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Isospin mass splittings of pseudoscalar charmed mesons

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The Cottingham formula is used to calculate the D^+-D^0 mass difference assuming that isospin-symmetry breaking arises entirely from electromagnetic interactions. Using the vector-meson-dominance model for electric and magnetic form factors and SU(4) coupling relations, we obtain a value of 12 MeV for this mass difference.

I. INTRODUCTION

The new particle at 1865 MeV discovered[1] at SPEAR is being interpreted as one of the charmed mesons $D^0(c\bar{u})$ or its antiparticle $\bar{D}^0(\bar{c}u)$. There is strong evidence[2] that the charged partners of these mesons $D^+(c\bar{d})$ or $D^-(\bar{c}d)$, of the same isospin doublet, have also been seen at 1876 ± 15 MeV through their decay in the exotic channel $K^{\mp}\pi^{\pm}\pi^{\pm}$ as expected of a charged charmed meson. It was observed by De Rújula, Georgi, and Glashow[3] that exact isospin mass splitting may have important consequences for the production rate of various charmed particles. It has also been mentioned by Lane and Weinberg[4] that this splitting will provide an interesting test of our ideas about the origin of isospin nonconservation. The isospin symmetry breaking is believed by De Rújula, Georgi, and Glashow[3] and by Lane and Weinberg[4] to originate from two sources, namely, (a) through quark mass differences (u-d mass difference) and (b) through ordinary one-photon exchange. De Rújula, Georgi, and Glashow[3] estimated the D^+-D^0 mass difference to be about 15 MeV, whereas Lane and Weinberg[4] by a slightly different method of calculation estimated the mass difference to be about 6.7 MeV. The problem of D^+-D^0 mass difference has also been considered by Fritzsch,[5] Ono,[6] Lichtenberg,[7] and Celmaster[8] in a similar framework. In all cases D^+ is heavier than D^0.

In the present work we shall assume the isospin-symmetry breaking to arise entirely from electromagnetic interactions. Admittedly, the problem of the neutron-proton mass difference is not without pitfalls; however, a deeper understanding of the problem has been achieved by Harari,[9] who showed that the $\Delta I = 2$ mass differences are correctly obtained by introducing the Born terms of the virtual Compton scattering in the Cottingham[10] formula, while for the $\Delta I = 1$ mass differences an additional contribution should arise from the subtraction term for the $t_1(q^2, \nu)$ amplitude since its behavior at high values of ν is dominated by the A_2 Regge pole. Attempts to evaluate this contribution for the neutron-proton mass difference have not been very successful for a variety of reasons. For a detailed discussion we refer the readers to an excellent article by Harari and Elitzur.[11] For the n-p mass difference they found that the contribution is not large enough even for an overall sign reversal. This failure according to them may reflect one or more of the following possibilities:

(a) There may be a q^2-divergent term having $\Delta I = 1$.

(b) A fixed pole at $J = 0$ (Ref. 12) may contribute to ΔM. Its sign and magnitude are unknown and cannot be directly determined by inelastic electron-scattering experiments.

(c) The finite-Energy-sum-rule[13] (FESR) calculation of the A_2 residue function is misleading since it neglects possible contributions of lower trajectories and poles. Such contributions may be crucial to FESR but small in the expression for ΔM. The actual A_2 contribution may therefore be larger than the one indicated by the naive FESR calculation.

Bruccella, Cini, Maria, and Tirozzi[14] repeated the calculations for various isospin multiplets and found that at least in the case of pseudoscalar mesons, namely, K^+-K^0 mass difference, calculations of the residue neglecting fixed poles and saturating FESR by low-lying states (pseudoscalar and vector-meson states) give a surprisingly good result. They found that $\Delta M^{\text{Regge}}(K^+$-$K^0) = -5.3$ MeV, $\Delta M^{\text{res}}(K^+$-$K^0) = 2.55$ MeV, giving $\Delta M(K^+$-$K^0) = -3.8$ MeV as compared to the experimental value -3.99 ± 0.13 MeV. They then adopted the attitude of considering the determination of the A_2 residue function for K mesons as basically correct because of the simple structure of the meson spectrum and deduced from it the Regge residue for any hadron H from the relation

$$\beta_H(q^2) = \frac{\beta_{K^+}(q^2) - \beta_{K^0}(q^2)}{\gamma_{K^+K^-A_2} - \gamma_{K^0\bar{K}^0A_2}}\, \gamma_{HHA_2}\,.$$

Using this type of universality, they always got a contribution in the right direction which reverses the wrong sign in all cases and gives an overall good agreement with experimental data. We will adopt this framework in the present paper to estimate the D^+-D^0 mass difference. In Sec. II we will write down the contribution to the mass difference as a sum of three terms $(\Delta M)^{\text{resonance}}$, $(\Delta M)^{\text{continuum}}$, and $(\Delta M)^{\text{subtracted}}$. In Sec. III we will estimate the contribution to the D^+-D^0 mass difference. Section IV is devoted to the discussion of the results and the validity of the assumptions involved.

II. CONTRIBUTION TO MASS DIFFERENCES

The Cottingham formula for the electromagnetic self-mass is given by

$$\Delta M = -\frac{1}{4\pi}\int_0^\infty \frac{dq^2}{q^2}\int_{-q}^{q} d\nu (q^2-\nu^2)^{1/2} \times[3q^2 t_1(q^2, i\nu) - (q^2+2\nu^2) \times t_2(q^2, i\nu)] \ . \tag{1}$$

In the presence of fixed poles at $J=0$ the asymptotic behavior of $t_i(q^2,\nu)$ at large ν and fixed q^2 is given by

$$\begin{aligned} t_1(q^2,\nu) &\underset{\nu\to\infty}{\simeq} R_1(q^2) + \beta_1(q^2)\nu^{\alpha_{A_2}(0)} \ ,\\ t_2(q^2,\nu) &\underset{\nu\to\infty}{\simeq} R_2(q^2)\nu^{-2} + \beta_2(q^2)\nu^{\alpha_{A_2}(0)-2} \ . \end{aligned} \tag{2}$$

$\alpha_{A_2}(0)$ is the $t=0$ intercept of the leading Regge pole and is of the order of $\frac{1}{3}$.[15] $R_i(q^2)$ and $\beta_i(q^2)$ are the residues of the fixed pole and the Regge pole, respectively, for $i=1,2$.

Thus one writes an unsubtracted dispersion relation for t_2 and a once-subtracted dispersion relation for t_1 and further, if the contribution to $\text{Im}(t_i(q^2,\nu)$ in the resonance region is separated, one may write ΔM as a sum of three terms, namely,

$$\begin{aligned} \Delta M &= (\Delta M)^{\text{resonance}} + (\Delta M)^{\text{continuum}} + (\Delta M)^{\text{subtracted}} \\ (\Delta M)^{\text{subtracted}} &= (\Delta M)^{\text{Regge}} + (\Delta M)^{\text{fixed pole}} \ , \end{aligned} \tag{3}$$

where

$$(\Delta M)^{\text{res}} = -\frac{1}{4\pi M}\sum_{\text{res}}\int_0^\infty q\,dq^2 \left\{\left[\left(1+\frac{\nu_r^2}{q^2}\right)^{1/2} - \frac{2\nu_r}{q}\right] 3F_1^r(q^2) - \left[\left(1+\frac{\nu_r^2}{q^2}\right)^{1/2}\left(1-\frac{2\nu_r^2}{q^2}\right) + \frac{2\nu_r^3}{q^3}\right] F_2^r(q^2)\right\} \ ,$$

$$\nu_r = (M_r^2 - M^2 + q^2)/2M \ , \tag{4}$$

and the form of $F_i^r(q^2)$ for each intermediate state is obtained from

$$(q^2 g_{\mu\nu} - q_\mu q_\nu)F_1^r + \left[\nu^2 g_{\mu\nu} + \frac{q^2}{M^2}p_\mu p_\nu + \frac{\nu}{M}(p_\mu q_\nu + q_\mu p_\nu)\right]F_2^r = \pi(2\pi)^4 2k_0 \sum_{\text{spin}} \langle p|J_\mu|k\rangle\langle k|J_\mu|p\rangle \ , \tag{5}$$

$$(\Delta M)^{\text{continuum}} = \frac{1}{2\pi}\int_0^\infty dq^2 \int_{\nu_t}^\infty \nu d\nu \{3[1-(1+q^2/\nu^2)^{1/2}]\,\text{Im}\, t_1(q^2, i\nu) + [(1+\nu^2/q^2)^{1/2}(1-2\nu^2/q^2) + 2\nu^2/q^2]\,\text{Im} t_2(q^2, i\gamma)\} \ , \tag{6}$$

$$(\Delta M)^{\text{Fixed pole}} = -\tfrac{3}{8}\int_0^\infty q^2 dq^2 R_1(q^2) \ , \tag{7}$$

$$(\Delta M)^{\text{Regge}} = -\frac{3}{4\pi}\int_0^\infty \beta(q^2)\frac{\nu_0^{\alpha(0)}}{\alpha(0)} q^2 dq^2 \ . \tag{8}$$

In what follows we will assume that the contributions from the continuum states are negligible, and no attempt will be made to calculate $(\Delta M)^{\text{fixed}}$.

III. D^+-D^0 MASS DIFFERENCE

A. Calculation of $(\Delta M)^{\text{resonance}}$

The $(\Delta M)^{\text{resonance}}$ will be calculated as a sum of two terms: (i) contribution from the Born term and (ii) contribution from the vector-meson intermediate state.

(i) Born term. For a charmed meson we have from (5),

$$F_1^{\text{Born}}(q^2) = -\frac{\alpha}{2M_D}G^2(q^2), \quad F_2^{\text{Born}}(q^2) = \alpha\frac{2M_D}{q^2}G^2(q^2) \ . \tag{9}$$

The electric and magnetic form factors can be obtained by using the vector-dominance hypothesis. In the present calculation we use SU(4) symmetry to write the electromagnetic current, which in the Glashow-Iliopoulos-Maiani model[16] (charmed quark has charge $\frac{2}{3}$ and charm +1) is given by

$$J_\mu^{\text{em}} = J_\mu^3 + \frac{1}{\sqrt{3}}J_\mu^8 - (\tfrac{2}{3})^{1/2}\left(J_\mu^{15} - \frac{1}{\sqrt{3}}J_\mu^0\right) \ , \tag{10}$$

and saturate the various form factors by ρ, ω, ϕ, and $\psi(3100)$ mesons. The electric form factor comes out to be

$$G(q^2)=\tfrac{1}{2}\left[\frac{m_\rho^2}{q^2+m_\rho^2}\mp\tfrac{1}{3}\left(\sin^2\theta+\frac{1}{\sqrt{2}}\sin2\theta\right)\frac{m_\omega^2}{q^2+m_\omega^2}\mp\tfrac{1}{3}\left(\cos^2\theta-\frac{1}{\sqrt{2}}\sin2\theta\right)\frac{m_\phi^2}{q^2+m_\phi^2}\pm\tfrac{4}{3}\frac{m_\psi^2}{q^2+m_\psi^2}\right], \quad (11)$$

where the upper (lower) sign refers to D^+ (D^0), and θ is the ω-ϕ mixing angle.[17] With this we obtain

$$\Delta M^{\rm Born}(D^+)-\Delta M^{\rm Born}(D^0)=3\ {\rm MeV}. \quad (12)$$

(ii) Vector-meson contribution. The electromagnetic current matrix elements between D and D^* can be written in the form

$$\langle D(p_1)|J_\mu|D^*(p_2)\rangle=\frac{e}{(2\pi)^3}\frac{f_{\gamma DD^*}}{m_\pi}\frac{1}{(4\omega_1\omega_2)^{1/2}}\times\epsilon_{\mu\nu\lambda\sigma}(p_1)_\nu q_\lambda\epsilon_\sigma(p_2)G^*(q^2), \quad (13)$$

where $\epsilon_\sigma(p_2)$ is the polarization vector of the D^* meson. From (5) we get

$$F_1^V(q^2)=F_2^V(q^2)=\alpha\left(\frac{M_D f_{\gamma DD^*}}{m_\pi}\right)^2\frac{1}{2M_D}[G^*(q^2)]^2, \quad (14)$$

where the magnetic form factors for D^+ and D^0 are given by

$$G^*(q^2)=\left[-\frac{3}{2}\frac{m_\rho^2}{q^2+m_\rho^2}+\frac{1}{2}\left(\sin^2\theta+\frac{1}{\sqrt{2}}\sin2\theta\right)\frac{m_\omega^2}{q^2+m_\omega^2}+\frac{1}{2}\left(\cos^2\theta-\frac{1}{\sqrt{2}}\sin2\theta\right)\frac{m_\phi^2}{q^2+m_\phi^2}+2\frac{m_\psi^2}{q^2+m_\psi^2}\right],$$

$$G^*(q^2)=\frac{1}{4}\left[\frac{3}{2}\frac{m_\rho^2}{q^2+m_\rho^2}+\frac{1}{2}\left(\sin^2\theta+\frac{1}{\sqrt{2}}\sin2\theta\right)\frac{m_\omega^2}{q^2+m_\omega^2}+\frac{1}{2}\left(\cos^2\theta-\frac{1}{\sqrt{2}}\sin2\theta\right)\frac{m_\phi^2}{q^2+m_\phi^2}+2\frac{m_\psi^2}{q^2+m_\psi^2}\right], \quad (15)$$

respectively. $f_{\gamma DD^*}$ is related to $f_{\gamma\rho\pi}$ by the relation

$$f_{\gamma\rho\pi}=f_{\gamma D^+D^{-*}}=\tfrac{1}{4}f_{\gamma D^0\bar{D}^{0*}}.$$

The coupling $f_{\gamma\rho\pi}$ can be evaluated either in terms of the $\omega\to\pi\gamma$ experimental width (1.1 MeV) or by fitting with the $\rho\to\pi\gamma$ experimental decay width (35 ± 10 keV).[18] Using $f_{\gamma\rho\pi}=0.1$ we obtain

$$\Delta M^V(D^+)-\Delta M^V(D^0)=5.5\ {\rm MeV}. \quad (16)$$

This contribution is positive, in contrast to the K^+-K^0 mass difference, because of the different isospin structure of D mesons.

B. The subtraction contribution

This is a large and important contribution that changes the overall sign for the K^+-K^0 mass difference. We shall consider the evaluation of the Regge residue using the FESR to be correct for K^+-K^0 pseudoscalar mesons and write the Regge residue for D mesons from the relation

$$\beta_D(q^2)=\frac{\beta_{K^+}(q^2)-\beta_{K^0}(q^2)}{\gamma_{K^+K^-A_2}-\gamma_{K^0\bar{K}^0A_2}}\gamma_{DDA_2}. \quad (17)$$

From (8) we then write the D^+-D^0 mass difference in terms of the K^+-K^0 mass difference as

$$\Delta M^{\rm sub.}(D^+)-\Delta M^{\rm sub.}(D^0)=\left(\frac{M_K}{M_D}\right)^{\alpha(0)}\frac{\gamma_{D^+D^-A_2}-\gamma_{D^0\bar{D}^0A_2}}{\gamma_{K^+K^-A_2}-\gamma_{K^0\bar{K}^0A_2}}\times[\Delta M^{\rm sub.}(K^+)-\Delta M^{\rm sub.}(K^0)], \quad (18)$$

and we get

$$\Delta M^{\rm sub.}(D^+)-\Delta M^{\rm sub.}(D^0)=4.15\ {\rm MeV}. \quad (19)$$

IV. RESULTS AND DISCUSSION

Collecting the contributions from Secs. III A and III B, we compute the final value for the D^+-D^0 mass difference to be 12.65 MeV. It should, however, be mentioned that this value has been obtained by using vector dominance for the electromagnetic form factors of pseudoscalar mesons and by using a value of the coupling constant $f_{\gamma\rho\pi}=0.1$ determined from the $\omega\to\pi\gamma$ experimental width. When the same value of the coupling constant $f_{\gamma\rho\pi}$ is used to evaluate the $\rho\to\pi\gamma$ decay width, the width comes out to be 66 keV, to be compared with the experimental value of 35 ± 10 keV. Thus it may be possible that the estimate of the contribution to the mass difference from the vector-meson intermediate state is too large probably by a factor of two, thereby decreasing the D^+-D^0 mass difference to 10 MeV. On the other hand, if we use a universal dipole fit for the electromagnetic form factors, namely,

$$G(q^2)=\frac{(\bar{q}^2)^2}{(q^2+\bar{q}^2)^2}, \quad \bar{q}^2=0.71\ {\rm GeV}^2,$$

we get a value of about 24 MeV for the D^+-D^0 mass difference.

A remark may be in order here regarding the choice of the form factors. A definite form for these can be decided only by more experimental tests. For example, a precise measurement of the pion charge radius and a detailed study of the single-meson production by weak charged and neutral currents both at threshold and above may determine whether to use a universal dipole fit or vector-meson dominance. It may turn out that real form factors are different from the ones used here, but we hope that the results will not significantly change once a better understanding of coupling constants and residue functions is achieved.

Finally, we would like to point out that if scaling holds in deep-inelastic-scattering experiments, the use of subtraction may not be justified. However, at the moment, there is strong experimental evidence to the contrary (see for example Chen[19]). Also, from a theoretical point of view, gauge field theories predict, as is well known, a logarithmic divergence of scaling (Tung[20]). As regards the continuum contribution, we can estimate its value in the D^+-D^0 mass difference by following the calculations of Buccella *et al.*[14] for the K^+-K^0 mass difference. This contribution turns out to be very small compared to the resonance contribution. A similar situation obtains in K^+-K^0 mass-difference calculations also.

It thus seems that the electromagnetic interaction alone is responsible for the contribution to the D^+-D^0 mass difference, and it is of the order of 10 MeV if one uses vector-meson dominance. If a nonelectromagnetic isospin-breaking interaction exists, its contribution to the mass difference is in addition to the contribution from electromagnetism. We feel that such an interaction, if it exists, should make a very small contribution to the mass difference and is not likely to be required by isospin mass-splitting considerations for charmed mesons.

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