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Field-theoretic model of composite hadrons. II

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We consider here a relativistic generalization of a field-theoretic model of composite hadrons with quarks as constituents proposed earlier. The quarks are assumed to occupy fixed energy levels in hadrons at rest, with the hadron mass being given additively in terms of the quark energies. These quark field operators for hadrons at rest are next Lorentz-boosted to describe hadrons in motion, using the fact that quark operators are Dirac field operators with known transformation properties. Static properties of baryons are utilized to estimate the quark-field-operator parameters. The Dirac Hamiltonian for the quark field operators also has a nonvanishing expression for quark-pair-creation processes. The covariant generalization of this Hamiltonian is used to describe strong-interaction vertices. The quark-field-operator parameters and the harmonic-oscillator wave function are next utilized to describe quantitatively the pion-nucleon coupling constant, as well as $N^* \to N\pi$, $\rho \to 2\pi$, $\phi \to 2K$ and $K^* \to K\pi$. The results agree with experimental values reasonably well, indicating that the above Hamiltonian may be the dynamical origin of the three-particle vertices of hadronic strong interactions as well as an explanation of the Okubo-Zweig-Iizuka rule.

I. INTRODUCTION

Earlier[1] we had considered a field-theoretic model of hadrons described by quark operators taken as Dirac field operators. We had confined our attention to the rest frame of the hadrons. In this frame of reference, a simple ansatz was made for the quark field operators satisfying translational invariance and usual equal-time anticommutation relations. No attempt was made for Lorentz covariance, since we were confining our attention to a specific frame of reference with hadrons at rest. The Dirac Hamiltonian for quarks was seen to contain four components: the quark and antiquark Hamiltonians, and the quark-pair-creation and pair-annihilation Hamiltonians, where the latter two components were seen in general not to vanish within our ansatz. It was presumed that the particle and/or antiparticle Hamiltonians with some potential-like interaction yield hadrons as eigenstates of the total Hamiltonian, where we take the field-theoretic version of the potential.[2] The origin and the nature of the potential were kept arbitrary. Some of the eigenvalue equations for hadrons were given, from which conventional equations for the wave functions of the hadrons can be derived. In the nonrelativistic limit our eigenvalue equation would merely be pedagogic; but it really was a generalization of the same in the sense that the effects on the eigenvalue equation of both the "large" and "small" Dirac components were retained here. Thus if we have "relativistic" quarks inside the hadron (at rest), the effect of this is technically retained. With this approach, mainly the effects when hadrons were at rest were calculated.

It was also conjectured that the pair-creation component of the Dirac Hamiltonian may give rise to the strong decays of hadrons in the quark model consistent with the Okubo-Zweig-Iizuka rule.[3] This was used to estimate kaon decay of the ϕ meson. Also, the quark-pair-annihilation component with minimal electromagnetic coupling gave $\pi^0 \to 2\gamma$ and $\eta \to 2\gamma$ *without* the use of partial conservation of axial-vector current (PCAC) or vector dominance, in an agreeable manner.

However, this model was nonrelativistic since we confined our attention to hadrons at rest. In the present model we further generalize the concepts to describe hadrons in motion. For this purpose, we use Lorentz boosting in an essential manner. It had been pointed out by the author[4] that if we have nonrelativistic systems, we can in a suitable manner generate the class of corresponding relativistic systems by Lorentz boosting in a manner similar to the little group method of representation of the Lorentz group as given by Wigner[5] and used by Weinberg[6] in the context of field theory. In this paper we give a concrete realization of the above scheme[4] to describe hadronic dynamics.

In the context of the quark model, Lorentz boosting was considered earlier by many authors.[7] However, this was confined to wave functions only and was thus limited in scope. We carry out here a similar operation for field operators,[8] which very much enlarges the perspective for many possible calculations. The present model differs from I even when hadrons have arbitrarily small momenta through the introduction of $S(L(p))$, where p is the four-momentum of the hadron, $L(p)$ is the corresponding Lorentz transformation, and $S(L)$ is the corresponding 4×4 matrix $\mathfrak{D}(\frac{1}{2},0)\oplus\mathfrak{D}(0,\frac{1}{2})$ representation of the Lorentz group.[9] The present model and I agree only when the hadron is at rest.

While considering the Lorentz boosting of the quark field operators representing constituents of hadrons, we introduce some new concepts. We *assume* that the constituent-quark field operator has a specific frequency (energy) when it describes the hadron *at rest.* This assumption is made in the context of the Hartree-Fock self-consistent method of calculation of energy levels[10] where it will be an approximation; it will also be true e.g., in the MIT bag model[11] where quarks occupy fixed energy levels inside the bag. We do not know how far such an assumption will be exact: It will depend on the mechanism of the formation of the bound state. However, we assume this to be a good approximation, and use it in an essential manner throughout the present paper.

As in I, we shall have Dirac field operators describing constituent quarks of hadrons *at rest.* In Sec. II, by Lorentz boosting of these operators, we *generate* the quark field operators which describe quarks as constituents of hadrons in motion. In this manner, with different Lorentz boostings, we describe quarks as constituents of different hadrons in motion.

We next note that a field operator is a space-time parametrization of *all* the states of the "particle" for which it stands. Thus e.g., the quark field operators in the electromagnetic current may correspond to constituents of *any* hadron with *any* velocity. We assume that $Q^g(x)$ is such a quark field operator. We thus have three types of quark field operators: $Q(x)$, which describes it as a constituent of *some* hadron at rest, $Q^{L(p)}(x)$, which describes it as a constituent of some hadron with four-momentum p generated from $Q(x)$ by Lorentz boosting knowing that it is a Dirac field operator, and $Q^g(x)$, which represents any $Q^{L(p)}(x)$. Since quarks are in some manner universal, it is assumed that $Q(x)$ is universal to whichever hadron the quark Q may belong, *except* for the time dependence of the above operator, which depends on the hadron we take in a manner stated earlier.

To be definite, we assume that quarks are permanently confined to hadrons, owing to some mechanism which we do not at present understand.[12] We thus have an *unphysical* vector space spanned by the quark field operators. The *physical* vector space is the vector space of hadrons, obtained as eigenstates of the Hamiltonian as in I, and subsequently Lorentz-boosted as in Sec. II of the present paper. Thus we assume that $Q^g(x)$ has meaning *only* as some $Q^{L(p)}(x)$, i.e., quarks have no physical meaning except as constituents of hadrons. Thus the question of quark propagators does not enter into the picture; however, we can have *hadron* propagators through quark interactions. The theory can probably be generalized to include quark propagators, but we do not consider this here. In Sec. II we also mention in what way $Q^g(x)$ is related to $Q^{L(p)}(x)$.

When hadrons are in motion the details of the present paper differ from I. Hence in Sec. III we determine the parameters of the quark field operator again from static properties of baryons. For this purpose we use the charge radius, g_A/g_V, and the magnetic moments of the baryons. The parameters of $\mathcal{P}$ and $\mathfrak{N}$ quarks (i.e., the masses) remain unaltered and the λ quark becomes heavier. The agreement with experiments is better than usual although not as good as we had in I. Section III thus determines the quark parameters as well as the radius of the harmonic-oscillator wave function of the baryons, and to some extent, that of the mesons through pion charge radius.

In Sec. IV, we discuss the quark-pair-creation component of the Dirac Hamiltonian which we use to describe the strong interactions of hadrons. For this purpose, we first replace the expression for the quark-pair-creation term by a *relativistic* expression, which agrees in the rest frame of hadrons with the expression we had in I. In Sec. V, we use this expression to *calculate* $G_{NN\pi}$, and the widths for $N^*\to N\pi$, $\rho\to2\pi$, $\phi\to2K$, and $K^*\to K\pi$. We shall see that now the *only* free parameters available to us are the radius of the harmonic-oscillator wave function of the mesons and the energy level of the $\mathcal{P}$ quark in the K^+ meson, since the other parameters have been fixed in Sec. III, and the nature of the field operators has been fixed in Sec. II. The agreement with experimental values is very reasonable. However, both for baryons and mesons $R^2=15\ \text{GeV}^{-2}$ seems to be more appropriate to conventional assignments of the colored-quark model with hadrons as color singlets.

In Sec. VI we discuss various general aspects of the model as well as the possible limitations of the present calculations.

II. LORENTZ BOOSTING

Let $\psi_Q(x)$ be the Dirac field operator of a constituent quark Q of a hadron of mass m in the rest frame of the hadron. We then write, as in I,

$$\psi_Q(x)=Q(x)+\tilde{Q}(x)\ , \tag{2.1}$$

where $Q(x)$ annihilates the quark and $\tilde{Q}(x)$ creates the antiquark. For $x^0=t=0$, we next use the Fourier transforms

$$Q(x)=(2\pi)^{-3/2}\int u(\vec{k})Q_I(\vec{k})\exp(i\vec{k}\cdot\vec{x})d^3k \tag{2.2}$$

and

$$\tilde{Q}(x)=(2\pi)^{-3/2}\int v(\vec{k})\tilde{Q}_I(\vec{k})\exp(-i\vec{k}\cdot\vec{x})d^3k\ . \quad (2.3)$$

We have used the notation[1]

$$u(\vec{k})=\begin{pmatrix} f(\vec{k}^2) \\ g\vec{\sigma}\cdot\vec{k}\end{pmatrix} \quad (2.4)$$

and

$$v(\vec{k})=\begin{pmatrix} g\vec{\sigma}\cdot\vec{k} \\ f(\vec{k}^2)\end{pmatrix} \quad (2.5)$$

$Q_I(\vec{k})$ and $\tilde{Q}_I(\vec{k})$ are the two-component quark-annihilation and antiquark-creation field operators, which satisfy the anticommutation relations

$$[Q_{Ir}(\vec{k}),Q_{Is}^\dagger(\vec{k}')]_+=[\tilde{Q}_{Ir}(\vec{k}),\tilde{Q}_{Is}^\dagger(\vec{k}')]_+ = \delta_{rs}\delta(\vec{k}-\vec{k}')\ . \quad (2.6)$$

It was shown in I that in such a case the Dirac field operator $\psi_Q(x)$ satisfies the usual equal-time anticommutation relations *provided* we have

$$f^2(\vec{k}^2)+g^2\vec{k}^2=1\ . \quad (2.7)$$

In addition to (2.4) and (2.5), we also use the obvious notations

$$u_r(\vec{k})=u(\vec{k})u_{Ir}\ , \quad (2.8)$$

$$v_r(\vec{k})=v(\vec{k})v_{Ir}\ , \quad (2.9)$$

and recognize that

$$Q_I(\vec{k})=\sum_{r=\pm 1/2}Q_{Ir}(\vec{k})u_{Ir} \quad (2.10)$$

and

$$\tilde{Q}_I(\vec{k})=\sum_{r=\pm 1/2}\tilde{Q}_{Ir}(\vec{k})v_{Ir} \quad (2.11)$$

f and g could be arbitrary functions of $\vec{k}^2$ satisfying (2.7). However, taking g as a constant appeared to be a reasonable approximation regarding derivation of some results along with the identification $g=(2m_Q)^{-1}$, which we shall continue to take here.

In I we had next considered the Dirac Hamiltonian density

$$\mathcal{H}_D(x)=\psi_Q^\dagger(x)(-i\vec{\alpha}\cdot\vec{\nabla}+\beta m_Q)\psi_Q(x)\ . \quad (2.12)$$

The Hamiltonian above has quark and antiquark components which, along with some potential-like interaction, are expected to generate the hadronic states as the eigenstates of the total Hamiltonian, as described in I.

Now we want to Lorentz-boost the above field operators to describe quarks as constituents of hadrons in motion. Such a Lorentz boosting will require a knowledge of the spacetime dependence of the field operators. Equations (2.2) and (2.3) essentially describe the space behavior of these field operators. We next assume that the time dependence of *each* constituent field operator is given by a fixed frequency. This essentially associates with each constituent a fixed energy level. Thus, if Q_1, Q_2, Q_3 are three quarks forming a baryon *at rest*, then we *assume* that

$$Q_i(x)=Q_i(\vec{x})\exp(-i\lambda_i mt)\ , \quad (2.13)$$

where each λ_i is positive and $\lambda_1+\lambda_2+\lambda_3=1$. For a physical background of such an idea, we may recollect the self-consistent Hartree-Fock approximation, where each constituent has a fixed energy eigenvalue in the average potential of the others.[10] In such a picture clearly the above comment (2.13) is an approximation. In the MIT bag model, e.g., the quarks occupy specific energy levels inside the bag, and thus the above comment may be exact.[11] How good such an approximation may be will obviously depend on the dynamics of formation of hadrons as bound states of quarks. In the absence of adequate knowledge regarding this,[12] the approximation (2.13) can only have *a posteriori* justification. In the present model of Lorentz boosting this assumption enters into the dynamics in an essential manner.

Now the quark field operator $Q(x)$ *with time dependence given by (2.13)* describes the constituent quark of a *specific* hadron at rest. Let $k=(k^0,\vec{k})$, where $k^0=\lambda m$ is the fractional energy carried by the constituent quark Q for the above hadron. Let the hadron have four-momentum p, and let $L(p)$ be the corresponding Lorentz transformation given as,[9] for $\mu=0,1,2,3$ and $i,j=1,2,3$,

$$L_{0\mu}=L_{\mu 0}=p^\mu/m$$

and (2.14)

$$L_{ij}=\delta_{ij}+\frac{p^ip^j}{m(p^0+m)}$$

We then *define*,[4] with (2.2) and (2.13),

$$Q^{L(p)}(x)=U(L(p))S(L(p))Q(L(p)^{-1}x)U^{-1}(L(p)) \quad (2.15)$$

$$=(2\pi)^{-3/2}(p^0/m)^{1/2}\int u^{L(p)}(\vec{k})Q_I(L(p)k)\exp\{-i[L(p)k]\cdot x\}d^3k \quad (2.16)$$

In (2.16), we have taken

$$u^{L(p)}(\vec{k}) = S(L(p))u(\vec{k}) \tag{2.17}$$

and

$$U(L(p))Q_I(k)U^{-1}(L(p)) = (p^0/m)^{1/2}Q_I(L(p)k). \tag{2.18}$$

Also,[9]

$$S(L(p)) = [(p^0+m)/(2m)]^{1/2} + [2m(p^0+m)]^{-1/2}\vec{\alpha}\cdot\vec{p}. \tag{2.19}$$

We have taken the Lorentz-contraction factor $(p^0/m)^{1/2}$ in (2.18) so that we can take e.g.

$$[Q_{Ir}(Lk), Q^\dagger_{Is}(L'k')]_+ = \delta_{rs}\delta_3(Lk - L'k'), \tag{2.20}$$

which becomes consistent with

$$U(L)[Q_{Ir}(\vec{k}), Q^\dagger_{Is}(\vec{k}')]_+ U^{-1}(L) = \delta_{rs}\delta(\vec{k}-\vec{k}') \tag{2.21}$$

as needed from (2.6) for constituent quarks of a given hadron. We shall regard equations such as (2.20) as a generalization of the anticommutators (2.6) for *arbitrary* Lorentz boosting of quark field operators, which may even belong to different hadrons. Equation (2.20), though it looks innocuous, is obviously a substantial generalization of quark dynamics, and in the present context, is an assumption,[13] since it relates quark field operators of different hadrons in different Lorentz frames. It can describe so-called "spectator quarks" of the quark model.

We have thus used two types of constituent-quark field operators: $Q(x)$ and $Q^{L(p)}(x)$. We should also have a third type of quark field operator which *a priori* does not belong to any specific hadron. E.g. the quark field operators in the electromagnetic current can correspond to a quark of any hadron in any frame of reference. Such quark field operators we call $Q^g(x)$ and make a simple *prescription* as to how these are to be treated. We shall assume that these field operators break up as in (2.1), and further that, when it gets contracted with the quark field operator of a known hadron in a known frame of reference, we shall replace

$$Q^g(x) = \alpha Q^{L(p)}(x), \tag{2.22}$$

where p is the four-momentum of the above hadron and we shall determine that $\alpha = 1$ from some consistency requirements.

We thus assume that $Q^g(x)$ contains within itself components such that it can annihilate the Q quark of any hadron in any Lorentz frame, and during contraction with the constituent-quark field operator belonging to a hadron, appropriate $Q^{L(p)}(x)$ gets projected out, and all other components of $Q^g(x)$ become irrelevant. We now imagine hadronic dynamics as follows: The vectors spanned by quarks constitute an *unphysical* vector space. Only the vector space of hadrons constructed from quark field operators as in I and subsequently Lorentz-boosted using (2.18) form the physical vector space. Thus any quark field operator $Q^g(x)$ that occurs in any Hamiltonian must be replaced by $Q^{L(p)}(x)$ corresponding to *some* hadron of four-momentum p. Otherwise, $Q^g(x)$ has no meaning. This model obviously corresponds to permanently confined quarks.

We shall now consider Eq. (2.22) to determine α, and for this purpose, shall explicitly evaluate the matrix elements of the electromagnetic current between spin-$\frac{1}{2}$ proton states. As in I, we shall take the colored-quark model with baryons as color singlets, and thus[1]

$$|p_{1/2}(\vec{0})\rangle = \frac{\epsilon_{ijk}}{3\sqrt{2}}\int \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)u(\vec{k}_1,\vec{k}_2,\vec{k}_3)d^3k_1d^3k_2d^3k_3 \times[\mathcal{P}^i_{I(1/2)}(k_1)^\dagger\mathcal{P}^j_{I(1/2)}(k_2)^\dagger\mathfrak{N}^k_{I(-1/2)}(k_3)^\dagger - \mathcal{P}^i_{I(1/2)}(k_1)^\dagger\mathcal{P}^j_{I(-1/2)}(k_2)^\dagger\mathfrak{N}^k_{I(1/2)}(k_3)^\dagger]|\text{vac}\rangle, \tag{2.23}$$

where we have the normalization[14]

$$1 = \int \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)|u(\vec{k}_1,\vec{k}_2,\vec{k}_3)|^2d^3k_1d^3k_2d^3k_3. \tag{2.24}$$

With (2.18), we now *define*[5,6]

$$|p_{1/2}(\vec{p})\rangle = (m/p^0)^{1/2}U(L(p))|p_{1/2}(\vec{0})\rangle \tag{2.25}$$

$$= \frac{\epsilon_{ijk}}{3\sqrt{2}}(p^0/m)\int \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)u(\vec{k}_1,\vec{k}_2,\vec{k}_3)d^3k_1d^3k_2d^3k_3 \times[\mathcal{P}^i_{I(1/2)}(L(p)k_1)^\dagger\mathcal{P}^j_{I(1/2)}(L(p)k_2)^\dagger\mathfrak{N}^k_{I(-1/2)}(L(p)k_3)^\dagger - \mathcal{P}^i_{I(1/2)}(L(p)k_1)^\dagger\mathcal{P}^j_{I(-1/2)}(L(p)k_2)^\dagger\mathfrak{N}^k_{I(1/2)}(L(p)k_3)^\dagger]|\text{vac}\rangle. \tag{2.26}$$

We have taken the Lorentz-contraction factor in (2.25) such that, using (2.20) and (2.24), we get

$$\langle p_{1/2}(\vec{p}\,')\,|\,p_{1/2}(\vec{p})\rangle = (m/p^0)\delta(\vec{L}'^{-1}(\vec{p}-\vec{p}\,'))$$
$$= \delta(\vec{p}-\vec{p}\,') \ . \qquad (2.27)$$

In the above, and subsequently, $\vec{L}=[L_{ij}]$, the space part of the Lorentz transformation, L.

Now the electromagnetic current is given as

$$J^\mu(x) = \sum_{i,Q} e_Q \bar{\psi}_Q^g(x)\gamma^\mu \psi_Q^g(x) \ , \qquad (2.28)$$

where i is the color index. We shall now evaluate

$$\langle p_{1/2}(\vec{p})\,|J^\mu(0)\,|\,p_{1/2}(\vec{p})\rangle \ .$$

Thus, by (2.22), the current effectively becomes

$$J^\mu(0) \equiv |\alpha|^2 \sum_{i,Q} \bar{Q}^{iL(p)}(0)\gamma^\mu Q^{iL(p)}(0)$$
$$= (2\pi)^{-3}|\alpha|^2 \int \sum_{i,Q} e_Q \bar{Q}^{iL(p)}(k')\gamma^\mu \times Q^{iL(p)}(k)d^3k\,d^3k' \ . \qquad (2.29)$$

In (2.29), we have substituted from (2.16)

$$Q^{L(p)}(k) = (p^0/m)^{1/2}u^{L(p)}(\vec{k})Q_I(L(p)k) \ . \qquad (2.30)$$

We now use[9]

$$S^{-1}(L)\gamma^\mu S(L) = L_{\mu\nu}\gamma^\nu \qquad (2.31)$$

and that, for any fixed r, with (2.8),

$$\int \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\,|u(\vec{k}_1,\vec{k}_2,\vec{k}_3)|^2 \times \bar{u}_r(\vec{k}_1)\gamma^\nu u_r(\vec{k}_1)d^3k_1d^3k_2d^3k_3 = \delta_{\nu 0} \ . \qquad (2.32)$$

In (2.32) we have used that $u(\vec{k}_1,\vec{k}_2,\vec{k}_3)$ is even in $\vec{k}_i$. We then obtain, using (2.20), that

$$\langle p_{1/2}(\vec{p})\,|J^\mu(0)\,|\,p_{1/2}(\vec{p})\rangle = (2\pi)^{-3}|\alpha|^2 p^\mu/p^0 \ . \qquad (2.33)$$

Hence, with the normalization (2.27), we obtain that it is consistent to take in (2.22) $\alpha=1$, such that

$$Q^g(x) = Q^{L(p)}(x) \qquad (2.34)$$

becomes the general prescription in dealing with $Q^g(x)$, when $Q^g(x)$ gets contracted with the quark operator of a hadron of four-momentum p.

The procedure followed in (2.25) along with (2.18) will be the general description for hadrons with arbitrary momenta. As noted earlier, this technique has similarity with the little-group method of representation of the Lorentz group.[5,6]

III. STATIC PROPERTIES

The present model differs appreciably from I even when small momenta are involved for the hadrons, since the spin rotations involved in (2.15) had been earlier ignored. These spin rotations had been e.g. responsible for the determination of SU(6) mixing as discussed by Le Yaouanc *et al.*[7] We shall now consider the changes involved in the determination of quark parameters (i.e., the masses) in this context as compared to I.

For this purpose, in this section we shall consider (a) the charge radius of the proton, (b) the magnetic moments of the nucleons and Λ, and (c) the charge radius of the pion. Along with g_A/g_V derived earlier, these will be adequate to determine the quark parameters of the model and the radius of the harmonic-oscillator wave functions of the baryons and the π meson. This is the objective of the present section. Except for the charge radius of the proton and pion, where there is not too much disagreement, the above results agree quite well. For the charge radii, the disagreement is no worse than in other models.

We shall now explicitly take in (2.26)

$$u(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \left(\frac{3R^4}{\pi^2}\right)^{3/4} \exp\left[-\frac{R^2}{6}\sum_{i<j}(\vec{k}_i-\vec{k}_j)^2\right]. \qquad (3.1)$$

We note that for the electromagnetic current with $t=(p-p')^2$ [using Dirac spinors for a moment instead of (2.8)],

$$\langle p_{r'}(\vec{p}\,')\,|J^\mu(0)\,|\,p_r(\vec{p})\rangle = (2\pi)^{-3}\left(\frac{m^2}{p^0p'^0}\right)^{1/2}\bar{u}_{r'}(\vec{p}\,') \times[\gamma^\mu F_1(t)+i\sigma^{\mu\nu}q_\nu F_2(t)]u_r(\vec{p}) \ , \qquad (3.2)$$

such that we get in the Breit frame

$$G_E^p(t) = (2\pi)^3(p^0/m)\langle p_r(-\vec{p})\,|J^0(0)\,|\,p_r(\vec{p})\rangle \ , \qquad (3.3)$$

where $t=-4\vec{p}^{\,2}$ and

$$G_E^p(t) = F_1(t) + \frac{t}{2m}F_2(t) \ .$$

Similarly we obtain

$$\langle p_{r'}(-\vec{p})\,|J^i(0)\,|\,p_r(\vec{p})\rangle = \frac{m}{p^0}\frac{i}{(2\pi)^3}[\vec{\sigma}\times(-2\vec{p})]^i_{r'r}G_M^p(t) \ , \qquad (3.4)$$

where

$$G_M^p(t) = \frac{1}{2m}[F_1(t)+2mF_2(t)] \ .$$

We now use (2.26), (3.1), and (2.29) along with (2.20). This yields, from (3.3),

$$G_E^p(t) = \left(\frac{3R^2}{2\pi}\right)^{3/2}\int \exp\left[-\frac{3R^2}{4}(\vec{k}_1'^{\,2}+\vec{k}_1^{\,2})\right]d^3k_1 \times \bar{u}_{1/2}^{L^{-1}(p)}(\vec{k}_1')\gamma^0 u_{1/2}^{L(p)}(\vec{k}_1) \ . \qquad (3.5)$$

In (3.5) we have, with (2.20),

$$\vec{k}_1' = \vec{k}_1 - \frac{4m}{3p^0}\,\vec{p} . \tag{3.6}$$

Clearly, we have used that $k_i^0 = k_i'^0 = m/3$; m is the mass of the proton. We now use that $g_{\mathcal{P}} = g_{\mathfrak{N}} = g$ is a constant, as in I, and write with (2.7)

$$f(\vec{k}^2) \simeq 1 - \tfrac{1}{2}g^2\vec{k}^2 - \tfrac{1}{8}g^4|\vec{k}|^4 . \tag{3.7}$$

This will be a valid approximation when $g^2 \ll R^2$, as we shall later see to be true. Then the integration in (3.5) is straightforward and we obtain

$$G_E^p(t) = \left(1 + \frac{2g^2}{9}\tau + \frac{2g^4}{27R^2}\tau\right)\exp(\tfrac{1}{6}R^2\tau) , \tag{3.8}$$

where we have substituted

$$\tau = t\bigg/\left(1 - \frac{t}{4m^2}\right) . \tag{3.9}$$

Clearly

$$G_E^p(t) = G_E^{\rm NR}(\tau) , \tag{3.10}$$

where $G_E^{\rm NR}(t)$ stands for the "nonrelativistic" form factor. We may notice the similarity of (3.8) and (3.10) with the results derived by Licht and Pagnamenta[7] on the basis of Lorentz-boosted wave functions as well as the differences. Equation (3.8) includes terms involving g, which could not be included by Licht and Pagnamenta with a nonrelativistic quark model, where g vanishes. Further, (3.10) does not have an overall factor $[1 - t/(4m^2)]^{-1}$ which was present in the calculations of Licht and Pagnamenta. Although the basic ideas are similar, ours is an explicit use of Lorentz boosting *with* field-theoretic ideas in Sec. II and probably is more consistent. Except for this discrepancy, our result is a confirmation of the results of Licht and Pagnamenta where more heuristic ideas were used. In particular, we have carefully identified the transformation property of the current in (2.33) to give the identification (2.34) which, as far as we are able to understand, did not have a parallel in the approach of Licht and Pagnamenta.

From (3.8), we obtain that the charge radius of the proton is

$$R_{\rm ch}{}^2 = R^2 + \tfrac{4}{3}g^2 + \frac{4g^4}{9R^2} . \tag{3.11}$$

In the above, we have taken $g_{\mathcal{P}} = g_{\mathfrak{N}} =$ constant. We now note that the calculation of g_A/g_V in I for $n \to p + e + \bar{\nu}_e$ remains unaltered, since for this no Lorentz boosting is needed. Here we had[1]

$$|g_A/g_V| = \frac{5}{3}\left(1 - \frac{4g_{\mathcal{P}}g_{\mathfrak{N}}}{3R^2}\right) . \tag{3.12}$$

Substituting that[15] $|g_A/g_V| = 1.25$, we thus obtain

$$\frac{g_{\mathcal{P}}g_{\mathfrak{N}}}{R^2} = \tfrac{3}{16} . \tag{3.13}$$

We next proceed to calculate the magnetic moments of the proton and the neutron using (3.4). Taking the low-energy limit, we thus obtain for the magnetic moment of the proton, with (3.6),

$$\frac{i}{(2\pi)^3}[\vec{\sigma}\times(-2\vec{p})]^i_{(1/2)(1/2)}\,\mu_p = \langle p_{1/2}(-\vec{p})|J^i(0)|p_{1/2}(\vec{p})\rangle$$
$$= (2\pi)^{-3}\left(\frac{3R^2}{2\pi}\right)^{3/2}\int \exp\left[-\frac{3R^2}{4}(\vec{k}_1'^2 + \vec{k}_1^2)\right]\, d^3k_1\,\bar{u}_{1/2}^{L^{-1}(p)}(\vec{k}_1')\gamma^i u_{1/2}^{L(p)}(\vec{k}_1) . \tag{3.14}$$

Clearly, on the right-hand side of (3.14), we are to neglect all contributions involving $|\vec{p}|^2$, which results in some simplifications. We proceed with the calculations using (2.4), (2.8), and (2.17). Further, we retain only the first two terms on the right-hand side of (3.7), and perform the integration in (3.14) using (3.6). We then obtain

$$\mu_p = \frac{1}{2m}\left(1 - \frac{4g^2}{3R^2}\right) + \tfrac{2}{3}g . \tag{3.15}$$

Similarly, one can obtain directly or from symmetry that

$$\mu_n = -\tfrac{2}{3}\mu_p . \tag{3.16}$$

From (3.13) and (3.15) one obtains

$$g = 1.64\ {\rm GeV}^{-1} , \tag{3.17}$$

and from (3.13) and (3.16) one obtains

$$g = 1.70\ {\rm GeV}^{-1} . \tag{3.18}$$

In both (3.17) and (3.18) as well as (3.13) we have taken

$$R^2 = 15\ {\rm GeV}^{-2} . \tag{3.19}$$

Equations (3.17) and (3.18) reflect the error in taking $g_{\mathcal{P}} = g_{\mathfrak{N}}$. We shall usually take for $\mathcal{P}$ and $\mathfrak{N}$ quarks

$$g = 1.67\ {\rm GeV}^{-1} . \tag{3.20}$$

When we do not assume that $g_{\mathcal{P}} = g_{\mathfrak{N}}$, we obtain as in I

$$\mu_p = \frac{1}{2m}\left[1 - \frac{4}{27R^2}(8g_{\mathcal{P}}{}^2 + g_{\mathfrak{N}}{}^2)\right] + \tfrac{2}{27}(8g_{\mathcal{P}} + g_{\mathfrak{N}}) \tag{3.21}$$

and

$$\mu_n = \frac{1}{2m}\left[-\frac{2}{3}+\frac{4}{27R^2}(2g_{\mathcal{P}}^2+4g_{\mathfrak{N}}^2)\right] - \tfrac{2}{27}(2g_{\mathcal{P}}+4g_{\mathfrak{N}}). \quad (3.22)$$

We get reasonable agreement for (3.13), (3.21), and (3.22) if we take (3.19) and

$$g_{\mathcal{P}} = 1.62 \text{ GeV}^{-1} \quad (3.23)$$

and

$$g_{\mathfrak{N}} = 1.71 \text{ GeV}^{-1} . \quad (3.24)$$

Thus the values obtained here for the $\mathcal{P}$ and $\mathfrak{N}$ quarks are almost the same as in I. However, we note a comparatively worse agreement with the expression of the charge radius of the proton which becomes

$$R_{ch}^2 = 18.77 \text{ GeV}^{-2} ,$$

such that

$$R_{ch} = 0.86 \text{ fm} . \quad (3.25)$$

We note that the expression (3.25) is, after all, better than one usually gets in such models.

For the Λ magnetic moment, one similarly derives

$$\mu_\Lambda = -\frac{1}{3}\frac{1}{2m_\Lambda}\left(1-\frac{4g_\lambda^2}{3R^2}\right)-\frac{2}{9}g_\lambda , \quad (3.26)$$

such that we obtain, with $\mu_\Lambda = -0.67$ nuclear magnetons,

$$g_\lambda \simeq 1 \text{ GeV}^{-1} . \quad (3.27)$$

Equations (3.23), (3.24), and (3.27) with $m_Q = (2g_Q)^{-1}$ give us

$$m_{\mathcal{P}} = 308 \text{ MeV} ,$$
$$m_{\mathfrak{N}} = 292 \text{ MeV} , \quad (3.28)$$

and

$$m_\lambda = 500 \text{ MeV} .$$

Thus we notice that the λ quark appears to be heavier than what we had estimated in I, although $\mathcal{P}$- and $\mathfrak{N}$-quark parameters remain unchanged.

We shall now determine also the charge radius of the pion, which is a simpler problem. We take in the colored quark model in the same manner as (2.26),

$$|\pi^+(\vec{p})\rangle = \frac{1}{\sqrt{6}}(p^0/m_\pi)^{1/2}\int \delta(\vec{k}_1+\vec{k}_2)u_\pi(\vec{k}_1)d^3k_1 \times \mathcal{P}_I^{i}(L(p)k_1)^+ \times \mathfrak{N}_I^{i}(L(p)k_2)|\text{vac}\rangle, \quad (3.29)$$

and then estimate, similar to (3.5),

$$\langle\pi^+(-\vec{p})|J^0(0)|\pi^+(\vec{p})\rangle = (2\pi)^{-3}\left(\frac{R_\pi^2}{\pi}\right)^{3/2}\left(\frac{m_\pi}{p^0}\right)\int \exp\left[-\frac{R_\pi^2}{2}(\vec{k}_1'^2+\vec{k}_1^2)\right]d^3k_1\,[f(\vec{k}_1'^2)f(\vec{k}_1^2)+g^2\vec{k}_1'\cdot\vec{k}_1]. \quad (3.30)$$

In the above equation,

$$\vec{k}_1' = \vec{k}_1 - \frac{m_\pi}{p^0}\vec{p} \quad (3.31)$$

and we have taken

$$u_\pi(\vec{k}) = \left(\frac{R_\pi^2}{\pi}\right)^{3/4}\exp(-\tfrac{1}{2}R_\pi^2\vec{k}^2). \quad (3.32)$$

This yields, in the same way as (3.8),

$$G_{ch}^\pi(t) = (1+\tfrac{1}{8}g^2\tau')\exp(\tfrac{1}{16}R_\pi^2\tau') , \quad (3.33)$$

where

$$\tau' = t/[1-t/(4m_\pi^2)] . \quad (3.34)$$

Hence

$$\langle R_{ch}^2\rangle_\pi = \frac{3R_\pi^2}{8}+\tfrac{3}{4}g^2. \quad (3.35)$$

If we take also

$$R_\pi^2 = 15 \text{ GeV}^{-2} , \quad (3.36)$$

then we obtain

$$\langle R_{ch}^2\rangle_\pi = 0.31 \text{ fm}^2 , \quad (3.37)$$

which does not agree completely with recent analysis of the experimental value[16] of the parameter given as (0.5 ± 0.07) fm^2, but has similar disagreement as one obtains from vector dominance. Taking $R_\pi^2 = 22.5$ GeV^{-2} will make it agree better, and then we get

$$\langle R_{ch}^2\rangle_\pi = 0.4 \text{ fm}^2 . \quad (3.38)$$

We notice that there are two possible effects which may vitiate the above results and which have not been included above. A part of the baryon momentum may be carried by gluons, as appears to be the case from deep-inelastic scattering, which would change the results above. Also, the electromagnetic interactions can occur through neutral-vector-meson exchange, which has not been estimated or included above. We cannot have full confidence in the parameters until we estimate these. Thus the values are to be regarded as tentative and we proceed with them in that spirit only.

Our purpose in this section has been to calculate the quark-field-operator parameters and R^2 of the baryon and meson harmonic-oscillator wave functions for use in calculations involving strong decays. Because of the comments already made regarding other possible effects we do not take seriously the form factors derived in (3.8) and (3.33), since for large momentum transfers we are not very confident about the validity of the harmonic-oscillator wave function and we are not sure which other effects will start playing a role. In the above calculations we regard the charge radius as comparatively unreliable, since the charge radius depends on the square of the momentum transfer, whereas g_A/g_V and the magnetic moments depend on at best a linear contribution from momentum transfer.

IV. QUARK-PAIR-CREATION TERM

The quark-pair-creation component of the Dirac Hamiltonian is given from (2.12) as[1]

$$V_{Q^\dagger\tilde{Q}}(x) = Q^\dagger(x)(-i\vec{\alpha}\cdot\vec{\nabla}+\beta m_Q)\tilde{Q}(x) \ . \tag{4.1}$$

We shall first attempt to replace (4.1) by a covariant generalization of the same. For this purpose, we note that the quark-antiquark pair created in (4.1) may belong to different hadrons. As in (2.13), let the respective energy components of the quark and antiquark be k'^0 and k^0, and thus let k' and k be four-momenta of the quark and antiquark *in the rest frame* of the respective hadrons. When $V_{Q^\dagger\tilde{Q}}$ operates as a perturbation Hamiltonian, there will be overall energy-momentum conservation. Thus, with the remaining quarks as spectator quarks, we shall effectively have in the rest frame of hadrons

$$\vec{k}' = -\vec{k} \ . \tag{4.2}$$

Hence, in momentum space, (4.1) gives rise to the operator with e.g., $Q(k') = u(\vec{k}')Q_I(k')$,

$$Q^\dagger(k')(-\vec{\alpha}\cdot\vec{k}+\beta m_Q)\tilde{Q}(k) = \bar{Q}(k')(m_Q+\tfrac{1}{2}\gamma^\mu k_\mu - \tfrac{1}{2}\gamma^\mu k'_\mu)\tilde{Q}(k) \ . \tag{4.3}$$

In (4.3) we have used that, with (2.4), (2.5), and (4.2),

$$\bar{u}(\vec{k}')\gamma^0(k^0-k'^0)v(\vec{k}) = 0 \ .$$

Equation (4.3) suggests that we may replace $V_{Q^\dagger\tilde{Q}}(x)$ by

$$V_{Q^\dagger\tilde{Q}}(x) \equiv \bar{Q}(x)(m_Q - \tfrac{1}{2}i\gamma^\mu\vec{\partial}_\mu + \tfrac{1}{2}i\gamma^\mu\overleftarrow{\partial}_\mu)\tilde{Q}(x) \ . \tag{4.4}$$

We repeat that (4.1) and (4.4) are equivalent for a quark-antiquark pair-creation process with zero total momentum for the pair-created in the rest frame of the two hadrons, and further we note that it is symmetric in the pair-created process and is invariant in form.

Now we imagine a general quark-pair-creation process, with the pair created belonging to different hadrons in arbitrary Lorentz frames. For such a process, with the ideas presented before Eq. (2.22) in mind, we write the quark-antiquark pair-creation Hamiltonian as

$$V^g_{Q^\dagger\tilde{Q}}(x) = \bar{Q}^g(x)(m_Q - \tfrac{1}{2}i\gamma^\mu\vec{\partial}_\mu + \tfrac{1}{2}i\gamma^\mu\overleftarrow{\partial}_\mu)\tilde{Q}^g(x) \ . \tag{4.5}$$

For evaluation of matrix elements using (4.5), Eq. (2.34) with appropriate identifications must be utilized.

We shall now explicitly do so. Let us assume that the quark and the antiquark created in (4.5) are contracted with the quark and antiquark which are constituents of hadrons of four-momenta p' and p, respectively. Then by (2.34) and (2.15) we obtain, with $L=L(p)$ and $L'=L(p')$,

$$\begin{aligned} V^{L',L}_{Q^\dagger\tilde{Q}}(0) &= (2\pi)^{-3}\left(\frac{p^0p'^0}{mm'}\right)^{1/2}\int d^3k\,d^3k'\,Q_I^\dagger(L'k')\bar{u}(\vec{k}')S^{-1}(L')[m_Q+\tfrac{1}{2}\gamma^\mu(Lk)_\mu - \tfrac{1}{2}\gamma^\mu(L'k')_\mu]S(L)v(\vec{k})\tilde{Q}_I(Lk) \\ &= (2\pi)^{-3}\left(\frac{p^0p'^0}{mm'}\right)^{1/2}\int d^3k\,d^3k'Q_I^\dagger(L'k')\bar{u}(k')[m_QS^{-1}(L')S(L)+\tfrac{1}{2}S^{-1}(L')S(L)\gamma^\mu k_\mu \\ &\qquad - \tfrac{1}{2}\gamma^\mu k'_\mu S^{-1}(L')S(L)]\,v(\vec{k})\tilde{Q}_I(Lk) \ . \end{aligned} \tag{4.6}$$

We shall repeatedly need (4.6), and thus shall evaluate the same for *small* momenta of the hadrons. Then we have from (2.19), and with $\vec{p}' = -\vec{p}$,

$$S^{-1}(L')S(L) = \begin{pmatrix} 1 & b\vec{\sigma}\cdot\vec{p} \\ b\vec{\sigma}\cdot\vec{p} & 1 \end{pmatrix} \ , \tag{4.7}$$

where

$$b = (m+m')/(2mm') \ . \tag{4.8}$$

Further, by (2.4) and (2.5),

$$\gamma^{\mu}k_{\mu}v(\vec{k})=\begin{pmatrix}(k^0g-f)\vec{\sigma}\cdot\vec{k}\\ g\vec{k}^2-fk^0\end{pmatrix} \tag{4.9}$$

and

$$\bar{u}(\vec{k}')\gamma^{\mu}k'_{\mu}=[f'k'^0-g\vec{k}'^2,(k'^0g-f')\vec{\sigma}\cdot\vec{k}'] . \tag{4.10}$$

In (4.9) and (4.10), we have taken $f=f(\vec{k}^2)$ and $f'=f(\vec{k}'^2)$.

Hence from (4.6), (4.7), (4.9), and (4.10) we obtain that for *small* hadronic momenta,

$$\begin{aligned}V_{Q^+\bar{Q}}^{L',L}(0)=(2\pi)^{-3}\int d^3k\,d^3k'\,Q_I^{\dagger}(L'k')\tfrac{1}{2}\{&(\vec{\sigma}\cdot\vec{k})[f'(1-f)+g^2\vec{k}'^2+f'g(k^0-k'^0)]\\ &-(\vec{\sigma}\cdot\vec{k}')[f(1-f')+g^2\vec{k}^2+fg(k'^0-k^0)]\\ &+b(\vec{\sigma}\cdot\vec{p})[f'f(g^{-1}-k^0-k'^0)+g(f'\vec{k}^2+f\vec{k}'^2)]\\ &-gb(\vec{\sigma}\cdot\vec{k}')(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{k})[g(k^0+k'^0)+1-f-f']\}\tilde{Q}_I(Lk) .\end{aligned} \tag{4.11}$$

In (4.11), we have utilized the earlier identification that $m_Q=(2g)^{-1}$.

We note that in (4.11) there is *no* free parameter, except, sometimes k^0 and k'^0, the energy levels of the antiquark and the quark in the respective hadrons in their rest frames. We shall utilize (4.11) for our subsequent calculations through field-theoretic identifications at small momenta.

V. STRONG COUPLINGS

We shall now proceed to *generate* strong interactions of hadrons from our quark model without any new strong-interaction parameters. For this purpose we shall obviously utilize the Hamiltonian (4.5) or (4.11), generalized from the nonrelativistic expression (4.1) as given in I. Since the problem becomes extremely complicated, we shall adopt the following technique, often applied in quark-model calculations. The present model is also a substantial generalization of the earlier ideas of this type as described below since here we take mesons as composite objects instead of being radiative fields[18] for these interactions. Our ideas are similar to Ref. 14; only here we *do not* have an *ad hoc* pair-creation contribution in hadronic matter, but take the corresponding component of the pair-creation Hamiltonian obtained from the Dirac Hamiltonian (2.12) and generalized in the last section. The technique consists of calculating the contribution from field-theoretic Hamiltonians for *small* hadronic momenta and then identifying this contribution with the contribution using (4.11), which is the small momentum version of (4.5). This *determines* the field-theoretic coupling constants, which we shall subsequently use assuming that these coupling constants remain unaltered for momentum transfers relevant to the physical problems. Besides the complexity of the problem, we have two additional reasons for adopting this procedure. Firstly, although we have tried our best to be logical in Sec. II, we do not yet have adequate confidence that this model will describe relativistic hadrons. The second and more important reason is that we do not know how far the harmonic-oscillator wave functions will be valid when overlap integrals with large momentum transfers come into the picture, as will automatically happen in actual physical problems. Hence we have adopted the philosophy that these wave functions may be good enough when small momentum transfers are involved, and that the relativistic effects can be best taken care of by the field-theoretic methods. At least as a first approximation this attitude will be useful, and we adopt this here.

Now we shall proceed to generate strong-interaction dynamics with the above technique.

A. Pion-nucleon coupling constant

The field-theoretic Hamiltonian is given as

$$\mathcal{H}_I^F(x)=G\bar{N}(x)\gamma_5\vec{\tau}\cdot\vec{\pi}(x)N(x) , \tag{5.1}$$

which, when $|\vec{p}|\ll m_\pi$, gives the contribution

$$\langle n_{1/2}(-\vec{p})\pi^+(\vec{p})|\mathcal{H}_I^F(0)|p_{1/2}(\vec{0})\rangle = (2\pi)^{-9/2}\frac{G}{2m\sqrt{m_\pi}}u_{I(1/2)}^{\dagger}(\vec{\sigma}\cdot\vec{p})u_{I(1/2)} . \tag{5.2}$$

We shall obtain the value of G by identifying (5.2) with the corresponding expression derived from (4.11) in the quark model.

For this purpose, we take proton and π^+ states as in (2.26) and (3.29) and take, for small momenta,

$$|n_{1/2}(\vec{p})\rangle = \frac{\epsilon_{ijk}}{3\sqrt{2}}(p^0/m)\int \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)u(\vec{k}_1,\vec{k}_2,\vec{k}_3)d^3k_1d^3k_2d^3k_3$$
$$\times[\mathcal{P}^{i}_{I(1/2)}(L(p)k_1)^{\dagger}\mathfrak{N}^{j}_{I(1/2)}(L(p)k_2)^{\dagger}\mathfrak{N}^{k}_{I(-1/2)}(L(p)k_3)^{\dagger}-\mathcal{P}^{i}_{I(-1/2)}(L(p)k_1)^{\dagger}$$
$$\times\mathfrak{N}^{j}_{I(1/2)}(L(p)k_2)^{\dagger}\mathfrak{N}^{k}_{I(-1/2)}(L(p)k_3)^{\dagger}]|\text{vac}\rangle\ . \tag{5.3}$$

We note that for the present problem, in (4.6), L corresponds to Lorentz boosting of π^+ and L', the Lorentz boosting of the neutron. Hence, using (3.1) and (4.6), we obtain, after some integrations for momenta of spectator quarks,

$$\langle n_{1/2}(-\vec{p})\pi^+(\vec{p})|V^{g}_{Q^+\bar{Q}}(0)|p_{1/2}(\vec{0})\rangle \equiv \tfrac{1}{18}\times\frac{1}{\sqrt{6}}\times(-5i)\times 6\times\left(\frac{3R^2}{2\pi}\right)^{3/2}\left(\frac{R_\pi{}^2}{\pi}\right)^{3/4}(2\pi)^{-3}$$
$$\times\int d^3k_1\bar{u}_{1/2}(\vec{k}_1')[S^{-1}(L')S(L)m_{\mathfrak{N}}+\tfrac{1}{2}S^{-1}(L')S(L)\gamma^{\mu}k''_{2\mu}-\tfrac{1}{2}\gamma^{\mu}k'_{1\mu}S^{-1}(L')S(L)]$$
$$\times v_{-1/2}(\vec{k}_2'')\exp[-\tfrac{1}{2}R_\pi{}^2\vec{k}_1''^2-\tfrac{3}{4}R^2(\vec{k}_1'^2+\vec{k}_1{}^2)]\ . \tag{5.4}$$

We shall have some contributions of the type $\bar{u}_{-1/2}(\vec{k}_1')\cdots v_{1/2}(\vec{k}_2'')$, which arise from a different spin combination in quark space. These contributions have been *included* in the above, since as can be seen, these will finally give the same contribution. In (5.4), we have also utilized (2.20), repeatedly, which yields

$$\vec{k}_1' = \vec{k}_1 - \tfrac{2}{3}\vec{p}\ , \tag{5.5}$$

$$\vec{k}_1'' = \vec{k}_1 - \tfrac{1}{2}\vec{p}\ , \tag{5.6}$$

and also we have

$$\vec{k}_2'' = -\vec{k}_1'',\quad k_1'^0 = m/3,\quad k_2''^0 = m_\pi/2\ . \tag{5.7}$$

In writing the time components in (5.7), (2.13) has been utilized. Neglecting terms such as $\vec{p}^{\,2}$, the exponential in (5.4) is simplified as

$$\exp[-\tfrac{1}{2}R_\pi{}^2\vec{k}_1''^2-\tfrac{3}{4}R^2(\vec{k}_1'^2+\vec{k}_1{}^2)] = \exp\left[-\frac{3R^2+R_\pi{}^2}{2}(\vec{k}_1-\lambda\vec{p})^2\right]\ , \tag{5.8}$$

where

$$\lambda = \frac{2R^2+R_\pi{}^2}{2(3R^2+R_\pi{}^2)}\ . \tag{5.9}$$

One then uses simplifications leading to (4.11) and thus gets from (5.4)

$$\langle n_{1/2}(-\vec{p})\pi^+(\vec{p})|V^{g}_{Q^+\bar{Q}}(0)|p_{1/2}(\vec{0})\rangle = -\frac{5i}{3\sqrt{6}}\left(\frac{3R^2}{2\pi}\right)^{3/2}\left(\frac{R_\pi{}^2}{\pi}\right)^{3/4}(2\pi)^{-3}\tfrac{1}{2}$$
$$\times\int u^{\dagger}_{I(1/2)}\{(\vec{\sigma}\cdot\vec{k}_2'')[f_1'(1-f_2'')+g^2\vec{k}_1'^2+f_1'g(k_2''^0-k_1'^0)]$$
$$-(\vec{\sigma}\cdot\vec{k}_1')[f_2''(1-f_1')+g^2\vec{k}_2''^2+f_2''g(k_1'^0-k_2''^0)]$$
$$+b(\vec{\sigma}\cdot\vec{p})[f_1'f_2''(g^{-1}-k_1'^0-k_2''^0)+g(f_1'\vec{k}_2''^2+f_2''\vec{k}_1'^2)]$$
$$-bg(\vec{\sigma}\cdot\vec{k}_1')(\vec{\sigma}\cdot\vec{p})(\vec{\sigma}\cdot\vec{k}_2'')[g(k_1'^0+k_2''^0)+1-f_1'-f_2'']\}v_{I(-1/2)}$$
$$\times\exp\left[-\frac{3R^2+R_\pi{}^2}{2}(\vec{k}_1-\lambda\vec{p})^2\right]d^3k_1\ . \tag{5.10}$$

In (5.10), $f_1' = f(\vec{k}_1'^2)$, $f_2'' = f(\vec{k}_2''^2)$, and

$$b = (m+m_\pi)/(2mm_\pi)\ . \tag{5.11}$$

We now use (3.7) and make a straightforward integration, and neglect terms such as $\vec{p}^{\,2}$. We then obtain

$$\langle n_{1/2}(-\vec{p})\pi^+(\vec{p})|V^{g}_{Q^+\bar{Q}}(0)|p_{1/2}(\vec{0})\rangle = -\frac{5i}{3\sqrt{6}}\times(2\pi)^{-3}\times\left(\frac{3R^2}{3R^2+R_\pi{}^2}\right)^{3/2}\times\left(\frac{R_\pi{}^2}{\pi}\right)^{3/4} i\,\alpha u^{\dagger}_{I(1/2)}(\vec{\sigma}\cdot\vec{p})u_{I(1/2)}\ , \tag{5.12}$$

where

$$\alpha = \tfrac{1}{2}b(g^{-1} - k_1'^0 - k_2''^0) + \tfrac{1}{2}g(k_1'^0 - k_2''^0)(\lambda_1' - \lambda_2') + \frac{3g^2}{3R^2+R_\pi^2}\left[\frac{5}{4}(\lambda_1'+\lambda_2') + \frac{b}{3}(2g^{-1}+k_1'^0+k_2''^0) - \frac{1}{12}(\lambda_1'-\lambda_2')(k_1'^0-k_2''^0)\right] - \frac{g^4}{(3R^2+R_\pi^2)^2}\left[\frac{10b}{g} + \frac{35(\lambda_1'+\lambda_2')}{8}\right]. \tag{5.13}$$

In (5.13),

$$\lambda_1' = \tfrac{2}{3} - \lambda, \quad \lambda_2' = \tfrac{1}{2} - \lambda . \tag{5.14}$$

Hence, comparing (5.12) and (5.2), which should be the same, we get

$$G = \alpha \times \pi^{3/4} \times \frac{20}{3\sqrt{3}} \times \sqrt{m_\pi} \times m \times R_\pi^{3/2} \times \left(\frac{3R^2}{3R^2+R_\pi^2}\right)^{3/2} . \tag{5.15}$$

On taking $R^2 = R_\pi^2 = 15\ \text{GeV}^{-2}$ and $g = 1.67\ \text{GeV}^{-1}$, as earlier, we get $\alpha = 0.798$, such that (5.15) yields

$$G = 15.8 \times \alpha = 12.6 , \tag{5.16}$$

compared to the experimental value of 13.5. At this stage we shall be satisfied with the above rough agreement. We notice that this "determines" R_π^2, which value we had assumed earlier in (3.36), anticipating the results here.

B. $N^* \to N\pi$

We shall consider the above decay process with the quark-pair-creation Hamiltonian in a similar manner. However, since we are interested in a rough verification we shall take N^{*++} and p as nonrelativistic objects and thus for $N^{*++}_{3/2} \to p_{1/2} + \pi^+ (P \text{ wave})$, we shall take our field-theoretic Hamiltonian as

$$\mathcal{H}_I^F(x) = \frac{f}{m_\pi}\, \bar{p}_{1/2}(x)[\partial_1\pi^+(x) - i\partial_2\pi^+(x)]^* N^{*++}_{3/2}(x).$$

With this nonrelativistic form of the field theory we calculate the width of N^* and substituting $\Gamma = 0.115$ GeV, we obtain

$$\frac{|f|}{m_\pi} \simeq 10\ \text{GeV}^{-1} . \tag{5.17}$$

We next note that

$$\langle p_{1/2}(-\vec{p})\pi^+(\vec{p})|\mathcal{H}_I^F(0)|N^{*++}_{3/2}(\vec{0})\rangle = \frac{f}{m_\pi}(2\pi)^{-9/2}\frac{1}{(2m_\pi)^{1/2}}(-i)|\vec{p}|\sin\theta e^{i\phi} . \tag{5.18}$$

We shall compare (5.18) with

$$\langle p_{1/2}(-\vec{p})\pi^+(\vec{p})|V^g_{Q+\bar{Q}}(0)|N^{*++}_{3/2}(\vec{0})\rangle . \tag{5.19}$$

We note that in the quark model

$$|N^{*++}_{3/2}(\vec{0})\rangle = \frac{\epsilon_{ijk}}{6}\int \delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)u(\vec{k}_1,\vec{k}_2,\vec{k}_3)d^3k_1 d^3k_2 d^3k_3\, \mathcal{P}^i_{I(1/2)}(k_1)^\dagger \mathcal{P}^j_{I(1/2)}(k_2)^\dagger \mathcal{P}^k_{I(1/2)}(k_3)^\dagger|\text{vac}\rangle , \tag{5.20}$$

with $u(\vec{k}_1,\vec{k}_2,\vec{k}_3)$ given by (3.1). The contribution to (5.20) comes exactly in the same manner as in the previous subsection and we finally get, with α as in (5.13),

$$\langle p_{1/2}(-\vec{p})\pi^+(\vec{p})|V^g_{Q+\bar{Q}}(0)|N^{*++}_{3/2}(\vec{0})\rangle = (2\pi)^{-3}\frac{-i}{\sqrt{3}}\left(\frac{3R^2}{3R^2+R_\pi^2}\right)^{3/2}\left(\frac{R_\pi^2}{\pi}\right)^{3/4} \times \alpha u^\dagger_{I-1/2}\vec{\sigma}\cdot\vec{p}\, v_{I-1/2} . \tag{5.21}$$

We now compare (5.18) and (5.21), which yields

$$\frac{|f|}{m_\pi} = \alpha\pi^{3/4}\frac{4}{\sqrt{3}}\sqrt{m_\pi}\,R_\pi^{3/2}\left(\frac{3R^2}{3R^2+R_\pi^2}\right)^{3/2} \tag{5.22}$$

$$\simeq 8\ \text{GeV}^{-1} , \tag{5.23}$$

which is in rough agreement with (5.17).

C. $\rho \to 2\pi$

The field-theoretic Hamiltonian taken here is

$$\mathcal{H}_I^F(x) = f_\rho \epsilon_{ijk}\rho_i^\mu(x)\pi_j(x)\partial_\mu\pi_k(x) , \tag{5.24}$$

which, on substituting $\Gamma = 0.152$ GeV, yields

$$f_\rho \simeq 6.05 . \tag{5.25}$$

Further, for *small* momenta we have

$$\langle \pi^+(\vec{p})\pi^0(-\vec{p})|\mathcal{H}_I^F(0)|\rho^+(\vec{0})\rangle = (2\pi)^{-9/2}\frac{-if_\rho}{m_\pi(2m_\rho)^{1/2}}\frac{1}{\sqrt{2}}|\vec{p}|\sin\theta e^{i\phi} , \tag{5.26}$$

which will be the expression to be compared with the corresponding expression from the quark model to determine f_ρ from the quark model. We thus take[1]

$$|\rho_1^+(\vec{0})\rangle = \frac{1}{\sqrt{3}} \int \delta(\vec{k}_1 + \vec{k}_2) u_\rho(\vec{k}_1) d^3k_1\, d^3k_2 \times \mathcal{P}^i_{I(1/2)}(k_1)^+ \mathfrak{N}_{I(1/2)}(k_2)|\text{vac}\rangle \ . \quad (5.27)$$

In the quark model, for $\rho^+ \to \pi^+\pi^0$, the spectator $\mathcal{P}$ quark may go to π^+ or to π^0. Through (4.11) one can easily satisfy oneself that both these contributions will be identical. Taking into account this fact, one obtains, in the quark model,

$$\langle \pi^+(\vec{p})\pi^0(-\vec{p})|V^g_{Q^+\bar{Q}}(0)|\rho_1^+(\vec{0})\rangle = (2\pi)^{-3} \frac{1}{2\sqrt{6}} \left(\frac{R_\pi^{\,2}}{\pi}\right)^{3/4} \left(\frac{2}{3}\right)^{3/2} 2\alpha u^+_{I-1/2} \vec{\sigma}\cdot\vec{p}\, v_{I-1/2} \ , \quad (5.28)$$

where

$$\alpha = \frac{1}{2m_\pi}(g^{-1} - m_\pi) + \frac{1}{R_\pi^{\,2}}\left[\tfrac{5}{12} g^2 + \frac{g}{3m_\pi}(2 + g m_\pi)\right] - \frac{1}{R_\pi^{\,4}}\left(\frac{10g^3}{9m_\pi} + \frac{35 g^4}{16\times 27}\right) . \quad (5.29)$$

In (5.29), we have taken $u_\rho(\vec{k}) = u_\pi(\vec{k})$ and $g_{\mathcal{P}} = g_{\mathfrak{N}} = g$ as earlier. Also in (4.11), $k^0 = k'^0 = m_\pi/2$ has been used. Comparison of (5.28) with (5.26) yields

$$|f_\rho| = \alpha\pi^{3/4} \frac{8\sqrt{2}}{9} m_\pi \sqrt{m_\rho} R_\pi^{\,3/2} \ . \quad (5.30)$$

With $m_q = 0.3$ GeV or $g = 1.67$ GeV^{-1}, one gets $\alpha = 2.156$, such that (5.30) yields for f_ρ as determined from the quark model

$$|f_\rho| \simeq 5.95 \ . \quad (5.31)$$

The agreement between (5.31) and the experimental value (5.25) is quite good. We may further note that if we use equivalence of (5.28) and (5.25), and then use (5.24), we get

$$\Gamma(\rho \to 2\pi) = \frac{64\times\sqrt{\pi}\times m_\pi^{\,2}}{9\times 27} \times |\vec{p}|^3 \times \alpha^2 R_\pi^{\,3} \quad (5.32)$$

$$= 147 \text{ MeV} \ , \quad (5.33)$$

which is well within the experimental error for the width of ρ meson.

We note that in Secs. V B and V C, no new parameter has been used.

D. $\phi \to 2K$

We first remark that the present model through Lorentz boosting as in Sec. II differs quantitatively from what we had done in I although for both the starting point is the pair-creation Hamiltonian (4.1).

As in Sec. V C, here we obtain, on the basis of the quark model using (4.11), and taking the ϕ meson as a pure λ-quark-antiquark pair,

$$\langle K^+(\vec{p})K^-(-\vec{p})|V^g_{Q^+\bar{Q}}(0)|\phi_1(\vec{0})\rangle = (2\pi)^{-3} \frac{1}{2\sqrt{3}} \left(\frac{R_\pi^{\,2}}{\pi}\right)^{3/4} (\tfrac{2}{3})^{3/2} \alpha u^+_{I-1/2} \vec{\sigma}\cdot\vec{p}\, v_{I-1/2} \ , \quad (5.34)$$

where

$$\alpha = b(m_{\mathcal{P}} - \omega_{\mathcal{P}}) + \frac{1}{R_\pi^{\,2}}\left[\frac{5\lambda_2}{6} g_{\mathcal{P}}^{\,2} + \tfrac{2}{3} b g_{\mathcal{P}}(1 + g_{\mathcal{P}}\omega_{\mathcal{P}})\right] - \frac{1}{R_\pi^{\,4}}\left(\frac{10 b g_{\mathcal{P}}^{\,3}}{9} + \frac{35\lambda_2 g_{\mathcal{P}}^{\,4}}{8\times 27}\right) . \quad (5.35)$$

In (5.35),

$$b = m_K^{\,-1} \ , \quad (5.36)$$

$\omega_{\mathcal{P}}$ is the occupied energy level of the $\mathcal{P}$ quark in the K^+ meson at rest, and $\lambda_2 = 1 - \omega_{\mathcal{P}}/m_K$. We have also taken the harmonic-oscillator wave functions of all the mesons to be same as that of π meson. As in the last section, one then makes a field-theoretic identification, and then estimates the result corresponding to (5.32) as

$$\Gamma(\phi \to K^+K^-) = \frac{32\sqrt{\pi}\, m_K^{\,2}}{9\times 27 m_\phi} |\vec{p}|^3 \alpha^2 R_\pi^{\,3} \ . \quad (5.37)$$

Let us next consider the process $\phi \to K^0\bar{K}^0$. This yields, parallel to (5.37),

$$\Gamma(\phi \to K^0\bar{K}^0) = \frac{32\sqrt{\pi}\, m_K^{\,2}}{9\times 27 m_\phi} |\vec{p}\,'|^3 \alpha'^2 R_\pi^{\,3} \ , \quad (5.38)$$

where α' is defined in a manner similar to (5.35), with corresponding change of some parameters. In the present case, in contrast to the earlier cases, $\omega_{\mathcal{P}}$ for the fractional energy of the $\mathcal{P}$ quark in K^+ is not unambiguously defined. We first *assume* that the fractional energies are proportional to the respective quark masses. Then in (5.35) we obtain

$$\omega_{\mathcal{P}} = 0.188 \text{ GeV}, \quad \lambda_2 = 0.62 \ . \quad (5.39)$$

Substituting these, we obtain, for $R_\pi^{\,2} = 15$ GeV^{-2},

$$\alpha = 0.469 \, . \quad (5.40)$$

Similarly, with the corresponding parameters changed, one obtains

$$\alpha' = 0.459 \, . \quad (5.41)$$

Using the above values in (5.37) and (5.38), one derives

$$\Gamma(\phi \to K^0\bar{K}^0)/\Gamma(\phi \to K^+K^-) = 0.635 \ . \quad (5.42)$$

We note that the above value is much below the world average, which is[15] 0.74 ± 0.06. However, Kalbefleisch *et al.*[19] obtain the above ratio as

0.60 ± 0.06, and the value in (5.42) is well within this range.

One then obtains from (5.37) and (5.38) that

$$\Gamma(\phi \to K^+K^-) + \Gamma(\phi \to K^0\overline{K}^0) = 2.45 \text{ MeV} . \quad (5.43)$$

Clearly (5.43) is too small. We attribute this to the wrong value of $\omega_{\mathcal{P}}$ in K^+.

We note that the assumption that the fractional energy of the constituents is proportional to the quark mass as made above is likely to be true only when the binding energies are negligible. In the present case for mesons this is obviously not so. Let us for a moment imagine the hydrogen atom, with the proton as infinitely heavy. Then effectively the whole binding energy will "belong" to the electron, and $\omega_e = m_e - B$. Thus, the binding energy "shared" by constituents may even be inversely proportional to the respective masses of the constituents. However, in the absence of mass level spectroscopy generating the occupied energy levels for the constituents, it is futile to use any rule of thumb except the above qualitative comments. We note that the $\mathcal{P}$-quark energy level in K^+ enters the first term in (5.35) in a sensitive manner. Hence we now use that equation to determine $\omega_{\mathcal{P}}$. Thus if we take as a rough assignment

$$\omega_{\mathcal{P}} = 0.158 \text{ GeV} \quad (5.44)$$

such that

$$\lambda_2 = 0.68 = 1 - \lambda_1 , \quad (5.45)$$

we then obtain

$$\alpha = 0.538 \quad (5.46)$$

and, keeping λ_2 unaltered,

$$\alpha' = 0.514 . \quad (5.47)$$

This yields

$$\Gamma(\phi \to K^0\overline{K}^0)/\Gamma(\phi \to K^+K^-) = 0.61 , \quad (5.48)$$

which is within the value of Ref. 19. We then obtain, instead of (5.43),

$$\Gamma(\phi \to 2K) \simeq 3.2 \text{ MeV} , \quad (5.49)$$

which is in fairly good agreement with experiments.[19]

E. $K^* \to K\pi$

Here, with field-theoretic identifications as done earlier, parallel to Eqs. (5.32), (5.37), and (5.38), we obtain

$$\Gamma(K^{*+} \to K^+\pi^0) = \frac{16\sqrt{\pi}\, m_\pi m_K}{9 \times 27 m_{K^*}} \, |\vec{p}|^3 \alpha^2 R_\pi^{\ 3} \quad (5.50)$$

and

$$\Gamma(K^{*+} \to K^0\pi^+) = 2\Gamma(K^{*+} \to K^+\pi^0) , \quad (5.51)$$

where

$$\begin{aligned}\alpha = {} & \tfrac{1}{2} b(g^{-1} - \tfrac{1}{2} m_\pi - \lambda_1 m_K) + \tfrac{1}{2} g(\lambda_1 m_K - \tfrac{1}{2} m_\pi)(\tfrac{1}{2} - \lambda_1) \\ & + \frac{g^2}{R_\pi^{\ 2}} \left[\tfrac{5}{24}(1 + 2\lambda_2) + \tfrac{1}{3} b(\lambda_1 m_K + \tfrac{1}{2} m_\pi + 2g^{-1}) \right. \\ & \left. - \tfrac{1}{12} g(\lambda_1 m_K - \tfrac{1}{2} m_\pi)(\tfrac{1}{2} - \lambda_2)\right] \\ & - \frac{g^4}{R_\pi^{\ 4}}\left[\frac{10b}{9g} + \frac{35(1 + 2\lambda_2)}{16 \times 27}\right] . \end{aligned} \quad (5.52)$$

In (5.52) obviously

$$b = (m_\pi + m_K)/(2 m_\pi m_K) . \quad (5.53)$$

Also, λ_1 has been determined earlier in (5.44). One then obtains

$$\alpha = 1.22 , \quad (5.54)$$

such that we have

$$\Gamma(K^*) \simeq 56 \text{ MeV} , \quad (5.55)$$

which is in reasonable agreement with experimental value of about 50 MeV. We note that in this subsection we have no free parameters, since λ_1 has been determined in Sec. V D.

VI. DISCUSSIONS

Let us now first explicitly note the basic assumptions of the present theory. We have assumed that hadrons can be described by quark field operators in the rest frame of these hadrons, and a simple, but not conventional ansatz is made for these field operators in this frame of reference. We next assume that constituent quarks occupy fixed energy levels when the hadron is at rest, and the mass of the hadron is given additively in terms of these energies. Thus the quarks as *constituents do not* lie on a mass hyperbola. The above assumption, however, gives space and time dependence of quark field operators as constituents of hadrons. To obtain the corresponding field operators for hadrons in motion, we Lorentz-boost the above field operators, knowing that these are Dirac field operators and hence have known transformation properties under Lorentz transformations. In Eq. (2.20), we assume the conventional anticommutators for the corresponding field operators, which are defined through (2.18).

We next generalize the quark field operators to bring in $Q^g(x)$, which includes all the above type of field operators corresponding to the quark as constituent of any hadron in any frame of reference. E.g., this generalized quark field operator enters into the electromagnetic current. For specific interaction processes $Q^g(x)$ becomes equivalent to some $Q^{L(p)}(x)$; without this identification $Q^g(x)$ has no meaning. Thus in this theory, quarks enter only as constituents of hadrons, and can have no meaning otherwise.

We next use this theory to estimate the quark parameters through the nonrelativistic properties of baryons. In particular, g_A/g_V and the magnetic moments of baryons are used to fix these parameters as well as the radius of the harmonic-oscillator wave function of the baryons. The charge radius of the proton and of the pion have also been calculated, but have not been used to fix the parameters.

We next generalize the quark-pair-creation term of the Dirac Hamiltonian to a covariant form, which is then utilized to generate strong interactions. With the radius of the harmonic-oscillator wave function of the meson as the same as that of the baryon, this estimates $G_{NN\pi}$ fairly well. Without any more new parameters, $N^* \to N\pi$ and $\rho \to 2\pi$ are also predicted in a reasonable manner.

We next estimate the energy level of the $\mathcal{P}$ quark in K^+ to get correct $\Gamma(\phi \to 2K)$. With this parameter fixed, we calculate $\Gamma(K^*)$, which agrees reasonably with the experimental value.

We note that in this manner we have generated successfully five strong-interaction processes quantitatively in terms of *only* quark-model parameters and quark-model harmonic-oscillator wave functions. The success of the model indicates that broadly speaking, these ideas may be valid and may be at the basis of the Okubo-Zweig-Iizuka rule,[3] as was conjectured in I.

We note that in the context of the present model hadronic dynamics is extremely complicated. E.g., hadron-hadron scattering will have three possible types of contributions. (i) We can have exchange reactions for this scattering where constituents will get exchanged,[20] and we shall have overlap integrals of the hadron wave functions. (ii). The potential which binds hadrons and is saturated in a manner we do not know may have residual contributions for hadron-hadron scattering corresponding to residual potential for atom-atom collisions. (iii) The quark-pair-creation terms discussed in Secs. IV and V will give rise to normal hadron exchanges as in field theory, and will contribute to hadron-hadron scattering in a conventional manner. In field theory we only consider the third type of interactions. With a covariant form of the quark-pair creation in (4.5) this contribution is *expected* to have relativistic covariance as well as normal unitarity, analytically, and crossing-symmetry properties. Hence, for processes where the contribution (iii) dominates, the general model-independent analyses using the above properties only are expected to continue to be valid.

Let us next consider electromagnetic interactions of hadrons. It can be easily seen that such processes *can* take place with intermediate neutral vector mesons, since, as in I, $\langle \mathrm{vac} | J^\mu(0) \times | \rho_\lambda^0(\vec{k}) \rangle$ *does not* vanish. This will *not* be equivalent to vector dominance, but it may give similar contributions. It is possible that the lack of accuracy in charge radius of proton and pion as estimated earlier may be due to such a contribution. We expect that any correction to the form factor (3.8) has to vanish when $t = 0$ since this estimates the charge of the proton as a bound state, and in this sense the contribution through e.g., ρ^0 will be totally different from what one gets from the vector dominance model.

Although we have estimated the form factor of the proton and the pion, we do not take these expressions seriously. Firstly, the effect of intermediate vector mesons has been ignored as stated above. Further, large momentum transfers probe the hadronic wave functions in regions where the harmonic-oscillator approximation for these may not be valid. When we have adequate confidence in the model, we can use this model to estimate the large-momentum-transfer behavior of the hadron wave functions. It will also become worthwhile to estimate the effects of e.g. SU(6) mixing, in order to be able to generate static results better,[21] which we have ignored here.

We should finally like to remark that although the model has mathematical complications, the physical ideas are mostly simple and straightforward. It is nice to see that these simple ideas can generate strong-interaction processes in a quantitative manner, predicting strong-interaction coupling constants in terms of quark-model parameters and the harmonic-oscillator wave functions of the hadrons only, and without any parameters outside the quark model. All the same, we would like to comment that because of complications of hadron dynamics as envisaged here, we consider the ideas of the present paper as more relevant than the agreement with the experimental results that has been achieved.

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