{b}', y)$ is the interaction probability for a single parton.

The cross sections as a function of impact parameter are determined by moments of the absorption spectrum according to Eqs. (5)–(7). Because the wee-parton distributions have been assumed to be independent, these moments take on a simple eikonal form:

$$\langle t \rangle = 1 - e^{-G^2 \langle \tau \rangle}, \quad (12)$$

$$\langle t^2 \rangle - \langle t \rangle^2 = e^{-2G^2 \langle \tau \rangle} (e^{+G^2 \langle \tau^2 \rangle} - 1), \quad (13)$$

where

$$\langle \tau^n \rangle = \int d^2\vec{b}_1 dy_1 |C(\vec{b}_1, y_1)|^2 [\tau(\vec{b}_1 + \vec{b}, y_1)]^n \quad (14)$$

are moments of the single-particle interaction probability at impact parameter \vec{b} .

The parton states, which are eigenstates for diffraction, are the same at all values of the overall impact parameter \vec{b} . Equations analogous to (6) and (7) therefore hold for the momentum-space amplitudes, which are two-dimensional Fourier transforms of the impact-parameter amplitudes. In this way, we obtain

$$d\sigma_{\text{el}}/d^2\vec{q} = \frac{1}{4\pi^2} \langle \bar{t} \rangle^2, \quad (15)$$

$$d\sigma_{\text{diff}}/d^2\vec{q} = \frac{1}{4\pi^2} (\langle \bar{t}^2 \rangle - \langle \bar{t} \rangle^2), \quad (16)$$

where

$$\langle \bar{t}^j \rangle = \sum_{N=0}^{\infty} e^{-G^2} (G^{2N}/N!) \prod_{i=1}^N \int d^2\vec{b}_i dy_i |C(\vec{b}_i, y_i)|^2 \left\{ \int d^2\vec{b} e^{i\vec{q}\cdot\vec{b}} \left[1 - \prod_{k=1}^N (1 - \tau(\vec{b}_k + \vec{b}, y_k)) \right]^j \right\}. \quad (17)$$

The averages needed for Eqs. (15) and (16) again take an eikonal form, and can be simplified to

$$\langle \vec{t} \rangle = \int d^2 \vec{b} e^{i \vec{q} \cdot \vec{b}} \langle t \rangle, \quad (18)$$

where $\langle t \rangle$ is given by Eqs. (12) and (14), and

$$\langle \vec{t}^2 \rangle - \langle \vec{t} \rangle^2 = \int d^2 \vec{b} d^2 \vec{b}' e^{i \vec{q} \cdot (\vec{b} - \vec{b}')} \exp\{-G^2[\langle \vec{\tau}(\vec{b}) \rangle + \langle \vec{\tau}(\vec{b}') \rangle]\} \{\exp[G^2 \langle \vec{\tau}(\vec{b}) \vec{\tau}(\vec{b}') \rangle] - 1\}, \quad (19)$$

where $\tau(\vec{b})$ is given by Eq. (13) and

$$\begin{aligned} \langle \tau(\vec{b}) \tau(\vec{b}') \rangle &= \int d^2 \vec{b}_1 d y_1 |C(\vec{b}_1, y_1)|^2 \\ &\times \tau(\vec{b}_1 + \vec{b}, y_1) \tau(\vec{b}_1 + \vec{b}', y_1). \end{aligned} \quad (20)$$

Hence we can calculate the differential cross section $\int d\sigma/(dtdm^2)dm^2$ —except for possible t_{\min} effects which we neglect.

This completes the formalism of the optical model. Given G^2 , which specifies the average number of wee partons; $|C(\vec{b}, y)|^2$, which specifies the single-parton probability distribution; and $\tau(\vec{b}, y)$, which specifies the single-parton interaction probability, we can calculate the elastic and diffractive cross sections versus impact parameter, using Eqs. (5), (6), (12), and (13), or versus momentum transfer, using Eqs. (15), (16), (18), and (19).

III. SPECIFIC MODEL

In this section, we choose explicit forms for the functions which characterize our optical model. We choose the numerical constants in these forms according to physical arguments, and in accord with the known elastic amplitude. With all parameters of the theory thus determined, we *predict* the inelastic diffractive cross section, and find that it agrees with the observed magnitude and momentum transfer dependence.

A detailed fit to elastic and inelastic diffractive data would be inappropriate, in view of the simplifying assumptions built into the optical potential approach. Our purpose here is rather to investigate whether the parton viewpoint correctly accounts for the behavior of inelastic diffraction "semiquantitatively," without the aid of adjustable parameters or unmotivated assumptions. The parameters in our model are entirely determined from the parton point of view, together with information from *elastic* scattering.

To describe the single-wee-parton probability distribution at a given energy, we choose

$$|C(\vec{b}, y)|^2 = K \exp(-|y|/\lambda - \vec{b}^2/\beta) \quad (21)$$

in Eq. (9a). The normalization $K = 1/(2\pi\beta\lambda)$ is required by Eq. (9b). Thus we have three parameters to describe the wee-parton distribution: G^2 , the average number; λ , the width in rapidity; and β , the mean square distance in impact parameter

from the hadron which produced them.

To describe the interaction probability of a single wee parton, we choose

$$\tau(\vec{b}, y) = A \exp(-|y|/\alpha - \vec{b}^2/\gamma) \quad (22)$$

in Eq. (11). Thus we have three more parameters: A , the maximum interaction probability, where $0 < A < 1$; α , the range in rapidity; γ , the range in impact parameter.

Our model appears to have six parameters, but actually has only five because, when the integrals over parton rapidities are carried out, the parameters α and λ enter only as their ratio. We choose the parameters as follows. We set $A = 1$, its maximum possible value, because the short-range parton interaction is expected to be very strong. We set $\alpha/\lambda = 2.0$ and $\gamma/\beta = 2.0$ because the range in y and b of the optical potential includes effects due both to the target particle wave function and due to the parton-parton interaction, so it should be somewhat larger than the range of the beam particle wave function alone. We shall later vary the values of these three parameters over a wide range and study how our results depend on these parameters.

We choose the remaining two parameters of the model to obtain $\sigma_{\text{tot}} = 43$ mb and $\sigma_{\text{el}} = 8.7$ mb, which are appropriate to pp scattering at $\sqrt{s} = 53$ GeV.¹⁰ In this way, we find $\beta = 6.0$ GeV⁻² and $G^2 = 2.93$.

With the model parameters thus determined, we calculate the cross section for beam dissociation and find 3.2 mb. This result is large enough to agree with experiments, which report 2.5–3.5 mb (i.e., 5–7 mb for beam dissociation + target dissociation + double dissociation).¹¹

The impact-parameter dependences of the cross sections are shown in Fig. 1. We see that the cross section for inelastic diffraction is strikingly different in shape from that of elastic scattering. The elastic cross section is roughly Gaussian, while the inelastic diffractive cross section is much more spread out and actually peaks away from $b = 0$. We also note that the diffractive cross section lies everywhere below the unitarity bound $\frac{1}{2}\sigma_{\text{tot}}(b) - \sigma_{\text{el}}(b)$. This was of course to be expected, since our model is based on the very unitarity considerations which lead to the bound.

Inelastic diffraction arises from fluctuations in the cross sections for the diagonal states at each

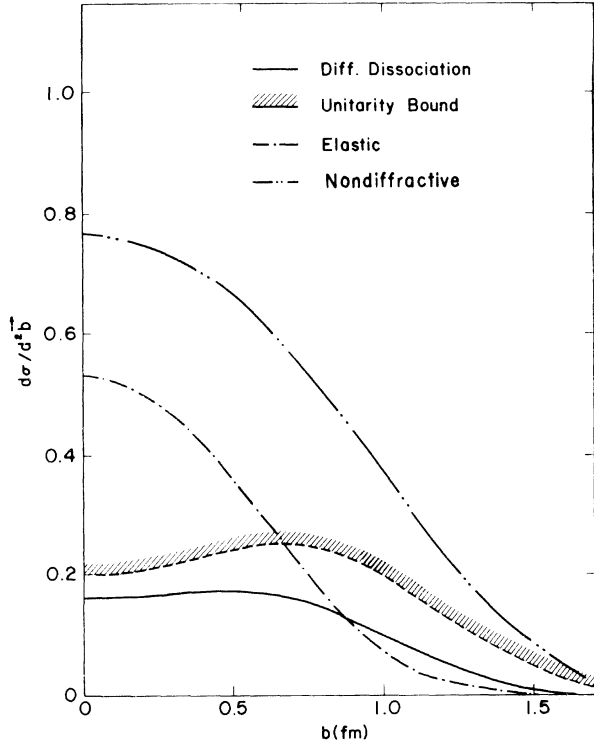


FIG. 1. The impact-parameter dependence of inelastic diffraction predicted by our model (solid curve). It corresponds to an integrated cross section $\sigma_{\text{diff}} = 6.5$ mb, which is consistent with experiment (see Ref. 4). It is also consistent with the bound $\sigma_{\text{diff}}(b) \leq \frac{1}{2} \sigma_{\text{tot}}(b) - \sigma_{\text{el}}(b)$ (/////), which follows from unitarity. Observe that inelastic diffraction is much more peripheral in impact parameter than elastic scattering (---) or the nondiffractive cross section $\sigma_{\text{tot}}(b) - \sigma_{\text{el}}(b) - \sigma_{\text{diff}}(b)$ (-·-·-·-). The parameters of the model were chosen so that $d\sigma_{\text{el}}/dt$ approximates the known elastic scattering at $\sqrt{s} = 53$ GeV.

impact parameter. These fluctuations result from variations in the number N , rapidities y_i , and impact parameters \vec{b}_i of the wee partons. We can use our model to estimate the relative contributions from these three sources of fluctuations. This is done as follows:

(1) To observe the effect of the y_i fluctuations, we look at what happens when they are removed. This is easily done by averaging over y_i before calculating the dispersion in the diagonal cross sections—i.e., using

$$\langle \tau^n(b) \rangle \rightarrow \int d^2\vec{b}_i \left[\int dy_1 |C(\vec{b}_1, y_1)|^2 \tau(\vec{b}_1 + \vec{b}, y_1) \right]^n \quad (23)$$

in place of Eq. (14). This leaves the first moment $\langle \tau(b) \rangle$, and with it the elastic amplitude, unchanged. But it reduces σ_{diff} from 6.47 to 5.67 mb. Thus the y_i fluctuations contribute only about 12% to σ_{diff}

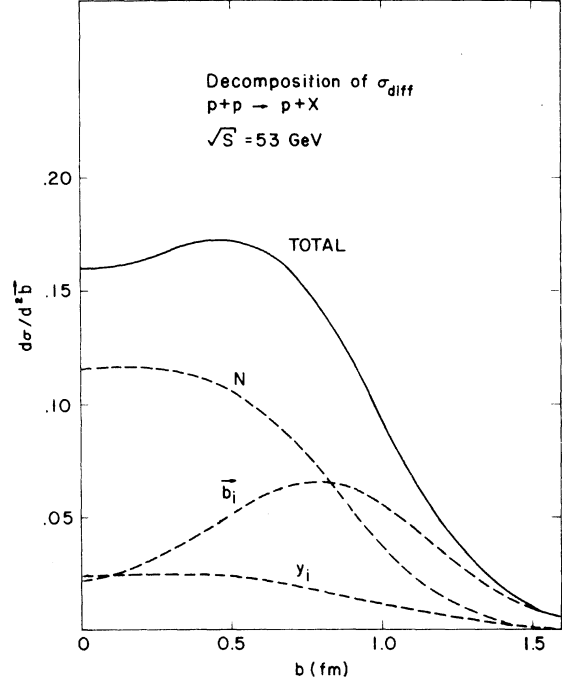


FIG. 2. The diffractive cross section as a function of impact parameter, repeated from Fig. 1, and the contributions to it from fluctuations in the number (N), rapidities (y_i), and relative impact parameters (\vec{b}_i) of the wee partons. The \vec{b}_i fluctuations are responsible for the very peripheral nature of $\sigma_{\text{diff}}(b)$, and contribute about $\frac{1}{2}$ of the integrated value 6.5 mb.

in this model. By subtracting the modified $d\sigma_{\text{diff}}/d^2\vec{b}$ from the complete one, we can infer the b dependence of the y_i -fluctuation effect. This is done in Fig. 2. We see that the cross section due to y_i fluctuations is more centrally distributed than the total diffractive inelastic cross section, although not quite as central as the elastic cross section shown in Fig. 1.

(2) The effect of the \vec{b}_i fluctuations can similarly be observed by averaging over \vec{b}_i before calculating the dispersion. That is, we replace Eq. (14) by

$$\langle \tau^n(b) \rangle \rightarrow \int dy_1 [d^2\vec{b}_1 |C(\vec{b}_1, y_1)|^2 \tau(\vec{b}_1 + \vec{b}, y_1)]^n. \quad (24)$$

This reduces σ_{diff} from 6.47 to 3.47 mb. Thus the \vec{b}_i fluctuations account for about 46% of σ_{diff} . To estimate the b dependence of the \vec{b}_i -fluctuation contribution we proceed as we did with the y_i fluctuations, and subtract the modified $d\sigma_{\text{diff}}/d^2\vec{b}$ from the complete one. The result, shown in Fig. 2, is an extremely peripheral distribution which peaks near $b = 0.8$ fm.

(3) The contribution of N fluctuations can be seen by averaging over both \vec{b}_i and y_i before calculating the dispersion, so that $\langle \tau^n(b) \rangle$ is replaced by $\langle \tau(b) \rangle^n$.

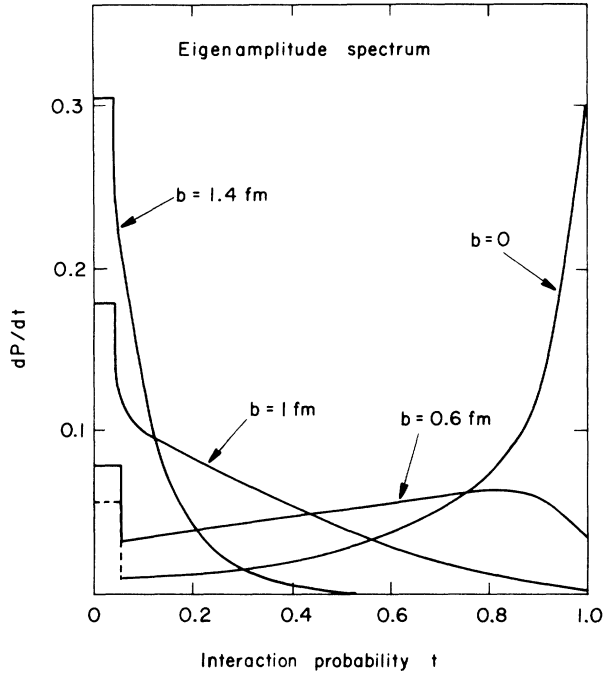


FIG. 3. The distribution of interaction probabilities at various impact parameters. The large dispersion in these probabilities is responsible for the large cross section for beam dissociation, which is observed experimentally. Note that these "eigenamplitudes" have already been averaged over the target configurations. In our model, there is a component of the parton wave function which contains zero wee partons, and which therefore does not interact. It accounts for $e^{-\sigma^2} = 5.4\%$ of the wave function. In order to display its effect, these curves are averaged in the region from 0 to 0.05.

This again preserves the fit to elastic scattering. It reduces σ_{diff} to 3.04 mb. Thus the N fluctuations contribute about 47% of the inelastic diffractive cross section. The b distribution of the N fluctuation, shown in Fig. 2, is quite central in shape.

(4) The above contributions to σ_{diff} are not strictly additive—even though we assume no correlations between \vec{b}_i , y_i , and N in the wave function. Our analysis based on eliminating their contributions one at a time is nevertheless reasonable. As a check of this, we note that the three contributions estimated above add up to 105%, which is reasonably close to 100%. Furthermore, we have repeated the calculations for various permutations of the order in which the various fluctuations were eliminated, and obtained essentially no changes in the results. We thus conclude that the contribution to diffraction due to y_i , \vec{b}_i , and N fluctuations is approximately 10%, 45%, and 45%, respectively.

(5) A comparison of the y_i -, \vec{b}_i -, and N -fluctuation components to dissociation shown in Fig. 2 teaches us that the peripheral shape of the total

dissociation cross section is mainly due to the large and very peripheral \vec{b}_i -fluctuation component.

The spectrum of absorption probabilities for various impact parameters is shown in Fig. 3. Note that these probabilities have already been averaged over the target configurations (this was implicitly done when we replaced the target by a homogeneous optical potential). If both the beam and the target were described in terms of partons, the full nonsmeared probabilities would then be even more spread out and e.g. the $b = 0.6$ -fm distribution would be strongly double-peaked in agreement with the analysis of Ref. 6.

The distribution of eigenchannel cross sections is shown in Fig. 4. One sees that the 43-mb cross section which is observed in pp scattering at $\sqrt{s} = 53$ GeV is actually an average over cross sections for parton configurations which vary enormously. Now Eq. (16), together with the optical theorem, gives us

$$d\sigma_{\text{diff}}/d^2\vec{q}|_{|\vec{q}|=0} = \langle\sigma^2\rangle - \langle\sigma\rangle^2. \quad (25)$$

From Fig. 4 we see that the main contribution to the very large dispersion of the total cross-section spectrum comes from the variation in the number of wee partons. We may thus anticipate that the small- t dissociation is mainly due to N fluctuations.

The momentum-transfer distribution of the beam-dissociation cross section is shown in Fig. 5. The data, which come from a CERN ISR experiment,⁴ have been integrated over the x range $0.95 < |x| < 1.0$ of the target proton. We see that the model prediction is in excellent agreement with experiments, both in magnitude and in shape. The predicted value for the forward slope is about 6.9 GeV^{-2} .

The decomposition of the diffractive cross section $d\sigma_{\text{diff}}/d^2\vec{q}$ into its various components can be carried out in the same way as for $d\sigma_{\text{diff}}/d^2\vec{b}$. From the result shown in Fig. 5 we learn two important lessons. Firstly, the \vec{b}_i -fluctuation component is very broad and dominates the total spectrum at large momentum transfer. It peaks around $t = -0.1 \text{ GeV}^2$ and nearly vanishes in the forward direction. The reason for this forward dip will be discussed in the next section. Secondly, small- t dissociation is seen to be dominated by the large and very steep (slope $\approx 12.2 \text{ GeV}^{-2}$) N fluctuation, in agreement with our expectation based on the eigen cross-section spectrum of Fig. 3. (The reader may be surprised that the \vec{b}_i fluctuations make both $d\sigma_{\text{diff}}/d^2\vec{b}$ and $d\sigma_{\text{diff}}/dt$ more broad; however, these cross sections contain sums of squares of many amplitudes, and are therefore not related directly by Fourier transformation.)

We shall now examine how sensitively our re-

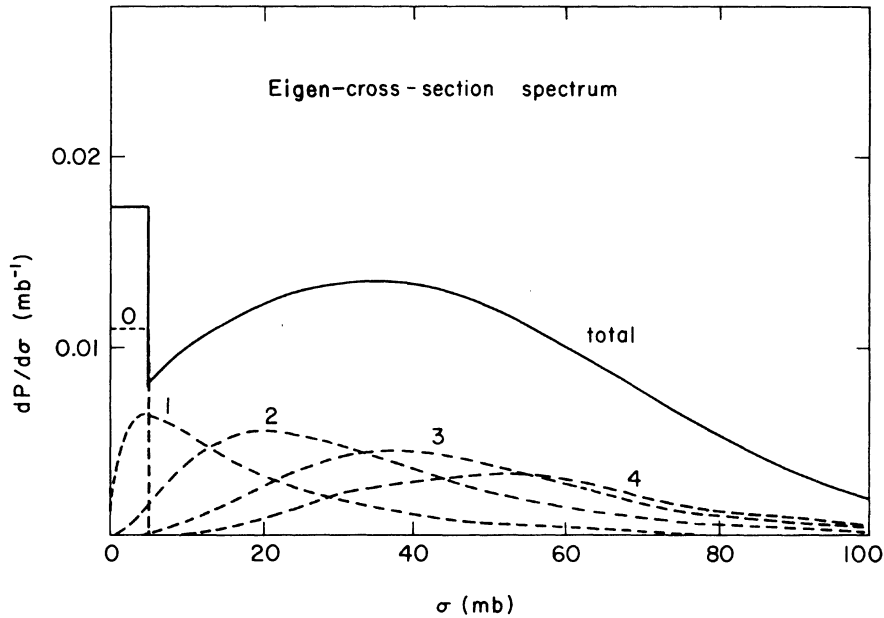


FIG. 4. The distribution of total cross sections for the parton states, which are the eigenstates of diffraction in our model (solid curve), and the contributions to it from wave-function components with 0, 1, 2, 3, or 4 wee partons (dashed curves). In the region 0–5 mb, we display the average for the sum in order to make visible the $\sim 5.4\%$ fraction with $\sigma=0$.

sults depend on the values chosen for the parameters. Let us begin by looking at some extremes. By choosing $G^2 = 2.6$, $A = 0.75$, $\beta = 6 \text{ GeV}^{-2}$, $\gamma = 12 \text{ GeV}^{-2}$, $\lambda/\alpha = 0$, we obtain a model which has no y_i fluctuations, but which has elastic and diffractive inelastic cross sections which are nearly identical to those of our original choice. Hence the y_i fluctuations are not essential to produce agreement with experiment. This is, of course, not a surprising result since we have seen that the y_i fluctuations contributed no more than about 10% of the total dissociation cross section and, furthermore, the y_i -fluctuation component did not dominate the spectra in any t range.

Next, by choosing $G^2 = 1.53$, $\gamma = 18 \text{ GeV}^{-2}$, $\beta = 0$, $\lambda/\alpha = 0$, we obtain a model which has the same elastic amplitude and the same integrated cross section $\sigma_{\text{diff}} = 6.47 \text{ mb}$ as our original model, but which has no y_i or \vec{b}_i fluctuations. This model disagrees violently with experiment, however, and thereby demonstrates that \vec{b}_i fluctuations must not be ignored. This disagreement appears mainly in the shape of $d\sigma_{\text{diff}}/dt$: The small- t slope becomes $A = 12 \text{ GeV}^{-2}$, which is nearly twice the experimental value.

As a final extreme, we could replace the Poisson distribution in the number of wee partons by a fixed number N . With $\beta = 0$, we could then obtain a model which has only \vec{b}_i fluctuations. It would disagree with experiment in the opposite way: The

diffractive inelastic cross section would have a dip at zero momentum transfer which is not observed.

These considerations demonstrate that *the \vec{b}_i fluctuations are responsible for about $\frac{1}{2}$ of the inelastic diffractive cross section*. Models which attempt to approximate this fraction by 0 or 1 will certainly fail.

Let us now discuss less extreme variations of the model parameters. We repeated all our calculations for numerous combinations of the values of the parameters, varying A between 0.7 and 1, γ/β between 1.5 and 3.0, and α/λ between 1.5 and 3.0. In each case the remaining two parameters of the model were set by fitting elastic scattering. The average number of wee partons varied between 2.3 and 4.1. The predicted diffractive cross section varied between 7.5 and 5.5 mb. The relative contributions of the three types of fluctuations into dissociation proved also to be quite stable. From this exercise we can assign the following values and errors for these contributions: y_i , \vec{b}_i , and N fluctuations contribute $10 \pm 5\%$, $45 \pm 10\%$, and $45 \pm 10\%$, respectively. The predicted differential cross section $d\sigma_{\text{diff}}/dt$ stayed always in rough agreement with experiment. Altogether, the results turned out to be surprisingly stable with respect to the variation of the model parameters not determined by elastic scattering. This is an extremely gratifying result since it means that the very good agreement between our predictions and

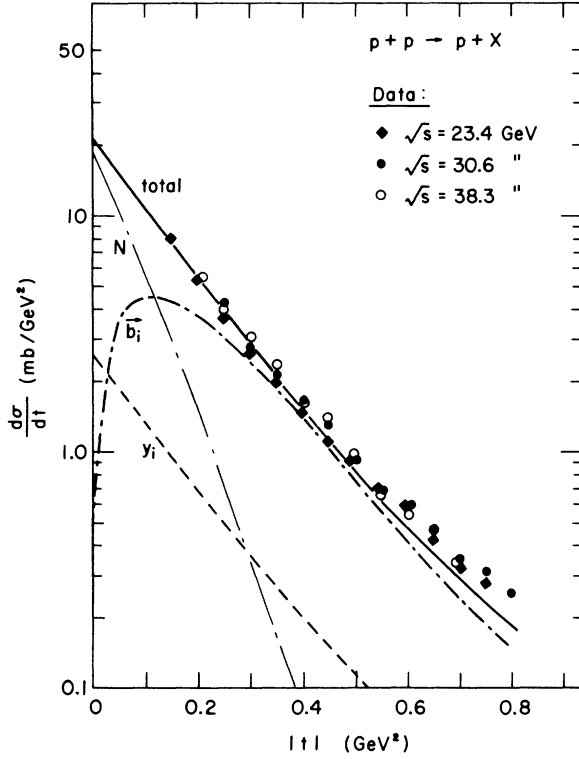


FIG. 5. The momentum-transfer dependence of beam dissociation, $\int (d\sigma/dt dm^2) dm^2$ for $pp \rightarrow p^*p$, predicted by our model. The experimental data are from Ref. 4. Also shown is a decomposition of the full cross section into contributions due to fluctuations in the number (N), rapidities (y_i), and relative impact parameters (\vec{b}_i) of the wee partons. The N -fluctuation contribution is seen to dominate near $t=0$, and the \vec{b}_i -fluctuation component at large- t values.

experiment is not a "numerical accident" due to a clever choice of parameter values but follows from the general structure of the model.

IV. THE NATURE OF FORWARD DISSOCIATION

The behavior of inelastic diffraction spectra near the forward direction has received much theoretical attention in the past. Before conclusive high-energy data became available, many theorists anticipated that the cross section of diffraction dissociation should vanish (or at least turn over) in the forward direction. The issue received special attention in 1971–1973 in relation to the Mueller-Regge formulation of diffraction theory and to the question of decoupling of the triple Pomeron. However, as the reader may well recall, the forward zero was predicted already earlier from much simpler theoretical arguments. It follows e.g. in the quark-model approaches to diffraction in the single-scattering approximation to Glauber theory.¹¹ When experiments showed no dip but rather

a large forward peak, it was thus an embarrassment not only to Regge-pole theorists, but also to followers of the additive-quark-model approach combined with the Glauber theory.¹²

In the Sec. III we saw that the differential cross section predicted by our model was in good agreement with experiment, *including the absence of a dip near $t=0$* . Furthermore, we observed that in the model, the large forward cross section was due to rapidity and multiplicity fluctuations of the wee partons. The impact-parameter fluctuations gave rise to a contribution which turned over and was very small at $t=0$. We shall now elaborate on this point and clarify why the additive-quark-model approach goes completely astray in its predictions concerning the t dependence of the dissociation spectra.

Let us briefly recall the theoretical argument for the forward zero in the additive quark model. To minimize inessential complications consider scattering of a composite system of N constituents in an external field. In the Born approximation to nonrelativistic Glauber theory, the amplitude for the reaction state $A \rightarrow$ state C is

$$T_{AC}(\vec{q}_\perp) = \sum_{k=1}^N T_k(\vec{q}_\perp) F_{AC}^{(k)}(\vec{q}), \quad (26)$$

where $T_k(\vec{q}_\perp)$ is the elastic scattering amplitude of the constituent k , and

$$F_{AC}^{(k)}(\vec{q}_\perp) = \langle \psi_C | e^{i\vec{q}_\perp \cdot \vec{b}_k} | \psi_A \rangle \quad (27)$$

is the overlap integral of the states A and C . In the forward direction, $\vec{q}_\perp = 0$, and the overlap integral reduces to the orthogonality integral of the two states

$$F_{AC}^{(k)}(\vec{q}_\perp = 0) = \langle \psi_C | \psi_A \rangle = \delta_{CA}, \quad (28)$$

which is zero for two orthogonal states.

Although the above derivation is greatly simplified, it does grasp the essential physics that underlies the forward zero in the framework of the additive quark model. The literature abounds with arguments that relativistic effects (or, equivalently, t_{min} effects) would invalidate the above derivation and remove the forward zero,¹³ but it is fairly easy to see that these arguments are based on inconsistent uses of relativity. This point was emphasized by Bell.¹⁴

An analysis of the additive quark model within our formalism sheds more light on the physical origin of the forward zero. Consider the scattering of a nucleon consisting of three quarks with fractional momenta x_1, x_2, x_3 and impact parameters $\vec{b}_1, \vec{b}_2, \vec{b}_3$, respectively. Since the number of constituents is fixed ($N=3$), the wave function has no N fluctuations. The wave function does

have fluctuations in the longitudinal momenta but, in the approximation of energy-independent scattering of the quarks, the absorption is independent of the x_i 's and thus longitudinal fluctuations do not give rise to *dispersion* in absorption and to diffraction dissociation. Hence we see that, in this model, all diffraction dissociation originates from \vec{b}_i fluctuations. But, in the Born approximation, the total cross section of a particular wave-function configuration is *independent* of the impact parameters of the quarks and equal to the number of quarks times the total quark cross section

$$\sigma_{\text{tot}}^{\text{BA}} = \int d^2 \vec{b} \sigma_{\text{tot}}(|\vec{b}_1 - \vec{b}|, \dots) = 3\sigma_{\text{tot}}^q. \quad (29)$$

In other words, the eigen-cross-section spectrum in the single-scattering approximation consists of a single δ peak at $\sigma = 3\sigma_{\text{tot}}^q$. Equation (25) tells us that the forward dissociation cross section is proportional to the dispersion squared of the eigen-cross-section spectrum. The width of a δ peak is zero, and the forward dissociation cross section thus vanishes.

Now consider our approach. As we saw in Sec. III, the contribution to dissociation due to the \vec{b}_i fluctuations peaked away from $t=0$ and was very small in the forward direction. *This forward dip is nothing but the above orthogonality zero*, slightly filled in by multiple-scattering contributions. However, the total dissociation cross section is large and peaks near $t=0$, since the contributions due to the N and y_i fluctuations present in our model, but absent in the additive quark model, are sizable and sharply peaked near $t=0$.

We strongly believe that our above discussion grasps the essence of the failure of the additive quark model in describing diffraction dissociation. The wave function of a fast-moving hadron is much more complicated than the naive (nonrelativistic as well as relativistic) quark model asserts. Since impact-parameter fluctuations cannot give rise to a sizable forward dissociation cross section, *the experimentally observed large cross section proves that the absorption also depends strongly on degrees of freedom in the wave function which are other than the transverse ones.*

An interesting feature of the dissociation contribution generated by the \vec{b}_i fluctuations is that its impact-parameter distribution is much more peripheral than those of the y_i - and N -fluctuation contributions and that of elastic scattering. At first sight one might think that this peripherality is generated by multiple scattering contributions. A closer look at the problem, however, shows that this is not so: *The peripherality is already present at the Born-term level.* In Fig. 6 we present a simple *Gedankenexperiment* which illustrates the

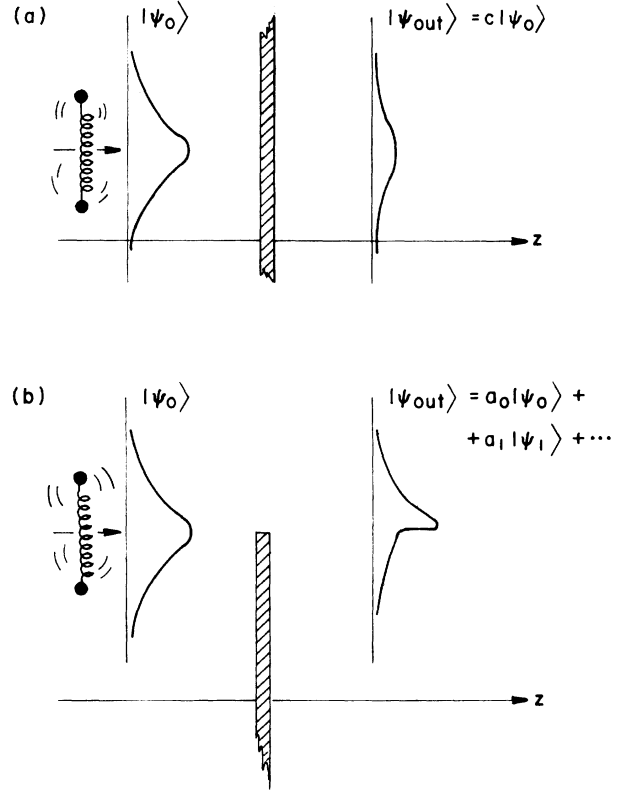


FIG. 6. A *Gedankenexperiment* illustrating the physical origin of the peripherality of the \vec{b}_i -fluctuation component. A harmonic oscillator, initially in its ground state $|\psi_0\rangle$, scatters from a very wide and very thin screen of hadronic matter. Because the screen is very thin, only single-scattering contributions are important. In (a), the oscillator hits the interior of the screen. If the scattering probability is independent of energy, then the wave function is absorbed uniformly. Hence no diffraction dissociation occurs. In (b), the oscillator hits the edge of the screen. The final-state wave function is distorted, and therefore contains excited components, i.e., diffraction dissociation takes place.

point. One sees from this figure that, while elastic scattering is large where the *magnitude* of the absorption is large, diffraction dissociation due to the \vec{b}_i fluctuations is large where the *derivative* of the absorption is large. Since the absorption is most rapidly varying at the edge of the scattering region, the peripheral distribution of the \vec{b}_i -fluctuation contributions follows.

We should add one more explanation about the properties of the \vec{b}_i -fluctuation contribution to dissociation. We concluded above that, although the \vec{b} -space distribution of this contribution should be *peripheral*, its momentum-space distribution *dips* near $t=0$. Such a behavior seems to contradict the intuitive idea that peripheral \vec{b} -space distributions should correspond to momentum-space dis-

tributions which are sharply peaked and maximal in the forward direction. The solution to this paradox is very simple. The Fourier-Bessel transformations between impact-parameter space and momentum space are of course transformations of amplitudes and not of cross sections. The dissociation amplitudes generated by the \vec{b}_i fluctuations peak at the edge of the interaction region (and thus give rise to a peripheral cross section), but they are rapidly varying and *change sign* in nearby regions. These oscillations of the \vec{b} -space amplitudes give rise to large cancellations in the Fourier-Bessel integrals and produce the small- t dip.

V. CONCLUSIONS AND DISCUSSION

In the parton approach pursued in this paper, the parton-parton interactions were assumed to be dominantly short-range interactions in rapidity space. From this assumption, and from some general knowledge of the parton distribution functions, two great simplifications of our analysis resulted. Firstly, it was seen that *the global properties of diffraction depended on the distribution and interactions of the very slow "wee" partons only*. Thus, no assumptions were needed about the structure of the hadrons' wave functions in the dynamically more complicated finite- x region. Secondly, we found that, as long as we were attempting to calculate elastic and total inelastic diffractive cross sections only, no detailed knowledge of the parton-parton forces was needed. It is a most beautiful property of the shadow dynamics that *these two cross sections depend on the first two moments of the absorption only*. Thus, as long as we have included the correct degrees of freedom in the wave functions and described the interparton forces properly in the average, our analysis should provide reasonable results.

We adopted simple phenomenological parametrizations for the wee-parton distributions and the parton interaction probability, fixed two of the five model parameters by fitting σ_{tot} and σ_{el} , and carried out the calculations for various values of the remaining three parameters. The most important results of our analysis can be summarized as follows:

(A) The model describes elastic scattering fairly well. This is not surprising, since $d\sigma_{\text{el}}/dt$ is experimentally nearly exponential, and thus can be described rather well in terms of the two parameters σ_{tot} and σ_{el} , whose measured values were used as input to our analysis.

(B) The model predicts 2.5–3.5 mb for the beam-dissociation cross section in proton-proton scattering in the Fermilab-CERN ISR energy range. This result is in excellent agreement with experi-

ments.

(C) The \vec{b} -space distribution of the inelastic diffractive cross section is found to be much more peripheral than that of elastic scattering. It is peaked away from $b=0$, near $b=0.5$ fm.

(D) The inelastic diffractive cross section has been separated into contributions due to rapidity, impact-parameter, and multiplicity fluctuations of the wee partons. The relative contributions of these three types of fluctuations are approximately 10%, 45%, and 45%, respectively. The \vec{b} -space distributions of the y_i - and N -fluctuation contributions are found to be central, whereas the \vec{b}_i -fluctuation contribution is strongly peripheral.

(E) We observed that the peripherality of the \vec{b}_i -fluctuation component was not primarily due to multiple scattering effects but rather a property of single scattering amplitudes. A simple *Gedankenexperiment* was presented which demonstrated why this is the case.

(F) The forward value and the slope of $d\sigma_{\text{diff}}/dt$ are correctly predicted by the model. This agreement is not at all trivial, since the slope of $d\sigma_{\text{diff}}/dt$ ($A \approx 6 \text{ GeV}^{-2}$) is much smaller than that of elastic scattering ($A \approx 11 \text{ GeV}^{-2}$).

(G) We decomposed $d\sigma_{\text{diff}}/dt$ into contributions due to the y_i , \vec{b}_i , and N fluctuations. The \vec{b}_i -fluctuation contribution was found to dominate at large momentum transfers, to peak around $t = -0.1 \text{ GeV}^2$, and to be very small in the forward direction. The N - and y_i -fluctuation contributions peaked sharply in the forward direction and dominated the scattering in the region near $t=0$.

(H) We clarified the reason for the catastrophic failure of the additive quark models (relativistic as well as nonrelativistic) in predicting the t dependence of diffractive production.

Lest the reader become overly encouraged by our good results, we now discuss some of the limitations of the model, as it now stands. First, all our calculations have been *inclusive*, i.e., we have summed over all the channels of diffractive production. This was required by the unitarity formalism developed in Sec. II. Thus, we cannot predict the internal properties of the diffractively produced states. Even the basic question of the mass spectrum of the excited states is not addressed—to say nothing of more detailed properties of the produced states such as the mass-slope correlation, spin and helicity dependence, etc.

A second limitation of the present formulation of the model is that we parametrized the wee-parton distribution and the parton-parton interactions independently. However, Lorentz invariance requires that what in one Lorentz frame appears as an interaction between a parton belonging to the beam hadron and another belonging to the target

hadron will in another Lorentz frame become an interaction between two partons belonging both to the wave function of the same hadron. Thus the wee-parton distribution and the parton-parton amplitude are not independent quantities. In principle, it should be possible to *derive* the wee-parton distribution corresponding to any given parton-parton amplitude. Since we do not know how to do this, but have a fairly clear idea what the answer must look like, we guessed the answer directly and provided a phenomenological parametrization for it. We do not know if our wee-parton distribution and parton-parton amplitude are theoretically consistent. However, since our results are quite independent of the values of the parameters of the model, we believe that this is not a serious objection to our model.

A third difficulty of the present formulation of our model, which is related to the previous one, is the following: We described the target hadron by a homogeneous optical potential. It should be possible to derive this potential from a convolution of the target parton distribution with the parton-parton amplitude. We have done this, and learned that our assumption that the wee partons are uncorrelated runs into trouble either with Lorentz invariance or with exact s -channel unitarity. However, we believe that a more realistic parametrization of the wee-parton distribution, which would include correlations between partons, would remove the theoretical inconsistency without influencing our numerical results significantly.

We wish to mention here a more detailed—but highly speculative—picture which could perhaps underlie our work. In that picture, a fast-moving nucleon consists of three valence quarks together with multiperipheral chains of seapartons, which are emitted at random by the valence quarks with coupling constant $\sim G$ (see Fig. 7). It may be meaningful to neglect the interactions between the chains—including processes in which one chain splits into two, or in which two fuse into one—and assume that each chain supplies at most one wee parton. In this way, the number of wee partons and the rapidity distribution for a single parton would correspond to independent fluctuations, as assumed in our model.

Next we briefly compare our analysis with a recent study carried out by Fiałkowski and Van Hove.¹⁵ Inspired by the results of Ref. 6, these authors calculated inelastic diffraction in a model in which the internal structure of a fast-moving nucleon was described in terms of three valence quarks and gluons, and the gluons were taken to be the active element in nondiffractive hadronic collisions at high energy.¹⁶ The absorption probability was assumed to depend on the fractional

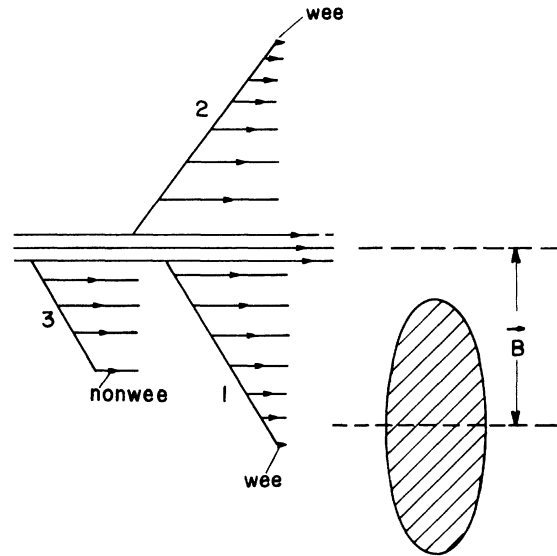


FIG. 7. Schematic illustration of a fast-moving proton approaching an optical potential based on a multiperipheral point of view. The proton structure is described in terms of three valence quarks plus multiperipheral chains of sea partons. The full wave function is a superposition of components with varying number of chains, each of which contains a varying number of partons at various impact parameters and rapidities. The total impact parameter of the collision, \vec{B} , is determined by the fast partons. Out of the three chains shown, only chain number 1 has a chance to interact with the target. Chain number 2 will pass the target out of its range, and chain number 3 contains non-wee partons only and will thus pass through the target without being capable of interacting with it.

momenta x_g, x'_g , and the relative impact parameter \vec{b}_g of the two colliding gluon states as follows:

$$t(x_g, x'_g, \vec{b}_g) = 1 - \exp[-\Omega(x_g, x'_g, \vec{b}_g)], \quad (30a)$$

where

$$\Omega(x_g, x'_g, \vec{b}_g) = \lambda(\vec{b}_g) x_g x'_g. \quad (30b)$$

The probability distributions for x_g and x'_g were assumed to be flat, and the impact-parameter distributions of the gluon center of momentum relative to the particle center of momentum were described by Gaussians. The function $\lambda(\vec{b}_g)$ was then determined by fitting the model to CERN ISR data on pp elastic scattering. The total inelastic diffractive cross section predicted by the model ranges from 2.7 to 5.1 mb, to be compared with experimental estimates of 5–8 mb.

It should be clear that our analysis is quite similar both in spirit and in its practical formulation to that of Fiałkowski and Van Hove. The \vec{b}_g -fluctuation component of dissociation is nearly identically described by the two approaches. In the longitudinal and density fluctuations, however, the

two models differ in an important way. In the approach of Fiałkowski and Van Hove, the total probability of absorption was assumed to depend strongly on the total momentum carried by the gluons [see Eq. (30)]. Since $x_g = 1 - x$, where x is the total momentum of the three valence quarks, we see that the probability of absorption depends strongly on the momenta of the (fast) valence quarks and that the model thus contains *strong long-range correlations*. Our model, on the other hand, is built upon the assumption that hadrons' wave functions are dominantly short-range-correlated, and thus the fluctuations in the wee region do not depend strongly on how the total momentum is shared between the valence quarks and the fast sea partons which carry the remaining momentum. Our model does contain some long-range correlations, but their amount is much smaller than in the approach of Fiałkowski and Van Hove.

The present analysis can be extended in many different directions. We mention here a few possibilities:

(i) By varying the radii and densities of the beam and target hadrons, one may investigate the factorization properties of diffraction scattering.

(ii) The model may be applied to scattering on nuclei. In particular, one may address questions such as: How important are inelastic shadowing effects in scattering on nuclei? How "black" are heavy nuclei? From studies of the A dependence of diffraction dissociation in hadron-nucleus collisions, estimates have been obtained for the total cross sections of unstable hadronic states scattering on nucleons. How much do these extracted "cross sections" have to do with the real excited state-nucleon cross sections? We have analyzed some of these problems, and our results will be forthcoming.

(iii) How to extend the model to study the excitation of *exclusive* final states? It seems to us that, in order to solve this problem, one must develop a much better understanding of the parton wave functions for fast-moving hadrons than what we now possess—including some understanding of their phases. Even a crude solution of this problem would be very useful. One could study how the dissociation spectra depend on the mass, spin, and helicity of the excited states, what is the physics underlying the mass-slope correlation, etc., and

as a result, would obtain important new insights into the dynamics of diffraction dissociation.

(iv) On the more theoretical side, many possibilities for further research are open. It would be useful to obtain better understanding of our somewhat intuitive hypothesis that the parton basis is the eigenbasis of diffraction. A clarification of how the wee-parton distribution depends on the parton-parton amplitude would be instructive. One could try to derive the parton amplitudes and the parton distribution functions from a microscopic dynamical theory of hadronic matter such as quantum chromodynamics. Finally, an understanding of the Lorentz transformation properties of hadronic wave functions would be very useful in connection with the above problem (iii).

In conclusion, we wish to reemphasize the most important aspect of our analysis. By studying the scattering in terms of normalized wave functions and unitary absorption probabilities, we make direct use of s -channel unitarity, i.e., of the shadow-scattering origin of diffraction. Our approach is thus quite different from conventional approaches to diffraction, in which detailed models are built for the "Born term" amplitudes, and these amplitudes are then "unitarized" by some iteration prescription. We have shown in this paper that s -channel unitarity plus a rather modest amount of dynamical input about the internal hadron structure and the nature of the constituent forces provides a good description of the global properties of diffraction scattering.

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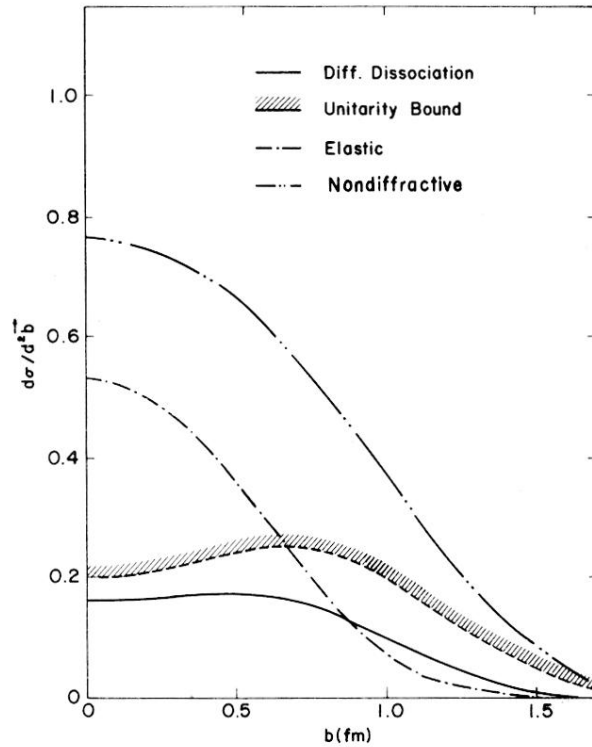


FIG. 1. The impact-parameter dependence of inelastic diffraction predicted by our model (solid curve). It corresponds to an integrated cross section $\sigma_{\text{diff}} = 6.5$ mb, which is consistent with experiment (see Ref. 4). It is also consistent with the bound $\sigma_{\text{diff}}(b) \leq \frac{1}{2} \sigma_{\text{tot}}(b) - \sigma_{\text{el}}(b)$ (/////), which follows from unitarity. Observe that inelastic diffraction is much more peripheral in impact parameter than elastic scattering (---) or the nondiffractive cross section $\sigma_{\text{tot}}(b) - \sigma_{\text{el}}(b) - \sigma_{\text{diff}}(b)$ (·-·-·-·). The parameters of the model were chosen so that $d\sigma_{\text{el}}/dt$ approximates the known elastic scattering at $\sqrt{s} = 53$ GeV.

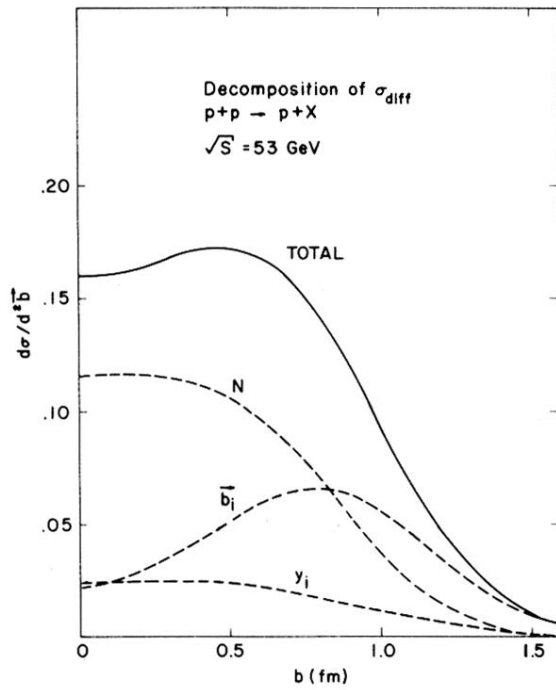


FIG. 2. The diffractive cross section as a function of impact parameter, repeated from Fig. 1, and the contributions to it from fluctuations in the *number* (N), *rapidities* (y_i), and *relative impact parameters* (\vec{b}_i) of the wee partons. The \vec{b}_i fluctuations are responsible for the very peripheral nature of $\sigma_{\text{diff}}(b)$, and contribute about $\frac{1}{2}$ of the integrated value 6.5 mb.

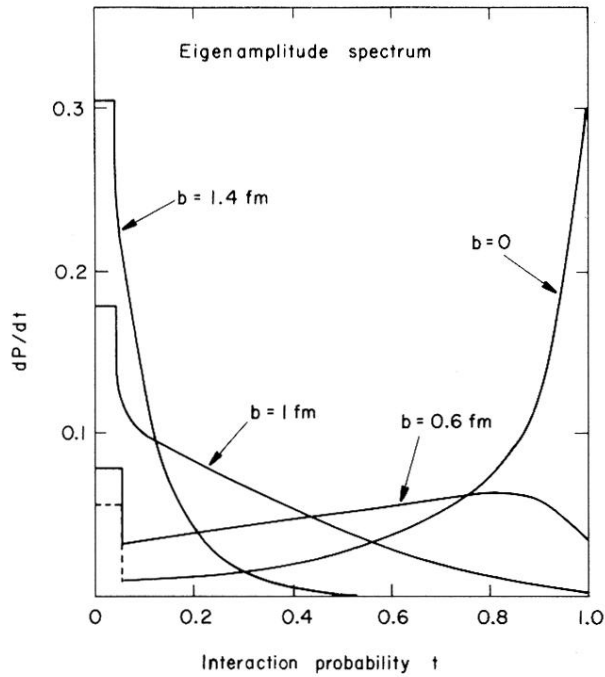


FIG. 3. The distribution of interaction probabilities at various impact parameters. The large dispersion in these probabilities is responsible for the large cross section for beam dissociation, which is observed experimentally. Note that these "eigenamplitudes" have already been averaged over the target configurations. In our model, there is a component of the parton wave function which contains zero wee partons, and which therefore does not interact. It accounts for $e^{-G^2} = 5.4\%$ of the wave function. In order to display its effect, these curves are averaged in the region from 0 to 0.05.

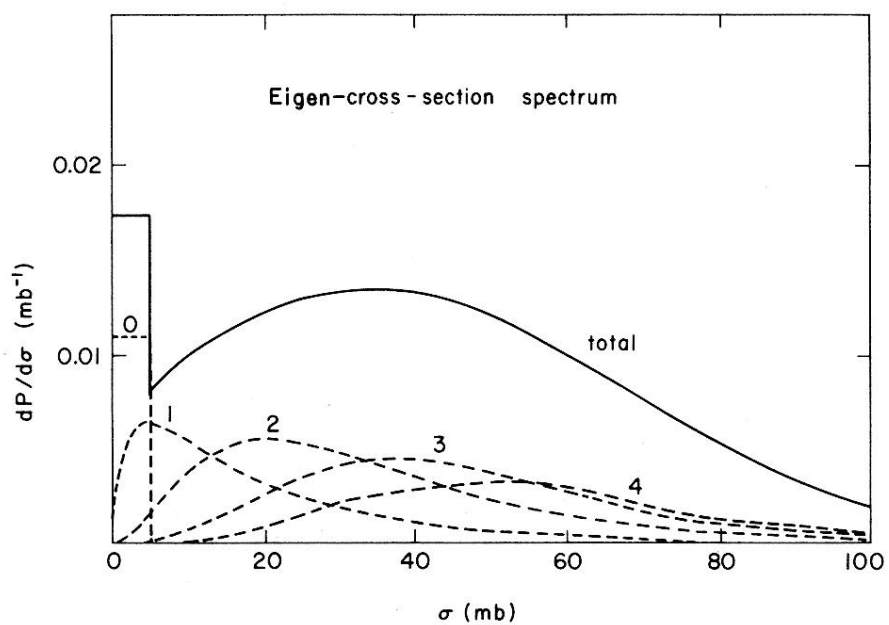


FIG. 4. The distribution of total cross sections for the parton states, which are the eigenstates of diffraction in our model (solid curve), and the contributions to it from wave-function components with 0, 1, 2, 3, or 4 wee partons (dashed curves). In the region 0–5 mb, we display the average for the sum in order to make visible the $\sim 5.4\%$ fraction with $\sigma=0$.

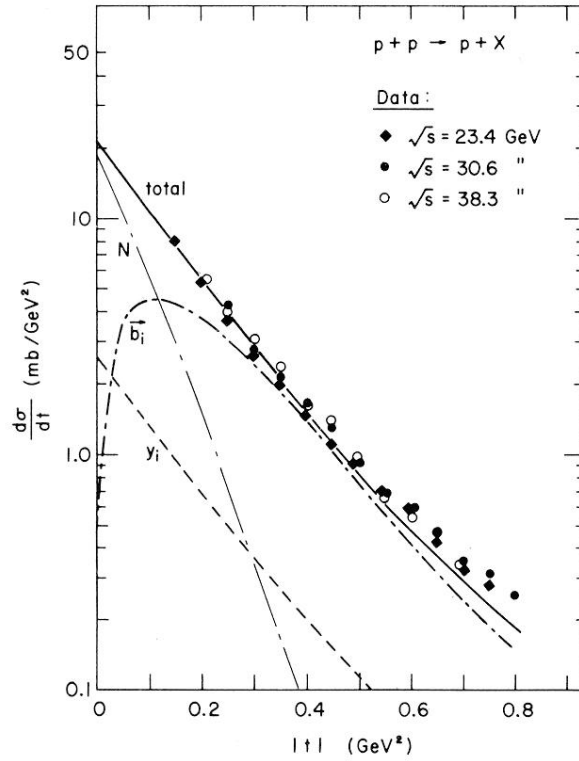


FIG. 5. The momentum-transfer dependence of beam dissociation, $\int (d\sigma/dtdm^2)dm^2$ for $pp \rightarrow p^*p$, predicted by our model. The experimental data are from Ref. 4. Also shown is a decomposition of the full cross section into contributions due to fluctuations in the number (N), rapidities (y_i), and relative impact parameters (\vec{b}_i) of the wee partons. The N -fluctuation contribution is seen to dominate near $t=0$, and the \vec{b}_i -fluctuation component at large- t values.

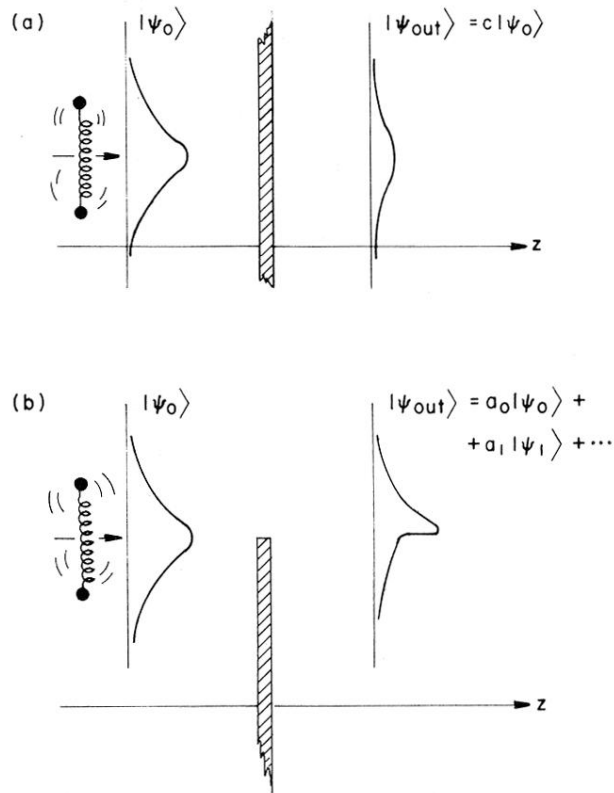


FIG. 6. A *Gedankenexperiment* illustrating the physical origin of the peripherality of the \vec{b}_i -fluctuation component. A harmonic oscillator, initially in its ground state $|\psi_0\rangle$, scatters from a very wide and very thin screen of hadronic matter. Because the screen is very thin, only single-scattering contributions are important. In (a), the oscillator hits the interior of the screen. If the scattering probability is independent of energy, then the wave function is absorbed uniformly. Hence no diffraction dissociation occurs. In (b), the oscillator hits the edge of the screen. The final-state wave function is distorted, and therefore contains excited components, i.e., diffraction dissociation takes place.

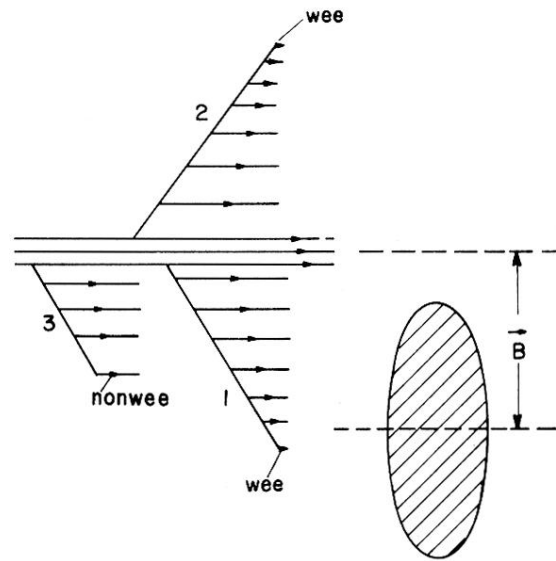


FIG. 7. Schematic illustration of a fast-moving proton approaching an optical potential based on a multi-peripheral point of view. The proton structure is described in terms of three valence quarks plus multi-peripheral chains of sea partons. The full wave function is a superposition of components with varying number of chains, each of which contains a varying number of partons at various impact parameters and rapidities. The total impact parameter of the collision, \vec{B} , is determined by the fast partons. Out of the three chains shown, only chain number 1 has a chance to interact with the target. Chain number 2 will pass the target out of its range, and chain number 3 contains non-wee partons only and will thus pass through the target without being capable of interacting with it.