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# Even-wave harmonic-oscillator theory of baryonic states. IV. Structure functions versus slope of neutron charge form factor

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The structure of a mixed nucleon facilitated by the even-wave harmonic-oscillator model, which was shown to be quite successful in explaining certain important low-energy parameters such as $G_A/G_V$, $\Delta \to N\pi$ width, $G_{NN\pi}$, and a few hitherto unresolved cases of baryon photocouplings, is employed, without extra parameters, to study the shape of the $n,p$ structure functions as well as the neutron charge radius squared ($\langle r_n{}^2\rangle$). A mixing angle close to the ideal ($\cot\theta = \sqrt{2}$) (determined from earlier investigations) is found to give a good account of the ratio $R^{np}$ ($= F_2^{en}/F_2^{ep}$) except for small values of the scaling variable ($z$) as well as of the difference $\Delta^{pn}$ ($= F_2^{ep} - F_2^{en}$) over the entire range of $z$. However, the same angle ($\theta > 0$) gives the wrong sign for $\langle r_n{}^2\rangle$. The incompatibility of the sign of $\langle r_n{}^2\rangle$ with the behavior of $R^{np}$, as found by A. Le Yaouanc *et al.*, thus remains unresolved in this simplest version of the even-wave model without further assumptions.

## I. INTRODUCTION

A mixed nucleon is required on several independent counts. In the static limit (small $q^2$) the most obvious need arises from the problem of mutual compatibility of $G_A/G_V$, the $\Delta \to N\pi$ width, and the $NN\pi$ coupling constant. Photoproduction[1] of some difficult cases, such as the Roper resonance, also leads to a similar conclusion.

The simplest but nontrivial candidates for mixing at the constituent (strong) SU(6) level are the $\underline{8}_d$ members of $\underline{56}$ and $\underline{70}$, so that

$$|N\rangle_{\text{phy}} = |56\rangle_N \cos\theta + |70\rangle_N \sin\theta\,. \qquad (1)$$

This formally breaks "strong" SU(6), but such breaking must be distinguished from breaking due to the relativistic motion of quarks inside the nucleon.[2] The latter apparently can be related[3] to the effect of SU(6)-current mixing[4] which had been considered in the mid 1960's, but which was subsequently found by Melosh[5] to be basically compatible with almost pure states in terms of the SU(6) strong classification. Most of the data in the low-$q^2$ limit, such as noted above, do not determine the sign of the mixing angle. On the other hand, there are phenomena, viz., the slope $b$ of the neutron charge form factor ($G_E^n$) as measured in the limit $q^2 \to 0$, and the ratio $R^{np}(= F_2^{en}/F_2^{ep})$ of the neutron and proton structure functions measured as functions of the scale variable $z(= q^2/2m\nu)$ in deep-inelastic scattering, which are sensitive to the sign as well as the magnitude of the angle $\theta$. The unmixed SU(6) nucleon wave function when used in the conventional harmonic-oscillator (h.o.) models (relativistic[6] or nonrelativistic[7]) predicts $b$ to be zero. Even when the internal motion of quarks is taken into account, no significant departure from the value $b = 0$ seems to occur.[8,9]

Therefore, a significant value of $b$ provides a fairly unambiguous signal for a mixed nucleon through SU(6) breaking,[10] with a sensitive dependence of $b$ on the sign of the mixing angle $\theta$. For the ratio $R^{np}$, the *unmixed* nucleon (with or without internal quark motion) predicts $R^{np} \geq \frac{2}{3}$.[11-13] Experimentally,[14] $R^{np}$ decreases monotonically with the variable $x$ from $R^{np} \simeq 1$ at $z = 0$ to $R \simeq \frac{1}{3}$ at $z = 0.8$. Therefore, one is led to ask the interesting question as to whether a strong SU(6) symmetry breaking can bring about an understanding of these diverse features.

Many attempts have been made to explain the departure of $R^{np}$ from its SU(6) value of $\frac{2}{3}$. Close[15,16] considered two possible mechanisms, (i) a breaking of isospin symmetry[15] which allows one of the two mixed symmetric isospin wave functions in the ($\underline{56}$, $L = 0$) multiplet to dominate the other in the deep-inelastic region as $z \to 1$, and (ii) a chiral-configuration-mixing effect[16] brought about by the Melosh transformation.[5] Altarelli *et al.*,[17] on the other hand, considered a ($\underline{56}, 0^+$) and ($\underline{70}, 1^-$) parity mixing in the $P_z = \infty$ frame, and found a five-parameter fit (including the angle $\theta$) to the deep-inelastic data, including gluons and $q\bar{q}$ pair contributions. However, these authors apparently were not interested in the low-energy data, in particular the parameter $b$.

Le Yaouanc *et al.*[9] have made the first detailed attempt to understand both $b$ and $R^{np}$ in terms of a mixed nucleon, by using the conventional framework of the h.o. model for the calculation of the structure functions.[11,12] They have developed an explicit formalism which relates the $qqq$ wave functions in the nucleon rest frame to one in the frame $P_z = \infty$ via the Licht-Pagnamenta transformation[18] and thus have given a concrete realization of the Bjorken scaling phenomenon[19] within the basic tenets of the simple quark model. For SU(6) mixing, they considered the most obvious choice within the h.o. framework, viz., $(\underline{56}, 0^+)$ and $(\underline{70}, 0^+)$ of $N=0$ and 2, respectively. However, they introduced some extra parameters to simulate the effects of internal quark motion. They were successful on almost all counts but could not get the correct sign for the slope parameter $b$[20] of the neutron charge form factor $G_E^n$.

The purpose of this paper is to examine the twofold issue of $R^{np}$ versus the sign of the neutron charge radius squared in the even-wave h.o. model,[21,22] in view of the notable success of the latter in understanding several baryonic phenomena from the mass spectrum[21,22] to photocouplings[1] and pseudoscalar couplings.[23] The natural facility of a mixed nucleon that the even-wave model provides[24,25] comes from the prediction of a low-lying $(\underline{70}, 0^+)$ as the new ground state of the $\underline{70}$ spectrum, which seems to be the most natural counterpart of the $(\underline{70}, 0^+)_{N=2}$ state in the usual h.o. model, appropriately considered in Ref. 9. Such a breaking of SU(6) provides a simultaneous understanding of several low-energy parameters, especially $G_A/G_V$, $\Delta \to N\pi$ width, and the $NN\pi$ coupling constant,[24] as well as certain "difficult" cases of photocouplings among baryonic resonances.[1] The latter phenomena are insensitive to the sign of the mixing angle, which is now hoped to be determined through the shape of $R^{np}$ and the parameter $b$.

Now the vastly different orbital structure of $(\underline{70}, 0^+)$ in the even-wave h.o. model from its full-wave counterpart would *a priori* seem to hold out fresh prospects of reconciliation of the shape of $R^{np}$ and sign of the $\langle r_n{}^2\rangle$. However, as will be seen further below, there is a reason to believe that the sign discrepancy found in Refs. 9 and 20 may be a more general feature than would appear to be the case because of its derivation in the usual h.o. model. The clue to the sign anomaly lies in the phase change that occurs between the momentum and coordinate representations of the $(\underline{70}, 0^+)_{N=2}$ wave function in the usual h.o. model.[9] In the even-wave model, it would appear off-hand that the quantity $\lambda$ has only a passive role in the corresponding wave function[22,24] and hence would not have to carry the burden of a phase change between the coordinate and momentum-space representations. However, a fairly general theorem,[26] which makes use of the Gaussian structure of the $\underline{56}$ wave function, indicates the result

$$\langle r_n{}^2\rangle = -R^4 \langle p_n{}^2\rangle\,, \tag{2}$$

where the expectation values on both sides are taken between $\psi^s$ and $\psi''$ in the appropriate coordinate system. Therefore, in order to conform to this theorem it would be necessary to take account of the phase change between the coordinate $(x, y)$ and the momentum $(p, q)$ representations of the $(\underline{70}, 0^+)$ wave functions.[24] [A direct calculation with $\lambda$ expressed in terms of $\xi, \eta$ variables, namely $\tan\lambda = 2\vec{\xi}\cdot\vec{\eta}/(\xi^2-\eta^2)$, also leads to the same conclusion.] Thus in the even-wave model too the above-mentioned theorem seems to ensure that the problem of the sign anomaly is just as serious as in the usual h.o. model. On the other hand, the results for $R^{np}$ and $\langle r_n{}^2\rangle$ in the even-wave model would appear to be worth recording physically because of its several other known successes,[1,23,24] and mathematically because of the different nature of its dynamical assumptions.

For the calculations to be presented in Secs. II and III we use an invariant volume element $d\tau$ which takes account of the constraints on the variables $x_i, y_i$ which had been neglected earlier.[22,24] However, the more refined volume element used here affects the earlier calculations only marginally, while it is crucial for the present investigation. This derivation is summarized in the Appendix. In Sec. II we sketch the derivation of the neutron charge form factor in the even-wave model, drawing attention to the correction needed on the overlap integrals described in paper II as a result of the above constraints. Section III describes the corresponding calculations for $R^{np}$ but without the inclusion of gluons and $q\bar{q}$ pairs. Section IV gives a discussion of the present results in relation to the above theorem as well as a summary of our conclusions.

## II. CHARGE FORM FACTOR OF THE NEUTRON

The evaluation of orbital matrix elements in the even-wave model within a sort of "zero-order approximation" which gives rise to certain orbital selection rules has been described in paper II. The selection rule of immediate relevance to the present calculation, viz., $\langle\psi^s, \psi''\rangle = 0$, for a $(\underline{70}, 0^+) \to (\underline{56}, 0^+)$ transition is, however, based on the assumed independence of the variables $(x_i, y_i)$ defined by[22]

$$x_i = \xi_i \cos\tfrac{1}{2}\lambda + \eta_i \sin\tfrac{1}{2}\lambda\,, \quad y_i = \xi_i \sin\tfrac{1}{2}\lambda - \eta_i \cos\tfrac{1}{2}\lambda\,, \tag{3}$$

$$\vec{x}\cdot\vec{y} = 0\,, \quad x^2 \geq y^2\,, \tag{4}$$

as well as a hidden character of the cyclic variable $\lambda$. This selection rule is adequate for most "coarse-grained" quantities such as $G_A/G_V$, pionic widths, etc., which have been considered in papers II and III, but not for the "fine-grained" quantities under the present investigation which depend crucially on a nonvanishing value of $\langle\psi^s, \psi''\rangle$. This can be achieved within our formalism by incorporating the constraints implied by Eq. (4) on the invariant volume element which now reads as (see appendix)

$$d\tau = d^3x\, d^3y\, d\lambda\, \delta(2\vec{x}\cdot\vec{y})\theta(x^2-y^2)(x^2-y^2). \tag{5}$$

Accordingly, the wave functions in the coordinate space are

$$\psi^s = N_s e^{-(x^2+y^2)/2}, \tag{6}$$

$$\psi'' = N_0[\tfrac{1}{2}\cos\lambda(e^{-(x^2+\sqrt{3}y^2)/4} + e^{-(\sqrt{3}x^2+y^2)/4}) - \tfrac{1}{2}\cos 2\lambda(e^{-(x^2+\sqrt{3}y^2)/4} - e^{-(\sqrt{3}x^2+y^2)/4})], \tag{7}$$

where

$$N_s^{-2} = \tfrac{1}{2}\pi^3, \quad N_0^{-2} = \tfrac{32}{3}\pi^3(2-\sqrt{3}).$$

The "mixed" nucleon wave function including the spin-isospin wave function is given by Eq. (1), where[27]

$$|56\rangle_N = (1/\sqrt{2})(x'\phi' + x''\phi'')\psi^s, \tag{8}$$

$$|70\rangle_N = \tfrac{1}{2}[(\phi' x'' + \phi'' x')\psi' + (\phi' x' - \phi'' x'')\psi'']. \tag{9}$$

The neutron charge form factor now works out as[9]

$$\begin{aligned} G_E^n(q^2) &= -\sqrt{2}\,\sin\theta\cos\theta \times \int d\tau\, \psi''\psi^s \exp[2i\vec{q}\cdot(\vec{x}\sin\tfrac{1}{2}\lambda - \vec{y}\cos\tfrac{1}{2}\lambda)] \\ &= -\sqrt{2}\,\sin\theta\cos\theta \times 0.86q^2 \text{ (for small } q^2). \end{aligned} \tag{10}$$

With a value of $\cot^2\theta = 2$ which fits $G_A/G_V$, etc.,[24] as well as resonance photocouplings,[1] Eq. (10) predicts

$$-6\langle r_n^2\rangle = \frac{dG_E^n(q^2)}{dq^2}\bigg|_{q^2=0} = b = -0.57\text{ GeV}^{-2}\times\epsilon(\theta), \tag{11}$$

where $\epsilon(\theta)$ is the sign of the angle $\theta$. It is clear that the magnitude of $b$ found above compares rather well with the observed value[28] of $(0.50 \pm 0.01)$ GeV$^{-2}$. As to the sign of $\theta$ which was not quite determined through the earlier even-wave investigations,[1,23,24] a positive value, which corresponds to the choice of $\phi<0$ of LOPR,[9] gives the wrong sign of $b$. Of course we also have the option of choosing $\theta<0$ which is equally compatible with our earlier results on $G_A/G_V$, etc.,[23,24] as well as photocouplings.[1] But this choice happens to give a wrong behavior for $R^{np}$, as we shall find in Sec. III.

## III. CALCULATIONS OF THE STRUCTURE FUNCTIONS

For the calculation of the structure functions in deep-inelastic scattering, we must resort to momentum-space wave functions, i.e., the Fourier transforms of $\psi^s$ and $\psi''$ defined in paper II, and proceed as in LOPR. We therefore write the invariant volume element in momentum space, viz., $d^3p'\,d^3p''$, in terms of longitudinal variables $z', z''$ appropriate to the infinite-momentum frame[9] and transverse variables $p_\perp$ and $q_\perp$ which are more in tune with the structure of the $(\underline{70}, 0^+)$ wave functions in our model. The derivation, which is sketched in the Appendix, leads to the following expression:

$$\begin{aligned} dv &= d^3p'\,d^3p'' \\ &= 3dz'\,dz''\,dz_1\,\delta(\sqrt{6}\,z'' + 3z_1 - 1)d^2p_\perp\,d^2q_\perp\,d\lambda_\perp \\ &\quad\times\delta(2p_\perp\cdot q_\perp)(p_\perp^2 - q_\perp^2)\theta(p_\perp^2 - q_\perp^2), \end{aligned} \tag{12}$$

where

$$\sqrt{2}\,z' = z_3 - z_2, \quad \sqrt{6}\,z'' = -2z_1 + z_2 + z_3, \tag{13}$$

and $p_\perp$ and $q_\perp$ are given by

$$\begin{aligned} p_\perp &= p'_\perp\cos\tfrac{1}{2}\lambda_\perp + p''_\perp\sin\tfrac{1}{2}\lambda_\perp, \\ q_\perp &= p'_\perp\sin\tfrac{1}{2}\lambda_\perp - p''_\perp\cos\tfrac{1}{2}\lambda_\perp. \end{aligned} \tag{14}$$

To define the momentum-space wave functions we take partial advantage of the hidden character of the cyclic variable $\lambda$ of Eqs. (6) and (7) to introduce a similar variable *ab initio* in momentum space as in Ref. 24, without, however, implying the equality of the corresponding variable in the coordinate description. However, we must insert separately a negative phase factor in the definition of $\psi', \psi''$ to tally with the results of Ref. 9, as well as the theorem[26] mentioned in the Introduction. With these precautions the momentum-space wave functions, listed in Ref. 24, may be rewritten in terms of the $(z', z'')$, $(p_\perp, q_\perp)$ variables as

$$\tilde{\psi}^s = \tilde{N}\exp[-1/2(z'^2 + z''^2 + p_\perp^2 + q_\perp^2)], \tag{15}$$

$$\tilde{\psi}' \approx (-)\tilde{N}_M(\sin\lambda F_+ + \sin 2\lambda F_-), \tag{16}$$

$$\tilde{\psi}'' \approx (-)\tilde{N}_M(\cos\lambda F_+ - \cos 2\lambda F_-), \tag{17}$$

where

$$\begin{aligned} F_\pm &= \tfrac{1}{2}[\exp(-p_\perp^2 - z'^2 - z''^2 - q_\perp^2/\sqrt{3}) \\ &\quad\pm\exp[-q_\perp^2 - (z'^2 + z''^2 + p_\perp^2)/\sqrt{3}]], \\ \tilde{N}_s^{-2} &= \pi^3/2, \\ \tilde{N}_M^{-2} &= \frac{\pi^3}{8}\left[\frac{\epsilon^2(\epsilon^2+1)}{4} + \frac{(\epsilon^2-1)\epsilon}{4}(\epsilon^2\tan^{-1}\epsilon - \tan^{-1}1/\epsilon)\right], \\ \epsilon^2 &= \sqrt{3}, \end{aligned} \tag{18}$$

$$\tan\lambda = \frac{2z'z'' + 2p'_\perp\cdot p''_\perp}{z'^2 - z''^2 + p'^2_\perp - p''^2_\perp}. \tag{19}$$

The difference between the exponents of Eq. (18) from those of Ref. 24 is based on the following approximate, but in practice accurate, representation:

$$p^2 - q^2 \approx p_\perp{}^2 - q_\perp{}^2 + z'^2 + z''^2 . \tag{20}$$

The same approximation yields the following simplified expression for $\cos\lambda$ in terms of the variables of Eq. (12), one which is particularly useful for the evaluation of the structure functions:

$$\cos\lambda \approx \frac{z'^2 - z''^2 + (p_\perp{}^2 - q_\perp{}^2)\cos\lambda_\perp}{p_\perp{}^2 - q_\perp{}^2 + z'^2 + z''^2} . \tag{21}$$

We now define certain spatial distribution functions $A(z)$, $B(z)$, and $C(z)$, viz.,

$$A(z) = 3\sqrt{6} \int dv\, |\psi^s|^2 \delta(z - z_1), \tag{22}$$

$$C(z) = \frac{3\sqrt{6}}{2} \int dv\, [|\bar{\psi}'|^2 + |\bar{\psi}''|^2] \delta(z - z_1), \tag{23}$$

$$B(z) = 3\sqrt{6} \int dv\, \bar{\psi}'' \delta(z - z_1) \bar{\psi}^s , \tag{24}$$

which obey the normalizations

$$\int dz\, A(z) = \int dz\, C(z) = 1 , \quad \int B(z) dz = 0 . \tag{25}$$

The functions $A(z)$, etc., are related to the proton- and neutron-type quark distributions $\mathcal{P}(z)$ and $\mathcal{N}(z)$ inside the proton[9,11] by

$$\mathcal{P}(z) = 2A \cos^2\theta + 2C \sin^2\theta + \sqrt{2}\, B \sin\theta \cos\theta , \tag{26}$$

$$\mathcal{N}(z) = A \cos^2\theta + C \sin^2\theta - \sqrt{2}\, B \sin\theta \cos\theta , \tag{27}$$

whence the structure functions $\nu W_2 \equiv F_2(z)$ in the scaling limit work out as

$$z^{-1} F_2^{ep} = [A \cos^2\theta + C \sin^2\theta + (\sqrt{2}/3) B \sin\theta \cos\theta] , \tag{28}$$

$$z^{-1} F_2^{en} = \tfrac{2}{3}[A \cos^2\theta + C \sin^2\theta - (\sqrt{2}/3) B \sin\theta \cos\theta] , \tag{29}$$

## IV. DISCUSSION OF RESULTS AND SUMMARY

In Fig. 1 we have plotted against experiment[14] the ratio $R^{np}$ of $F_2^{en}$ and $F_2^{ep}$ as a function of $z$ ($0 \leq z \leq 1$) for the same magnitude of $\theta$, viz., $\cot\theta = \sqrt{2}$ as determined earlier for photocouplings[1] and other resonance parameters,[23,24] and a sign $\theta > 0$ which corresponds to LOPR's $\phi < 0$. (Recall from Sec. II that this sign of $\theta$ yields the wrong sign but the correct magnitude for $b$.) In this sense our fit is parameter-free, since the mixing angle has been determined in advance. Further, since we have not considered the effect of the $q\bar{q}$ sea as well as gluons[9,15] we should not expect agreement near small $z$ (where these effects play a dominant role). Subject to this limitation of our model, the qualitative fit appears quite reasonable for $z \gtrsim \frac{1}{3}$. In particular, we predict

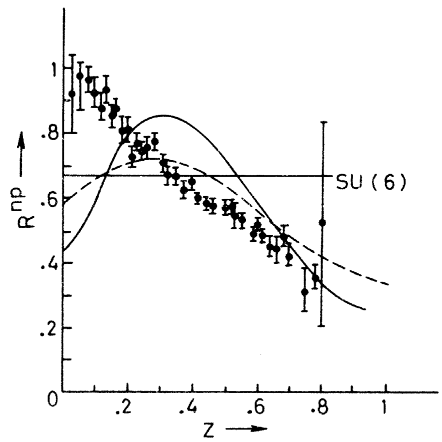


FIG. 1. The ratio $R^{np}$ as a function of $z$. Present results: solid line; LOPR results: dashed line. Data are taken from Ref. 12.

$$R^{np}(\tfrac{1}{3}) = 0.85(\sim 0.75) , \quad R^{np}(1) = 0.3(\sim 0.3) , \tag{30}$$

which compares extremely well with the experimental figures (in parentheses) for $z$ near unity and reasonably well for $z$ near $\frac{1}{3}$. Considering the extreme simplicity of the model and an almost total absence of any adjustable parameters, this is not a trivial achievement. Of course our mixing angle $\theta$ is appreciably larger than LOPR's $\phi$, but a substantial decrease in $\theta$ to provide a better fit to the $R^{np}$ curve would have to be at the cost of disagreement on the other low-energy parameters,[24] since in our model, unlike LOPR's, we do not have any separate parametric handles characteristic of internal quark motion.

A good fit is also obtained for the quantity $\Delta^{pn} = F_2^{ep} - F_2^{en}$ which is shown in Fig. 2, along with the data[29] and the LOPR fit. In this case too, the good agreement with experiment seems to be achieved with the same value of $\theta$, viz. $\cot\theta = +\sqrt{2}$ . Understandably the neglect of $q\bar{q}$-sea and gluon contributions (which should cancel out for the difference $\Delta^{pn}$) does not appear to be important for this quantity, as inferred from the quality of the fit without these contributions.

Against this encouraging background must be reckoned the negative result on the slope of the neutron charge form factor which, curiously enough, agrees in magnitude but not in sign. According to the theorem (Ref. 26) mentioned in Sec. I, it is apparently impossible to obtain the correct signs of both the mean square radii characterizing the neutron charge distribution in momentum *and* coordinate spaces, at least within the framework of this valence-quark model. It is entirely possible that $q\bar{q}$-sea contributions may change this result significantly.[30] However, in this paper we have re-

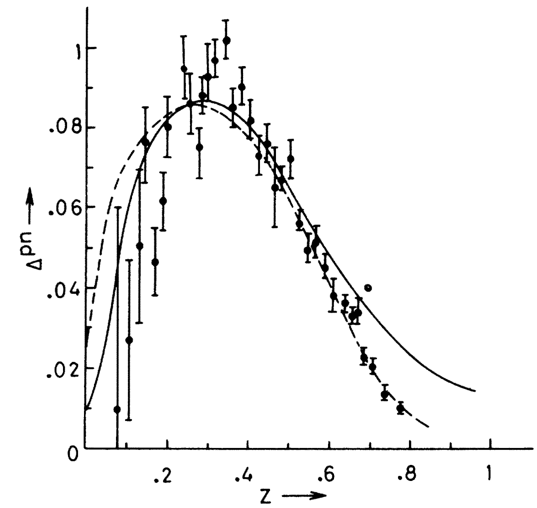


FIG. 2. The difference $\Delta^{pn} = F_2^{ep} - F_2^{en}$ as a function of $z$. Present results: solid line ($\theta = 35.5°$); LOPR results: dashed line. Data are taken from Ref. 29.

frained from such an exercise, which would in any case need some fresh parametrization over the simple parameter-free premises of our model, thus tending to obscure a clear-cut physical inference in this regard.

To summarize, our purpose in the investigation has been not so much to produce high-quality fits to low- and high-energy data with the inclusion of details of various effects[9,15] as to examine in the even-wave h.o. model the behavior of $R^{np}$ in the limit $z \to 1$, the sign of the $b$ parameter, and the sign of the mixing angle $\theta$, which had remained undetermined in the earlier investigations with the even-wave model.[1,23,24] For this purpose we have not attempted any comprehensive parametrization of various effects (which are presumably needed for high-quality fits), but rather have depended on a single effective parameter $\theta$ for a simple overall description of both high- and low-energy effects. Since, on the other hand, a given choice of the sign of $\theta$ is incompatible with both $b$ and $R^{np}$, we have adopted the choice $\theta > 0$ which effectively reproduces $R^{np}$, though at the cost of a wrong sign for $b$. The same is also the conclusion of LOPR.[9,20] To this extent we have not succeeded in differentiating between the full-wave and even-wave h.o. models. However, the present results, together with those of Refs. 1, 23, and 24, should help establish a phenomenological viability of the even-wave model in favorable comparison to the usual h.o. model, in addition to a better theoretical *raison d'être* for a mixed nucleon provided by its prediction of $(\underline{70}, 0^+)$ as the ground state of the $\underline{70}$ series.


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## APPENDIX A

We outline here the transformation connecting the volume elements in the $\xi_i, \eta_i$ variables and $x_i, y_i$ variables, as well as for their momentum-space counterparts. The variables $x_i$ and $y_i$ are related to the $\xi_i$ and $\eta_i$ variables in the following way[22]:

$$x_i = \xi_i \cos\tfrac{1}{2}\lambda + \eta_i \sin\tfrac{1}{2}\lambda\,, \tag{A1}$$

$$y_i = \xi_i \sin\tfrac{1}{2}\lambda - \eta_i \cos\tfrac{1}{2}\lambda\,, \tag{A2}$$

with the constraints

$$\vec{x}\cdot\vec{y} = 0\,, \quad x^2 \geq y^2\,. \tag{A3}$$

In order to decouple $\lambda$ from $\vec{x}, \vec{y}$ and to regard $\vec{x}$ and $\vec{y}$ as effectively independent, the constraint $\vec{x}\cdot\vec{y} = 0$ requires the introduction of a seventh variable

$$u = 2\vec{x}\cdot\vec{y}\,. \tag{A4}$$

The Jacobian $J$ of transformation connecting $\vec{\xi}, \vec{\eta}, u$ variables with $\vec{x}, \vec{y}, \lambda$ is given as under

$$J = |x^2 - y^2|\,, \tag{A5}$$

therefore, the transformation becomes

$$d\vec{\xi}\, d\vec{\eta}\, du = d\vec{x}\, d\vec{y}\, d\lambda\, |x^2 - y^2|\,, \tag{A6}$$

which can be further written in the form

$$\begin{aligned} d\vec{\xi}\, d\vec{\eta} &\to d\vec{x}\, d\vec{y}\, d\lambda\, |x^2 - y^2| \delta(2\vec{x}\cdot\vec{y}) \\ &\to d\vec{x}\, d\vec{y}\, d\lambda\, (x^2 - y^2)\theta(x^2 - y^2)\delta(2\vec{x}\cdot\vec{y})\,. \end{aligned} \tag{A7}$$

The variables $p', p''$ are canonically conjugate to $\xi, \eta$ variables, so the volume element in momentum space with $p', p''$ variables can be written as

$$dv = dp'_{\parallel}\, dp''_{\parallel}\, d^2p'_{\perp}\, d^2p''_{\perp}\,. \tag{A8}$$

Lorentz contraction of the $p_{\parallel}$ variables and a normalization similar to Ref. 9 leads to the expression

$$dv = 3dz'\, dz''\, dz_1\, \delta(\sqrt{6}\, z'' + 3z_1 - 1) d^2p'_{\perp}\, d^2p''_{\perp}\,, \tag{A9}$$

where the constraint $z_1 + z_2 + z_3 = 1$ has been incorporated in the $\delta$ function and $z', z''$ are as defined in Eq. (13) of the text. To transform $p'_\perp, p''_\perp$ we define two related quantities $p_\perp, q_\perp$ as in Eq. (14) of the text, and proceed as in the derivation of Eq. (A7) to obtain

$$d^2p'_\perp\, d^2p''_\perp = d^2p_\perp\, d^2q_\perp\, d\lambda_\perp\, \delta(2p_\perp\cdot q_\perp)(p_\perp^2 - q_\perp^2) \times \theta(p_\perp^2 - q_\perp^2)\,. \tag{A10}$$

This finally yields the full momentum-space volume element as given in Eq. (12) of the text.


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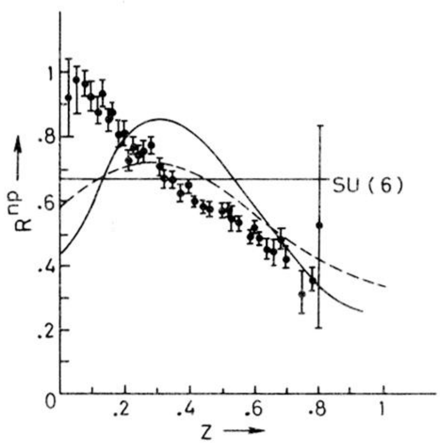


FIG. 1. The ratio $R^{np}$ as a function of $z$. Present results: solid line; LOPR results: dashed line. Data are taken from Ref. 12.

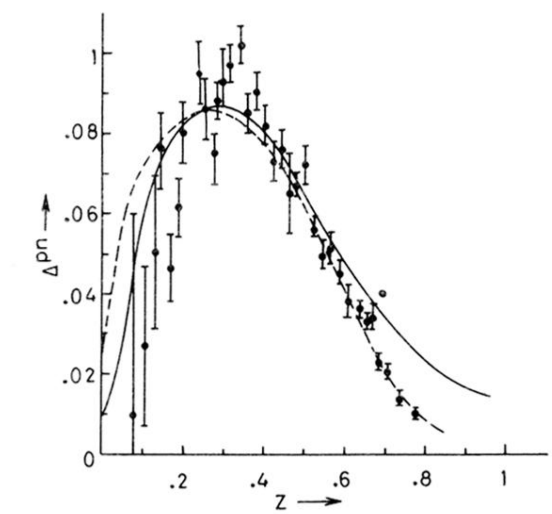


FIG. 2. The difference $\Delta^{pn} = F_2^{ep} - F_2^{en}$ as a function of $z$. Present results: solid line ($\theta = 35.5°$); LOPR results: dashed line. Data are taken from Ref. 29.