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# Do axions exist?

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We critically examine various existing experiments which could provide evidence for the axion. Although our conclusions regarding the existence of this particle are somewhat pessimistic, we discuss other possible experiments which could throw additional light on this question.

Recently, Weinberg[1] and Wilczek[2] suggested that there may well exist a very light, long-lived, pseudoscalar boson which they called an axion. If this particle actually exists, it would constitute strong evidence in favor of the gauge-theory nature of the fundamental interactions. Thus an experimental resolution of the axion's existence is extremely important. In his paper, Weinberg[1] discusses some possible experiments which bear on axions. Although he reserves judgement on the matter, the evidence appears to be predominantly against the existence of the axion.

In this paper we would like to pursue the issue by reexamining the experiments discussed by Weinberg[1] and by considering other evidence as well. We shall see that the picture that emerges remains quite bleak for the axion. Nevertheless, we think it is useful to present, in the latter part of this paper, a brief discussion of some other possible experiments that could be done to search for the axion.

The plan of this paper is as follows: In Sec. I, we discuss the theoretical motivation for considering the axion and detail some of its expected properties, including its coupling to leptons and nucleons. In Sec. II, we calculate the rate for a nucleus to deexcite by axion emission and compare it to the analogous photon rate. Section III is devoted to an analysis of axion production in reactor experiments. Here we discuss principally the limits imposed by the experiments of Reines and collaborators. In Sec. IV, we analyze a beam-dump experiment performed at SLAC. We estimate the expected number of events which should have been produced by axions through pair production and hadron interaction, and we compare these to the experimental rates. Section V contains a discussion of some other experiments which could be pursued in search of the axion, together with estimates of expected rates. Finally, in Sec. VI, we present our conclusions. Some more technical matters are relegated to Appendixes.

## I. THEORETICAL BACKGROUND

The motivation for the axion comes from considering the consequences of a theoretical picture in which the weak and electromagnetic as well as the strong interactions are based on underlying non-Abelian gauge theories. The gauge theory of strong interactions is assumed to be quantum chromodynamics (QCD) which is based on an exact SU(3) color symmetry of quarks and gluons.[3] The weak and electromagnetic interactions are supposed to stem from a spontaneously broken gauge theory based on a weak group which, in the simplest example, is taken to be $SU(2)\times U(1)$.[4] Recently, it has been realized that the existence of instanton solutions[5] for non-Abelian gauge theories raised a potential problem for this theoretical picture. In QCD, the presence of these solutions allows for the appearance of additional terms in the Lagrangian which produce strong $P$, $T$, and $CP$ symmetry violations that contradict experimental observations. A natural mechanism for eliminating these terms obtains if either[6]

(1) at least one of the quarks in the theory is massless;

(2) the Lagrangian of the *full* theory has an overall global chiral U(1) symmetry.

The first option above is inconsistent with standard current-algebra estimates.[7] A consequence of the second can be shown to be that the axion should exist.

The axion is light for two reasons. In the absence of nonperturbative instanton effects, the axion would be a Goldstone boson associated with the spontaneously broken chiral U(1) symmetry of the theory. In reality the symmetry is broken and one therefore expects that the axion acquires a small mass, of the order of the $\pi$ mass. However, the mass scale characterizing the broken weak symmetry is very much larger than the one associated with ordinary chiral symmetry breaking and consequently the axion's mass is further reduced. These remarks have been rendered quantitative by Weinberg[1] and by Bardeen and Tye[8] who applied current-algebra methods to deduce the mass of the axion. They find a mass in the range of 100–200 keV, depending on the value of various model parameters. If the axion is that light, it can only decay into two photons, and its lifetime should be around $10^{-1}$ sec (Refs. 1, 2, 8). (More precise estimates will be given below.)

The prediction of the axion comes from a rather complicated interrelation between the gauge theories of the weak and electromagnetic interactions, on the one hand, and those of the strong interactions, on the other. Thus finding the axion would constitute *prima facie* evidence for the validity of the gauge-theory description of all the fundamental interactions. This circumstance, however, has a certain drawback: The detailed properties of the axion depend on which model of the weak interaction one adopts. For definiteness we shall adopt a rather standard version of the Weinberg-Salam SU(2)× U(1) model[4] with $N$ left-handed quark doublets. It should be kept in mind, however, that in other weak-interaction models the existing experimental evidence which appears to indicate that the axion does not exist may not be so damning.

Having adopted as our weak-interaction model, essentially the model discussed in Ref. 6, it is straightforward to compute the expected coupling of the axion to leptons and quarks. Furthermore, the use of current-algebra techniques will give us the strength of the coupling of the axion to nucleons. To impose the necessary overall chiral U(1) symmetry to the full Lagrangian one needs to introduce two Higgs doublets in the model.[6] Quarks, leptons, and the intermediate boson of the theory acquire a mass because the Higgs doublets are assumed to possess nonzero vacuum expectation values $\lambda_i$, $i=1,2$. The Fermi constant $G$ is related to the $\lambda_i$ by

$$f_\phi=(\lambda_1{}^2+\lambda_2{}^2)^{1/2}=(\sqrt{2}\;G)^{-1/2}\approx 250\text{ GeV}. \tag{1}$$

The ratio of the expectation values of the Higgs fields is arbitrary and, following Bardeen and Tye,[8] we shall denote this ratio by $X$. By definition $X$ is a positive number and there is no particular reason to expect it to have a value much different from unity. As we shall see, unless $X$ is very large or very small, the axion should have a light mass.

The chiral-U(1) current, of which the axion is a psuedo-Goldstone boson, can easily be written down in the model

$$\tilde{J}^{\mu}=f_\phi\partial^\mu\phi+X\sum_i\overline{Q}_i\tfrac{1}{2}\gamma^\mu\gamma_5Q_i+\frac{1}{X}\sum_i\overline{q}_i\tfrac{1}{2}\gamma^\mu\gamma_5q_i+\frac{1}{X}\sum_i\overline{l}_i\tfrac{1}{2}\gamma^\mu\gamma_5l_i\,, \tag{2}$$

Here $\phi$ is the axion field, $Q_i$ are charge $\frac{2}{3}$ quarks $(u,c,\dots)$, $q_i$ are charge $-\frac{1}{3}$ quarks $(d,s,\dots)$, and $l_i$ are charge $-1$ leptons $(e,\mu,\tau,\dots)$. This current is not suitable for current-algebra manipulations because it has an anomaly.[9] Nevertheless, a soft current can be constructed by subtracting an appropriate piece from $\tilde{J}^\mu$. This has been done by Weinberg[1] and by Bardeen and Tye[8] in more detail. We quote the answer found by the latter authors for the anomaly-free chiral current denoted by $J^\mu$:

$$J^\mu=\tilde{J}^\mu-\frac{N(X+1/X)}{1+Z}\left(\overline{u}\,\frac{\gamma^\mu\gamma_5}{2}\,u+Z\,\overline{d}\,\frac{\gamma^\mu\gamma_5}{2}\,d\right). \tag{3}$$

Here $u$, $d$ are the usual (light) up and down quarks, $Z=m_u/m_d\approx 0.56$ (Ref. 10) and $N$ is the number of quark doublets in the theory.

Utilizing the soft current, Eq. (3), one can now estimate the coupling of axions to nucleons by standard current-algebra techniques. In particular, we shall be interested in the matrix element of $J^\mu$ between nucleon states. Thus it is a reasonable approximation to neglect all but the $u$ and $d$ pieces in $J^\mu$ in Eq. (3). Having done this, it proves convenient to decompose the $u$ and $d$ pieces into isoscalar and isovector axial-vector current contributions:

$$A^s_\mu=\overline{u}\,\frac{\gamma_\mu\gamma_5}{2}\,u+\overline{d}\,\frac{\gamma_\mu\gamma_5}{2}\,d\,, \tag{4a}$$

$$A^3_\mu=\overline{u}\,\frac{\gamma_\mu\gamma_5}{2}\,u-\overline{d}\,\frac{\gamma_\mu\gamma_5}{2}\,d\,. \tag{4b}$$

Then the effective chiral current is

$$J^{\text{eff}}_\mu=-\left(\frac{N-1}{2}\right)\left(X+\frac{1}{X}\right)A^s_\mu+\left[\frac{X}{2}\left(1-N\,\frac{(1-Z)}{(1+Z)}\right)-\frac{1}{2X}\left(1+N\,\frac{(1-Z)}{(1+Z)}\right)\right]A^3_\mu\,. \tag{5}$$

Examining the matrix element of the divergence of this current, taken between nucleon states, at zero momentum transfer one obtains, by a method analogous to that of Goldberger and Treiman,[11] the coupling of the axion to nucleons. If we write an effective Lagrangian for the coupling of the axion to the nucleon field $\psi=\binom{p}{n}$ as

$$\mathcal{L}_{\text{eff}}=i\overline{\psi}\gamma_5(g^{(0)}+g^{(1)}\tau_3)\psi\phi\,, \tag{6}$$

we find that

$$g^{(0)}=-\frac{M_N}{f_\phi}\left[\left(\frac{N-1}{2}\right)\left(X+\frac{1}{X}\right)\ F_A^{(0)}\right], \tag{7a}$$

$$g^{(1)}=\frac{M_N}{f_\phi}\left[\frac{X}{2}\left(1-N\ \frac{(1-Z)}{(1+Z)}\right)\right.$$
$$\left.-\frac{1}{2X}\left(1+N\ \frac{(1-Z)}{(1+Z)}\right)\right]F_A^{(1)}\ . \tag{7b}$$

Here $M_N$ is the mass of the nucleon, while $F_A^{(T)}$, $T=0,1$, are the weak coupling constants associated with currents $A_\mu^s$ and $A_\mu^3$, respectively. We know experimentally that $F_A^{(1)}\approx-1.23$ while a quark model estimate[12] yields $F_A^{(0)}=\frac{3}{5}F_A^{(1)}$.

We should make a number of observations at this stage. First, the quantities in the square brackets in Eqs. (7a) and (7b) are numbers which, presumably, are of order one. Thus, as an order-of-magnitude estimate the relevant coupling is typified by

$$g_{aNN}=\frac{M_N F_A^{(1)}}{f_\phi} \tag{8}$$

Using the Goldberger-Treiman relationship[11] we have

$$g_{aNN}{}^2=g_{\pi NN}{}^2\left(\frac{f_\pi}{f_\phi}\right)^2\approx 1.45\times10^{-7}\ g_{\pi NN}{}^2\ . \tag{9}$$

Thus the coupling of the axion to nucleons is very much smaller than the corresponding pion-nucleon coupling. Secondly, if we examine Eqs. 7(a) and 7(b) in detail we note that, with positive $X$, the isoscalar axion-nucleon coupling can never vanish (with $N\geq 2$ of course). In contrast, if $N$ is not too large there is always a value of $X$ for which the isovector coupling is zero. For instance, if $N=2$ and $Z\approx 0$ for $X\approx 2$.

We quote below the predictions of Weinberg[1] and Bardeen and Tye[8] for the mass of the axion and its decay rate into $2\gamma$'s in this same model. These parameters will be of use to us in what follows. They find

$$m_a=m_\pi\left(\frac{f_\pi}{f_\phi}\right)N\left(X+\frac{1}{X}\right)\ \frac{Z^{1/2}}{1+Z} \tag{10}$$

and

$$\tau_{a\to 2\gamma}\approx 0.4Z^{-1}\left(\frac{100\text{ keV}}{m_a}\right)^5\ . \tag{11}$$

We note that Eq. (10) indicates the reason why the axion mass is so light. The axion mass is similar to the pion mass—a typical Goldstone-boson mass—times the ratio $f_\pi/f_\phi$ which typifies the difference in chiral breaking versus weak breaking. Since this ratio is small the axion mass is also small. Numerically, using $Z\approx0.56$ we find

$$m_a\approx 25N\left(X+\frac{1}{X}\right)\text{ keV}. \tag{12}$$

Since we have evidence that at least two quark doublets exist, $m_a$ is at least 100 keV. With this mass, Eq. (11) yields a lifetime of about a second. However, the lifetime is very sensitive to the exact value of $m_a$. Moreover, if the axion is heavier than a MeV, the process $a\to e^+e^-$ is allowed, and the lifetime would be much shorter. The coupling of the axion to electrons, or to charged leptons, is easily calculated from the underlying Lagrangian. It is obviously similar in form to the axion-nucleon coupling with $M_N$ being replaced by the mass of the electron. In detail one finds that the axion couples to charged leptons as

$$\mathcal{L}_{\text{coupl}}=ig_l\,\bar{l}\gamma_5 l\,\phi\ , \tag{13}$$

where

$$g_l=\frac{m_l}{f_\phi}\ \frac{1}{X}\ , \tag{14}$$

with $m_l$ being the lepton's mass. Using Eqs. (13) and (14) we obtain

$$\tau_{a\to e^+e^-}=\frac{8\pi X^2 f_\phi{}^2}{m_a^2(m_a^2-4m_e^2)^{1/2}}\ . \tag{15}$$

If we take $X\approx1$ and $m_a$ of the order of a few MeV's we find a lifetime around $10^{-8}$–$10^{-9}$ sec.

## II. NUCLEAR DEEXCITATION VIA AXIONS

We have seen in the preceding section that the axion is a very light boson coupled weakly to nucleons. Because the axion is so light it is possible for an excited nuclear state $|J_1^{\pi_1}\,T_1\rangle$ to decay to its ground state $|J_2^{\pi_2}T_2\rangle$ via axion emission. Furthermore, since the relevant couplings are weak we may estimate this decay by lowest-order perturbation theory. The calculation of the decay probability due to axion emission is quite analogous to the one for photon decay. As Weinberg[1] has observed, the axion should behave, because it is a pseudoscalar object, as a "magnetic" photon. That is, the allowed values of angular momentum and parity carried away by the axion in such nuclear deexcitation are $0^-$, $1^+$, $2^-$, $3^+$, ... . By using standard multipole techniques[13,14] involving a nonrelativistic reduction of the relevant nucleon matrix elements [for axions, Eq. (6)], one can write the decay rate due to photon emission as

$$\omega_\gamma = \frac{8\pi\alpha k}{(2J_1+1)} \sum_{J\geq 1} \left\{ \left| \sum_{T=0,1} \begin{pmatrix} T_2 & T & T_1 \\ -M_2 & M & M_1 \end{pmatrix} \langle J_2; T_2 \| \hat{T}^{\rm el}_{J;T}(k) \| J_1; T_1 \rangle \right|^2 + \left| \sum_{T=0,1} \begin{pmatrix} T_2 & T & T_1 \\ -M_2 & M & M_1 \end{pmatrix} \langle J_2; T_2 \| \hat{T}^{\rm mag}_{J;T}(k) \| J_1; T_1 \rangle \right|^2 \right\}. \tag{16}$$

Here $\hat{T}^{\rm el}$ and $\hat{T}^{\rm mag}$ are the usual electric and magnetic multipole operators (see below and Appendix A). The caret is used to denote a second-quantized operator; the symbols $\|$ denote matrix elements reduced in both angular momentum and isospin.

A similar calculation yields for axion emission

$$\omega_a = \frac{8\pi\tilde{\alpha} k_a}{(2J_1+1)} \sum_{J\geq 0} \left| \sum_{T=0,1} \begin{pmatrix} T_2 & T & T_1 \\ -M_2 & M & M_1 \end{pmatrix} \times \langle J_2; T_2; \| \hat{M}^a_{J;T}(k_a) \| J_1; T_1 \rangle \right|^2 , \tag{17}$$

where $\hat{M}^a$ is the relevant nuclear multipole operator (see below and Appendix A).

In the above formulas $k = E_1 - E_2$ and $k_a = [(E_2 - E_1)^2 - m_a{}^2]^{1/2}$ are the momenta carried, respectively, by the photon and the axion, while $\alpha = e^2/4\pi$ and $\tilde{\alpha} = g_{aNN}/4\pi$ are the appropriate coupling constants. The multipole operators appearing in Eqs. (16) and (17) can be written as

$$\hat{T}^{\rm el}_{JM;T0} = \frac{k}{M_N} \{ F_1^{(T)} \hat{\Delta}'^{M;0}_{J;T} + \tfrac{1}{2}\mu^{(T)} \hat{\Sigma}^{M;0}_{J;T} \} \quad (J \geq 1) , \tag{18a}$$

$$i\hat{T}^{\rm mag}_{JM;T0} = \frac{k}{M_N} \{ F_1^{(T)} \hat{\Delta}^{M;0}_{J;T} - \tfrac{1}{2}\mu^{(T)} \hat{\Sigma}'^{M;0}_{J;T} \} \quad (J \geq 1), \tag{18b}$$

$$i\hat{M}^a_{JM;T0} = \frac{k_a}{M_N} \{ \tfrac{1}{2}\rho^{(T)} \hat{\Sigma}''^{M;0}_{J;T} \} \quad (J \geq 0) , \tag{18c}$$

where $\hat{\Delta}$, $\hat{\Delta}'$, $\hat{\Sigma}$, $\hat{\Sigma}'$, and $\hat{\Sigma}''$ are specific nuclear multipole operators[13,14] discussed in detail in Appendix A. The form factors $F_1^{(T)}$, $\mu^{(T)}$, and $\rho^{(T)}$ have the values, at zero momentum transfer,

$$F_1^{(T)} = 1 \quad (T = 0,1) , \tag{19a}$$

$$\mu^{(0)} \approx 0.88 , \tag{19b}$$

$$\mu^{(1)} \approx 4.71 , \tag{19c}$$

$$\rho^{(0)} = -(N-1)\left(X + \frac{1}{X}\right) \frac{F_A^{(0)}}{F_A^{(1)}} , \tag{19d}$$

$$\rho^{(1)} = X\left(1 - N\,\frac{(1-Z)}{(1+Z)}\right) - \frac{1}{X}\left(1 + N\,\frac{(1-Z)}{(1+Z)}\right). \tag{19e}$$

In general we will be interested in transitions in which $k$ and $k_a \ll k_F$, the typical nuclear Fermi momentum ($k_F \approx 250$ MeV). In this case we may evaluate these expressions in the long-wavelength limit (LWL). Let us define the ratio of the convection-current contribution to the magnetization-current contribution for the matrix elements of $\hat{T}^{\rm mag}_{JM;T0}$ to be $\eta_J^{(T)}$:

$$\eta_J^{(T)} \equiv \langle \hat{\Delta}^{M;0}_{J;T} \rangle / \tfrac{1}{2} \langle \hat{\Sigma}'^{M;0}_{J;T} \rangle , \quad J \geq 1 , \tag{20a}$$

where, using Eqs. (A5) in Appendix A, we see that $\eta_J^{(T)}$ is independent of $k$ in the LWL. The transition matrix elements of $\hat{T}^{\rm mag}$ then may be written

$$\hat{T}^{\rm mag}_{JM;T0} = \frac{ik}{2M_N} \hat{\Sigma}'^{M;0}_{J;T} \left( \mu^{(T)} - \eta_J^{(T)} \right) . \tag{20b}$$

Using the long-wavelength relationship (A6) in Appendix A we obtain

$$\hat{M}^a_{JM;T0} = -\left(\frac{k_a}{k}\right)\left(\frac{J}{J+1}\right)^{1/2} \frac{\rho^{(T)}}{\mu^{(T)} - \eta_J^{(T)}} \hat{T}^{\rm mag}_{JM;T0} . \tag{21}$$

That is, the axion indeed behaves as a "magnetic" photon. Let us now focus on the dominant multipole having $J^\pi = 1^+$ (the axion analog of $M1$ $\gamma$ decay). For isoscalar ($T=0$) transitions we may employ Eq. (A7) in Appendix A to deduce that $\eta_1^{(0)} = \frac{1}{2}$ and hence from Eqs. (16) and (17) find that

$$\omega_a^{(J=1,\,T=0)} = \frac{1}{2}\,\frac{\tilde{\alpha}}{\alpha} \left(\frac{k_a}{k}\right)^3 \left(\frac{\rho^{(0)}}{\mu^{(0)} - 1/2}\right)^2 \omega_\gamma^{(M1,\,T=0)} . \tag{22a}$$

On the other hand for isovector ($T=1$) transitions no relationship such as (A7) exists in general. However, typically $\eta_1^{(1)} \approx \eta_1^{(0)}$ and, since $\mu^{(1)}$ is so large [Eq. (19c)], we may to a good approximation neglect the convection-current contribution entirely (i.e., take $|\eta_1^{(1)}/\mu^{(1)}| \ll 1$). In this case we find that

$$\omega_a^{(J=1,\,T=1)} \approx \frac{1}{2}\,\frac{\tilde{\alpha}}{\alpha} \left(\frac{k_a}{k}\right)^3 \left(\frac{\rho^{(1)}}{\mu^{(1)}}\right)^2 \omega_\gamma^{(M1,\,T=1)} . \tag{22b}$$

We note for reference that $\tilde{\alpha}/\alpha \approx 2.33\times 10^{-4}$. For other values of $J > 1$ we expect similar relations to emerge; the case of nuclear deexcitation via the emission of a $J^\pi = 0^-$ axion will be discussed in Sec. V.

Equations (22) allow us to convert from $\gamma$-decay rates to axion emission rates, at least for the dominant dipole decay mode. To determine quantitatively the rates we require the zero-momentum transfer values of the axion form factors $\rho^{(T)}$, $T=0,1$. Because $\rho^{(1)}$ can vanish for appropriate values of $X$ and $N$, *it may be that all isovector nuclear deexcitations by axions are forbidden.* On the other hand, for $N \geq 2$, $\rho^{(0)}$ is nonzero for all values of $X$ and isoscalar transitions should be possible in general. Note that, in contrast to $\gamma$ decay where isoscalar electromagnetic transitions are considerably hindered relative to isovector transitions, in decay via axion emission isoscalar transitions go with their full allowed strength and, indeed, may be stronger than isovector transitions.

## III. AXIONS IN REACTOR NEUTRINO EXPERIMENTS

Weinberg[1] has noted that data from reactor neutrino experiments may be relevant to axions. The crucial issue in the interpretation of the experiment is the magnitude of the expected axion flux from reactors. The antineutrino flux from the Savannah River reactor at the neutrino experimental area is estimated at $2\times10^{13}\bar{\nu}/\text{cm}^2\,\text{sec}$ (Ref. 15). Estimates of the prompt $\gamma$-ray flux based on experiments[16] give about one prompt $\gamma$ per $\bar{\nu}$ from decays of excited fission fragments. Since axion emission should compete with electromagnetic decay (particularly $M1$ decay), the reactor may produce a significant axion flux and Weinberg[1] estimated $\approx 2\times10^7\, a/\text{cm}^2$ sec. This estimate is obtained by assuming that prompt $\gamma$ rays are predominantly $E1$ and that there is an $M1$ component in the spectrum reduced in intensity by $(v/c)^2 \sim 10^{-2}$ where $v$ is the typical nucleon velocity. (This ratio of $M1$ to $E1$ in nuclei is roughly valid.) The axion decay rate is reduced further by a factor $\frac{1}{2}(\tilde{\alpha}/\alpha)\sim10^{-4}$ relative to $M1$ decay and hence this yields $\sim10^{-6}$ axions per $\bar{\nu}$. This estimate, although crude, does not seem unreasonable at first sight.

We have looked carefully into this matter to try to obtain a more reliable number. This has been proven quite difficult, and the analysis presented below serves mostly to point out the uncertainties involved in trying to obtain such an estimate.

Using the results of Sec. II we have for $J=1$ transitions

$$\omega_a/\omega_\gamma^{\text{mag}} \approx \frac{1}{2}\left(\frac{\tilde{\alpha}}{\alpha}\right)\left(\frac{k_a}{k}\right)^3\left(\frac{\rho^{(1)}}{\mu^{(1)}}\right)^2, \quad T=1\ , \tag{23a}$$

$$\omega_a/\omega_\gamma^{\text{mag}} = \frac{1}{2}\left(\frac{\tilde{\alpha}}{\alpha}\right)\left(\frac{k_a}{k}\right)^3\left(\frac{\rho^{(0)}}{\mu^{(0)}-1/2}\right)^2 \quad T=0\ . \tag{23b}$$

Without any knowledge of $\rho^{(1)}$ and $\rho^{(0)}$ the estimate of axion to $M1$ $\gamma$ decay of $\frac{1}{2}(\tilde{\alpha}/\alpha)\sim10^{-4}$ is sensible. However, in the decays of fission fragments we expect isovector $M1$ decays to predominate and with $\mu^{(1)}=4.7$ a ratio smaller than $10^{-5}$ is probably more reasonable.

To estimate the axion flux reliably one needs to know further what fraction of photons in the $\gamma$-ray spectrum from fission are due to $M1$ transitions. A search of the literature[17] gave no satisfactory answer. The products of fission are left in high angular momentum states,[18] typically $J\sim 6\text{-}10\hbar$ and the nuclei are highly deformed. Deexcitation by $E2$ (quadrupole) $\gamma$ emission is likely, being enhanced by collective effects. The experimental angular distributions and the observations of rotational spectra are consistent with this view.[18-20] These enhanced $E2$ decays generally give rise to low-energy photons ($E_\gamma \lesssim 1$ MeV). In the present case the nature of high-energy $\gamma$ rays come from dipole decays,[21] but equally good fits to experiment are obtained by assuming quadrupole decays. Moreover, none of the analyses distinguished between magnetic and electric radiation or isovector and isoscalar transitions. We note in addition that axion decay involves the spin operator and thus these rates will not be enhanced by collective effects. Clearly the problem is very complex and it may be that a truly reliable estimate is not possible. In summary, on the basis of the isovector nature of the $M1$ decays, we believe that the estimate of $\sim10^{-6}$ for the expected yield of axions relative to $\bar{\nu}$ of Weinberg[1] should be reduced to $\sim10^{-7}$. Furthermore the expectation that $M1$ deexcitation is uncommon following fission suggests one should reduce this estimate even further. For the purposes of this paper we shall adopt the value $\sim10^{-8}$.

In the experiment of Reines *et al.*,[15] designed to detect the process $\bar{\nu}_e + e \to \bar{\nu}_e + e$, axions might have been detected as well. The most likely signal involves the detection of photons from $a\to 2\gamma$ or recoil electrons or photons from the reaction $a+e\to\gamma+e$. In each case the observed axion energy spectrum would depend on the details of the flux from the reactor. If we assume that the axion spectrum can be simply obtained from the photon spectrum by scaling, then from the data of Verbinski *et al.*[16] on prompt $\gamma$ rays from fission we deduce that the average energy of the emitted axions is approximately 1 MeV. With an effective area $A=1.64\times10^3$ cm$^2$ we expect $3\times10^8$ axions/sec incident on the detector. The total number of axion decays per day is now readily calculated using $l=60$ cm as the effective decay length:

$$N_{a\to2\gamma} = N_0\left(\frac{l}{c}\right)\frac{1}{\gamma\tau_{a\to2\gamma}}(8.64\times10^4)$$

$$\approx 7\times10^3\left(\frac{m_a}{100\ \text{keV}}\right)^6\ . \tag{24}$$

We note how sensitive this number is to the axion's mass. In Ref. 15 the estimated $\gamma$ background for $E_\gamma > 1.5$ MeV is given as $-160 \pm 260$ events/day. About $\frac{1}{5}$ of the prompt $\gamma$ rays have energies above 1.5 MeV and with this same factor one might expect 1400 detected axion decays, well above the observed limit. We have been informed by Reines[22] that for $E_\gamma$ below 1.5 MeV the $\gamma$ background is also consistent with zero but within a larger standard deviation of $\sim 10^3$ counts/day, again below our estimate. Given the uncertainty in the axion flux, no strong statement can be made here. We note, however, that if the axion is much heavier than 100 keV the expected rate is very much larger and this would be in clear conflict with experiment.

We should note that axion Compton scattering could also be a source of $\gamma$ rays. The cross section for this process differs from the Klein-Nishina formula because the axion is a $0^-$ particle, and, of course, because the axion couples very much more weakly than the photon. One finds (neglecting the axion mass, which is a reasonable approximation here)

$$\sigma_{ae\to e\gamma} = \frac{2\pi\alpha\alpha_a}{m_e^{\,2}}\,\frac{1}{X^2}\left[\frac{m_e}{k}\ln\left(1+\frac{2k}{m_e}\right) - 2\,\frac{(1+3k/m_e)}{(1+2k/m_e)^2}\right]. \tag{25}$$

Here

$$\alpha_a = \frac{(m_e/f_\phi)^2}{4\pi}$$

and

$$\frac{\alpha_a}{\alpha} \approx 4\times 10^{-11}\ .$$

We note that $\sigma_{ae\to e\gamma}$ vanishes at threshold, while at high energy $k/m_e \gg 1$, it is asymptotic to

$$\sigma_{ae\to e\gamma} \approx \frac{\pi\alpha\alpha_a}{m_e k}\,\frac{1}{X^2}\left(\ln\frac{2k_e}{m_e} - \frac{3}{2}\right). \tag{26}$$

Figure 1 is a plot of

$$\sigma_{ae\to e\gamma}\left(\frac{\pi\alpha\alpha_a}{m_e^{\,2}}\,\frac{1}{X^2}\right)^{-1}$$

and of the corresponding scaled photon Compton cross section $\sigma_{\gamma e\to e\gamma}/(\pi\alpha^2/m_e^{\,2})$.

For $\langle E_a\rangle \approx 1$ MeV the cross section for axion Compton scattering is approximately maximal and

$$\sigma_{ae\to\gamma e}\,(1\ \mathrm{MeV}) \approx \frac{2.35\times 10^{-36}}{X^2}\ \mathrm{cm}^2.$$

Using this value and an electron density of $3\times 10^{23}$ electrons/cm$^3$ we predict from this process $\sim 1.1\times 10^3/X^2$ additional counts. As it was for $2\gamma$ decay, less than $\frac{1}{5}$ of these events would register

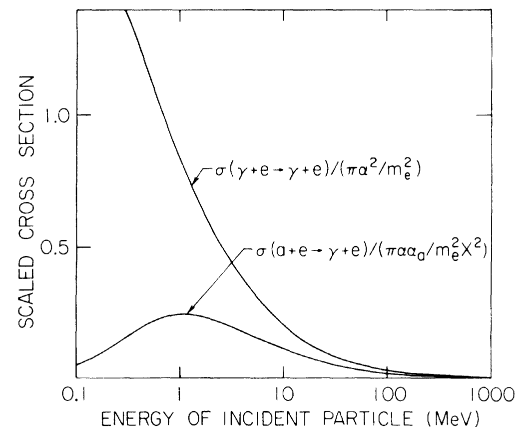


FIG. 1. The "Compton" scattering cross section for axions and photons. The axion-induced scattering cross section is enhanced in the graph by a factor $(\alpha/\alpha_a)X^2$ relative to that for photons to aid in comparison.

as counts with energy above 1.5 MeV in the NaI detector. If $X$ were very small, this rate could be significant. However, if $X$ is too small then $m_a \gg 100$ keV and in general one expects the count rate from axion decays to give a significantly larger signal in the experiment of Reines *et al.*

Another reactor experiment performed by Reines and collaborators[23] might also have been able to detect axions. The experiment was designed to detect the neutral-current neutrino disintegration of the deuteron $\bar{\nu}d\to pn\bar{\nu}$. Axions could mimic this process. Weinberg[1] has estimated an expected axion rate of about $4\times 10^5$ neutron counts/day. Our predicted rate would be about a factor of $10^2$ less. Experimentally,[23] one observes a reactor-associated rate of $-2.9\pm 7.2$ events/day. Thus, it appears that this experiment could be strong evidence that the axion does not exist. However, there are two circumstances that may change this conclusion. First, only axions of energy greater than 2.2 MeV can initiate this reaction. Thus this estimate rests even more strongly on the expectation that high-energy axions are produced by the reactor. It may well be that the distribution of $M1$ $\gamma$ strength and thus most of the axion decay strength is concentrated at low energies. Secondly and perhaps more importantly, the axion disintegration of the deuteron proceeds essentially exclusively through an isovector transition. Thus, if $\rho^{(1)} \approx 0$ the expected deuteron disintegration rate would be seriously reduced. Indeed in this circumstance the axion flux from the reactor would also be reduced, since most of the $M1$ $\gamma$ strength could well be isovector. Note that this would also imply that the expected count rate from the other reactor experiment is also reduced.

Our analysis of the evidence from the reactor experiments regarding the axions is not too encouraging. However, we should stress that there are many issues which have significant bearing on our predictions for which our assumptions are at best educated guesses. Nevertheless, the indications are suggestive that if the axion exists it should be a predominantly isoscalar object of mass not much larger than 100 keV ($X \approx 1-2$).

## IV. THE SLAC BEAM-DUMP EXPERIMENT

The reactor experiments seem to make the existence of the axion less believable. Yet, as we have seen, there are many uncertainties associated with the interpretation of the experiments. A potentially cleaner experiment was performed at SLAC.[24] The experiment was designed to detect highly penetrating particles that could have been produced by high-energy electrons interacting in the SLAC beam dump.

Some relevant experimental details are discussed in Appendix B. Briefly a large detector consisting of optical spark chambers and trigger counters viewed the SLAC beam dump through approximately 55 m of dirt. During three runs a total of ~40 Coulombs of high-energy electrons were stopped in the beam dump.

Because the axion couples to electrons, axions can be produced by a process analogous to photon bremsstrahlung. The cross sections for axion and photon bremsstrahlung are simply related for relativistic energies. We find

$$\left(\frac{d\sigma}{dK}\right)_a = \left(\frac{d\sigma}{dK}\right)_\gamma \frac{1}{2}\,\frac{\alpha_a}{\alpha}\,\frac{1}{X^2}\left(\frac{K^2}{p^2+p'^2-\frac{2}{3}pp'}\right). \qquad (27)$$

Here $K$ is the axion momentum, $p\,(p')$ is the initial (final) electron momentum and $\alpha_a$ is defined after Eq. (25). We should note that in the above formula we have neglected the axion's mass. To obtain the spectrum of axions produced we must fold the spectrum of degraded electrons (through ordinary bremsstrahlung) with the above cross section. This is done in detail in Appendix B. A good approximation for the axion's spectrum is provided by

$$\left(\frac{dN_a}{dK}\right) = \frac{1}{6}\,\frac{\alpha_3}{\alpha}\,\frac{1}{X^2}\,N_0\,\frac{1}{K}$$
$$= 4\times 10^7\,\frac{1}{X^2}\,\frac{1}{K}\Big/\text{Coulomb}. \qquad (28)$$

We should note that because of the high energies involved the axions will all go forward into an angle much smaller than the solid angle of the detector.

One easily identifiable signal of the axion, that would have triggered the spark chambers in this experiment, is a pair of muons generated in the shielding dirt in front of the detector. The cross section for pair production by axions is again calculable, and in the relativistic limit, it is simply related to the corresponding photon process. We find that

$$\left(\frac{d\sigma}{dp_+}\right)_a = \left(\frac{d\sigma}{dp_+}\right)_\gamma \frac{\alpha_a'}{\alpha}\,\frac{1}{X^2}\left(\frac{K^2}{p_+^2+p_-^2+\frac{2}{3}p_+p_-}\right). \qquad (29)$$

Note that in the above

$$\alpha_a' = \frac{(m_\mu/f_\phi)^2}{4\pi}\ . \qquad (30)$$

However, since $(d\sigma/dp_+)_\gamma \sim 1/m_\mu^2$ there is no dependence on the muon mass. That is, the cross sections for an axion to produce $\mu$ pairs or $e$ pairs are identical, neglecting the difference in shielding factors. (For the energies of $\mu$'s produced in this experiment, shielding is not an important effect.) Using the axion spectrum (28), and Eq. (29), we may now calculate the expected number of pairs detected. This calculation is done in detail in Appendix B and we find [Eq. (B9)],

$$N_{\text{pairs}} \approx 5.5\,\frac{1}{X^4}\ , \qquad (31)$$

for the number of pairs expected during the entire experiment. These events are easy to recognize, involving a pair of essentially parallel long tracks nearly normal to the face of the detector. No events of this type are evident in the data. Unfortunately, the statistics are poor and other than an indication that $X$ is not small no definite conclusions are possible. We emphasize, however, that this experiment is free of the serious uncertainties associated with the reactor experiments. In principle, with more data, such an experiment could provide a strong bound on $X$ and therefore on the mass of the axion.

Axions may also interact hadronically and could have generated hadronic showers in the optical spark chamber. A hadronic interaction involving a high-energy axion would probably have produced an event with high multiplicity in the aluminum plates of the spark chamber. Furthermore, such an event would have to take place nearly in the center of the chamber, since the opening angle for bremsstrahlung is so small. As noted in Appendix B, the sensitivity of the SLAC experiment to purely hadronic events is somewhat uncertain. Thus although the chambers contained 2.7 hadronic interaction lengths of material, it is likely that only a region about 1 interaction length long was active for these types of events. An examination of the spark-chamber photographs indicates that there were 24 events with three or more prongs. However, of these, no more than three events were clearly hadronic in nature, with no obvious muon track visible.

It is difficult to estimate reliably the number of hadronic events that axions will produce. Just as a knowledge of the $\pi$-nucleon coupling constant $g_{\pi NN}$ is not sufficient to calculate the high-energy $\pi$-nucleon cross section, so will our formula for $g_{aNN}$, Eq. (9), not suffice to yield a value for the axion hadronic cross section. Nevertheless, as a rough order-of-magnitude estimate, we may suppose that an axion interacts with hadrons $(\frac{1}{2}f_\pi/f_\phi)^2\, B_\pi{}^2 \sim 3.6\times10^{-8}B_\pi{}^2$ as strongly as a pion would, with $B_\pi$ of order unity. We may use Eq. (28) to compute the total number of axions produced in this experiment, with total energy above a minimum cut off energy ($E_{\min}\approx 500$ MeV). Since about one hadronic interaction length was effective in this experiment, we estimate the number of axion induced hadronic events as

$$N_{\text{hads}} \approx N_a\times 3.6\times10^{-8}\times B_\pi{}^2\times 1 \approx 200\left(\frac{B_\pi}{X}\right)^2 \tag{32}$$

If $(B_\pi/X)$ is of order unity, we see that this number is far above what was observed. We must hasten to add, however, that the assumption that $B_\pi$ is of order unity may not be true (for $m_a$ not to be too far above 100 keV, $X$ must be near unity). For instance, as Bardeen, Tye, and Vermaseren[25] have observed, if the axion were to couple at high energy with a coupling proportional to the "current-algebra" mass of the quarks ($m\sim 10$ MeV) rather than with a coupling proportional to the "constituent" mass ($m\sim 300$ MeV) then $B_\pi{}^2\approx 10^{-3}$. This point reemphasizes the importance of concentrating on axion processes involving leptons, where the physics is less uncertain.

## V. POSSIBLE EXPERIMENTS TO SEARCH FOR AXIONS

Our analysis of the SLAC beam-dump experiment, coupled to the analysis of the reactor experiments done by Weinberg[1] and by us, does not bode too well for the axion. Similar negative results were obtained by Ellis and Gaillard[26] in analyzing the Gargamelle beam-dump experiment.[27] Nevertheless, it is fair to say that large uncertainties remain. Thus it appears to us worthwhile to discuss possible experiments which may be done *specifically* to look for the axion.

In a recent paper, Bardeen, Tye, and Vermaseren[25] have discussed two possible ways of looking for axions:

(1) Detect axions produced in high-energy proton scattering by looking for axion-induced $\mu$ pairs.

(2) Produce axions by intense low-energy electron beams and detect them by looking for the $2\gamma$ decay mode.

Wilczek[2] and Weinberg[1] have discussed the prospects of detecting axions in radiative decays of heavy vector mesons, such as the $\psi$ or $\Upsilon$. Many authors[28] have discussed detecting axions in weak kaon decay. Here we would like to concentrate on possible nuclear physics experiments which bear on axions. First, however, we want to briefly comment on the above suggestions.

The production of axions in hadronic experiments suffers from the uncertainty in actually being able to estimate the axion's flux reliably. The authors of Ref. 25 estimate that axions produced by $10^{17}$ protons at 400 GeV should produce about $30(B_\pi/X)^2$ $\mu$ pairs. If we use the results of our analysis of the SLAC beam-dump experiment to set a bound on $(B_\pi/X)^2 \lesssim \frac{1}{60}$ we see that the number of $\mu$ pairs predicted is of order unity. This may render this experiment marginal.

The other experiment discussed by Bardeen, Tye, and Vermaseren[25] is potentially more fruitful. This is because both the production process (axion bremsstrahlung) and the detection process ($2\gamma$ decay) are free of ambiguities, save for the parameter $X$, which is directly related to the mass of the axion. Furthermore, the number of axions produced per day by intense electron beams is comparable to that produced by high-energy protons, with $B_\pi\approx 1$. Low-energy machines used for radiation-damage studies or the superconducting accelerator (SCA) at Stanford can be run effectively at currents equivalent to $10^{20}$–$10^{21}$ electrons/day. Hence, it is not inconceivable to assume that one may produce nearly $10^{10}(1/X^2)$ axions/day. Assuming, for the sake of argument, that the average energy of an axion produced by a low-energy electron-beam machine is around $\langle E_a\rangle\sim 1$ MeV one would expect, with an effective decay length of 3 m, to see approximately

$$N_d \approx \frac{10}{X^2}\left(\frac{m_a}{100\text{ keV}}\right)^6 \tag{33}$$

$2\gamma$ decays/day. This number is just illustrative. More reliable numbers can be calculated, and have been discussed in Ref. 25, without any physical uncertainties, save for $X$ or equivalently the mass of the axion. Note how rapidly dependent is $N_d$ on the mass of the axion. We should mention also that at higher electron beam energies, electron pair production by axions may be a useful means of detection. Experiments of this sort are now under consideration at Stanford.

Finally we should comment briefly on the detection of axions in decay experiments. The process $K\to\pi a$ discussed in Ref. 28 appears to be quite model dependent. Of more interest is the decay of heavy vector mesons, particularly $\psi\to\gamma a$. Because the $\psi$ is a $c\bar{c}$ composite, the rate for $\psi$ decay into axions should be proportional to $X^2$ rather than $1/X^2$, as is the case with lepton production. Hence if lepton experiments are seen to necessitate $X$ to be small to "save" the axion, the rate of $\psi \to a\gamma$ would then be increased substantially. Using the estimate of Wilczek[2] and incorporating the factor of $X^2$ one has

$$\frac{\Gamma(\psi \to a\gamma)}{\Gamma(\psi \to e^+e^-)} \approx \frac{G_F}{\sqrt{2}\pi\alpha}\, m_c^2 X^2 = 7\times 10^{-4} X^2 . \qquad (34)$$

A number of nuclear-physics experiments can be envisaged to search for the axion. We shall discuss three possibilities. In a nucleus, a $0^+ \to 0^-$ transition is forbidden to go by first-order electromagnetic processes. However, it may occur by two photon emission or electron conversion. Now because the axion is a $0^-$ object, such a transition can occur directly by axion emission. Using the formalism of Sec. II and Appendix A, it is easy to estimate the expected rate for $0^+ - 0^-$ transitions due to axion emission. As an order-of-magnitude estimate we obtain

$$\omega_a^{(J=0)} \approx \frac{2\pi\tilde{\alpha}k_a^{\,5}}{(2T_1+1)M_N^2Q^2}\,|a_0\rho^{(0)} + a_1\rho^{(1)}|^2 , \qquad (35)$$

where we have employed Eq. (A8) in Appendix A. The factors $a_0$ and $a_1$ involve nuclear matrix elements of $\tilde{m}_0'$ and are of order unity unless forbidden by isospin conservation and $Q \approx k_F \approx 250$ MeV. One finds that this rate is comparable to the second-order electromagnetic process. Thus if one could find nuclei for which a pair of $0^+$-$0^-$ levels occurred adjacently (with nothing but high-spin states in between), it would be eminently sensible to look for axions there. We have not been able to find any such levels in the nuclear tables; however, nothing in principle forbids such a level sequence from occurring.[29]

Axions could also be detected directly in nuclear-fluorescence experiments. For $M1$ isoscalar transitions, not suppressed by phase space, our estimates of Sec. II yield an axion-to-$\gamma$ ratio

$$R^{(0)} \approx \frac{1}{2}\frac{\tilde{\alpha}}{\alpha}\left(\frac{\rho^{(0)}}{\mu^{(0)}-\frac{1}{2}}\right)^2 = 8.15\times 10^{-4}(N-1)^2(X+1/X)^2\left(\frac{F_A^{(0)}}{F_A^{(1)}}\right)^2 . \qquad (36)$$

Using the quark model estimate,[12]

$$\frac{F_A^{(0)}}{F_A^{(1)}} = \frac{3}{5},$$

we deduce

$$R^{(0)} \approx 1.2\times 10^{-3}(N-1)^2[\tfrac{1}{2}(X+1/X)]^2 , \qquad (37)$$

which is certainly not negligible. Similar considerations follow for isovector $M1$ decays but here $R^{(1)}$ is at least two orders of magnitude smaller and more model dependent. Suitable isoscalar candidate transitions are the 12.71-MeV $1^+0 \to 0^+0$ transition in $^{12}$C and the 7.03-MeV $2^+0 \to 1^+0$ transition in $^{14}$N. It is an open question, however, if sufficient population can be achieved in the excited states of these nuclei so that axion deexcitation can actually be detected via the $2\gamma$ decay mode. In this respect Mössbauer resonance techniques are usually limited to quite nearby low-lying levels.

We would like to mention a final nuclear possibility which is quite spectacular. This involves axion production in a nuclear explosion.[30] As an order-of-magnitude estimate we may suppose that the amount of axions produced in a nuclear explosion should be comparable to the one expected in a few days running in the reactor neutrino experiments. Thus one may perhaps hope to observe around $10^4$–$10^5$ photons from axion decay with an apparatus similar to that of Reines,[15] located at approximately the same distance from a nuclear device. After detonation these axions should appear promptly, ahead of the residual slow neutrons. Thus with fast timing and sufficient shielding to veto the prompt $\gamma$ rays, it may be feasible to detect the axion's by products in this way. Needless to say, this would be a one-shot experiment.

## VI. CONCLUDING REMARKS

We have reanalyzed the reactor neutrino experiments of Reines and collaborators[15,23] and a SLAC beam-dump experiment[24] to try to determine if axions exist. Our analysis was done in a standard version of the SU(2)×U(1) model of the weak interactions with an additional U(1) chiral symmetry.[6] Our conclusions regarding axions are rather pessimistic. However, these experiments would not have detected axions if: (1) they are mostly isoscalar, (2) their mass is close to 100 keV ($X \approx 1-2$), (3) their high-energy coupling to hadrons is small ($B_\pi \ll 1$). We feel that the combination of all the above conditions, while not impossible, may be procrustean. Nevertheless, we also believe that it is important that other more definitive tests of the axion be undertaken. Some such experiments are discussed in Sec. V.

It is perhaps worthwhile to comment briefly on what are the theoretical alternatives if axions are actually found not to exist.[31] The simplest possibility is that $m_u = 0$. Although this contradicts current-algebra estimates, there are a number of approximations employed in these estimates which can be questioned. (In particular, one may wonder if it is valid to take $\langle \bar{u}u\rangle = \langle \bar{d}d\rangle = \langle \bar{s}s\rangle$ and $f_\pi = f_K$.[31]) A more intriguing possibility, which is under investigation is whether in a different weak-interaction model [such as SU(2)×SU(2)×U(1), for example] the phenomenology of imposing an extra U(1) invariance is substantially different from what has been discussed here. Finally, and this is much more challenging theoretically, it may be that our ideas of weak symmetry breaking via Higgs bosons are not correct and that in a theory with dynamical symmetry breaking the axion need never exist.

## ACKNOWLEDGMENTS

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## APPENDIX A: NUCLEAR MULTIPOLE OPERATORS

The multipole operators introduced in Eqs. (18) may all be written in first quantization in the form

$$\theta^{M;0}_{J;T}(q) = \int d\vec{x} \sum_{i=1}^{A} \delta(\vec{x}-\vec{x}_i)\theta^M_J(q\vec{x}_i) I^0_T(i)\ , \quad \text{(A1)}$$

where the sum runs over the $A$ nucleons in the nucleus. The operators $\theta^M_J(i)$, which depend only on the spatial and spin coordinates, are[13,14]

$$\Delta^M_J(q\vec{x}_i) \equiv \vec{M}^M_{JJ}(q\vec{x}_i)\cdot \frac{1}{q}\vec{\nabla}_i\ , \quad \text{(A2a)}$$

$$\Delta'^M_J(q\vec{x}_i) \equiv \left[-\left(\frac{J}{2J+1}\right)^{1/2} \vec{M}^M_{JJ+1}(q\vec{x}_i) + \left(\frac{J+1}{2J+1}\right)^{1/2} \vec{M}^M_{JJ-1}(q\vec{x}_i)\right]\cdot\frac{1}{q}\vec{\nabla}_i\ , \quad \text{(A2b)}$$

$$\Sigma^M_J(q\vec{x}_i) \equiv \vec{M}^M_{JJ}(q\vec{x}_i)\cdot\vec{\sigma}(i)\ , \quad \text{(A2c)}$$

$$\Sigma'^M_J(q\vec{x}_i) \equiv \left[-\left(\frac{J}{2J+1}\right)\vec{M}^M_{JJ+1}(q\vec{x}_i) + \left(\frac{J+1}{2J+1}\right)\vec{M}^M_{JJ-1}(q\vec{x}_i)\right]\cdot\vec{\sigma}(i)\ , \quad \text{(A2d)}$$

$$\Sigma''^M_J(q\vec{x}_i) \equiv \left[\left(\frac{J+1}{2J+1}\right)^{1/2} \vec{M}^M_{JJ+1}(q\vec{x}_i) + \left(\frac{J}{2J+1}\right)^{1/2} \vec{M}^M_{JJ-1}(q\vec{x}_i)\right]\cdot\vec{\sigma}(i)\ , \quad \text{(A2e)}$$

where we define

$$\vec{M}^M_{JL}(q\vec{x}) \equiv j_L(qx)\vec{Y}^M_{JL1}(\Omega_x)\ . \quad \text{(A3)}$$

The isospin dependence is contained in the operator $I^0_T(i)$:

$$I^0_T(i) \equiv \begin{cases} \frac{1}{2}\ (T=0,\ \text{isoscalar})\ , \\ \frac{1}{2}\tau_3(i)\ (T=1,\ \text{isovector third component}). \end{cases} \quad \text{(A4)}$$

Our interest in the present work is not in evaluating the matrix elements of these operators with specific nuclear wave functions, but only in discussion their general nature in the long-wavelength limit (LWL). In particular, we are interested in comparing the operators $\hat{T}^{\text{mag}}$ and $\hat{M}^a$ [Eq. (18)] and so restrict our attention to $\Delta$, $\Sigma'$ and $\Sigma''$. Upon expanding the spherical Bessel functions [Eq. (A3)] for small values of $qx$ and keeping only the leading order we find for $J \geq 1$

$$\Delta^M_J(q\vec{x}_i) \xrightarrow{\text{LWL}} (q/Q)^{J-1} m_J(Q\vec{x}_i)\ , \quad \text{(A5a)}$$

$$\Sigma'^M_J(q\vec{x}_i) \xrightarrow{\text{LWL}} (q/Q)^{J-1} m'_J(Q\vec{x}_i)\ , \quad \text{(A5b)}$$

where $Q$ is a typical nuclear momentum transfer, $Q \sim k_F \sim 1/R$ where $R$ is a typical nuclear radius. The operators $m$ and $m'$ are discussed in detail in Ref. 14; the only important fact for our present purposes is that their matrix elements are of order unity. Finally, the operator $\Sigma''$ may be related to $\Sigma'$ in the long-wavelength limit:

$$\Sigma''^M_J(q\vec{x}_i) \xrightarrow{\text{LWL}} \left(\frac{J}{J+1}\right)^{1/2} \Sigma'^M_J(q\vec{x}_i)\ . \quad \text{(A6)}$$

Thus two operators, $\Delta$ and $\Sigma'$ characterize $\hat{T}^{\text{mag}}$ and $\hat{M}^a$ in the long wavelength limit. For $J=1$ a special relationship between inelastic isoscalar matrix elements of $\Delta$ and $\Sigma'$ may be derived (see Ref. 14) by writing the total-angular-momentum operator as $\vec{J}=\vec{L}+\vec{S}$ and using the fact that inelastic matrix elements of $\vec{J}$ vanish,

$$4\left\langle \hat{\Delta}^{M;0}_{1;0}\right\rangle = \left\langle \hat{\Sigma}'^{M;0}_{1;0}\right\rangle\ \text{(LWL)}\ . \quad \text{(A7)}$$

Here we use the facts that $\Delta_1$ can be related to the orbital-momentum operator and $\Sigma'_1$ to the spin-angular-momentum operator in the LWL.

The only remaining case to be considered is $\Sigma''$ when $J=0$,

$$\Sigma''^0_0(q\vec{x}_i) \xrightarrow{\text{LWL}} (q/Q)\,\tilde{m}'_0(Q\vec{x}_i)\ , \quad \text{(A8)}$$

where matrix elements of $\tilde{m}'_0$ are of order unity (see Ref. 14).

## APPENDIX B: THE SLAC BEAM-DUMP EXPERIMENT

### 1. Description of the experiment

The SLAC beam-dump experiment is described in detail in the thesis of Rothenberg.[24] In this appendix we point out some details of the experiment that are relevant to the detection of axions.

Figure 2 shows the overall view of the experiment. The detector was placed ~55 m from the beam dump at End Station A at SLAC. Beam stops of various materials were used during the experiments, but for the question of axion production by a bremsstrahlunglike process the details of the beam stop are irrelevant. The detector was in direct line with the stopping electron beam; another detector closer to the beam dump detected prompt muons and served as a prompt trigger on the electron beam.

The main detector shown in Fig. 3 consisted of four large aluminum optical spark chambers and three banks of plastic scintillator trigger counters. Each spark chamber consisted of eleven 2.54-cm-thick aluminum plates; the chambers were 2.4 m square. The total thickness of the detector corresponded to ~2.7 hadronic interaction lengths. The trigger requirement was two counters, one from each of two adjacent scintillator banks (i.e., A and B or B and C in Fig. 3). The bias was below minimum ionizing in the trigger counters.

In order to evaluate the signal form axion production of $\mu$ pairs in the shielding in front of the detector some properties of the shielding material are important. Most of the shielding was 55 m of rock in the hill between the beam dump and the detector. The rock was predominantly miocene (70% quartz and 30% feldspar) which we take to be simply $SiO_2$. The density of the rock was 2.0 g/cm$^3$, the average $Z$ was 10 and the average $A$ was 20. The rock and the other shielding was enough to insure that a 20-GeV muon produced at the beam dump would stop at least 12 m from the detector.

The most energetic muons are displaced less than 45 cm by multiple scattering while penetrating their maximum range (~40 m) and the corresponding displacement is less for less penetrating muons. Thus, since the detector is larger than this, we can safely assume that all muons energetic enough to penetrate would have struck the detector.

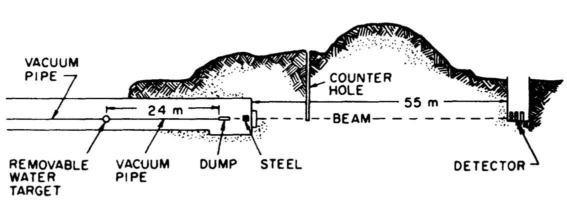


FIG. 2. Schematic of the SLAC beam-dump experiment showing the location of the detector and shielding in relation to the end station A beam dump.

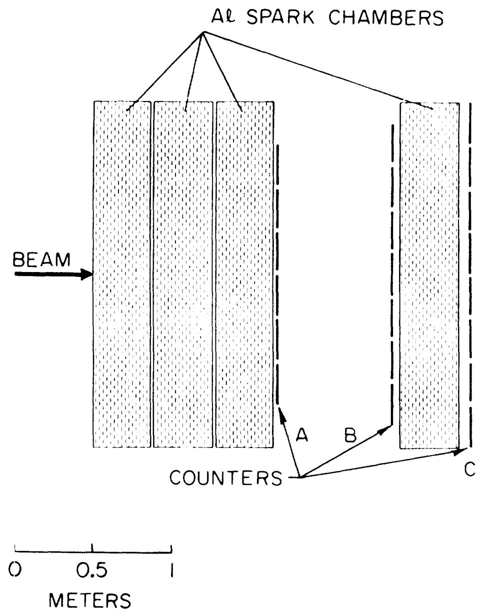


FIG. 3. Schematic of the detector showing the optical spark chambers and scintillation counters. Each of the four spark-chamber modules consisted of eleven 2.5-cm-thick aluminum plates.

The efficiency for detection of hadrons produced in and around the detector is more difficult to estimate. Clearly, events originating in the fourth spark chamber in Fig. 3 would not be expected to produce the required coincidence trigger in the scintillator banks, but hadronic events generated in the first three chambers and perhaps in a volume of shielding directly in front of the detector could generate valid triggers. We take the conservative estimate that only 1 hadronic interaction length directly in front of the $A$ counter bank was active. We should note, however, that the active region could well be larger than this estimate.

### 2. The axion spectrum

To calculate the axion spectrum in the SLAC experiment, we must calculate the effective bremsstrahlung cross section for production of axions by fast electrons in matter. Because a monoenergetic beam of electrons is degraded in energy when it enters material, the production cross section for axions can change significantly from its value *in vacuo*. Tsai and Whitis[32] have calculated the distribution $I(t, E)$ of first-generation electrons after the beam has traveled $t$ photon radiation lengths into material. In Ref. 32 it was found that second-generation electrons change the photon spectrum very little at higher energies for 2 radiation lengths. We can safely assume this result to hold for axions as well and, since the high-energy spectrum is of primary interest, we consider only first-generation electrons. The results of Ref. 32 for first-generation electrons can be parametrized as

$$I(t,E)=\frac{1}{E_0}\,\frac{(\ln E_0/E)^{4t/3-1}}{\Gamma(4t/3)} \tag{B1}$$

$I(t,E)$ has the two required properties, namely: (1) It is normalized to unity $\int_0^\infty I(t,E)dE=1$ for $E_0\to\infty$ and (2) $I(t,E)$ will give approximately the correct decay law for the average beam energy, $\langle E\rangle=E_0 2^{-4t/3}\approx E_0 e^{-t}$. We can now calculate the effective axion-production cross section by folding this distribution with the probability for a relativistic electron of energy $E$ to bremsstrahlung an axion of momentum $K$. Using Eq. (27), in the limit of complete shielding, we find that the number of axions per unit momentum is

$$\frac{dN_a}{dK}=\frac{N_0}{2X^2}\left(\frac{\alpha_a}{\alpha}\right)\frac{K}{E_0^2}\left[\int_0^\infty dt'\,\frac{1}{\Gamma(4t'/3)}\int_K^{E_0}\frac{dE}{E_0}\left(\ln\frac{E_0}{E}\right)^{(4/3)t'-1}\right]. \tag{B2}$$

Going to the dimensionless variable $y=\ln E_0/E$ we may rewrite Eq. (B3) as

$$\frac{dN_a}{dK}=\frac{N_0}{2X^2}\left(\frac{\alpha_a}{\alpha}\right)\frac{K}{E_0^2}\int_0^\infty dt'\,\frac{1}{\Gamma(4t'/3)}\int_0^{\ln(E_0/K)}dy\,e^y y^{(4/3)t'-1}. \tag{B.3}$$

Expanding the exponential and integrating term by term yields

$$\frac{dN_a}{dK}=\frac{N_0}{2X^2}\left(\frac{\alpha_a}{\alpha}\right)\frac{K}{E_0^2}\left(\sum_{N=0}^{\infty}\frac{1}{N!}\int_0^\infty dt'\,\frac{(\ln E_0/K)^{(4/3)t'+N}(4t'/3)}{\Gamma(1+4t'/3)(N+4t'/3)}\right). \tag{B4}$$

The results of numerical integration are shown in Fig. 4 where we have plotted $K/E_0$ times the summation above versus $K/E_0$, as well as the function $E_0/3K$ versus $K/E_0$. The factor in the bracket was calculated using Simpson's rule with a mesh of 0.1. For $K$ in the range $0.1\le K/E_0\le 0.9$, the term $N=7$ contributes less than $\frac{1}{2}\%$ to the sum and the series was terminated there. In practice, although the integral over $t'$ has an infinite range, we cut off the integral at $t'=10$. This is an excellent approximation for the given range of $K$ studied.

It is clear from Fig. 4 that the effective axion spectrum looks not like $K$ but like $K^{-1}$, i.e., like photons, apart from numerical factors. Hence, in considering the number of axions produced in thick target bremsstrahlung, it is important to include the effects of beam degradation. As shown in Fig. 4, the axion spectrum is well approximated analytically by the formula

$$\frac{dN_a}{dK}=\frac{N_0}{6X^2}\left(\frac{\alpha_a}{\alpha}\right)\frac{1}{K} \tag{B5}$$

### 3. Number of axion-produced $\mu$ pairs

Having determined the spectrum of axions produced in this experiment, it is now a simple matter to consider the number of $\mu$ pairs that they will produce. For these purposes we shall assume that the muons are always minimum-ionizing. Then the number of $\mu$ pairs is given by folding the axion spectrum with the cross section for muon pair production multiplied by the density of scatterers times the effective length that the muons travel,

$$N_{\text{pair}}=\int_{K_{\min}}^{E_0}dK\left(\frac{dN_a}{dK}\right)\times\int_{K_{\min}}^{K}dP_+\left(\frac{d\sigma}{dP_+}\right)\rho_{\text{At}}\,\frac{P_+}{(dP_+/dx)_{\min}} \tag{B6}$$

To be conservative, we shall take $K_{\min}=1$ GeV. Furthermore, it will suffice to use the relativistic formula for the axion pair production cross sec-

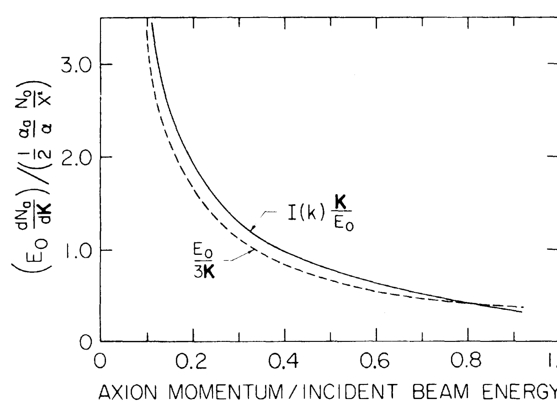


FIG. 4. The approximation to the axion bremsstrahlung production rate used in the text. The quantity $E_0(dN_a/dK)/(\frac{1}{2}(\alpha_a/\alpha)(N_0/X^2))$ is plotted against the ratio of axion momentum to the beam energy. The dotted line shows the approximation in the text—$E_0/3K$. The solid curve is the result of a calculation using the first-generation electron spectrum from Ref. 30 and Eq. (B 1).

tion, Eq. (29), provided we restrict $\delta K \leq P_+ \leq (1-\delta) K (\delta=0.1)$. Clearly a more accurate calculation can be done, but we expect the above analysis to be reasonably reliable and sufficient for our present purposes. Using

$$\rho_{At} = 6\times 10^{22}\ \text{cm}^{-3},$$

$$\left(\frac{dP_+}{dx}\right)_{\min} = 3.2\times 10^3\ \text{GeV/cm},$$

and defining $\epsilon = K_{\min}/E_0$ we find ($E_0$ in GeV)

$$N_{\text{pair}} = \frac{3.32\times 10^{-3}}{X^4}\, E_0 \left\{(1-2\delta)(1-\epsilon)\left[\ln\left(2\frac{E_0}{m_\mu}\right) - \frac{7}{2}\right] + \epsilon(1-2\delta)\ln\frac{1}{\epsilon} + 2(1-\epsilon)[(1-\delta)\ln(1-\delta) - \delta\ln\delta]\right\}. \tag{B7}$$

In the SLAC experiment approximately 40 Coulombs of electrons were dumped. Runs were made at various energies: ~12.5-GeV electrons for ~2.7 Coulombs; ~15-GeV electrons for ~2.2 Coulombs; ~18.5-GeV electrons for ~9.2 Coulombs; and ~19-GeV electrons for ~26.7 Coulombs. Using Eq. (B8) we expect

$$N_{\text{pair}} \simeq \frac{5.5}{X^4} \tag{B8}$$

$\mu$ pairs to have been detected during the experiment.

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[29]Peter Jackson at TRIUMF has made a systematic and intensive search for such $0^+$-$0^-$ levels in connection with the work of N. Isgur [Nucl. Phys. B98, 329 (1975)] on neutral currents. He informs us that unfortunately no such sequence is presently known.

[30]F. Villa at SLAC has looked into the sensitivity of pre-

vious experiments looking for prompt products in nuclear test for detecting axions. He concluded that the sensitivity of these experiments was not sufficient for their detection (private communication to one of us, R. D. P.).

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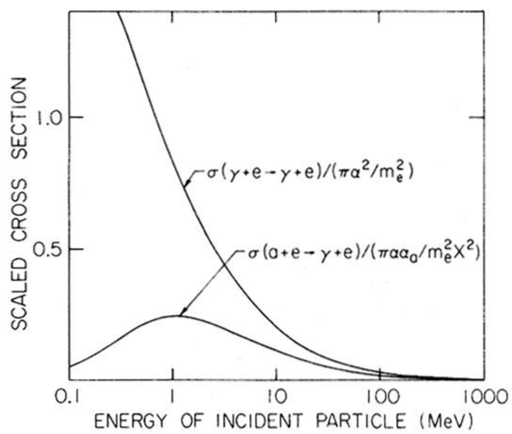


FIG. 1. The "Compton" scattering cross section for axions and photons. The axion-induced scattering cross section is enhanced in the graph by a factor $(\alpha/\alpha_a)X^2$ relative to that for photons to aid in comparison.

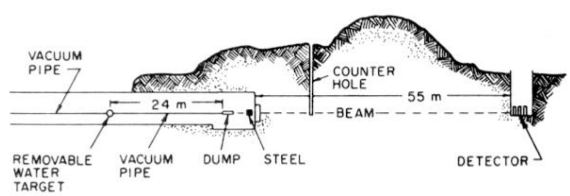


FIG. 2. Schematic of the SLAC beam-dump experiment showing the location of the detector and shielding in relation to the end station A beam dump.

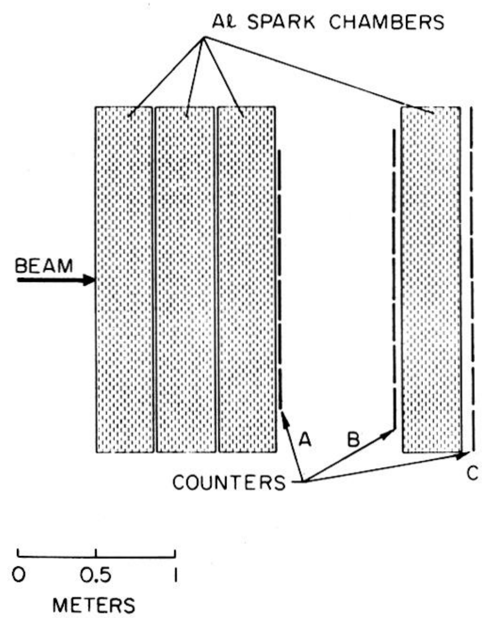


FIG. 3. Schematic of the detector showing the optical spark chambers and scintillation counters. Each of the four spark-chamber modules consisted of eleven 2.5-cm-thick aluminum plates.

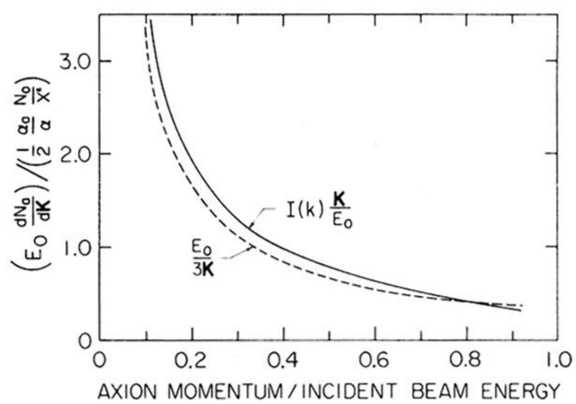


FIG. 4. The approximation to the axion bremsstrahlung production rate used in the text. The quantity $E_0(dN_a/dK)/(\frac{1}{2}(\alpha_a/\alpha)(N_0/X^2)$ is plotted against the ratio of axion momentum to the beam energy. The dotted line shows the approximation in the text—$E_0/3K$. The solid curve is the result of a calculation using the first-generation electron spectrum from Ref. 30 and Eq. (B 1).