Renormalization effects from superheavy Higgs particles

D. Toussaint
Joseph Henry Laboratories, Princeton University, Princeton, N.J. 08540
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Using a modification of the standard Weinberg-Salam model, we compute renormalization corrections to the W and Z masses proportional to differences in the Higgs-particle mass squared. This contradicts the expectation that superheavy particles should be undetectable in low-energy phenomena.

I. INTRODUCTION

Weak-interaction theories with spontaneously broken gauge symmetry include scalar Higgs particles. There is no experimental evidence for such particles, although their detection is notoriously difficult. (See, for example, the review by Gaillard, Ellis, and Nanopoulos.[1]) One might attempt to exorcise these particles by assuming their masses to be extremely large. Here we point out a phenomenon in perturbation theory that allows one to put constraints on the way in which Higgs-particle masses are made large. In particular, we examine the renormalization contributions to the W and Z masses as the Higgs particles become heavy in a Weinberg-Salam model[2] with two Higgs doublets. We calculate terms in the vector-meson propagators which are proportional to the squared mass of the Higgs particles, and show that when some of the Higgs-particle masses are made very large there are physical renormalization effects of this order in one-loop perturbation theory. The standard model, with one scalar doublet, has effects which grow with the Higgs-particle mass as $\ln M_{\text{Higgs}}^2$, not nearly as fast as M_{Higgs}^2.

One might expect that as the mass of any particle became large it would decouple from the low-energy theory in the manner demonstrated by Appelquist and Carazzone.[3] The presence of a physical effect on the W and Z masses proportional to M_{Higgs}^2 indicates that this is not necessarily true. We emphasize that this is a physical effect and cannot be removed by redefining the parameters of the theory.

In brief, the argument of Appelquist and Carazzone is that convergent graphs containing an internal heavy particle will be proportional to an inverse power of the large mass and hence will vanish as the mass becomes large. The effects of internal heavy particles on divergent graphs may be removed by adjusting the counterterms used to remove the infinities, which is equivalent to redefining the parameters of the theory. This argument depends upon our ability to subtract each primitively divergent Green's function independently. The difficulty is that in broken-symmetry theories there are more divergent Green's functions to which heavy particles can contribute than there are counterterms available. We are assured by the underlying symmetry that the infinite parts of these Green's functions can be removed, but there will be finite physical renormalization effects left over.

A general feature of spontaneously-broken-symmetry theories is that mass ratios are proportional to coupling-constant ratios. When Higgs-particle masses are made large with the W and Z masses and the gauge coupling constants kept fixed, the scalar self-coupling constant λ becomes large. However, the computation presented here does not involve scalar vertices, and so does not explicitly involve λ.

Veltman has demonstrated a similar effect on the W and Z propagators when the theory contains a superheavy lepton with a light neutrino.[4] In this case it is a Yukawa coupling which becomes large, but again the coupling does not appear explicitly in the computation.

In Sec. II we sketch the model used in the calculation. Section III describes the calculation and contains some comments on the results.

II. MODEL

We use the Weinberg-Salam model, except that we include two complex doublet scalar fields Φ_1 and Φ_2, both with hypercharge 1. This is a simple extension of the standard model, but it has a richer Higgs phenomenology. In order to simplify the scalar potential we require a discrete symmetry $\Phi_2 \to -\Phi_2$ and $d_R^a \to -d_R^a$, where d_R^a are the right-handed charge $-\frac{1}{3}$ quark fields. This discrete symmetry also ensures the condition derived by Glashow and Weinberg for the absence of flavor-changing neutral currents mediated by scalars, namely, that quarks of a given electric charge couple only to a single Higgs field.[5]

The most general potential for the scalars consistent with this symmetry is

$$V(\Phi_1,\Phi_2)=m_1^2\Phi_1^\dagger\Phi_1+m_2^2\Phi_2^\dagger\Phi_2+\lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\tfrac{1}{2}\lambda_4[(\Phi_1^\dagger\Phi_2)^2+(\Phi_2^\dagger\Phi_1)^2] + \lambda_5(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\,. \quad (1)$$

If m_1^2, m_2^2, λ_4 and λ_5 are negative and $\lambda_3+\lambda_4+\lambda_5$ is not too large, this potential will have a minimum at

$$\langle\Phi_1\rangle=\begin{pmatrix}0\\x\end{pmatrix},\quad \langle\Phi_2\rangle=\begin{pmatrix}0\\y\end{pmatrix},$$

where

$$\begin{aligned}x^2&=\frac{1}{4\lambda_1\lambda_2-(\lambda_3+\lambda_4+\lambda_5)^2}\times[-2\lambda_2 m_1^2+(\lambda_3+\lambda_4+\lambda_5)m_2^2],\\ y^2&=\frac{1}{4\lambda_1\lambda_2-(\lambda_3+\lambda_4+\lambda_5)^2}\times[-2\lambda_1 m_2^2+(\lambda_3+\lambda_4+\lambda_5)m_1^2].\end{aligned}\tag{2}$$

To find the physical Higgs particles we expand Φ_1 and Φ_2 about the minimum of the potential, and find the eigenvectors of the quadratic terms in the shifted potential. We parametrize the fields as

$$\begin{aligned}\Phi_1&=\begin{pmatrix}0\\x\end{pmatrix}+\begin{pmatrix}0\\\phi_0/\sqrt{2}\end{pmatrix}+\frac{y}{v}\begin{pmatrix}\chi\\i\chi_0/\sqrt{2}\end{pmatrix}+\frac{x}{v}\begin{pmatrix}\alpha\\i\alpha_0/\sqrt{2}\end{pmatrix},\\ \Phi_2&=\begin{pmatrix}0\\y\end{pmatrix}+\begin{pmatrix}0\\\eta_0/\sqrt{2}\end{pmatrix}-\frac{x}{v}\begin{pmatrix}\chi\\i\chi_0/\sqrt{2}\end{pmatrix}+\frac{y}{v}\begin{pmatrix}\alpha\\i\alpha_0/\sqrt{2}\end{pmatrix},\end{aligned}\tag{3}$$

where $v=(x^2+y^2)^{1/2}$. χ and α represent complex charged fields, while the other fields are real and neutral. α and α_0 are unphysical scalars which mix with the $W^\pm$ and the Z mesons, respectively. χ is a charged Higgs particle and χ_0 is a neutral pseudoscalar. If $\lambda_4=0$ the discrete symmetry imposed on V becomes a continuous symmetry $\Phi_2\to e^{i\theta}\Phi_2$, $d_R^a\to e^{-i\theta}d_R^a$. The χ_0 is then a Goldstone boson: the axion of Wilczek and Weinberg.[6] Φ_0 and η_0 are not eigenvectors of the quadratic potential, but mix together to form two neutral scalar particles. These physical particles are

$$\phi=c\phi_0+s\eta_0,\quad \eta=-s\phi_0+c\eta_0,$$

where $c=\cos\xi$, $s=\sin\xi$, and

$$\tan\xi=\frac{-\lambda_1x^2+\lambda_2y^2+[(\lambda_1x^2-\lambda_2y^2)^2+x^2y^2(\lambda_3+\lambda_4+\lambda_5)^2]^{1/2}}{xy(\lambda_3+\lambda_4+\lambda_5)}.\tag{4}$$

The masses of the physical scalars are

$$\begin{aligned}m_{\chi_0}^2&=-2\lambda_4v^2,\\ m_\chi^2&=-(\lambda_4+\lambda_5)v^2,\\ m_{\phi,\eta}^2&=2\lambda_1x^2+2\lambda_2y^2\\ &\quad\pm[(2\lambda_1x^2-2\lambda_2y^2)^2+4x^2y^2(\lambda_3+\lambda_4+\lambda_5)^2]^{1/2}.\end{aligned}\tag{5}$$

The Feynman rules for the interaction of the scalars with the gauge mesons are found from the scalar covariant derivative part of the Lagrangian,

$$\begin{aligned}(D_\mu\Phi_1)^\dagger(D_\mu\Phi_1)+(D_\mu\Phi_2)^\dagger(D_\mu\Phi_2)&=\left(\partial_\mu\Phi_1^\dagger+\frac{ig'}{2}B_\mu\Phi_1^\dagger+\frac{ig}{2}\Phi_1^\dagger\tau^iA_\mu^i\right)\\ &\quad\times\left(\partial^\mu\Phi_1-\frac{ig'}{2}B^\mu\Phi_1-\frac{ig}{2}\tau^jA^{\mu j}\Phi_1\right)\\ &\quad+(\Phi_1\to\Phi_2).\end{aligned}\tag{6}$$

We substitute the expanded form of Φ_1 and Φ_2 [Eq. (3)] into this expression and perform some tedious algebra to find the Feynman rules for the physical particles. Figure 1 and Table I contain the Feynman rules for vertices involving $W^\pm$ or Z and physical Higgs particles.

III. CALCULATION AND COMMENTS

We need to examine only the graphs in the vector-meson propagators involving physical Higgs particles, since these are the only ones that could be proportional to $M_{\rm Higgs}^2$. There are three classes of such diagrams, shown in Figs. 2, 3, and 4. The tadpole diagrams of Fig. 2 will not change the Z-to-W mass ratio, since the couplings of ϕ and η to the $W^\pm$ and Z are proportional to the squares of the tree-approximation masses of the W and Z. This is equivalent to saying that the tadpoles may be removed by redefining x and y. The diagrams of Fig. 3 contain explicit factors of M_Z^2 and M_W^2 from the vertices and are only logarithmically divergent, so as the Higgs-particle masses go to infinity they will have parts proportional to $M_{W,Z}^2\ln M_{\phi,\eta}^2$. This logarithmic effect is also present in the standard model with one physical scalar.

We are left with the diagrams of Figs. 4(a) and 4(b), in which H represents a physical or an unphysical scalar. The integrals for these diagrams can be easily evaluated by dimensional regularization, and we call the results I_1 and I_2:

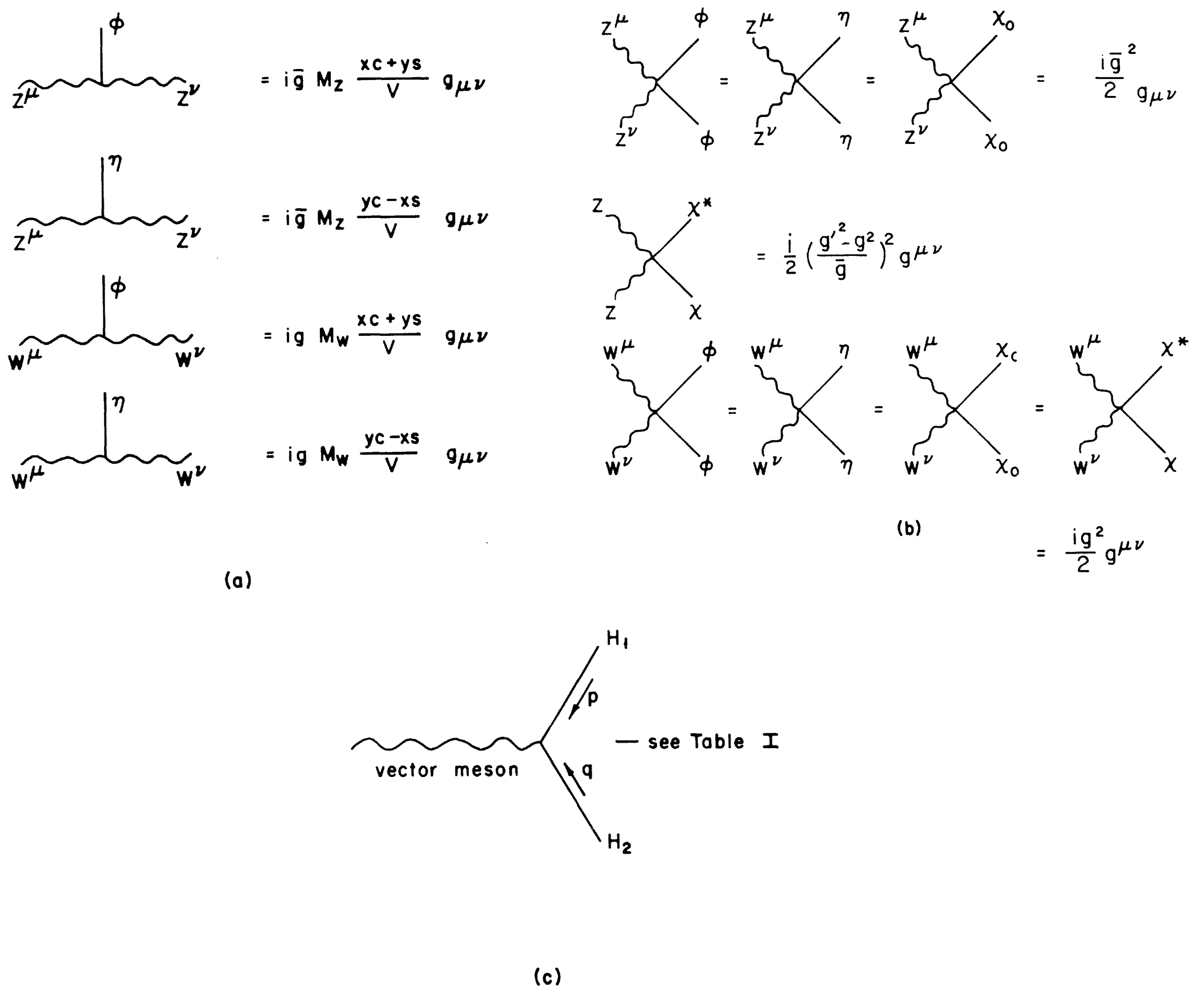

FIG. 1. Feynman rules for the two-Higgs-particle model.

$$I_1 = \int \frac{d^n k}{(2\pi)^n}\,\frac{1}{k^2-m^2} = \frac{i}{(4\pi)^2}\,(m^2 - m^2 \ln m^2)\,, \tag{7}$$

$$\begin{aligned} I_2 &= \int \frac{d^n k}{(2\pi)^n}\,\frac{(2k+p)^\mu (2k+p)^\nu}{(k^2-m_1{}^2)[(k+p)^2-m_2{}^2]} \\ &= \frac{i}{(4\pi)^2}\,(-\tfrac{1}{3}p^2 + m_1{}^2 + m_2{}^2)\,g^{\mu\nu} + \frac{i}{(4\pi)^2}\,\tfrac{1}{3}p^\mu p^\nu \\ &\quad - \frac{i}{(4\pi)^2}\int_0^1 d\alpha[(-2\alpha(1-\alpha)p^2 + 2(1-\alpha)m_1{}^2 + 2\alpha m_2{}^2)\,g^{\mu\nu} \\ &\qquad + (1-2\alpha)^2 p^\mu p^\nu]\ln[-\alpha(1-\alpha)p^2 + (1-\alpha)m_1{}^2 + \alpha m_2{}^2]\,. \end{aligned} \tag{8}$$

In Eqs. (7) and (8) we have subtracted the part proportional to Euler's constant γ_E as well as the infinite part, which amounts to a trivial shift in the arbitrary renormalization mass μ. Euler's constant arises in one-loop integrals through the expression $(4\pi\mu^2)^{\epsilon/2}\Gamma(\epsilon/2)$, where $\epsilon = 4-n$, and μ is defined by writing the coupling constant in n dimensions as $g\mu^{\epsilon/2}$. This expression is just $2/\epsilon - \gamma_E + \ln 4\pi\mu^2$, and Euler's constant and the factor of 4π may be removed by introducing a rescaled renormalization mass μ' such that $\ln\mu'^2 = \ln 4\pi\mu^2 - \gamma_E$. Therefore, dropping γ_E and $\ln 4\pi$ from one-loop integrals just amounts to making a finite change in the counterterms.

In the limit where m_1 or m_2 is much larger than M_W or M_Z, we may drop the p^2 and the $p^\mu p^\nu$ terms in Eq.

(8) and evaluate the integral over α. In this approximation

$$I_2=\frac{i}{(4\pi)^2}\,g^{\mu\nu}[m_1^{\,2}+m_2^{\,2}+f(m_1^{\,2},m_2^{\,2})]\,,$$

where

$$f(m_1^{\,2},m_2^{\,2})=\frac{m_2^{\,4}\ln m_2^{\,2}-m_1^{\,4}\ln m_1^{\,2}}{m_1^{\,2}-m_2^{\,2}}+\tfrac{1}{2}(m_1^{\,2}+m_2^{\,2})\,. \tag{9}$$

Note that when $m_1=m_2$, $f(m_1^{\,2},m_2^{\,2})$ is equal to $-2m_1^{\,2}\ln m_1^{\,2}$ and I_2 is proportional to I_1. However, if m_1 is large and m_2 is not, f becomes $-m_1^{\,2}\ln m_1^{\,2}+\frac{1}{2}m_1^{\,2}$. For the unbroken theory, or for sets of diagrams where both the scalar masses are equal, the diagrams of Fig. 4(a) cancel against those of Fig. 4(b), and we get no contribution to the vacuum polarization proportional to m^2, as is required by gauge invariance. However, if there are large mass differences among the Higgs particles, the structure of f is such that the cancellation will not be complete; there will be terms of order m^2 left over.

We compute in the Feynman gauge, so the unphysical scalars have the same masses as the W and Z. For the graphs of the type in Fig. 4(b) where one of the particles is unphysical, we set $f(m_H^{\,2},M_W^{\,2})=-m_H^{\,2}\ln m_H^{\,2}+\frac{1}{2}m_H^{\,2}$. For physical Higgs-particle masses that are large relative to the W and Z masses, the m^2 terms in the vacuum polarizations of the W become

$$\begin{aligned}\Pi_W^{\mu\nu}=\frac{i}{(4\pi)^2}\,g^{\mu\nu}\frac{g^2}{4}\bigg[&\left(\frac{yc-xs}{v}\right)^2 f(m_\phi^{\,2},m_\chi^{\,2})+\left(\frac{xc+ys}{v}\right)^2 f(m_\eta^{\,2},m_\chi^{\,2})\\&+f(m_{\chi_0}^{\,2},m_\chi^{\,2})+\left(\frac{xc+ys}{v}\right)^2(-m_\phi^{\,2}\ln m_\phi^{\,2}+\tfrac{1}{2}m_\phi^{\,2})\\&+\left(\frac{yc-xs}{v}\right)^2(-m_\eta^{\,2}\ln m_\eta^{\,2}+\tfrac{1}{2}m_\eta^{\,2})+2m_\chi^{\,2}\ln m_\chi^{\,2}\\&+m_\phi^{\,2}\ln m_\phi^{\,2}+m_\eta^{\,2}\ln m_\eta^{\,2}+m_{\chi_0}^{\,2}\ln m_{\chi_0}^{\,2}\bigg]\,.\end{aligned} \tag{10}$$

For the Z propagator we find

$$\begin{aligned}\Pi_Z^{\mu\nu}=\frac{i}{(4\pi)^2}\,g^{\mu\nu}\frac{\bar g^2}{4}\bigg[&\left(\frac{yc-xs}{v}\right)^2 f(m_\phi^{\,2},m_{\chi_0}^{\,2})+\left(\frac{ys+xc}{v}\right)^2 f(m_\eta^{\,2},m_{\chi_0}^{\,2})\\&+\left(\frac{ys+xc}{v}\right)^2(-m_\phi^{\,2}\ln m_\phi^{\,2}+\tfrac{1}{2}m_\phi^{\,2})+\left(\frac{yc-xs}{v}\right)^2(-m_\eta\ln m_\eta^{\,2}+\tfrac{1}{2}m_\eta^{\,2})\\&+m_\phi^{\,2}\ln m_\phi^{\,2}+m_\eta^{\,2}\ln m_\eta^{\,2}+m_{\chi_0}^{\,2}\ln m_{\chi_0}^{\,2}\bigg]\,,\end{aligned} \tag{11}$$

where

$$\bar g=(g^2+g'^2)^{1/2}\,.$$

In the tree approximation the squared masses of the W and Z are $\frac{1}{2}g^2v^2$ and $\frac{1}{2}\bar g^2v^2$, respectively. A contribution to the W and Z masses that is equivalent to a change in v would be unobservable, unless we could measure the three- and four-Higgs-particle couplings to independently measure v. Therefore, the quantity of interest is the change in the mass ratio of the W and Z. This change is given by

$$\begin{aligned}\Delta\!\left(\frac{M_Z^{\,2}}{M_W^{\,2}}\right)&=\frac{-1}{M_W^{\,2}}\Pi_W+\frac{g^2}{\bar g^2}\frac{1}{M_W^{\,2}}\Pi_Z\\&=\frac{-i}{4}\left(\frac{g}{4\pi}\right)^2\frac{1}{M_W^{\,2}}\bigg\{\left(\frac{yc-xs}{v}\right)^2[f(m_\phi^{\,2},m_\chi^{\,2})-f(m_\phi^{\,2},m_{\chi_0}^{\,2})]\\&\qquad+\left(\frac{xc+ys}{v}\right)^2[f(m_\eta^{\,2},m_\chi^{\,2})-f(m_\eta^{\,2},m_{\chi_0}^{\,2})]+f(m_{\chi_0}^{\,2},m_\chi^{\,2})+2m_\chi^{\,2}\ln m_\chi^{\,2}\bigg\}\,.\end{aligned} \tag{12}$$

We see immediately that if the mass of the charged Higgs particle χ is the same as that of the pseudoscalar χ_0, there will be no effects proportional to $m_{\text{Higgs}}^{\,2}$. However, if the mass difference between χ and χ_0 becomes large, $\Delta(M_Z^{\,2}/M_W^{\,2})$ may be large. In particular, if m_χ becomes very large but the masses of all other Higgs particles are near the W and Z mass scale, the expres-

TABLE I. Feynman rules for diagrams of Fig. 1(c).

Vector meson	H_1	H_2	Vertex
W_μ^-	ϕ	χ	$-\frac{ig}{2}\frac{yc-xs}{v}(p-q)^\mu$
W_μ^-	η	χ	$\frac{ig}{2}\frac{xc+ys}{v}(p-q)^\mu$
W_μ^-	χ_0	χ	$-\frac{g}{2}(p-q)^\mu$
W_μ^-	ϕ	α	$-\frac{ig}{2}\frac{xc+ys}{v}(p-q)^\mu$
W_μ^-	η	α	$-\frac{ig}{2}\frac{yc-xs}{v}(p-q)^\mu$
W_μ^+	ϕ	χ^*	$\frac{ig}{2}\frac{yc-xs}{v}(p-q)^\mu$
W_μ^+	η	χ^*	$-\frac{ig}{2}\frac{xc+ys}{v}(p-q)^\mu$
W_μ^+	χ_0	χ^*	$-\frac{g}{2}(p-q)^\mu$
W_μ^+	ϕ	α^*	$\frac{ig}{2}\frac{xc+ys}{v}(p-q)^\mu$
W_μ^+	η	α^*	$\frac{ig}{2}\frac{yc-xs}{v}(p-q)^\mu$
Z^μ	χ^*	χ	$-i\frac{g'^2-g^2}{2\bar{g}}(p-q)^\mu$
Z^μ	ϕ	χ_0	$\frac{\bar{g}}{2}\frac{yc-xs}{v}(p-q)^\mu$
Z^μ	η	χ_0	$-\frac{\bar{g}}{2}\frac{xc+ys}{v}(p-q)^\mu$
Z^μ	ϕ	α_0	$-\frac{\bar{g}}{2}\frac{xc+ys}{v}(p-q)^\mu$
Z^μ	η	α_0	$-\frac{\bar{g}}{2}\frac{yc-xs}{v}(p-q)^\mu$

sion in curly brackets in Eq. (12) approaches m_χ^2, since it follows directly from the definitions of v, c, and s that

$$\left(\frac{xc+ys}{v}\right)^2+\left(\frac{yc-xs}{v}\right)^2=1\,. \tag{13}$$

Curiously, if ϕ, η, and χ have the same superheavy mass and χ_0 is light, there is no effect, but if χ and only one of the scalars are heavy, there is an effect.

We hasten to note that the experimental consequences of this phenomenon are not large until the mass differences become truly enormous. For example, if the mass of the charged Higgs particle is 1000 GeV and all other Higgs particles have masses near the W mass, there is about a 3% correction to the Z-to-W mass ratio. This means that for small momentum transfers the strength of the neutrino neutral-current interactions, $\bar{g}^2/M_Z^2$, would be 6% larger than the strength of the charged currents, g^2/M_W^2. However, these effects become significant before the Higgs self-coupling becomes too enormously large. Using the rough formula $M_{\text{Higgs}}^2/M_W^2 \sim \lambda/g^2$, for a Higgs-particle mass of order 750 GeV, we find $\lambda/(4\pi)^2 \approx 0.25$.

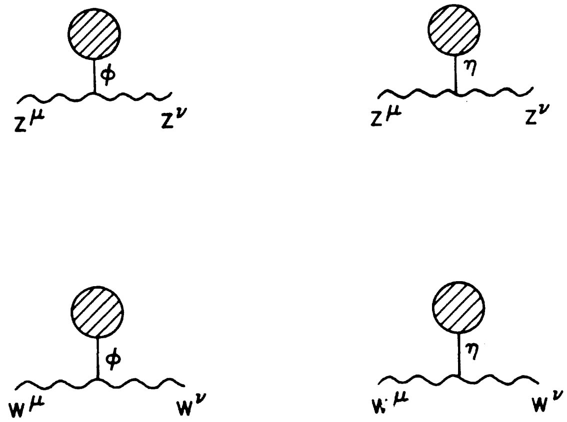

FIG. 2. Tadpole diagrams.

The relations among masses and coupling constants may be used to write the vacuum polarizations of the W and Z in an intriguing way. If the model has large mass splittings among the Higgs particles, and fermion doublets with large mass splittings as in Ref. 4, we have seen that the vacuum polarizations take the form

$$\Pi \approx g^2C_1+g^2C_2\frac{m_{\text{Higgs}}^2}{M_W^2}+g^2C_3\frac{m_{\text{fermion}}^2}{M_W^2}\,, \tag{14}$$

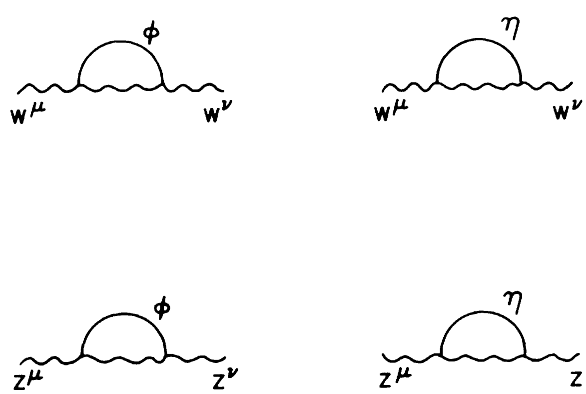

FIG. 3. Logarithmically divergent diagrams.

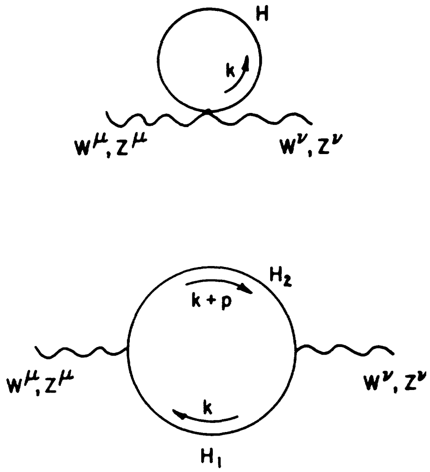

FIG. 4. Quadratically divergent diagrams.

where the C_i are numerical constants and g is a gauge coupling constant. Using the relations

$$\frac{m_{\mathrm{Higgs}}^{\;2}}{M_W^{\;2}} \approx \lambda/g^2\,, \quad \frac{m_{\mathrm{fermion}}^{\;2}}{M_W^{\;2}} \approx \frac{h^2}{g^2} \tag{15}$$

where h is a Yukawa coupling constant, this may be rewritten in the form

$$\Pi \approx g^2 C_1 + \lambda C_2 + h^2 C_3\,. \tag{16}$$

The one-loop corrections to the W and Z propagators contain terms of order λ and h^2, as well as order g^2. We recall that in becoming massive via the Higgs mechanism, the vector mesons absorbed three scalar particles, and scalar particles have self-couplings and Yukawa couplings as well as gauge couplings. In some sense, these couplings reappear in one-loop corrections to the masses.

The one-loop corrections to the W and Z propagators place some not very stringent constraints on weak-interaction model building. In addition, they illustrate that in a theory with spontaneously broken symmetry, where renormalizability depends upon relations among coupling constants and masses, the decoupling of a particle from low-energy phenomena as its mass becomes large may not be taken for granted.

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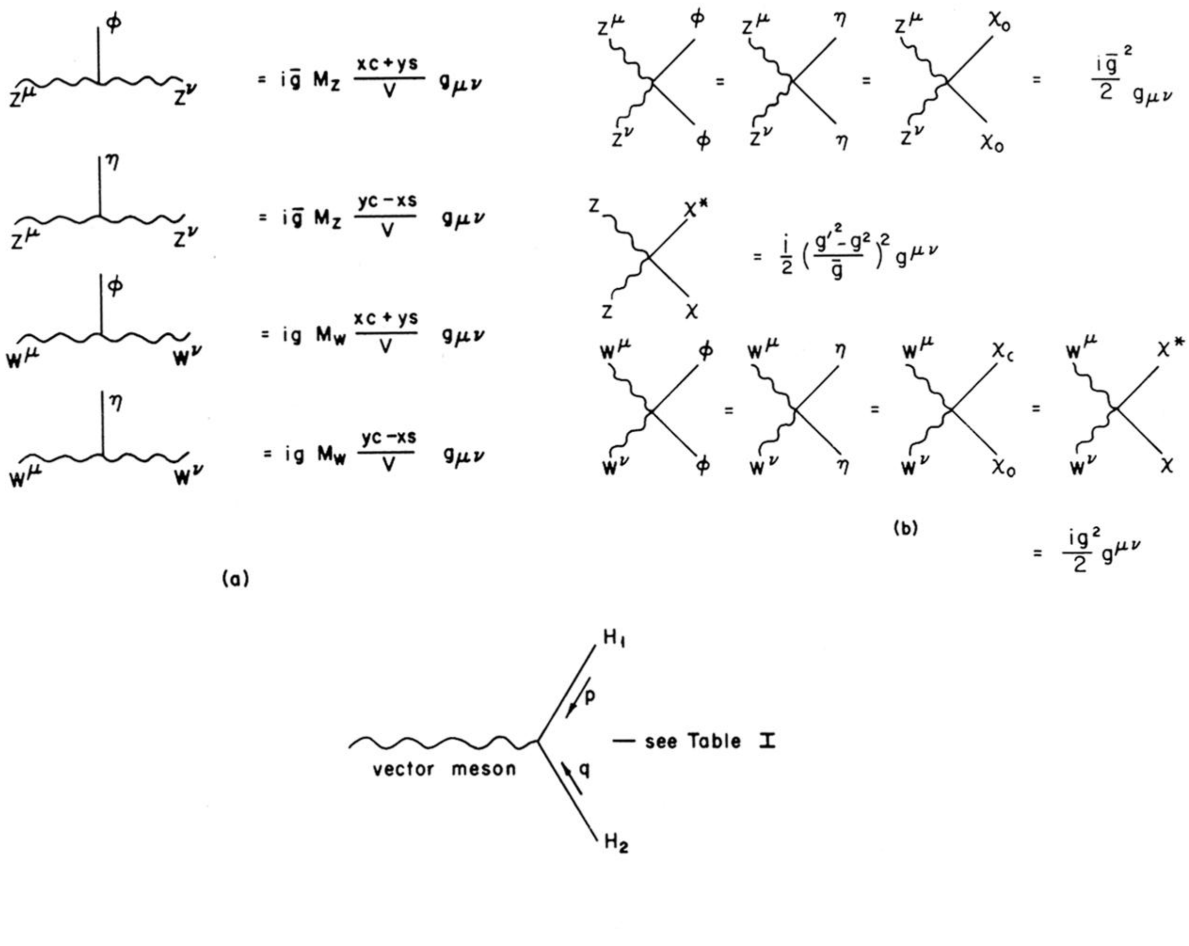

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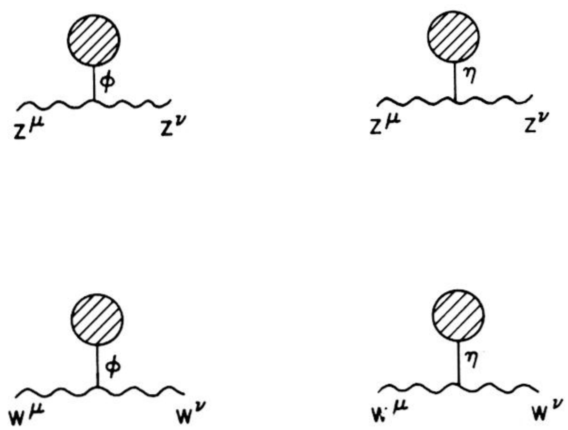

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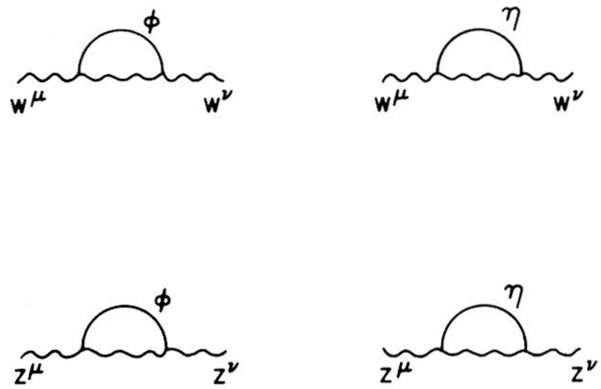

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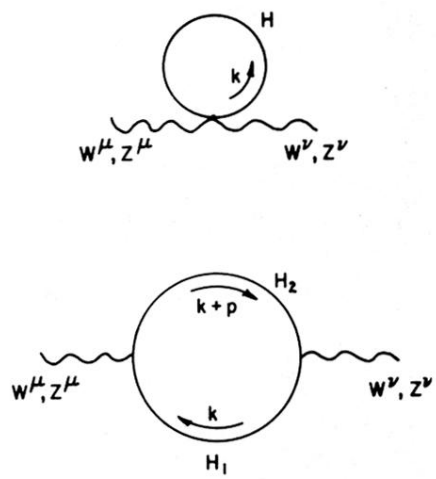

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