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Phenomenological SU(6) breaking of baryon wave functions and the chromodynamic spin-spin force

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A contradiction is found between two successful models of SU(6) breaking. A quark-model mixing scheme $(56,0^+)+(70,0^+)$ for the baryon octet has been devised to explain the ratio $F_2^{en}(x)/F_2^{ep}(x)$ in the valence-quark region and explains naturally other departures from the usual SU(6) predictions. On the other hand, the gluon-exchange model of SU(6) breaking accounts satisfactorily for the hadron spectrum splittings. The spin-spin contribution from this chromodynamic force is indeed shown to generate a $(56,0^+)+(70,0^+)$ mixing of the octet. However, it yields a wrong sign for the mixing angle, thus pointing to a contradiction between spin-spin forces of one-gluon-exchange type and the deep-inelastic structure functions in the valence-quark region. Other spin-spin potentials, giving the right sign for the mixing angle, are shown to be also in difficulty, because of the hyperfine structure of excited levels. Finally, a careful discussion is made of the subtle Σ-Λ effect in both approaches.

I. INTRODUCTION

Ideas about the nature of the SU(6)-breaking forces inside the hadrons have been suggested by De Rújula, Georgi, and Glashow[1] (hereafter DGG), followed by other authors.[2] Invoking the asymptotic freedom of color-gauge theories, they propose that the small-distance quark-quark forces should be dominated by one-gluon exchange; making a v/c expansion, an SU(6)-breaking potential appears: the well-known Breit-Fermi interaction. On the other hand, the large-distance confining force should be SU(6) symmetric.

These suggestions are very interesting since the earlier approaches to the SU(6)-breaking forces[3] were purely phenomenological. Here, instead, we have a much more theoretical derivation, and the form of the potential is well defined, although the strength of the force α_s is, in principle, momentum dependent. A particularly striking result derived by DGG is the relation between two mass differences within the $\underline{56}$ baryon ground state: Σ-Λ and Δ-N, which turn out to be both of hyperfine type. In the work of DGG, the wave functions remain unknown since they do not assume a specific long-range potential. In Ref. 2, explicit wave functions are given corresponding to a linearly rising potential.

Another series of manifestations of SU(6) breaking can be described by an SU(6) configuration mixing of the baryon octet. We have shown[4] that the remarkable behavior of the ratio F_2^{en}/F_2^{ep} in the valence-quark region, which deviates strongly from the $\frac{2}{3}$ prediction of the $\underline{56}$ model, can be ascribed to a well-defined mixing between the usual $(56,0^+)$ ground state of the harmonic-oscillator model[5] and an upper level with radial excitation $(70,0^+)$:

$$\Psi_8=\cos\varphi(56,0^+)_0+\sin\varphi(70,0^+)_2. \quad (1.1)$$

The decuplet is assumed to be unmixed for simplicity. The $(70,0^+)$ is the unique configuration at the level $N=2$ which can produce the desired effect. Other configurations would produce no effect on the ratio F_2^{en}/F_2^{ep} except at the cost of exceedingly large mixing angles. [Note that at this step SU(3) is conserved, so that no octet-decuplet mixing is allowed, in contrast with the $(56,0^+)+(56,2^+)$ mixing of Lipkin[6].] Moreover, the mixing angle is unambiguously fixed to be

$$\varphi\simeq-20°. \quad (1.2)$$

A crucial step in this analysis is the connection established between the usual rest-frame wave functions of the naive quark model, and the structure functions, naturally introduced in the $P=\infty$ frame by quark-parton models. This connection relies on the simple boost prescription

$$\psi_{P=\infty}(\{x_i,\vec{p}_{iT}\})=\psi_0(\{(p_{iz}-m_i)/m_N,\vec{p}_{iT}\}) \quad (1.3)$$

for the spatial wave function, and a Lorentz transformation of the Dirac spinors in the spin part.[4,7]

The above SU(6)-mixing interpretation of the ratio F_2^{en}/F_2^{ep} is partly in the spirit of an earlier work by Altarelli, Cabibbo, Maiani, and Petronzio.[8] They introduced a mixing $(56,0^+)+(70,1^-)$ at $P=\infty$. We have preferred a $P=0$ approach, which is more powerful when one deals with the static properties. Both these approaches traduce in a quantitative manner the qualitative fact, emphasized by Close,[9] that the isosinglet-diquark state is enhanced in the nucleon wave function for $x\gtrsim0.5$.

What seems to us very important to emphasize for the following discussion is the fact that the SU(6)-broken behavior of the structure functions is *not only* a *very* short-distance ($x\approx 1$) phenomenon. It extends from $x\sim 0.3$ to $x\sim 0.8$, the points beyond being not well measured. Therefore, *it appears unsatisfactory to explain it only by something happening at* $x\approx 1$, as in Farrar and Jackson.[10] And also, it is likely that an important part of the phenomenon can be analyzed in the frame of the naive quark model—with nonrelativistic or semirelativistic assumptions—rather than with the ultrarelativistic hard-parton methods of Ref. 10. In our previous papers, we have spoken of the behavior of F_2^{en}/F_2^{ep} "at $x\to 1$" and this was misleading, all the more since our boost prescription is not expected to work at $x=1$, but only for moderate x. $F(x)$ does not vanish as it should for $x\geq 1$.[11]

This remark helps us to understand why the SU(6)-broken behavior of the structure functions teaches something about low-energy, small-momentum-transfer properties of the baryons, which test a rather *peripheral* region of the wave function. Indeed, a most remarkable issue of the mixing (1.1) concerns the SU(6) predictions for static properties. It does not affect the good predictions like $\mu_p/\mu_n=-\frac{3}{2}$ and $F/D=\frac{2}{3}$, and it cures the not-so-good predictions like $\mu^*/\mu_p=2\sqrt{2}/3$ and $G^*/G_A=\frac{4}{3}$, yielding the desired $+40\%$ correction. It also yields a Σ-Λ splitting effect of 70 MeV.[12] On the other hand, the explanation of hyperfine splittings like Δ-N is outside the scope of this approach, and requires a knowledge of the SU(6)-breaking *forces*.

An obvious question now is the following: Is the mixing (1.1) explainable in terms of the DGG SU(6)-breaking potential? The Schrödinger perturbation theory establishes a very simple relation between a perturbing potential and the perturbation of the wave function [see (2.1) below]. It comes out that the Fermi spin-spin contact term precisely generates a $(56,0^+)_0+(70,0^+)_2$, but with the *wrong sign*. After having shown this fact (Sec. II), we see if one can avoid this disappointing conclusion by a change in the spatial behavior of the potential. We find a definite obstacle, because it is then impossible to maintain the DGG explanation of excited-level hyperfine splittings (Sec. III).

The remarkable prediction of the subtle Σ-Λ splitting by both approaches then raises the following question: Why do they both yield the same correct sign, while they disagree on the mixing-angle sign? We answer this delicate question in Sec. IV, and discover at the same time a correction to the DGG calculation of the gluon-exchange effect.

II. PERTURBATION DUE TO THE BREIT-FERMI POTENTIAL: SU(6) CONFIGURATION MIXING OF THE GROUND STATE

We adopt as unperturbed wave functions both for baryons and mesons those predicted by the harmonic-oscillator quark model.[5] This means that we take the harmonic oscillator for the confining force. It is a particular case of the SU(6)-invariant growing potential considered in DGG, who leave it unspecified. Some authors[13] consider a linear potential as being more theoretically founded. In fact, one can compare the spectrum of these respective potentials by treating their difference as a perturbation. This has been done by Gromes and Stamatescu.[14] There appears a splitting of the $N=2$ oscillator levels. However, it is not very relevant to the study of the SU(6) breaking. The harmonic oscillator seems on the whole an acceptable zero-order approximation for the study of the ground state.

Our starting point will be the Schrödinger perturbation method, limited to the first order in a perturbing potential which we call V,

$$|\Psi\rangle=|\Psi_0\rangle+\sum_{n\neq 0}\frac{|\Psi_n\rangle\langle\Psi_n|V|\Psi_0\rangle}{E_0-E_n}\,, \tag{2.1}$$

and, moreover, limited to the first allowed excited level. We do not mean that V is indeed truly small or that higher excited states do not contribute significantly. On the contrary one knows that the effective quark-gluon coupling may be large for the ground state. But we hope that this simple calculation will give the general trend of the effect. The perturbation V is the one considered by DGG, *neglecting for the moment* the SU(3) breaking arising from the m_λ-m_φ mass difference (the common quark mass is then denoted by m):

$$V=k\alpha_s\sum_{i<j}S_{ij}\,, \tag{2.2}$$

where $k=-\frac{4}{3}$ for mesons and $-\frac{2}{3}$ for baryons, and

$$S_{ij}=S^0_{ij}+S^{SS}_{ij}+S^{SO}_{ij}+S^T_{ij}, \tag{2.3}$$

where S^0_{ij} is the spin-independent part,

$$S^0_{ij}=\frac{1}{|\vec r|}-\frac{1}{2m^2}\left[\frac{\vec p_i\cdot\vec p_j}{|\vec r|}+\frac{\vec r\cdot(\vec r\cdot\vec p_i)\vec p_j}{|\vec r|^3}\right]-\frac{\pi}{m^2}\delta^3(\vec r); \tag{2.4}$$

S^{SS}_{ij} is a spin-spin force,

$$S^{SS}_{ij}=-\frac{8\pi}{3m^2}\delta^3(\vec r)\vec S_i\cdot\vec S_j\,; \tag{2.5}$$

S^{SO}_{ij} is a spin-orbit force,

$$S^{SO}_{ij}=-\frac{1}{2m^2|\vec r|^3}[(\vec r\times\vec p_i)\cdot\vec S_i-(\vec r\times\vec p_j)\cdot\vec S_j+2(\vec r\times\vec p_i)\cdot\vec S_j-2(\vec r\times\vec p_j)\cdot\vec S_i]\,; \tag{2.6}$$

and S^T_{ij} is a tensor force,

$$S^T_{ij}=\frac{1}{m^2|\vec{r}|^3}\left[\vec{S}_i\cdot\vec{S}_j-3\,\frac{(\vec{S}_i\cdot\vec{r})(\vec{S}_j\cdot\vec{r})}{|\vec{r}|^2}\right]. \qquad (2.7)$$

Note that everywhere a nonrelativistic expansion is made in v/c, up to v^2/c^2. This is in the spirit of the usual quark-model calculations. But one must recall that v/c is not truly small, at least for nonstrange quarks. In Sec. IV, we discuss some weaknesses in the assumptions adopted here. Some of them may be crucial.

Let us examine the effect of the various terms in (2.3) on the $(56,0^+)$ baryon state wave function. First, we make explicit the octet and decuplet wave function (omitting the color-singlet function)

$$\Psi_0(8,\tfrac{1}{2}^+)=\frac{1}{\sqrt{2}}(\phi'\chi'+\phi''\chi'')\psi_0^s, \qquad (2.8)$$

$$\Psi_0(10,\tfrac{3}{2}^+)=\phi^s\chi^s\psi_0^s, \qquad (2.9)$$

where ϕ,χ,ψ denote, respectively, the SU(3), spin, and space wave functions, with the various types of symmetry under quark exchange.[15] Due to J^P conservation, V will mix Ψ_0 only to states of the same J at the second excited oscillator level $N=2$. Let us enumerate the various $(8,\frac{1}{2}^+)$ and $(10,\frac{3}{2}^+)$ at this level. For the $(8,\frac{1}{2}^+)$ we have the following states:

$$|56,0^+,N=2,S=\tfrac{1}{2},8,\tfrac{1}{2}^+\rangle=\frac{1}{\sqrt{2}}(\phi'\chi'+\phi''\chi'')\psi^s_{N=2}, \qquad (2.10)$$

$$|70,0^+,N=2,S=\tfrac{1}{2},8,\tfrac{1}{2}^+\rangle$$
$$=\tfrac{1}{2}[(\phi'\chi'-\phi''\chi'')\psi''_{N=2}+(\phi'\chi''+\phi''\chi')\psi'_{N=2}], \qquad (2.11)$$

$$|70,2^+,N=2,S=\tfrac{3}{2},8,\tfrac{1}{2}^+\rangle$$
$$=\frac{1}{\sqrt{2}}\{\phi'[\chi^s,\psi'_{L=2}]_{1/2}+\phi''[\chi^s,\psi''_{L=2}]_{1/2}\}, \qquad (2.12)$$

$$|20,1^+,N=2,S=\tfrac{1}{2},8,\tfrac{1}{2}^+\rangle$$
$$=\frac{1}{\sqrt{2}}\{\phi'[\chi'',\psi^a_{L=1}]_{1/2}-\phi''[\chi',\psi^a_{L=1}]_{1/2}\}, \qquad (2.13)$$

and, for the decuplet $(10,\frac{3}{2}^+)$,

$$|56,0^+,N=2,S=\tfrac{3}{2},10,\tfrac{3}{2}^+\rangle=\phi^s\chi^s\psi^s_{N=2}, \qquad (2.14)$$

$$|56,2^+,N=2,S=\tfrac{3}{2},10,\tfrac{3}{2}^+\rangle=\phi^s[\chi^s,\psi^s_{L=2}]_{3/2}, \qquad (2.15)$$

$$|70,2^+,N=2,S=\tfrac{1}{2},10,\tfrac{3}{2}^+\rangle$$
$$=\frac{1}{\sqrt{2}}\phi^s\{[\chi',\psi'_{L=2}]_{3/2}+[\chi'',\psi''_{L=2}]_{3/2}\}. \qquad (2.16)$$

The notation $[\chi,\psi]_J$ taken from Mitra and Ross[16] denotes the quark-spin–quark-orbital-momentum coupling to the total angular momentum J.

The various spatial wave functions are given in Table I, where $\vec{\lambda}$ and $\vec{\rho}$ are the relative coordinates

$$\vec{\rho}=\frac{1}{\sqrt{2}}(\vec{r}_1-\vec{r}_2),$$
$$\vec{\lambda}=\frac{1}{\sqrt{6}}(\vec{r}_1+\vec{r}_2-2\vec{r}_3), \qquad (2.17)$$

and the normalization is made with the respect to the measure

$$\int\prod_{i=1}^{3}d^3\vec{r}_i\,\delta\Big(\frac{1}{3}\sum_i\vec{r}_i\Big).$$

Let us now look for the nonzero matrix elements of V between the ground state and these various $N=2$ degenerate levels. To make the discussion simpler, one can use the overall symmetry of the baryon wave functions:

$$\langle\Psi_n|V|\Psi_0\rangle=3\langle\Psi_n|V_{12}|\Psi_0\rangle, \qquad (2.18)$$

where V_{12} is the potential between quarks 1 and 2, and the wave functions have been chosen to be either symmetric (label s or double prime) or antisymmetric (label a or prime) for the exchange of 1 and 2 in each separate contribution of the form $\phi\chi\psi$. It is then sufficient to look at the symmetry properties of V_{12} with respect to spin and space.

We get the following conclusions:

(i) The operator S^0 mixes the ground state and the multiplet $(56,0^+)_{N=2}$ for both the octet and the decuplet.

(ii) The spin-spin force S^{SS} mixes the octet with the $(56,0^+)_{N=2}$ and the $(70,0^+)_{N=2}$; on the other hand, the decuplet is mixed with the $(56,0^+)_{N=2}$ only.

TABLE I. Spatial wave functions of the $N=2$ levels. $\psi_0=(1/3\sqrt{3}\,R^6\pi^3)^{1/2}\exp[-(\vec{\rho}^{\,2}+\vec{\lambda}^2)/2R^2]$.

$$\psi^s(56,0^+)=\frac{1}{\sqrt{3}R^2}[3R^2-(\vec{\rho}^{\,2}+\vec{\lambda}^2)]\psi_0$$

$$\psi^s(56,2^+)=\left(\frac{8\pi}{15}\right)^{1/2}\frac{1}{R^2}[Y_2^M(\vec{\rho})+Y_2^M(\vec{\lambda})]\psi_0$$

$$\psi'(70,0^+)=\frac{1}{\sqrt{3}R^2}(-2\vec{\lambda}\cdot\vec{\rho})\psi_0$$

$$\psi''(70,0^+)=\frac{1}{\sqrt{3}R^2}(\vec{\lambda}^2-\vec{\rho}^{\,2})\psi_0$$

$$\psi'(70,2^+)=\left(\frac{8\pi}{3}\right)\frac{1}{R^2}\sum_m\langle 1,1;M-m,m|2,M\rangle\,Y_1^m(\vec{\rho})Y_1^{M-m}(\vec{\lambda})\psi_0$$

$$\psi''(70,2^+)=\left(\frac{8\pi}{15}\right)^{1/2}\frac{1}{R^2}[Y_2^M(\vec{\rho})-Y_2^M(\vec{\lambda})]\psi_0$$

$$\psi^a(20,1^+)=\left(\frac{8}{3}\right)^{1/2}\frac{1}{R^2}(\vec{\rho}\times\vec{\lambda})\psi_0$$

(iii) The tensor force S^T mixes the octet with the $(70, 2^+)_{N=2}$, and the decuplet with the $(56, 2^+)_{N=2}$ and the $(70, 2^+)_{N=2}$.

(iv) The spin-orbit force does not generate any mixing, and therefore we have the following mixing schemes:

$$\Psi(8, \tfrac{1}{2}^+) = |56, 0^+\rangle_{N=0} + \alpha\,|56, 0^+\rangle_{N=2} + \beta\,|70, 0^+\rangle_{N=2} + \epsilon\,|70, 2^+\rangle_{N=2}, \quad (2.19)$$

$$\Psi(10, \tfrac{3}{2}^+) = |56, 0^+\rangle_{N=0} + \gamma\,|56, 0^+\rangle_{N=2} + \delta\,|56, 2^+\rangle_{N=2} + \eta\,|70, 2^+\rangle_{N=2}, \quad (2.20)$$

where, of course, the multiplets must be projected on the respective SU(3) components.

Let us compare this with our own mixing scheme (1.1) and (1.2). The $(56, 0^+)_{N=2}$ which appears in (2.19) and (2.20) is irrelevant to the discussion of SU(6) breaking; it only slightly modifies the starting spatial wave function. The $(70, 0^+)_{N=2}$ in (2.19) is just what we have suggested on phenomenological grounds. The $(56, 2^+)_{N=2}$ in (2.20) was not considered in our approach–which was suggested by the nucleon structure functions. It is worth investigating the effect of such an additional mixing for the calculation of $\mu^*(\Delta \to N\gamma)$ and G^*.

We can now calculate the coefficient β, which comes from the spin-spin potential, through (2.1)

$$\beta = -\frac{1}{M_2 - M_0}\left\langle (70, 0^+)_{N=2}, 8, \tfrac{1}{2}^+ \middle| \sum_{i<j} V_{ij}^{SS} \middle| (56, 0^+)_{N=0}, 8, \tfrac{1}{2}^+ \right\rangle, \quad (2.21)$$

where M_0, M_2 are the ground-state and second-excited-level mean masses. We obtain then

$$\beta = \frac{1}{M_2 - M_0}\,\frac{2\pi\alpha_s}{3m^2}\,\langle \psi''_{N=2}|\delta^3(\vec{\rho})|\psi^s_{N=0}\rangle \quad (2.22)$$

using the spin matrix elements

$$\langle \chi'|\vec{S}_1\cdot\vec{S}_2|\chi'\rangle = -\tfrac{3}{4}, \qquad \langle \chi''|\vec{S}_1\cdot\vec{S}_2|\chi''\rangle = \tfrac{1}{4}. \quad (2.23)$$

With the wave funtion $\psi''_{N=2}$ given in Table I, we get

$$\langle \psi''_{N=2}|\delta^3(\vec{\rho})|\psi^s_{N=0}\rangle > 0, \quad (2.24)$$

and then $\beta > 0$ in contradiction with our phenomenological determination $\varphi < 0$. This latter sign was determined by the x behavior of the structure function $F_2^{en}(x)/F_2^{ep}(x)$; since x is the longitudinal-momentum fraction of a quark, we defined in Ref. 4 φ using momentum-space wave functions. To have consistent configuration-space wave functions, one must carefully perform the Fourier transform; in Table I, there corresponds to the combination $\tilde{\psi}''(70, 0^+) \sim (\vec{p}_\rho^{\,2} - \vec{p}_\lambda^{\,2})$ (Ref. 4) the combination

$$\psi''(70, 0^+) \sim (\vec{\lambda}^2 - \vec{\rho}^2).$$

The predicted positive sign of β comes from the positive $\vec{\lambda}^2$ term. On the other hand, the negative sign of φ was crucial in interpreting the *decrease* with x of $F_2^{en}(x)/F_2^{ep}(x)$. A positive sign would imply an *increase* of this ratio above $\frac{2}{3}$ for large x.

Concerning the magnitude of β, it can be related to the hyperfine splitting Δ -N which is caused by the same spin-spin force

$$|\beta| \simeq \frac{\Delta - N}{M_2 - M_0}.$$

More precisely, we have

$$\Delta - N = \frac{2\sqrt{2}\,\pi\alpha_s}{3m^2}\,\langle \psi^s_{N=0}|\delta^3(\vec{\rho})|\psi^s_{N=0}\rangle, \quad (2.25)$$

and therefore

$$\beta = \frac{\Delta - N}{M_2 - M_0}\,\frac{\sqrt{3}}{2\sqrt{2}}. \quad (2.26)$$

In conclusion, we see that the Breit-term potential indeed generates a mixing of the nucleon with the $(70, 0^+)_{N=2}$ level through the spin-spin force, but the sign of the mixing angle is opposite to the phenomenological determination from the nucleon structure functions.[16] This is a very serious problem. To escape this situation, there seem to be at least two possibilities: either we keep to the Breit-Fermi potential and look for weakness in our treatment, or we look for other spin-spin potentials.

III. DISCUSSION OF THE SPIN-SPIN POTENTIAL

Although the SU(6)-breaking scheme proposed by DGG is very appealing, there is some controversy in the literature about the statement that the spin dependence arises only from one-gluon exchange. Other conjectures have been stimulated by the failure of the ψ-η_c hyperfine splitting predicted by DGG,

$$\psi - \eta_c \simeq 27\ \text{MeV}.$$

This value is much too small as compared to the tentative experimental assignment $\eta_c(2800)$. Schnitzer[17] suggests substituting the Coulomb potential by the full linear plus Coulomb potential in the Breit-Fermi expression. This substitution gives a $1/r$ spin-spin potential. It could give a satisfactory splitting by giving up the usual radial-excitation assignment of the $\psi'(3700)$. Duncan[18] suggests still more complicated spin-spin effective interactions due to high-order gluon corrections. Some of them are phenomenologically included by Schnitzer,[19] in particular an anomalous magnetic-moment coupling of the gluon to quarks, which yields also $1/r$ spin-spin force. Without necessarily adopting their conclusions, we draw the conclusion that the nature of the spin-spin force is still obscure.

Therefore we look, from a purely empirical point of view, for the mixing effect and Δ-N splitting coming from various spin-spin potentials. In analogy with (2.26), we find the following:

(i) $1/r$ spin-spin potential

$$\beta = \frac{\Delta - N}{M_2 - M_0}\,\frac{1}{2\sqrt{6}}\,, \tag{3.1}$$

which is of the wrong sign as in (2.26).

(ii) On the contrary an increasing potential $|\vec{r}|^n$ yields

$$\beta = -\frac{\Delta - N}{M_2 - M_0}\,\frac{n}{2\sqrt{6}} < 0\,. \tag{3.2}$$

Thus we see that the spin-spin potentials suggested by gauge theories, either in $\delta^3(\vec{r})$ or $1/r$, yield a wrong sign for the mixing parameter. On the contrary, an increasing spin-spin potential gives a right sign and would give a correct order of magnitude for $2 \leqslant n \leqslant 4$.

However, independently of its lack of theoretical basis, such a spin-spin potential increasing with distance encounters very serious difficulties in explaining the $L = 1$ levels.

From (2.21) and $V_{12} = f(\vec{\rho})\vec{S}_1 \cdot \vec{S}_2$, one sees that $\beta < 0$ implies

$$\int_0^\infty \int_0^\infty \lambda^2 \rho^2 (\lambda^2 - \rho^2) f(\rho) \exp\left(-\frac{\lambda^2 + \rho^2}{R^2}\right) d\rho\, d\lambda < 0 \tag{3.3}$$

and then

$$\int_0^\infty \rho^4 f(\rho) \exp\left(-\frac{\rho^2}{R^2}\right) d\rho > \tfrac{3}{2} R^2 \int_0^\infty \rho^2 f(\rho) \exp\left(-\frac{\rho^2}{R^2}\right) d\rho\,. \tag{3.4}$$

This implies a hyperfine splitting for $L = 1$ levels, ${}^2 10$-${}^4 8$ at least of the same order of magnitude as for the ground state, in contradiction with experiment. Note that the right-hand side is positive in (3.4) to get Δ-$N > 0$.

On the contrary, the contact interaction is in fair agreement with the $L = 1$ hyperfine splitting both for mesons and baryons—it predicts a degeneracy between ${}^4 8$ and the ${}^2 10$ levels of the $(70, 1^-)$ multiplet and moreover ${}^2 8$ lower in mass.

Gromes and Stamatescu[14] have made a calculation with the $\delta^3(\vec{r})$ and $1/r$ spin-spin potentials. They find, respectively,

$${}^4 8 : {}^2 10 : {}^2 8 = 1 : 1 : -1\,,$$

$${}^4 8 : {}^2 10 : {}^2 8 = 5 : -3 : -5\,.$$

One sees that the $1/r$ hypothesis is already in a bad position. The increasing potential is still worse:

$${}^4 8 : {}^2 10 : {}^2 8 = (n+6) : -(3n+6) : -(n+6)\,.$$

We finally encounter two rather general difficulties:

(i) The contradiction between the hyperfine structure of the spectrum and the sign of the mixing angle.

(ii) For the spectrum itself, there is a contradiction between what is needed to explain the hyperfine splitting of the conventional hadrons (contact interaction) and the ψ-η_c splitting (dominant $1/r$ spin-spin potential).

The point (i) means that we cannot obtain an interpretation of our phenomenological mixing scheme in terms of a perturbation by color *two-body* forces.

IV. CRITICAL DISCUSSION OF THE CALCULATIONS OF THE Σ-Λ SPLITTING

We are now faced with the following situation:

(i) On the one hand, we have a phenomenological mixing scheme for the SU(6) breaking of the baryon wave function, which provides an explanation of a certain number of phenomena: the F_2^{en}/F_2^{ep} behavior for large x, the ratios μ^*/μ_p and G^*/G_A, and the Σ-Λ mass difference.[12] However, we cannot explain the origin of this mixing by a two-body (spin-spin) potential *a fortiori* by the contact Fermi term suggested by DGG. Then the problem of predicting the hyperfine structure of the hadron spectrum is left completely open in this scheme.

(ii) On the other hand, the Breit-Fermi potential proposed by DGG yields a number of encouraging results on the meson and baryon spectra, in particular the right sign and order of magnitude of the Σ-Λ splitting. However, DGG do not try to understand the SU(6) breaking in the ratios

$$\mu^*/\mu_p,\quad G^*/G_A,\quad F_2^{en}/F_2^{ep}\,.$$

One notices that there is a phenomenon where both approaches seem to meet: the explanation of Σ-Λ splitting. But it is rather paradoxical since the sign of β determined by the Fermi force is opposite to the phenomenological value in the scheme (i), and in this case the sign of Σ-Λ is given by the sign of β ($\beta<0$). What happens is that in the perturbation theory, the Σ-Λ splitting is a rather subtle phenomenon, involving first order both in the SU(3) breaking and a spin-dependent perturbation, i.e., it is an interference term in the second-order perturbation. As we shall see, there are two different contributions of the same order: one (I) is due to the variation of the wave function with the SU(3) mass breaking in the kinetic energy, and the second (II) is due to the variation of the spin-dependent potential with the SU(3) mass breaking. The two approaches neglect either (I) or (II). In our scheme, we start from an SU(6)-broken but SU(3)-symmetric wave function and we calculate the contribution (I). The contribution (II) is not present in our approach, by assumption. DGG start from an SU(6)-symmetric wave function and they calculate the contribution (II). They could also calculate the contribution (I). We have done this calculation and we find indeed for (I) a *negative* contribution to the Σ-Λ splitting, instead of a *positve* one in our scheme. But this contribution (I) turns out to be smaller than (II). Then both approaches yield a positive sign for Σ-Λ.

A. Nonrelativistic perturbation theory

Let us set this discussion in a quantitative manner. We consider the Hamiltonian in its nonrelativistic form:

$$H=\sum_i T_i(m_i)+\sum_{i<j}U_{ij}+\sum_{i<j}V_{ij}(m_i,m_j)\,, \qquad (4.1)$$

where we make explicit the mass dependence, crucial in the discussion of the Σ-Λ splitting. $T_i(m_i)$ is the kinetic energy of the ith quark; U_{ij} is an SU(6)-invariant potential which represents the main part of the potential; in our approach it is the harmonic-oscillator potential; in DGG it is the *SU(6)-invariant confining force*, whose form is left undetermined; V_{ij} is an eventually mass-dependent SU(6)-breaking force; in DGG it is just the Breit-Fermi potential, and the mass dependence comes only from the reduction to Pauli spinors. As for us, we have assumed an SU(6)-breaking force independent of the quark masses without any theoretical prejudice.

The unperturbed Hamiltonian is defined to be

$$H_0=\sum_i T_i(m_{i0})+\sum_{i<j}U_{ij}\,, \qquad (4.2)$$

where $m_{10}=m_{20}=m_{30}$ and the corresponding eigenfunctions are denoted by $\psi_n^{(0)}$, and we consider now the effect of introducing the potential and introducing a small change in the quark masses $m_{i0}\to m_{i0}+\delta m_i$ [in fact, we break SU(3) by making $m_\lambda\neq m_{\mathcal{P}}=m_{\mathcal{N}}$]. Then the perturbation is given by

$$\begin{aligned}W&\equiv\sum_i\delta T_i(m_i)+\sum_{i<j}V_{ij}(m_i,m_j)\\&=\sum_i\delta T_i(m_i)+\sum_{i<j}V_{ij}(m_{i0},m_{j0})\\&\quad+\sum_{i<j}\delta V_{ij}(m_i,m_j)\,,\end{aligned} \qquad (4.3)$$

where δ denotes the variation with respect to the mass change

$$m_{i0}\to m_{i0}+\delta m_i\,.$$

In W the first term is first order in SU(3) breaking, the second is first order in the spin-dependent force, the third is first order in both. The energy perturbation to the (overall) second order is denoted by

$$\Delta E=\langle\psi_0^{(0)}|W|\psi_0^{(0)}\rangle+\sum_{n\neq0}\frac{|\langle\psi_0^{(0)}|W|\psi_n^{(0)}\rangle|^2}{E_0-E_n}\,. \qquad (4.4)$$

From (4.4) and by using (4.3) we pick up the terms of first order both in the SU(3) breaking and in the spin-dependent potential, which are those contributing to the Σ-Λ splitting:

$$\begin{aligned}(\Delta E)_{\Sigma\text{-}\Lambda}&=\langle\psi_0^{(0)}|\sum_{i<j}\delta V_{ij}(m_i,m_j)|\psi_0^{(0)}\rangle\\&\quad+2\sum_{n\neq0}\frac{\langle\psi_0^{(0)}|\sum_{i<j}V_{ij}(m_{i0},m_{j0})|\psi_n^{(0)}\rangle\langle\psi_n^{(0)}|\sum_i\delta T_i(m_i)|\psi_0^{(0)}\rangle}{E_0-E_n}\,.\end{aligned} \qquad (4.5)$$

In the frame of DGG, it is easy to see that the two terms in (4.5) have indeed the same order of magnitude, say $\alpha_s\delta m/m$, where $\delta m/m$ is the strength of the SU(3) breaking and α_s is the strength of the spin-dependent potential. In the frame of the harmonic oscillator, we have the following estimates:

$$T(m)\sim\vec{p}^{\,2}/2m\sim1/mR^2\sim\omega\,,$$

$$\delta T(m)\sim\frac{\delta m}{m}\omega\,,\quad V(m)\sim\alpha_s\,,$$

$$\delta V(m)\sim\frac{\delta m}{m}\alpha_s\,,$$

$$E_0-E_2\sim\omega\,.$$

The first term corresponds to what we have called contribution (II), the only one calculated by DGG. It is not present in our approach. The second term is not considered by DGG, although it has exactly the same order. It corresponds to the variation of the original wave function due to the SU(3) breaking in the kinetic energy

$$\delta|\psi_0^{(0)}\rangle=\sum_{M\neq 0}\frac{|\psi_n^{(0)}\rangle\langle\psi_n^{(0)}|\sum_i\delta T_i(m_i)|\psi_0^{(0)}\rangle}{E_0-E_n}\,. \quad (4.6)$$

This is what we have called contribution (I). Then

$$\begin{aligned}(\Delta E)_{\Sigma-\Lambda}&=\langle\psi_0^{(0)}|\sum_{i<j}\delta V_{ij}(m_i,m_j)|\psi_0^{(0)}\rangle\\&\quad+2\langle\delta\psi_0^{(0)}|\sum_{i<j}V_{ij}(m_{i0},m_{j0})|\psi_0^{(0)}\rangle\\&=(\mathrm{II})+(\mathrm{I})\,. \end{aligned}\quad (4.7)$$

Equation (4.7) shows that $(\Delta E)_{\Sigma-\Lambda}$ comes from the *total* variation of

$$\langle\psi_0^{(0)}|\sum_{i<j}V_{ij}(m_i,m_j)|\psi_0^{(0)}\rangle \quad (4.8)$$

with respect to the mass change $m_{i0}\to m_{i0}+\delta_{mi}$. On the other hand (I) can also be written in the form

$$2\langle\psi_0^{(1)}\Big|\sum_i\delta T_i(m_i)\Big|\psi_0^{(0)}\rangle \quad (4.9)$$

with

$$|\psi_0^{(1)}\rangle=\sum_{M\neq 0}\frac{|\psi_n^{(0)}\rangle\langle\psi_n^{(0)}|\sum_{i<j}V_{ij}(m_{i0},m_{j0})|\psi_0^{(0)}\rangle}{E_0-E_n}\,, \quad (4.10)$$

as can be seen from the second term of (4.5).

One recognizes in (4.9) our calculation of the Σ-Λ splitting, provided that the mixing is truly generated by the spin-dependent and SU(3)-symmetric potential $\sum_{i<j}V_{ij}(m_{i0},m_{j0})$. The paradox raised above is then solved in the following way. If V_{ij} is the Breit-Fermi potential, then (II) > 0 according to DGG and (I) < 0 since $\beta>0$ according to (2.26) (we assume harmonic-oscillator wave functions and we retain only $N=2$ levels). On the contrary, with the phenomenological value of β, $\beta<0$, we get (I) > 0 and then $(\Delta E)_{\Sigma-\Lambda}>0$ in both approaches. It remains now to compute quantitatively (I) in the case of a Breit-Fermi potential, using (2.26), to compare the relative weights of (II) and (I). In the Appendix, we solve exactly the problem of three particles of unequal masses interacting through a harmonic-oscillator potential. Here we give only the approximate results:

$$(\Sigma-\Lambda)_{(\mathrm{I})}=-\tfrac{1}{4}(\Delta-N)(1-m_\varphi/m_\lambda)\,, \quad (4.11)$$

while

$$(\Sigma-\Lambda)_{(\mathrm{II})}=\tfrac{2}{3}(\Delta-N)(1-m_\varphi/m_\lambda) \quad (4.12)$$

according to DGG. The total contribution is still positive,

$$\Sigma-\Lambda=\tfrac{5}{12}(\Delta-N)(1-m_\varphi/m_\lambda)\,, \quad (4.13)$$

and still of the right order of magnitude,

$$\Sigma-\Lambda\simeq 50\text{ MeV}\,, \quad (4.14)$$

using the DGG values for m_φ and m_λ.

B. Relativistic perturbation theory

We have explained the difference between our approach and that of DGG for the Σ-Λ splitting by showing that they correspond to two different contributions in a nonrelativistic perturbation theory. The role of these two contributions appears in a very transparent way if we treat the problem in the frame of relativistic quantum mechanics with two-body forces. This is the natural frame for one-gluon-exchange forces as considered by DGG. As for us, we do not know if our phenomenological mixing can be explained in such a frame. (See the end of this subsection and also Sec. III.)

We now write the Hamiltonian in the form

$$H_D=H_{D0}+V_D\,, \quad (4.15)$$

where H_{D0} includes the free-particle Dirac Hamiltonians plus the confining potential, and V_D represents the SU(6)-breaking force which dominates the small distance. In its relativistic form, V_D will only be a combination of γ matrices acting on pairs of quarks,

$$V_D=\sum_{i<j}V_{Dij}\,.$$

For instance, for DGG $V_{Dij}\sim\vec\alpha_i\cdot\vec\alpha_j$. It thus may not depend explicitly on the quark masses. The dependence on the masses displayed in (4.1) then *comes only of the reduction from Dirac to Pauli spinors.*

We then introduce the SU(3) breaking in the quark masses:

$$m_{i0}\to m_{i0}+\delta m_{i0}\,.$$

This gives rise to a second perturbation:

$$V_{\mathrm{su}(3)}=\sum_i\beta_i\delta m_i\,. \quad (4.16)$$

The second-order perturbation on the energy, of first order in both V_D and $V_{\mathrm{su}(3)}$, which is responsible for the Σ-Λ splitting, is denoted by

$$(\Delta E)^D_{\Sigma-\Lambda}=2\sum_{n\neq 0}\frac{\langle\psi_{D0}^{(0)}|\sum_{i<j}V_{Dij}|\psi_{Dn}^{(0)}\rangle\langle\psi_{Dn}^{(0)}|V_{\mathrm{su}(3)}|\psi_{D0}^{(0)}\rangle}{E_0-E_n}\,, \quad (4.17)$$

to be compared with the nonrelativistic expression (4.5). The first contribution in (4.5) is now in-

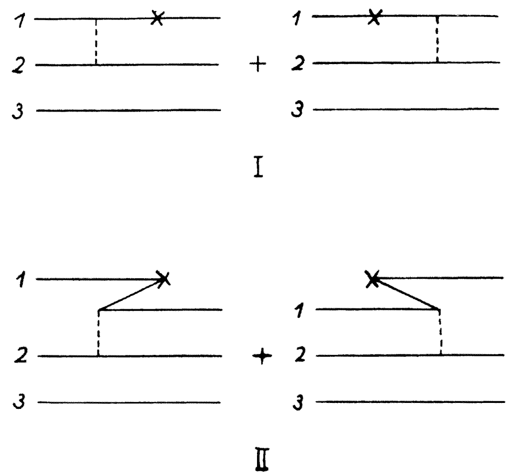

FIG. 1. The direct (I) and Z-graph (II) contributions to the Σ-Λ mass difference. The cross represents the SU(3) breaking $\beta\delta m$, and the dashed line the spin-dependent potential V_D, as described in the text.

cluded in the second-order relativistic perturbation. What happens is that now the $\{\psi^{(0)}_{Dn}\}$ are a complete set of solutions of the *relativistic* Hamiltonian H_{D0}. In terms of old-fashioned perturbation theory diagrams, (4.17) corresponds to two different types of diagrams [Fig. 1, (I) and (II)], which represent, respectively, the two contributions mentioned above.

If we calculate the graph of Fig. 1 (I) by inserting as intermediate state only the nearest excited level of the baryons, we just get the contribution (I) as defined by the second term in (4.5), once we make an expansion of Dirac spinors in terms of Pauli spinors up to order v^2/c^2: $V_{Dij}\sum_i\beta_i\delta m_i$ leads to $\sum_i\delta T_i(m_i)$ (see our paper in Ref. 12).

Let us now consider the graph of Fig. 1 (II). We do not know a complete set of eigenvectors of H_{D0}. Let us try then to approximate the intermediate states by free-quark states, *one with negative energy*. We select in (4.17) the term

$$2\sum_{n\neq 0}\frac{\langle\psi^{(0)}_{D0}|V_{D12}|\psi^{(0)}_{Dn}\rangle\langle\psi^{(0)}_{Dn}|\beta_1\delta m_1|\psi^{(0)}_{D0}\rangle}{E_0-E_n} \tag{4.18}$$

with

$$V_{D12}\sim\vec{\alpha}_1\cdot\vec{\alpha}_2 \tag{4.19}$$

in the case of the Breit-Fermi potential. In the approximation of free-quark intermediate states and, neglecting the kinetic energy,

$$E_0-E_n\simeq 3m-(2m-m)=2m\,. \tag{4.20}$$

Then (4.18) can be written

$$\frac{1}{2m}2\langle\psi^{(0)}_{D0}|V_{D12}\Big(\sum_{n\neq 0}|\psi^{(0)}_{Dn}\rangle\langle\psi^{(0)}_{Dn}|\Big)_{\text{free}}\beta_1\delta m_1|\psi^{(0)}_{D0}\rangle \tag{4.21}$$

with

$$\Big(\sum_{n\neq 0}|\psi^{(0)}_{Dn}\rangle\langle\psi^{(0)}_{Dn}|\Big)_{\text{free}}=\Lambda_-^{(1)}(\vec{p}_1)\otimes\Lambda_+^{(2)}(\vec{p}_2)\otimes\Lambda_+^{(3)}(\vec{p}_3) \tag{4.22}$$

and with

$$\Lambda_\pm(\vec{p})=\frac{|E|\pm(\vec{\alpha}\cdot\vec{p}+\beta m)}{2|E|}\,. \tag{4.23}$$

Let us recall that the relativistic wave functions $\psi^{(0)}_{D0}$ are assumed to have the ordinary SU(6) form with Pauli spinors substituted by free Dirac spinors $u_i(\vec{p}_i)$. The effect of the Λ_+'s is trivial. On the other hand,

$$(\Lambda_-^{(1)}(\vec{p}_1)\beta_1)\,u_1=\frac{\beta_1|E|-m}{|E|}\,u_1\simeq 2m_1\frac{\delta u_1}{\delta m_1} \tag{4.24}$$

neglecting $\vec{p}_1^{\,2}/m_1^{\,2}$ terms. Then (4.21) becomes

$$2\delta m_1\langle\psi^{(0)}_{D0}|V_{D12}|\frac{\delta}{\delta m_1}\psi^{(0)}_{D0}\rangle \tag{4.25}$$

In (4.25) the variation refers only to the Dirac spinors and *not* to the spatial wave function. (4.25) can still be transformed to

$$\delta m_1\frac{\delta}{\delta m_1}\langle\psi^{(0)}_{D0}|V_{D12}|\psi^{(0)}_{D0}\rangle\,. \tag{4.26}$$

Reducing $\langle\psi^{(0)}_{D0}|V_{D12}|\psi^{(0)}_{D0}\rangle$ to Pauli spinors up to order v^2/c^2, one just obtains

$$\left\langle\psi^{(0)}_0\left|\delta m_1\frac{\delta V_{12}}{\delta m_1}\right|\psi^{(0)}_0\right\rangle. \tag{4.27}$$

In the case of the Σ-Λ splitting, the only effective term is the Fermi contact term. *This is just the term calculated by DGG.*

V. CONCLUSION

Our general conclusion would be that the status of dynamical SU(6) breaking is still unclear. By dynamical we mean the breaking of the spectrum and of the wave functions and *not* the so-called SU(6)$_W$ breaking in strong decays and current matrix elements which is by now rather well understood.[20]

In the latter, one has to not assume a breaking of the spectrum of SU(6) wave functions, but rather to take into account the kinematical effect of internal quark motion.

The Breit-Fermi potential of DGG seems to account fairly for the gross features of the spectrum, even if some difficulties appear on the phenomenological side (ψ-η_c splitting),[17] or on the theoretical side.[17-19,21] The sign of the Δ-N and Σ-Λ splittings appear quite naturally. Moreover, the short range of the spin-spin force is welcomed for the excited levels.

On the other hand, phenomenological mixing schemes as ours, at $\vec{P}=0$, describing at the same time deep-inelastic structure functions and static properties of the octet and decuplet baryon ground state, or the one of Cabibbo and collaborators at $P_z \to \infty$, which relates the behavior of $F_2^{en}(x)/F_2^{ep}(x)$ to the Σ-Λ mass difference, also meet fair successes.[22] Unhappily, there appears to be a contradiction between the mixing induced by the DGG *spin-spin* force, and the one found empirically. A similar contradiction has been recently found by Barbieri, Gatto, and Kunszt[23] for the Q_1-Q_2 mixing: the sign of the mixing angle was found opposite to the one induced by the DGG *spin-orbit* force. No simple way has been found to maintain at the same time the explanation of the hyperfine splittings and the description of other SU(6)-breaking phenomena by configuration mixing. In particular, a simple modification of the spatial dependence of the spin-spin force, while giving a good sign for the mixing angle and the Δ-N splitting, will fail for the hyperfine splittings of excited levels.

Apart from this main conclusion of our paper, there is an interesting byproduct of the discussion of the Σ-Λ splitting. In the frame of a chromodynamic SU(6)-breaking scheme, we have found a negative counterterm to the DGG calculation of Σ-Λ, which finally comes out to be half the value quoted by DGG. This smaller value of Σ-Λ is the one found by the MIT bag model,[24] which also finds Σ-Λ too small by a factor $\frac{1}{2}$, having taken into account the full variation of the wave function with δm_λ. Also, our discussion emphasizes the importance of Z-graph contributions in gluon-exchange corrections: the Z graphs may indeed be more important than direct contributions.

APPENDIX

To see directly the SU(3)-breaking effect *on the spatial wave function* (the term omitted by DGG in the calculation of the Σ-Λ splitting), let us solve here the problem of three particles of *unequal masses* interacting through a *SU(3)-symmetric confining force* (as assumed by DGG) plus a perturbation: *a mass-dependent Breit-Fermi potential*. We take as the confining potential a two-body harmonic oscillator.

Consider three particles, two equal masses $m_1 = m_2 = m$, and a mass $m_3 \neq m$. The total Hamiltonian can be written as

$$H = H_0 + V, \tag{A1}$$

$$H_0 = \frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} + \frac{\vec{p}_3^{\,2}}{2m_3} + \tfrac{1}{2}\kappa\omega^2 \sum_{i<j} (\vec{r}_i - \vec{r}_j)^2, \tag{A2}$$

$$V = -\tfrac{2}{3}\alpha_s \sum_{i<j} S_{ij}, \tag{A3}$$

where S_{ij} is of the Breit-Fermi form,

$$S_{12} = X_{12} + Y_{12}, \tag{A4}$$

where we have separated the Fermi contact spin-spin term

$$X_{12} = -\frac{8\pi}{3}\delta(\vec{r}_{12})\frac{1}{m_1 m_2}\vec{S}_1\cdot\vec{S}_2, \tag{A5}$$

which contributes to the Δ-N and Σ-Λ splittings.

In (A2), $\kappa\omega^2$ is an SU(3)-symmetric (mass-independent) constant.

Defining the center of mass and relative coordinates and momenta,

$$\begin{aligned} \vec{R} &= \frac{m\vec{r}_1 + m\vec{r}_2 + m_3\vec{r}_3}{2m+m_3}, \\ \vec{\rho} &= \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \\ \vec{\lambda} &= \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3), \end{aligned} \tag{A6}$$

$$\begin{aligned} \vec{P} &= \vec{p}_1 + \vec{p}_2 + \vec{p}_3, \\ \vec{p}_\rho &= \frac{1}{\sqrt{2}}(\vec{p}_1 - \vec{p}_2), \\ \vec{p}_\lambda &= \frac{\sqrt{6}}{2}\left[\frac{m_3}{2m+m_3}(\vec{p}_1+\vec{p}_2) - \frac{2m}{2m+m_3}\vec{p}_3\right], \end{aligned} \tag{A7}$$

we get

$$\sum_{i<j}(\vec{r}_i - \vec{r}_j)^2 = 3(\vec{\rho}^{\,2} + \vec{\lambda}^2), \tag{A8}$$

$$\frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} + \frac{\vec{p}_3^{\,2}}{2m_3} = \frac{\vec{P}^2}{2(2m+m_3)} + \frac{\vec{p}_\rho^{\,2}}{2m} + \frac{2m+m_3}{6mm_3}\vec{p}_\lambda^{\,2}. \tag{A9}$$

We see that, although the harmonic-oscillator force is SU(3) symmetric, the unperturbed wave function will be dependent on the mass breaking $m_\lambda - m_\rho$, since the kinetic energy term is mass dependent.

The ground-state spatial wave function will be of the form

$$\psi_0 = N_0 \exp\left[-\left(\frac{\vec{\rho}^{\,2}}{2R_\rho^{\,2}} + \frac{\vec{\lambda}^2}{2R_\lambda^{\,2}}\right)\right], \tag{A10}$$

where, since the harmonic-oscillator force is SU(3) invariant, R_ρ and R_λ satisfy the relation

$$mR_\rho^{\,4} = \frac{3mm_3}{2m+m_3}R_\lambda^{\,4}, \tag{A11}$$

so that

$$\frac{R_\lambda^{\,2}}{R_\rho^{\,2}} = \left(\frac{2m+m_3}{3m_3}\right)^{1/2} \tag{A12}$$

Let us compute the Δ-N and Σ-Λ splittings using the perturbation V and the unperturbed mass-dependent wave functions (A10). We obtain

$$\Sigma-\Lambda=\tfrac{1}{3}(y_1-y_2)(-\tfrac{2}{3}\alpha_s)\,,\qquad \Delta-N=\tfrac{1}{2}y_3(-\tfrac{2}{3}\alpha_s)\,,\tag{A13}$$

where

$$y_1=-\frac{8\pi}{3}\frac{1}{m^2}I(R_\lambda=\alpha R_\rho)\,,\qquad y_2=-\frac{8\pi}{3}\frac{1}{mm_\lambda}J(R_\lambda=\alpha R_\rho)\,,\qquad y_3=-\frac{8\pi}{3}\frac{1}{m^2}I(R_\lambda=R_\rho)\,,\tag{A14}$$

where, from (A12),

$$\alpha=\left(\frac{2m+m_\lambda}{3m_\lambda}\right)^{1/4},\tag{A15}$$

and I and J are the spatial integrals

$$I=\langle\psi_0|\delta(\vec{r}_1-\vec{r}_2)|\psi_0\rangle=\frac{1}{(2\pi)^{3/2}}R_\rho^{-3}\,,\tag{A16}$$

$$J=\langle\psi_0|\delta(\vec{r}_1-\vec{r}_3)|\psi_0\rangle=\frac{1}{(2\pi)^{3/2}}R_\rho^{-3}\left(\frac{1}{4}+\frac{3}{4}\frac{R_\lambda^2}{R_\rho^2}\right)^{-3/2}\tag{A17}$$

Finally we obtain

$$\Sigma-\Lambda=\tfrac{2}{3}(\Delta-N)\left\{1-\frac{m}{m_\lambda}\left[\frac{1}{4}+\frac{3}{4}\left(\frac{2m+m_\lambda}{3m_\lambda}\right)^{1/2}\right]^{-3/2}\right\}\tag{A18}$$

instead of the expression found by DGG,

$$(\Sigma-\Lambda)_{\rm DGG}=\tfrac{2}{3}(\Delta-N)\left(1-\frac{m}{m_\lambda}\right).\tag{A19}$$

We obtain, approximately, expanding (A17) up to first order in powers of $\delta m/m$,

$$(\Sigma-\Lambda)\simeq\tfrac{5}{8}(\Sigma-\Lambda)_{\rm DGG}\,.\tag{A20}$$

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The same intuitive argument also leads to an excess of u quarks in the central region of the *neutron* in the presence of the chromodynamic force, and therefore to a *negative* neutron charge radius, as noted by R. D. Carlitz, S. D. Ellis, and R. Savit, [Phys. Lett. 68B, 443 (1977)], and this time in agreement with experiment—contrary to our mixing with $\varphi<0$ [see the erratum to Ref. 4, Phys. Rev. D 13, 1519 (E) (1976)]. This illustrates a rather general inability of simple three-quark models to predict ***at the same time*** the sign of $u-2d$ in the valence-quark region and the sign of $\langle r^2\rangle_n$, as we shall show in a forthcoming comment.

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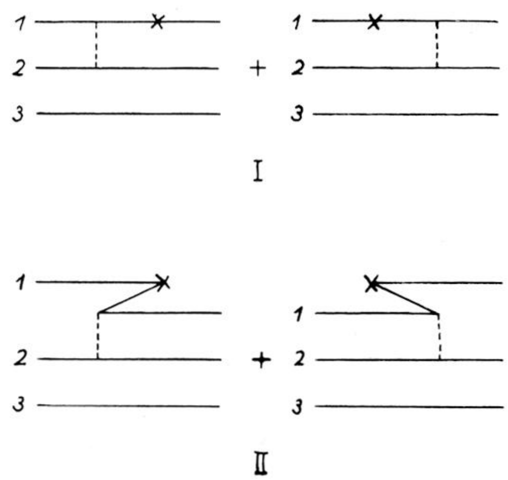

FIG. 1. The direct (I) and Z-graph (II) contributions to the Σ-Λ mass difference. The cross represents the SU(3) breaking $\beta\delta m$, and the dashed line the spin-dependent potential V_D, as described in the text.