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Field-theoretic model of composite hadrons. I

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We discuss here a field-theoretic model of composite hadrons with quarks. The quark field operators are assumed to be broken up into particle and antiparticle components at any time t similar to the large and small components of a free-Dirac-field operator. This assumption is made consistent with equal-time anticommutators. This implies that the Dirac Hamiltonian for quark field operators has four components: the particle, antiparticle, pair-creation, and pair-annihilation components. For a simple ansatz for the field operators, the latter two components need not vanish. The particle and antiparticle components along with some potential-like interactions are assumed to generate the hadrons as composite states. With the usual form of weak and electromagnetic currents, this yields some corrections to the Van Royen-Weisskopf relations and gives excellent agreement for the static properties of the nucleons. It is seen that the pair-creation component of the Hamiltonian can generate $\phi \to KK$ decay, with a correct branching ratio for $\Gamma(\phi \to K^+K^-)/\Gamma(\phi \to K^0\bar{K}^0)$. Thus, the pair-creation Hamiltonian seems to be the dynamical explanation of the Okubo-Zweig-Iizuka rule. Further, the pair-annihilation component of the Hamiltonian with minimal electromagnetic interaction also generates $\Gamma(\pi^0 \to 2\gamma)$. With the mixing angle obtained from the quadratic mass formula, $\Gamma(\eta \to 2\gamma)$ also seems to have a reasonable prediction. We have considered only nonrelativistic hadrons hoping that a potential-like description is valid in such a frame of reference.

I. INTRODUCTION

We shall consider here a field-theoretic model of composite hadrons described in terms of quarks. We first consider the quark field operators as four-component Dirac field operators which need not satisfy the free Dirac equation. We then break up these field operators into particle and antiparticle components in a manner to be specified. An ansatz for this breakup of the field operators is made which is consistent with equal-time anticommutators of the Dirac field and with translational invariance. The "large" and "small" components of these operators are consistently retained without assuming that we have the free Dirac equation.

We next consider the Dirac Hamiltonian corresponding to these operators. This Hamiltonian can have four components: the particle Hamiltonian, the antiparticle Hamiltonian, and the pair-creation and pair-annihilation Hamiltonians. For the free-Dirac-field Hamiltonian, the two latter components vanish. However, we note that, consistent with our ansatz, these need not vanish in general. The particle and/or antiparticle Hamiltonians, together with a potential we do not know, or with a potential-like interaction we do not understand, are assumed to yield composite hadrons as eigenstates of the Hamiltonian. For an interaction capable of being written with an effective potential, conventional field-theoretic techniques[1] using only equal-time anticommutation relations will yield the equations for the wave functions of composite hadrons. This can give rise to the usual mass-eigenvalue equations of the nonrelativistic quark models.[2] Here in this paper we do not enter into this aspect of hadron spectroscopy since we regard this as a very difficult problem, in spite of the many fairly successful attempts.[2] On the other hand, we assume that suitable solutions exist, and attempt to exploit the consequences.

Here we note that we are essentially using the Schrödinger equation. We prefer to do so because we want to obtain the composite hadronic state in terms of constituent-quark field operators at any fixed time t. We consider that the composite state should be capable of being described on a space-like surface as a functional of the constituent field operators on that surface. For simplicity we have taken them as equal-time surfaces. Partovi[3] has given a covariant formulation of bound-state problems which in the c.m. of the bound state yields a Schrödinger-type equation. For covariance we have essentially such a picture in mind, instead of the usual Bethe-Salpeter equation,[4] where the time degree of freedom is known to yield extra energy eigenvalues[5] and the problem of physical interpretation of the wave functions.[6]

The above comments, considered in the context of the present paper, do not constitute an essential assumption. What is really assumed is that a potential-like interaction for the quark field operators exists in the rest frame of the hadrons. Hence we do not attempt to maintain covariance for the quark field operators. Instead, we try to retain the whole Dirac field operator which could describe the quarks as constituents of hadrons at rest, including relativistic motion of quarks inside the hadron.[7] We are looking for an essential parametrization of this field operator that will yield some known properties of the hadrons.

We use these quark field operators in the usual form for weak and electromagnetic currents. This gives the usual Van Royen–Weisskopf relations[8] with corrections coming from "small" Dirac components. Also, $\pi^+ \to \pi^0 + e^+ + \nu_e$ is uniquely determined in this model with no arbitrary constant involved and is consistent with the usual symmetry property of the conserved vector current.[9]

It is known that the three-dimensional harmonic-oscillator wave function describes the composite hadrons fairly well,[10] particularly for baryons. We note that g_A/g_V, magnetic moments, and the charge radius of the proton come out extremely well with a simple parametrization of the Dirac field operator for the quarks.

With this parametrization, the quark-pair-creation term of the Hamiltonian does not vanish. It is noted that this term alone can predict a $\Gamma(\phi \to K^+K^-)$ width for a reasonable harmonic-oscillator wave function if we do not take colored quarks. Even when colored quarks are there, the disagreement can be understood in a qualitative manner. This also yields a correction to $\Gamma(\phi \to K^+K^-)/\Gamma(\phi \to K^0\bar{K}^0)$ as would otherwise be determined from phase space only, and is in better agreement with experiments. The fact that the quark-pair-creation term in the Hamiltonian does not vanish can be a dynamical explanation of the Okubo-Zweig-Iizuka rule.[11] Any violation of the same will be a higher-order effect (for which we have at present no prescription), or, more likely, is due to mixing. In the latter case it will be closely related to hadron spectroscopy of mass levels, which we have not yet discussed.

With minimal electromagnetic coupling, the pair-annihilation term of the Hamiltonian also generates a Hamiltonian which describes $\Gamma(\pi^0 \to 2\gamma)$ in a reasonable manner, when the correction term for the Van Royen-Weisskopf relation is ignored in the pion weak decay. If the wave function at the space origin for pseudoscalar mesons remains unaltered, this also correctly predicts $\Gamma(\eta \to 2\gamma)$. However, these results seem to involve a modification of our ideas on gauge transformations, and thus, in the context of a unified picture of electromagnetic and weak interactions, need further investigations.

In all cases only first-order calculations are made using equal-time anticommutators of the quark field operators along with translational invariance. The agreement, although mostly good for the cases investigated, must be regarded as tentative and qualitative. However, this gives an opening through field theory to hadronic dynamics which would be useful. In fact it is nice to see that a simple field-theoretic model with only equal-time anticommutators and translational invariance is as successful as seems to be the case here. This method through field theory not only gives better insight into the conventional way of retaining large components in wave functions as has been done by many authors,[12] but gives a much broader perspective to hadronic dynamics.

II. GENERAL THEORY

We shall consider here a spin-$\frac{1}{2}$ four-component field operator for quarks in the presence of a potential. We first consider a simple ansatz regarding the quark field operators as follows. We assume that the quark field operator $\psi_Q(x)$ can be broken into two parts,

$$\psi_Q(x) = Q(x) + \tilde{Q}(x)\,, \tag{1}$$

where $Q(x)$ stands for annihilation of the quark and $\tilde{Q}(x)$ stands for creation of the antiquark. We next take the two-component form of the quark field operators as follows:

$$Q(x) = \begin{pmatrix} f(-\vec{\nabla}^2) \\ -i\vec{\sigma}\cdot\vec{\nabla}g(-\vec{\nabla}^2) \end{pmatrix} Q_I(x) \tag{2}$$

and

$$\tilde{Q}(x) = \begin{pmatrix} i\vec{\sigma}\cdot\vec{\nabla}g(-\vec{\nabla}^2) \\ f(-\vec{\nabla}^2) \end{pmatrix} \tilde{Q}_I(x)\,. \tag{3}$$

In Eqs. (2) and (3), f and g are, as noted, differentiation operators. $Q_I(x)$ and $\tilde{Q}_I(x)$ are two-component field operators introduced so that they have translational invariance in the usual form. Thus $\psi_Q(x)$ has normal transformation under translation. Differentiation operators are chosen to maintain translational invariance. The above operators will describe "constituents" of hadrons in their rest frame or at nonrelativistic energies in an obvious manner which we shall specify later.

We next consider the operators at time $t=0$, and take their Fourier transforms as

$$Q(\vec{x}) = (2\pi)^{-3/2}\int \begin{pmatrix} f(\vec{k}^2) \\ \vec{\sigma}\cdot\vec{k}g(\vec{k}^2) \end{pmatrix} Q_I(\vec{k}) \times \exp(i\vec{k}\cdot\vec{x})d^3k \tag{4}$$

and

$$\tilde{Q}(\vec{x}) = (2\pi)^{-3/2}\int \begin{pmatrix} \vec{\sigma}\cdot\vec{k}g(\vec{k}^2) \\ f(\vec{k}^2) \end{pmatrix} \tilde{Q}_I(\vec{k}) \times \exp(-i\vec{k}\cdot\vec{x})d^3k\,. \tag{5}$$

For the two-component operators, we assume the anticommutators

$$[Q_{Ir}(\vec{x}),\, Q_{Is}^\dagger(\vec{y})]_+ = [\tilde{Q}_{Ir}(\vec{x}),\, \tilde{Q}_{Is}^\dagger(\vec{y})]_+ = \delta_{rs}\delta(\vec{x}-\vec{y})\,, \tag{6}$$

and that all the other anticommutators vanish. The corresponding anticommutators in "momentum space" for equal times will also be valid.

For a moment we now use the notation

$$Q_I(\vec{k}) \equiv \begin{pmatrix} Q_I(\vec{k}) \\ 0 \end{pmatrix}$$

as a trivially four-component object and rewrite Eq. (4) as

$$Q(\vec{x}) = (2\pi)^{-3/2} \int (f + g\vec{\alpha}\cdot\vec{k}) Q_I(\vec{k}) \exp(i\vec{k}\cdot\vec{x}) d^3k \tag{7}$$

Then from Eq. (6) in momentum space one obtains

$$[Q_\alpha(\vec{x}), Q_\beta^\dagger(\vec{y})]_+ = (2\pi)^{-3} \int \left[f^2 \frac{(1+\beta)}{2} + (g^2\vec{k}^2)\frac{(1-\beta)}{2} + fg\vec{\alpha}\cdot\vec{k} \right]_{\alpha\beta} \exp[i\vec{k}\cdot(\vec{x}-\vec{y})] d^3k. \tag{8}$$

We also use

$$\tilde{Q}_I(\vec{k}) = \begin{pmatrix} 0 \\ \tilde{Q}_I(\vec{k}) \end{pmatrix}$$

for a moment as a four-component object and rewrite Eq. (5) as

$$\tilde{Q}(\vec{x}) = (2\pi)^{-3/2} \int (f + g\vec{\alpha}\cdot\vec{k}) \tilde{Q}_I(\vec{k}) \exp(-i\vec{k}\cdot\vec{x}) d^3k. \tag{9}$$

This yields, again using (6) in momentum space,

$$[\tilde{Q}_\alpha(\vec{x}), \tilde{Q}_\beta^\dagger(\vec{y})]_+ = (2\pi)^{-3} \int \left[f^2 \frac{(1-\beta)}{2} + g^2\vec{k}^2 \frac{(1+\beta)}{2} + fg\vec{\alpha}\cdot\vec{k} \right]_{\alpha\beta} \times \exp[-i\vec{k}\cdot(\vec{x}-\vec{y})] d^3k. \tag{10}$$

Thus from (1), (8), and (10), one obtains

$$[\psi_{Q\alpha}(\vec{x}), \psi_{Q\beta}^\dagger(\vec{y})]_+ = (2\pi)^{-3} \int (f^2 + g^2\vec{k}^2)_{\alpha\beta} \times \exp[i\vec{k}\cdot(\vec{x}-\vec{y})] d^3k = \delta_{\alpha\beta}\delta(\vec{x}-\vec{y}), \tag{11}$$

even when interaction is present. Hence, in the ansatz (2) and (3), f and g will not be independent; rather, we must have

$$f^2(\vec{k}^2) + g^2(\vec{k}^2)\vec{k}^2 = 1, \tag{12}$$

or

$$f^2(-\vec{\nabla}^2) = 1 + g^2(-\vec{\nabla}^2)\vec{\nabla}^2. \tag{13}$$

Hence, in a formal manner Eqs. (2) and (3) are now rewritten as

$$Q(x) = \begin{pmatrix} (1 + g_Q^2\vec{\nabla}^2)^{1/2} \\ -i\vec{\sigma}\cdot\vec{\nabla} g_Q \end{pmatrix} Q_I(x) \tag{14}$$

and

$$\tilde{Q}(x) = \begin{pmatrix} i\vec{\sigma}\cdot\vec{\nabla} g_Q \\ (1 + g_Q^2\vec{\nabla}^2)^{1/2} \end{pmatrix} \tilde{Q}_I(x). \tag{15}$$

Equations (14) and (15) are consistent with equal-time anticommutators for the Dirac field $\psi_Q(x)$. g_Q may depend on the quark chosen, which we have now included in the notation.

We now consider the Dirac Hamiltonian density

$$\mathcal{H}_{\psi_Q}(x) = :\psi_Q^\dagger(x)(-i\vec{\alpha}\cdot\vec{\nabla} + \beta m_Q)\psi_Q(x): \equiv \mathcal{H}_Q(x) + \mathcal{H}_{\tilde{Q}}(x) + \mathcal{V}_{Q+\tilde{Q}}(x) + \mathcal{V}_{\tilde{Q}+Q}(x). \tag{16}$$

In (16), we have broken the Hamiltonian into four components: $\mathcal{H}_Q$ and $\mathcal{H}_{\tilde{Q}}$ are respectively the quark and antiquark Hamiltonian densities and $\mathcal{V}_{Q+\tilde{Q}}$ and $\mathcal{V}_{\tilde{Q}+Q}$ are respectively pair-creation and pair-annihilation terms which may not vanish identically. We shall subsequently see that these Hamiltonians can give relevant physical processes for composite hadrons.

We then use the breakup (1) with (14) and (15) and substitute in (16). Then, ignoring space-divergent contributions in the Hamiltonian densities, we obtain

$$\mathcal{H}_Q(x) = Q_I^\dagger(x)[m_Q(f_Q^2 + g_Q^2\vec{\nabla}^2) - 2f_Q g_Q\vec{\nabla}^2]Q_I(x), \tag{17}$$

$$\mathcal{H}_{\tilde{Q}}(x) = :\tilde{Q}_I^\dagger(x)[-m_Q(f_Q^2 + g_Q^2\vec{\nabla}^2) + 2f_Q g_Q\vec{\nabla}^2]\tilde{Q}_I(x):, \tag{18}$$

$$\mathcal{V}_{Q\dagger\tilde{Q}}(x) = Q_I^\dagger(x)[(-i\vec{\sigma}\cdot\vec{\nabla})(f_Q^2 + g_Q^2\vec{\nabla}^2 - 2m_Q g_Q f_Q)]\tilde{Q}_I(x), \tag{19}$$

and

$$\mathcal{V}_{\tilde{Q}\dagger Q}(x) = \tilde{Q}_I^\dagger(x)[(-i\vec{\sigma}\cdot\vec{\nabla})(f_Q^2 + g_Q^2\vec{\nabla}^2 - 2m_Q f_Q g_Q)]Q_I(x). \tag{20}$$

We note that for free-quark-field operators

$$g_Q = [2p^0(p^0 + m_Q)]^{-1/2}, \tag{21}$$

and thus

$$f_Q = [(p^0 + m_Q)/(2p^0)]^{1/2}, \tag{21'}$$

where

$$p^0 = (m_Q^2 - \vec{\nabla}^2)^{1/2}. \qquad (21'')$$

In fact, we write the particle component of the free Dirac field as

$$Q(x) = (2\pi)^{-3/2} \int \begin{pmatrix} f_Q \\ -i\vec{\sigma}\cdot\vec{\nabla}g_Q \end{pmatrix} Q_I(\vec{p}) \exp(-ipx) d^3p, \qquad (22)$$

where f_Q and g_Q are given by (21) and (21′) and

$$Q_I(\vec{p}) = \sum_{r=\pm 1/2} a_r(\vec{p}) u_{Ir}. \qquad (23)$$

In such a case, i.e., for free quark fields, one gets from (17) and (21)

$$\mathcal{H}_Q(x) = Q_I^\dagger(x)(m_Q^2 - \vec{\nabla}^2)^{1/2} Q_I(x). \qquad (24)$$

We have a similar expression for $\mathcal{H}_{\bar{Q}}(x)$, and the quark-pair-creation and quark-pair-annihilation components identically vanish. However, for quarks representing constituents of a hadron, strongly bound in a manner we do not understand, such an identification by taking Eq. (21) may be inappropriate. For *a posteriori* reasons, taking g_Q as a constant depending only on the quark appears to be more reasonable. Technically, our intention here is to keep g_Q as arbitrary, and see whether we can associate meaning to these as constants dependent on quarks and/or hadrons of which they happen to be constituents.

We shall now consider hadrons to be composite objects obtained as eigenstates from quark and/or antiquark components of the Dirac Hamiltonian (16) bound by some potential, the nature and the origin of which we do not at present understand. For example, let us consider the Hamiltonian densities $\mathcal{H}_{\mathcal{P}}(x)$, $\mathcal{H}_{\bar{\mathcal{P}}}(x)$, $\mathcal{H}_{\mathfrak{N}}(x)$, $\mathcal{H}_{\bar{\mathfrak{N}}}(x)$ and the spin-singlet and isospin-triplet potential

$$V_{st}(t) = \tfrac{1}{2} \int q_I^\dagger(\vec{x}, t)\tau_i \tilde{q}_I(\vec{y}, t) v_{st}(\vec{x} - \vec{y}) \times \tilde{q}_I^\dagger(\vec{y}, t)\tau_i q_I(\vec{x}, t) d^3x\, d^3y. \qquad (25)$$

In (25), we have adopted the notation

$$q_I(\vec{x}, t) = \begin{pmatrix} \mathcal{P}_I(\vec{x}, t) \\ \mathfrak{N}_I(\vec{x}, t) \end{pmatrix}. \qquad (26)$$

Let us now construct the state

$$a^\dagger(\pi_i(\vec{0}))|\text{vac}\rangle \equiv (2\pi)^{-3/2} \frac{1}{\sqrt{2}} \int q_I^\dagger(\vec{x}, t)\tau_i \tilde{q}_I(\vec{y}, t) \times u_\pi(\vec{x} - \vec{y}) d^3x\, d^3y\, |\text{vac}\rangle. \qquad (27)$$

We easily see that for any t, (27) is an eigenstate of

$$H_{\mathcal{P}}(t) + H_{\mathfrak{N}}(t) + H_{\bar{\mathcal{P}}}(t) + H_{\bar{\mathfrak{N}}}(t) + V_{st}(t)$$

with an eigenvalue m_π and with total momentum zero provided

$$[2m_q(f^2 + g^2\vec{\nabla}^2) - 4fg\vec{\nabla}^2 + v_{st}(\vec{x})]u_\pi(\vec{x}) = m_\pi u_\pi(\vec{x}). \qquad (28)$$

Equation (28) is the Schrödinger equation written down in a formal manner using (6) where both the large and small components of the constituent field operator have been retained. Equal-time anticommutators are adequate as long as we can approximate the interaction through a potential which we have assumed here for the rest frame of the hadrons. Equation (28) will give mass levels for hadron spectroscopy, which we can write down in a conventional form with heavy quarks with the usual approximation for g as in (21). The relativistic corrections for quarks have been retained in (28) in a formal manner. The advantage gained is that we can use the operator expressions for electromagnetic and weak interactions through field theory, and get suitable corrections from "small" components which we shall proceed to demonstrate. We regard pair-creation and pair-annihilation components of the Hamiltonian as "perturbations" which will yield decay amplitudes.

III. APPLICATIONS

A. Mesonic interactions and Van Royen-Weisskopf relations

Let us include a color degree of freedom and write the π^+ state at rest as

$$|\pi^+(\vec{0})\rangle = (2\pi)^{-3/2} \frac{1}{\sqrt{6}} \int \mathcal{P}_I^i(\vec{x}_1)^\dagger \tilde{\mathfrak{N}}_I^i(\vec{x}_2) u_\pi(\vec{x}_1 - \vec{x}_2) \times d^3x_1\, d^3x_2\, |\text{vac}\rangle, \qquad (29)$$

where i is is the color index. Similarly, we can write the $|\pi^0(\vec{0})\rangle$ state. For momenta $\vec{k}$ such that $|\vec{k}| \ll m_\pi$, we also have with (26)

$$|\pi^0(\vec{k})\rangle = (2\pi)^{-3/2} \frac{1}{2\sqrt{3}} \int u_\pi(\vec{x}_1 - \vec{x}_2) \exp[i\vec{k}\cdot(\vec{x}_1 + \vec{x}_2)/2]\, d^3x_1\, d^3x_2\, q_I^i(\vec{x}_1)^\dagger \tau_3 \tilde{q}_I^i(\vec{x}_2)\, |\text{vac}\rangle. \qquad (30)$$

With (6), Eq. (30) gives the usual normalization

$$\langle \pi^0(\vec{k}) | \pi^0(\vec{k}')\rangle = \delta(\vec{k} - \vec{k}'), \qquad (31)$$

provided we use the normalization

$$\int |u_\pi(\vec{x})|^2 d^3x = 1. \qquad (32)$$

With Fourier transforms as in (4) and (5), and

$$u_\pi(\vec{x}) = (2\pi)^{-3/2}\int u_\pi(\vec{k})\exp(i\vec{k}\cdot\vec{x})d^3k, \tag{33}$$

one obtains

$$|\pi^0(\vec{k})\rangle = \frac{1}{2\sqrt{3}}\int \delta(\vec{k}_1+\vec{k}_2)d^3k_1d^3k_2u_\pi(\vec{k}_1) \times q_I^i(\vec{k}_1+\tfrac{1}{2}\vec{k})^\dagger\tau_3\tilde{q}_I^i(\vec{k}_2+\tfrac{1}{2}\vec{k})|\text{vac}\rangle. \tag{34}$$

In the above, we construct the bound states with field operators at time $t=0$. In our applications we shall consider translational invariance of the field operators and the equal-time anticommutators (6) in coordinate or momentum space, with the field operators being represented by (14) and (15) for nonrelativistic hadrons.

1. $\pi^+ \to \pi^0 + e^+ + \nu_e$

The matrix element for the above decay process is known from the SU(2) property of the weak vector current. Since the quark model is an explicit realization of this symmetry, the same result is derived; however, we do the trivial calculation here to illustrate the method.

Taking the rest frame of the two pions, the relevant component of the weak current which will contribute to the above provess is $V_1^0(x) - iV_2^0(x)$, which has the necessary term

$$\bar{\mathfrak{N}}^i(x)\gamma^0\mathcal{P}^i(x) + \bar{\tilde{\mathfrak{N}}}^i(x)\gamma^0\tilde{\mathcal{P}}^i(x) = \mathfrak{N}_I^i(x)^\dagger[\overleftarrow{f}\overrightarrow{f} + (\vec{g}\overleftarrow{\nabla})\cdot(\vec{g}\overrightarrow{\nabla})]\mathcal{P}_I^i(x) + \tilde{\mathfrak{N}}_I^i(x)^\dagger[\overleftarrow{f}\overrightarrow{f} + (\vec{g}\overleftarrow{\nabla})\cdot(\vec{g}\overrightarrow{\nabla})]\tilde{\mathcal{P}}_I^i(x). \tag{35}$$

In deriving (35), (4) and (6) have been used, and the arrows indicate left differentiation or right differentiation, as the case may be. From (29), (30), and (35), one then obtains, using (6),

$$\langle\pi^0(\vec{0})|V_1^0(0) - iV_2^0(0)|\pi^+(\vec{0})\rangle = (2\pi)^{-3}\sqrt{2}, \tag{36}$$

where (32) has been used. This yields $(f_+)_\pi = \sqrt{2} = 1\cdot 41$, which is an agreeable result.

If we introduce the Cabibbo angle and the Cabibbo current, we then have[13]

$$(f_+)_\pi = \sqrt{2}\cos\theta$$

and (37)

$$(f_+)_K = \sqrt{2}\sin\theta.$$

2. $\pi^+ \to e^+ + \nu_e$

We shall take π^+ at rest. Then the necessary annihilation component of the weak current is

$$J_{5-}^0(x) = J_{51}^0(x) - iJ_{52}^0(x) \equiv \tilde{\mathfrak{N}}^i(x)\gamma^0\gamma_5\mathcal{P}^i(x). \tag{38}$$

We take Fourier transforms (4) and (5) in (38) at time $t=0$, and thus obtain

$$J_{5-}^0(0) = (2\pi)^{-3}\int \tilde{\mathfrak{N}}_I^{\dagger i}(\vec{k}')[f(\vec{k}'^2)f(\vec{k}^2) + g(\vec{k}'^2)g(\vec{k}^2)(\vec{\sigma}\cdot\vec{k}')(\vec{\sigma}\cdot\vec{k})] \times \mathcal{P}_I^i(\vec{k})d^3k'd^3k. \tag{39}$$

From (39), using (29) in momentum space we obtain

$$\langle\text{vac}|J_{5-}^0(0)|\pi^+(\vec{0})\rangle = (2\pi)^{-3}\sqrt{6}\int(f^2 - g^2\vec{k}^2)u_\pi(\vec{k})d^3k = (2\pi)^{-3/2}\sqrt{6}(1+2g^2\vec{\nabla}^2)u_\pi(\vec{0}) \tag{40}$$

$$= i(2\pi)^{-3/2}c_\pi(m_\pi)^{1/2} \tag{41}$$

We thus obtain

$$|(1+2g^2\vec{\nabla}^2)u_\pi(\vec{0})| = \frac{c_\pi(m_\pi)^{1/2}}{\sqrt{6}} \tag{42}$$

In Eqs. (40) and (42), $u_\pi(\vec{0})$ is the meson wave function at the *space* origin. We notice on the left-hand side of (42) a term dependent on the derivative of the wave function at the space origin as a correction to the nonrelativistic-quark-model result of Van Royen and Weisskopf coming from the small Dirac components. Substituting $c_\pi = 0.094$ GeV, we get

$$|(1+2g^2\vec{\nabla}^2)u_\pi(\vec{0})| = 0.0141\ \text{GeV}^{3/2} \tag{43}$$

In the above, we have taken $g = g_\mathcal{P} = g_\mathfrak{N}$. We shall have occasion to use (43) to estimate $\Gamma(\pi^0 \to 2\gamma)$ ignoring the correction term to the Van Royen–Weisskopf relation later.

3. Electromagnetic coupling of vector mesons

We take e.g. the ρ^0-meson state with spin 1 as

$$|\rho_1^0(\vec{0})\rangle = (2\pi)^{-3/2}\frac{1}{\sqrt{6}} \times \int u_\rho(\vec{x}_1 - \vec{x}_2)d^3x_1d^3x_2q_{I(1/2)}^i(\vec{x}_1)^\dagger \times \tau_3\tilde{q}_{I(1/2)}^i(\vec{x}_2)|\text{vac}\rangle. \tag{44}$$

The above state is assumed to be constructed in a way similar to (27) with no knowledge of the forces which bind to give rise to such states. Now the space part of the pair-annihilation component of the electromagnetic current is given, e.g., for the $\mathcal{P}$ quarks, as,

$$e_p \bar{\mathcal{P}}^i(x)^\dagger \vec{\alpha} \mathcal{P}^i(x) = e_p \bar{\mathcal{P}}_I^i(x)^\dagger \{\vec{\sigma}\vec{f}\vec{f} + i(\vec{\nabla}\times\vec{\nabla})\vec{g}\vec{g} + \vec{g}\vec{g}[\vec{\sigma}(\vec{\nabla}\cdot\vec{\nabla}) - \vec{\nabla}(\vec{\sigma}\cdot\vec{\nabla}) - (\vec{\sigma}\cdot\vec{\nabla})\vec{\nabla}]\}\mathcal{P}_I^i(x). \tag{45}$$

Thus (44) and (45) give us on simplification,

$$\langle \text{vac}\,|J^1(0)\,|\rho_1^0(\vec{0})\rangle = (2\pi)^{-3/2}\frac{1}{\sqrt{6}}3\int v_{I(1/2)}^\dagger\left[\sigma_1 + 2g^2(\vec{\sigma}\cdot\vec{\nabla})\frac{\partial}{\partial x}\right]u_{I(1/2)}\delta(\vec{x})u_\rho(\vec{x})d^3x$$

$$= (2\pi)^{-3/2}\frac{3i}{\sqrt{6}}\left(1 + \frac{2g^2}{3}\vec{\nabla}^2\right)u_\rho(\vec{0}). \tag{46}$$

With the field-current identity we have

$$\langle\text{vac}|J^1(0)|\rho_1^0(\vec{0})\rangle = \frac{m_\rho^2}{f_\rho}\times\frac{1}{\sqrt{2}}\times(2\pi)^{-3/2}\times\frac{1}{(2m_\rho)^{1/2}}. \tag{47}$$

From (46) and (47) one gets

$$|(1 + \tfrac{2}{3}g^2\vec{\nabla}^2)u_\rho(\vec{0})| = \frac{m_\rho^{3/2}}{\sqrt{6}f_\rho}, \tag{48}$$

which is the corrected Van Royen–Weisskopf relation with a correction coming from the space derivatives of the wave function at the origin. In fact, if we substitute experimental numbers, we obtain

$$(1 + \tfrac{2}{3}g^2\vec{\nabla}^2)u_\rho(\vec{0}) = 0.048\ \text{GeV}^{3/2}. \tag{48$'$}$$

Thus, the paradox of different values in (43) and (48′) is not really there. This comment is in addition to the fact that the singlet and triplet wave functions u_π and u_ρ may not be equal, as appears to be the case from a comparison of (43) and (86), which seems to indicate the corrections to be relatively small.

The corrected Van Royen–Weisskopf relations (42) and (48) can be more relevant if we have a determination of the meson wave functions in the neighborhood of the origin, i.e., valid for also large quark momenta. One can satisfy oneself that the harmonic-oscillator wave functions are not good approximations at the space origin in spite of its many agreements elsewhere.

B. Static properties of the nucleon

We note that with colored quarks, the SU(6) eigenstates for proton and neutron can be written in momentum space as

$$|p_{1/2}(\vec{0})\rangle = \frac{1}{3\sqrt{2}}\epsilon_{ijk}\int\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)u(\vec{k}_1,\vec{k}_2,\vec{k}_3)d^3k_1\,d^3k_2\,d^3k_3$$
$$\times[\mathcal{P}_{I(1/2)}^i(\vec{k}_1)^\dagger\mathcal{P}_{I(1/2)}^j(\vec{k}_2)^\dagger\mathfrak{N}_{I(-1/2)}^k(\vec{k}_3)^\dagger - \mathcal{P}_{I(1/2)}^i(\vec{k}_1)^\dagger\mathcal{P}_{I(-1/2)}^j(\vec{k}_2)^\dagger\mathfrak{N}_{I(1/2)}^k(\vec{k}_3)^\dagger]\,|\text{vac}\rangle \tag{49}$$

and

$$|n_{1/2}(\vec{0})\rangle = \frac{1}{3\sqrt{2}}\epsilon_{ijk}\int\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)u(\vec{k}_1,\vec{k}_2,\vec{k}_3)d^3k_1\,d^3k_2\,d^3k_3$$
$$\times[\mathcal{P}_{I(1/2)}^i(\vec{k}_1)^\dagger\mathfrak{N}_{I(1/2)}^j(\vec{k}_2)^\dagger\mathfrak{N}_{I(-1/2)}^k(\vec{k}_3)^\dagger - \mathcal{P}_{I(-1/2)}^i(\vec{k}_1)^\dagger\mathfrak{N}_{I(1/2)}^j(\vec{k}_2)^\dagger\mathfrak{N}_{I(1/2)}^k(\vec{k}_3)^\dagger]\,|\text{vac}\rangle. \tag{50}$$

The other states can be similarly written. The harmonic-oscillator wave function may be taken as[14]

$$u(\vec{k}_1,\vec{k}_2,\vec{k}_3) = \left(\frac{3R^4}{\pi^2}\right)^{3/4}\exp\left[-\tfrac{1}{6}R^2\sum_{i<j}(\vec{k}_i-\vec{k}_j)^2\right]. \tag{51}$$

1. $n\to p+e^-+\bar{\nu}_e$

We shall calculate g_A/g_V for the above process. We have, considering only the relevant component here,

$$\vec{J}_{5+}(x) = \bar{\mathcal{P}}^i(x)\vec{\gamma}\gamma_5\mathfrak{N}^i(x)$$
$$= \mathcal{P}_I^i(x)^\dagger\{\vec{f}\,\vec{\sigma}\,\vec{f} + \vec{g}\vec{g}[(\vec{\sigma}\cdot\vec{\nabla})\vec{\nabla} + \vec{\nabla}(\vec{\sigma}\cdot\vec{\nabla}) - \vec{\sigma}(\vec{\nabla}\cdot\vec{\nabla})]\}\mathfrak{N}_I^i(x). \tag{52}$$

We now consider $\vec{J}_{5+}(0)$ for $t=0$, take Fourier transforms (4) and (5), and explicitly evaluate $\langle p_{1/2}(\vec{0})|\vec{J}_{5+}(0)|n_{1/2}(\vec{0})\rangle$ using (49) and (50) and thus estimate g_A/g_V. We then obtain

$$g_A/g_V \equiv \tfrac{5}{3}\int\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)\,|u(\vec{k}_1,\vec{k}_2,\vec{k}_3)|^2d^3k_1\,d^3k_2\,d^3k_3$$
$$\times(f^2 - g^2\vec{k}_1^{\,2} + \tfrac{2}{3}g^2\vec{k}_1^{\,2}),$$

which, upon using (51), yields

$$g_A/g_V = \frac{5}{3}\left(1 - \frac{4}{3}\frac{g^2}{R^2}\right). \qquad (53)$$

In (53), *we have assumed that g is a constant.* This is an *ad hoc* assumption which seems to yield g_A/g_V, magnetic moments, and the charge radius of the proton simultaneously.

A remark here is worthwhile. Assuming that g is a constant for quarks inside hadrons along with (12) implies that quark momenta inside hadrons remain *strictly* finite. This is inconsistent with the wave function (51) where quark momenta have a Gaussian cutoff. We simultaneously make both assumptions which will be valid when only comparatively low quark momenta will be probed in integrals of the wave functions. We shall see that our estimates of g and R^2 will be consistent with this picture particularly for baryons.

2. Electromagnetic properties

We consider the electromagnetic interaction to be given by

$$\mathcal{H}_I(x) = -e_Q Q^\dagger(x)\vec{\alpha}\cdot\vec{A}Q(x). \qquad (54)$$

We note that for magnetic moments which we want to calculate now, we need matrix elements of electromagnetic currents between moving hadrons. In the following paper we develop these ideas which include relativistic boosing of the states as well as the quark field operators,[15] which thus include the effect of Wigner notations. However, here we proceed in the spirit of the nonrelativistic quark model.

Equation (54) implies that the magnetic moment is obtained from the effective Hamiltonian

$$\mathcal{H}_I^{(m)}(x) = -e_Q g_Q Q_I^\dagger(x)[\vec{f}\vec{\sigma}\cdot(\vec{\nabla}\times\vec{A})]Q_I(x). \qquad (55)$$

On taking $g_{\mathcal{P}} = g_{\mathfrak{N}} = g$, as for (53), one then obtains

$$\begin{aligned}\mu_p &= (2\pi)^3\left\langle p_{1/2}(\vec{0})\right|\sum_Q e_Q g_Q Q_I^\dagger(0)^\dagger(\vec{f}\sigma_3)Q_I^\dagger(0)\left|p_{1/2}(\vec{0})\right\rangle \\ &= g\left(\frac{3R^2}{2\pi}\right)^{3/2}\int \exp\left(-\frac{3R^2}{2}\vec{k}_1^{\,2}\right)f(\vec{k}_1^{\,2})\,d^3k_1 \\ &\simeq g\left(1-\frac{g^2}{2R^2}\right). \end{aligned} \qquad (56)$$

In the above, we have taken $f(\vec{k}_1^{\,2}) \simeq 1 - \frac{1}{2}g^2\vec{k}_1^{\,2}$. Further, as usual with SU(6), $\mu_n = -\frac{2}{3}\mu_p$. If we take $R^2 = 15\ \text{GeV}^{-2}$ and

$$g = 1.67\ \text{GeV}^{-1}, \qquad (57)$$

we obtain from (53) and (56) that

$$g_A/g_V = 1.25, \qquad (58)$$

$$\mu_p = 2.85 \text{ nuclear magnetons}, \qquad (59)$$

and

$$\mu_n = -1.90 \text{ nuclear magnetons}. \qquad (60)$$

The agreement above is good.[16] If we further take $g_{\mathcal{P}} \neq g_{\mathfrak{N}}$, we then have

$$\begin{aligned}\mu_p &= \tfrac{8}{9}g_{\mathcal{P}}\left(1-\frac{g_{\mathcal{P}}^2}{2R^2}\right)+\tfrac{1}{9}g_{\mathfrak{N}}\left(1-\frac{g_{\mathfrak{N}}^2}{2R^2}\right), \\ \mu_n &= -\tfrac{2}{9}g_{\mathcal{P}}\left(1-\frac{g_{\mathcal{P}}^2}{2R^2}\right)-\tfrac{4}{9}g_{\mathfrak{N}}\left(1-\frac{g_{\mathfrak{N}}^2}{2R^2}\right),\end{aligned} \qquad (61)$$

and

$$g_A/g_V = \frac{5}{3}\left(1-\frac{4}{3}\times\frac{g_{\mathcal{P}}g_{\mathfrak{N}}}{R^2}\right).$$

The above three equations are quite well satisfied experimentally if we take

$$g_{\mathcal{P}} = 1.63\ \text{GeV}^{-1},$$

$$g_{\mathfrak{N}} = 1.71\ \text{GeV}^{-1},$$

and (62)

$$R^2 = 15\ \text{GeV}^{-2}.$$

Also, if we make the usual identification for Λ, we obtain

$$\mu_\Lambda = -\tfrac{1}{3}g_\lambda\left(1-\frac{g_\lambda^2}{2R^2}\right),$$

so that, substituting $\mu_\Lambda = -0.67$ nuclear magnetons,[16] one obtains

$$g_\lambda = 1.12\ \text{GeV}^{-1}. \qquad (64)$$

If we identify $m_Q = (2q_Q)^{-1}$, we then obtain from (62) and (64)

$$m_{\mathcal{P}} = 307\ \text{MeV},$$

$$m_{\mathfrak{N}} = 292\ \text{MeV},$$

and (65)

$$m_\lambda = 446\ \text{MeV}.$$

It may be seen that the decuplet mass level spacing of about 146 MeV appears to be *completely* due to the λ quark being heavier. However, the isotopic-spin breaking of the $\mathcal{P}$ and $\mathfrak{N}$ quarks seem to be larger than expected, and the sign of the mass difference also does not appear to be correct on the basis of masses of isotopic multiplets of hadrons. We expect that these results may have only qualitative validity without the details being valid since from deep-inelastic electron scattering we know that for the octet a mixing of $\underline{56}$ and $\underline{70}$ of SU(6) is expected.[17] We include the results of Eq. (65) tentatively since these parameters will play a role in (19) for describing strong decays of hadrons, particularly in the following paper; however, we may regard the average mass of nonstrange quarks as reliable, and not so much as the isotopic breaking.

We can also obtain the charge radius of the proton. With $t=-4\vec{p}^2$, taking the Breit frame, for $-t \ll m_p^2$, we get from (51)

$$G_E^{(p)}(t)=(2\pi)^3\langle p_{1/2}(-\vec{p})|J^0(0)|p_{1/2}(\vec{p})\rangle = \left(\frac{3R^2}{2\pi}\right)^{3/2}\int \exp\left[-\frac{3R^2}{4}(\vec{k}_1-\tfrac{4}{3}\vec{p})^2 - \frac{3R^2}{4}\vec{k}_1^{\,2}\right]d^3k_1 = \exp(\tfrac{1}{6}R^2t)\,, \quad (66)$$

so that we get from (62)

$$\langle r^2\rangle_{\rm ch}^{1/2}=R=0.77\ \text{fm}. \quad (67)$$

If, instead of (66), we take the result of Licht and Pagnamenta[18] and write

$$G_E^{(p)}(t)=\left(1-\frac{t}{4m_p^2}\right)^{-1}\exp\left\{\frac{R^2t}{6[1-t/(4m_p^2)]}\right\}, \quad (68)$$

then we obtain

$$\langle r^2\rangle_{\rm ch}^{1/2}=\left(R^2+\frac{3}{2m_p^2}\right)^{1/2}=0.81\ \text{fm}. \quad (69)$$

The agreement seems good, but we would like to stick to the earlier comment that all results should be regarded as qualitative, and whatever understanding we gain here is incomplete. This is so because gluons seem to carry half the proton momenta, and in our approach, in the same manner as all other workers with similar approaches, we have ignored the gluons. Also Eq. (68) is to be viewed with reservations in the context of the present nonrelativistic model.

C. ϕ decay and the Okubo-Zweig-Iizuka rule

We have noted that the Dirac Hamiltonian (16) has associated with it a pair-creation Hamiltonian, which, as given in (19), does not vanish when g_Q is a constant. This Hamiltonian e.g., can generate strong decays, automatically associated with the Okubo-Zweig-Iizuka selection rule. To illustrate this, we first note that space integration over $\mathcal{V}_{Q^\dagger\tilde{Q}}(x)$ will imply that the quark pair created will have total zero momentum. Further, total energy-momentum also will be conserved since we are maintaining translational invariance. Hence the quark-antiquark pair created in decays due to $\mathcal{V}_{Q^\dagger\tilde{Q}}(x)$ will belong to *different* hadrons; otherwise, from kinematic considerations, the contribution vanishes. Hence the pair-creation component of the Dirac Hamiltonian can give rise to hadron decays which obviously becomes equivalent to the Okubo-Zweig-Iizuka rule.

Now, $\phi\to K^+K^-$ or $\phi\to K^0\bar{K}^0$ are kinematically nonrelativistic problems, so that the present model can be thought of as being useful without modifications or generalization. As in (65), we take $m_Q=(2g_Q)^{-1}$ and approximate (19) by

$$\mathcal{V}_{Q^\dagger\tilde{Q}}(x)\simeq \tfrac{3}{2}g_Q^{\,2}Q_I(x)^\dagger(-i\vec{\sigma}\cdot\vec{\nabla})\vec{\nabla}^2\tilde{Q}_I(x). \quad (70)$$

Equation (70) will be our interaction Hamiltonian, where we do not have any unknown constant to be determined. We shall take ϕ as a pure strange quark-antiquark pair, and thus write

$$|\phi_1(\vec{0})\rangle=\frac{1}{\sqrt{3}}\int\delta(\vec{k}_1+\vec{k}_2)u_\phi(\vec{k}_1)d^3k_1\,d^3k_2 \times\lambda^i_{I(1/2)}(\vec{k}_1)^\dagger\tilde{\lambda}^i_{I(1/2)}(\vec{k}_2)|\text{vac}\rangle. \quad (71)$$

Also, we take

$$|K^+(\vec{p})\rangle=\frac{1}{\sqrt{6}}\int\delta(\vec{k}_1+\vec{k}_2)u_K(\vec{k}_1)d^3k_1\,d^3k_2 \times\mathcal{P}^i_I(\vec{k}_1+\lambda_1\vec{p})^\dagger\tilde{\lambda}^i_I(\vec{k}_2+\lambda_2\vec{p})|\text{vac}\rangle. \quad (72)$$

In (72), λ_1 and λ_2 are respectively proportional to $\mathcal{P}$ and λ quark masses, with $\lambda_1+\lambda_2=1$. We can similarly write down the other K-meson states. Using now (70) as the interaction Hamiltonian and taking into account translational invariance, we now obtain

$$\langle K^+(\vec{p})K^-(\vec{p}\,')|S|\phi_1(\vec{0})\rangle=\delta_4(P_f-P_i)M_{fi}\,, \quad (73)$$

where

$$M_{fi}=-i(2\pi)^4\langle K^+(\vec{p})K^-(-\vec{p})|\mathcal{V}_{Q^\dagger\tilde{Q}}(\vec{0})|\phi_1(\vec{0})\rangle = 2\pi i\times\tfrac{3}{2}g_{\mathcal{P}}^2\times\frac{1}{\sqrt{3}}\times\tfrac{1}{6}\times 3 \times\int u_\phi(\vec{k}_1)u_K^*(\vec{k}_1+\lambda_2\vec{p})u_K^*(\vec{k}_1+\lambda_2\vec{p}) \times u^\dagger_{I-1/2}\vec{\sigma}\cdot(\vec{k}_1+\vec{p})(\vec{k}_1+\vec{p})^2v_{I-1/2}d^3k_1\,. \quad (74)$$

We next take *all* the meson wave functions as

$$u(\vec{k})=\left(\frac{R^2}{\pi}\right)^{3/4}\exp(-\tfrac{1}{2}R^2\vec{k}^2). \quad (75)$$

We then obtain from (74) on integration

$$M_{fi}=(i\pi)\times\frac{\sqrt{3}}{2}g_{\mathcal{P}}^2\times i|\vec{p}|\sin\theta e^{i\phi}\times\left(\frac{R^2}{\pi}\right)^{3/4} \times\frac{1+2\lambda_1}{3}\times(\tfrac{2}{3})^{3/2} \times\left[\frac{5}{3R^2}+\frac{(1+2\lambda_1)^2}{9}\cdot|\vec{p}|^2\right]\exp(-\tfrac{1}{3}R^2\lambda_2^2|\vec{p}|^2). \quad (76)$$

This yields, neglecting $|\vec{p}|^2$ terms,

$$\Gamma(\phi\to K^+K^-)=\frac{\sqrt{\pi}\times 50\times(1+2\lambda_1)^2m_\phi|\vec{p}|^3}{3\times 27\times 27}\ \frac{g_{\mathcal{P}}^4}{R}. \quad (77)$$

If we substitute $\lambda_1=0.41$ and $g_{\varphi}=1.63\ \text{GeV}^{-1}$, and use that[19] $\Gamma(\phi\to K^+K^-)\simeq 1.9$ MeV, then we obtain that

$$R^2\simeq 1\ \text{GeV}^{-2}. \tag{78}$$

We note that (78) is not acceptable in usual harmonic-oscillator models.

On the other hand, if one does not take colored quarks for mesons, then the right-hand side of (77) gets multiplied by 3, and thus (78) gets replaced by

$$R^2\simeq 9\ \text{GeV}^{-2}, \tag{79}$$

which is quite reasonable. In such a case we expect that mesons may have indistinguishable color isotopes. This speculation implies that e.g. we have three types of ϕ mesons, red, blue, and white, and a red ϕ meson decays into two red K mesons, and so forth. This will happen because $\mathcal{V}_{Q\dagger\bar{Q}}$ cannot change the color. However, for $N^*\to N\pi$, where N^* and N are color singlets, a pion of all three colors will result with equal probability. In such a picture (mostly) mesons will be mixtures of three color states instead of being a superposition of them, yielding a color singlet. We recognize that such a hypothesis is far from compulsory, but we keep in mind such a speculation since off hand we cannot reject it. On the basis of such a speculation, we obtain also, from (62),

$$\begin{aligned}\Gamma(\phi\to K^+K^-)/\Gamma(\phi\to K^0\bar{K}^0) &= \text{phase-space factor}\\ &\quad\times(g_{\varphi}/g_{\mathfrak{N}})^4\\ &=1.53\times 0.826\\ &=1.27.\end{aligned} \tag{80}$$

Equation (80) is an agreeable result in view of the fact that the above ratio is 1.33 ± 0.14. However, this value is far from settled.

However, we note that in (78) $R^2\propto g_{\varphi}^8$, and thus is extremely sensitive to slight changes in the value of g_{φ} as determined in (62). Thus, we may have the mesons as color singlets, and qualitatively we may have the ideas of the present section still vindicated by (80), where we imagine that a more complete determination will increase the value of g_{φ} and $g_{\mathfrak{N}}$. Also, we note that we have used a completely nonrelativistic form for the quark-pair-creation term, excluding any effect of the motion of K^+K^- pair, which may not be a valid approximation.

It is of course always true that the forces which bind may also be responsible for decay. But the appearance of a pair-creation term in (16) with a simple ansatz makes it aesthetically appealing that this may automatically describe the strong decays and be a dynamical explanation of the Okubo-Zweig-Iizuka rule. We shall also make use of the pair-annihilation component of the Hamiltonian in the next section to describe $\Gamma(\pi^0\to 2\gamma)$.

D. $\pi^0\to 2\gamma$ and $\eta\to 2\gamma$

We write the quark-pair-annihilation component of the Dirac Hamiltonian (16) as

$$\tilde{Q}_I^\dagger(x)(-i\sigma\cdot\vec{\nabla}g_Q,\vec{f})\begin{pmatrix} m_Q-i\vec{\sigma}\cdot\vec{\nabla} \\ -i\vec{\sigma}\cdot\vec{\nabla}-m_Q\end{pmatrix}\begin{pmatrix} f \\ -i\vec{\sigma}\cdot\vec{\nabla}g_Q\end{pmatrix}Q_I(x). \tag{81}$$

We introduce electromagnetic interactions through minimal electromagnetic coupling, replacing $\vec{\nabla}$ by $\vec{\nabla}-ie_Q\vec{A}$ in (81). This generates a two-photon component along with quark pair annihilation given as

$$g_Q^2e_Q^2\tilde{Q}_I^\dagger(x)Q_I(x)\vec{A}\cdot(\vec{\nabla}\times\vec{A}). \tag{82}$$

The term (82) can describe π^0 decay into two photons in the quark model. In fact one may easily satisfy oneself that this is the only term from (81) which can give a finite contribution to such a process. Thus, in the quark model with color we shall take

$$\mathcal{H}_{\text{em}}(x)\equiv\sum_{iQ} g_Q{}^2e_Q{}^2\tilde{Q}_I^i(x)^\dagger Q_I^i(x)\vec{A}(x)\cdot(\vec{\nabla}\times\vec{A}(x)). \tag{83}$$

Now from (30) we get

$$\langle\vec{k}_1,\vec{\epsilon}_1;\vec{k}_2,\vec{\epsilon}_2|S|\pi^0(\vec{0})\rangle=\delta_4(P_f-P_i)M_{fi}, \tag{84}$$

where, taking $g_{\varphi}=g_{\mathfrak{N}}=g$,

$$\begin{aligned}M_{fi}=-\frac{1}{(2\pi)^{1/2}}\times\frac{1}{2\sqrt{3}}\times 6\times\frac{1}{2k_1^0}u_\pi(\vec{0})g^2(e_{\varphi}{}^2-e_{\mathfrak{N}}{}^2)\\ \times[\vec{\epsilon}_1\cdot(\vec{k}_2\times\vec{\epsilon}_2)+\vec{\epsilon}_2\cdot(\vec{k}_1\times\vec{\epsilon}_1)]\ .\end{aligned}$$

We note that, summing over polarizations,

$$\sum|M_{fi}|^2=\alpha^2\frac{16\pi}{3}g^4|u_\pi(\vec{0})|^2,$$

so that

$$\Gamma(\pi^0\to 2\gamma)=\alpha^2\frac{16\pi}{3}g^4|u_\pi(\vec{0})|^2\frac{m_\pi{}^2}{4}. \tag{85}$$

We now compare (85) with (43), which is the corrected Van Royen–Weisskopf relation. In fact (85) yields, with (57), and $\Gamma=8$ eV.

$$|u_\pi(\vec{0})|=0.0153\ \text{GeV}^{3/2}. \tag{86}$$

The right-hand sides in (43) and (86) do not differ appreciably. Thus the correction in the Van Royen–Weisskopf relation is seen to be about 20%, and we may have reasonable confidence in estimating two photon decays from the Hamiltonian (83). We note that the framework of the quark model, for (43), $\Gamma(\pi^+ \to \mu^+ + \nu)$ and for (86), $\Gamma(\pi^0 \to 2\gamma)$, are the experimental inputs respectively. It has not been necessary to invoke the hypothesis of partially conserved axial-vector currents with the modifications suggested by many authors.[18] However, minimal electromagnetic coupling has been utilized starting from the quark pair-annihilation Hamiltonian in the form (81).

Another comment is worthwhile. The relationship between (43) and (86) will remain unaltered whether we take colored quarks or quarks without color, or whether mesons have color isotopes as was speculated in Sec. III C. Thus, with this way of looking at $\pi^0 \to 2\gamma$, when pion weak decay width is used, color does not seem to be necessary.[20]

We now consider $\eta \to 2\gamma$. Similar to (30), we now take

$$|\eta(\vec{0})\rangle = (2\pi)^{-3/2} \int \left[\frac{\cos\theta}{2\sqrt{3}} q_I^i(\vec{x}_1)^\dagger \tilde{q}_I^i(\vec{x}_2) + \frac{\sin\theta}{\sqrt{6}} \lambda_I^i(\vec{x}_1)^\dagger \tilde{\lambda}_I^i(\vec{x}_2)\right] u_\eta(\vec{x}_1 - \vec{x}_2) d^3x_1 d^3x_2 |\text{vac}\rangle \quad , \tag{87}$$

where θ is the mixing angle for nonstrange- and strange-quark-antiquark pairs. The interaction Hamiltonian is taken as (83) and we shall use (62) and (64). We then obtain

$$M_{fi}(\eta \to 2\gamma) = -\frac{e^2}{(2\pi)^{1/2}} \times 6 \times u_\eta(\vec{0}) \frac{1}{2k_1^0} [\vec{\epsilon}_1 \cdot (\vec{k}_2 \times \vec{\epsilon}_2) + \vec{\epsilon}_2 \cdot (\vec{k}_1 \times \epsilon_1)] \times \left[\frac{\cos\theta}{2\sqrt{3}} (\tfrac{4}{9} g_{\mathcal{P}}{}^2 + \tfrac{1}{9} g_{\mathfrak{N}}{}^2) + \frac{\sin\theta}{\sqrt{6}} \times \tfrac{1}{9} g_\lambda{}^2 \right]. \tag{88}$$

Hence we obtain

$$\Gamma(\eta \to 2\gamma)/\Gamma(\pi^0 \to 2\gamma) = \frac{m_\eta{}^2}{m_{\pi^0}{}^2} \frac{|u_\eta(\vec{0})|^2}{|u_\pi(\vec{0})|^2} \frac{[(4g_{\mathcal{P}}{}^2 + g_{\mathfrak{N}}{}^2)\cos\theta + \sqrt{2} g_\lambda{}^2 \sin\theta]^2}{(4g_{\mathcal{P}}{}^2 - g_{\mathfrak{N}}{}^2)^2} = 47 |u_\eta(\vec{0})|^2 / |u_\pi(\vec{0})|^2 \ . \tag{89}$$

In (89), we have taken $\theta = 25^\circ$, as obtained approximately from the quadratic mass for pseudoscalar mesons.[21] Thus we get, if $u_\eta(\vec{0}) = u_\pi(\vec{0})$.

$$\Gamma(\eta \to 2\gamma) = (376 \pm 25) \text{ eV} \ , \tag{90}$$

which is in reasonable agreement with experimental value[15] of 323 ± 48 eV. Alternatively, (89) may be taken as a way of determining $u_\eta(\vec{0})$.

We note that the present assignment agrees with the conventional mixing and not with η being almost completely a strange-quark-antiquark pair.[22]

IV. DISCUSSION AND OUTLOOK

We would first like to comment on some obvious limitations of the present model. Although we have taken "large" and "small" Dirac components, and thus the quarks inside hadrons can be "relativistic," we have really a nonrelativistic theory, which may explain phenomena with hadrons at rest. However, we have applied this theory to two cases where hadrons are not at rest: the magnetic moments of baryons and charge radius, and the ϕ decay. Even though hadrons have small velocities while in motion, Wigner rotation in quark space will come into the picture. Hence, in these two cases the present model is really incomplete. We have constructed a relativistic version of this model[15] which takes into account these features. In the present paper we ignore these effects, proceed in a manner strictly parallel to nonrelativistic quantum field theory, and examine the results. We consider it worthwhile to retain these results since what we have done in Ref. 15 may not be the actual description of hadrons for relativistic boosting. We note that in Ref. 15 the final results regarding magnetic moments remain broadly correct, but the description of ϕ decay gets substantially altered.

When we talk about the quark model of hadrons, we usually have three models in mind: (i) heavy quarks, (ii) comparatively lighter observable colored quarks,[23] and (iii) quarks that are permanently confined,[24] which thus could be as light as needed. We have not distinguished these models in our approach, since we have not raised this question and have specifically avoided mass eigenvalue equations beyond just giving such an equation for potential-like interactions. In view of the "known" magnetic moments of quarks, in models (i) and (ii) the effective mass of the quark inside the hadrons is always assumed to be small. The quarks are assumed to have one-third the baryon mass inside the baryons and half the meson mass inside the mesons. In such a case the agreement of (43) and

(86) is to be regarded as accidental, since in the later equations we have taken $g_{\mathcal{P}}$ and $g_{\mathfrak{N}}$ as determined from nuclear magnetic moments and have extrapolated these values for π mesons.

If we do not regard the results of Sec. III D as accidental, we are led to an interesting conclusion that the quarks inside the nucleon and the pion both have the same magnetic moments. This conclusion favors (almost) permanently confined quarks with common magnetic moments whether they belong to baryons or to mesons, and bound in a manner we do not understand. Chromodynamics with infrared slavery[25] may be a possible mechanism, but we are yet to understand how it really happens, since this mechanism should simultaneously generate the mass spectrum.[26] The fact that quarks have the same magnetic moment inside mesons as in nucleons is not a new observation as is known from electromagnetic decays of vector mesons.[27] This point of view gets further support here.

We note that if quarks are permanently confined, the vector space spanned by them will be an unphysical vector space. The physical vector space will consist of the vector space of known hadrons, and other particles. However, now there will be nonvanishing triangular vertices for three hadrons, arising from the quark-pair-creation Hamiltonian along with its Hermitian conjugate. Such vertices will be consistent with the prescriptions of the Okubo-Zweig-Iizuka rule. This dynamical explanation of the above rule seems to us to be an attractive feature of the present model. Further, the relativistic generalization of these triangular vertices may as well give the S-matrix theory with usual analyticity and, probably, crossing symmetry. Thus the relevance of such approaches for many physical processes can be understood.

When g_Q is a constant, as we have very often taken in our applications, (14) and (15) indicates that the quark must always lie inside a sphere of radius g_Q^{-1} in momentum space. Such a system evokes the memory of a degenerate Fermi system. We have not exploited any such idea here. However, intrinsically a finite-momentum bag does not appear to us to be any less physical than the finite-space MIT bag.[28] Further, here we have made it consistent with equal-time anticommutators and translational invariance, and thus here we have an automatically quantized system, which was not true of the MIT bag.

Obviously, in the applications the present model is a generalization of the *nonrelativistic* harmonic-oscillator quark model. It is a field-theoretic version of the same when "small" Dirac components are retained. We take the attitude that this should be suitably Lorentz boosted to describe hadrons in motion.[15] Feynman *et al.* have also considered a *relativistic* harmonic-oscillator model[29] on which a lot of work has been done. This has the time degree of freedom, which gives rise to difficulties of interpretation[29,30] as well as to too many eigenvalues.[31] Kim and Noz[32] suppress this degree of freedom by a subsidiary condition and are able to give a relativistic probability interpretation with such equations. However, the author believes that retaining the time degree of freedom to construct bound states is unphysical. This is particularly relevant in the context of a bound-state equation proposed by Partovi[3] which is a relativistic equation similar in its derivation to the Bethe-Salpeter equation, but which reduces to a Schrödinger-type equation in the rest frame of the bound particle. We have essentially such a picture in mind for the relativistic version of what we have done, in the rest frame of hadrons.

We believe that the nonrelativistic quark model is quite reasonable particularly since Kim and Noz have shown that most of these results can also be derived with covariant harmonic-oscillator wave functions with a relativistic probability interpretation.

Thus it seems to the author that the model proposed is rich in possibilities where we should be willing to try unfamiliar properties for quark field operators until we understand them better. Expermental results rather than our predispositions should decide what those properties are. A possible relativistic version of the present model is proposed in Ref. 15.

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