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Octet dominance of nonleptonic hyperon decays in a nonrelativistic quark model

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Extracting an effective Hamiltonian by taking the nonrelativistic limit of quark-quark scattering through W-boson exchange, it is shown that we obtain octet dominance for the matrix elements $\langle B_s|H_W^{pc}|B_r\rangle$, where B_r, B_s denote ordinary baryons. Further, it is shown that the above matrix elements are enhanced so as to compensate the Cabibbo suppression factor $\sin\theta_C$ to some extent.

It is well known[1] that the current-current picture of weak interactions does not automatically guarantee the most striking aspect of nonleptonic decays, namely the approximate validity of the $\Delta I=\frac{1}{2}$ rule for strangeness-changing decays. The situation is also more or less the same[2] for a gauge theory of weak and electromagnetic interactions at least in its minimal model unless one assumes additional pieces of currents. In particular, it is necessary to assume[3] an additional $V+A$ current involving the charmed quark and that the matrix elements of the strangeness-changing $(V-A)\times(V+A)$ interaction [such an interaction is automatically octet under SU(3) as the charmed quark is a singlet], even for ordinary baryons which have an extremely small contamination of charmed quarks, dominate over those of the $(V-A)^2$ interaction which does not involve a charmed quark and which contribute to both $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ parts.

The purpose of this paper is to show that in the quark model, when we deal with ordinary $(V-A)$ charged currents, the $\Delta I=\frac{1}{2}$ rule for matrix elements of the form $\langle B_s|H_W^{pc}|B_r\rangle$ follow in the leading nonrelativistic approximation, while $\langle B_s|H_W^{pv}|B_r\rangle$ vanish in the same approximation. Here B_r and B_s denote ordinary baryons. It is well known that in the current-algebra approach the question of the $\Delta I=\frac{1}{2}$ rule or octet dominance for nonleptonic decays of baryons hinges on the octet dominance of the above matrix elements both for the s-wave and p-wave amplitudes, for the latter such matrix elements enter through the baryon-pole approximation. We also show that the D/F ratio for the SU(3) parametrization of the above matrix elements comes out to be -1. Further, not only does the octet dominance follow in the above-mentioned approximation, but also there is an indication that the magnitude of the matrix elements is enhanced, that is to say that the suppression expressed by the Cabibbo factor $\sin\theta_C$ is compensated to some extent. It may be noted that previously the octet dominance and $D/F=-1$ has been shown in the quark model for Bose quarks[4,5] or octet dominance for the matrix elements $\langle B_s|H_W^{pc}|B_r\rangle$ for various color versions of triplet model of fermion quarks.[6] We wish to emphasize that in the latter case the additional degree of freedom of color is a necessary ingredient. We, of course, take quarks with spin-$\frac{1}{2}$, and in our approach color plays no part, nor does any $V+A$ current involving the charmed quark, the latter would be relevant only for nonleptonic decays of charmed hadrons. As is the case in the above type of models, we take the current and constituent quarks to be identical; the distinction between them may not be very meaningful in the nonrelativistic quark model we are considering. Finally, we wish to remark that our approach has some anology with that considered by De Rújula, Georgi, and Glashow[7] for hadron masses. Below we give details of our calculation.

We consider quark-quark scattering through the exchange of weak $W^{\pm}$ bosons for which the matrix elements are of the form

$$M \underset{m_W^2 \gg q^2}{\frown} \frac{1}{2}\,\frac{g_W^2 \sin\theta_C\cos\theta_C}{m_W^2}\left[\,\overline{U}(p_i')\gamma_\lambda(1+\gamma_5)\alpha_i^- U(p_i)\,\overline{U}(p_j')\gamma_\lambda(1+\gamma_5)\beta_j^+ U(p_j) + (i\longleftrightarrow j)\right], \tag{1}$$

where $q=p_i-p_i'=p_j'-p_j$. The U's are Dirac spinors in Dirac space but are column vectors involving u, d, s quarks in SU(3) space; α_i^- and β_j^+ are operators which respectively transform a u-like state into a d-like state and an s-like state to a u-like state. We take the nonrelativistic limit of the above matrix elements. In the leading nonrelativistic approximation, only γ_4 and $\gamma_i\gamma_5$ have nonzero limits. Thus only the parity-conserving (pc) part of the M survives in the leading nonrelativistic approximation, and we have in this limit

$$M^{\rm pc}\sim\frac{1}{2\sqrt{2}}G_F\sin\theta_C\cos\theta_C \times\sum_{i>j}(\alpha_i^-\beta_j^+ +\beta_i^+\alpha_j^-)(1-\vec{\sigma}_i\cdot\vec{\sigma}_j)\,, \tag{2}$$

where $G_F/\sqrt{2}=g_W^2/m_W^2$ and $\vec{s}_i=\frac{1}{2}\vec{\sigma}_i$ is the spin of the ith quark. Taking the Fourier transform of the above matrix elements we have

$$H_W^{\rm pc}=\left(\frac{1}{2\sqrt{2}}G_F\sin\theta_C\cos\theta_C \times\sum_{i>j}(\alpha_i^-\beta_j^+ +\beta_i^+\alpha_j^-)(1-\vec{\sigma}_i\cdot\vec{\sigma}_j)\right)\delta^3(\vec{r})\,. \tag{3}$$

Defining

$$d'=\langle\Psi_0|\delta^3(\vec{r})|\Psi_0\rangle\,, \tag{4}$$

it is easy to see that the relevant matrix elements

$$\langle B_s|H_W^{\rm pc}|B_r\rangle\sim\bar{u}\,a_{rs}\,u$$

are given by

$$a_{\Lambda n}=\frac{G_F}{2\sqrt{2}}\sin\theta_C\cos\theta_C\,d'(-\sqrt{6})\,, \tag{5a}$$

$$a_{\Sigma^+p}=\frac{G_F}{2\sqrt{2}}\sin\theta_C\cos\theta_C d'(-6)\,,$$
$$=\sqrt{2}\;a_{\Sigma^0n} \tag{5b}$$

$$a_{\Xi^0\Lambda^0}=\frac{G_F}{2\sqrt{2}}\sin\theta_C\cos_C d'(-2\sqrt{6}) \tag{5c}$$

$$a_{\Xi^-\Sigma^-}=0\,. \tag{5d}$$

The factors $-\sqrt{6}$, -6, $-2\sqrt{6}$ follow by writing the spin-unitary-spin wave functions of p, n, Σ^+, Σ^0, Λ^0, and Ξ^0 (see, for example, Ref. 8). The relation $a_{\Sigma^+p}=+\sqrt{2}\;a_{\Sigma^0n}$ expressed in (5b) ensures the $\Delta I=\frac{1}{2}$ rule (or octet dominance), and hence $A(\Sigma_+^+)=0$ in the current-algebra approach. Once the octet dominance for a_{rs} is established, we can parametrize a_{rs} in the SU(3) limit as

$$a_{rs}=\sqrt{2}\,(2Fif_{6rs}+2Dd_{6rs})\,. \tag{6}$$

Then the relation (5d) immediately gives

$$D/F=-1\,. \tag{7}$$

Now using the current-algebra relations[1] for the s-wave amplitudes one has

$$A(\Lambda\to p\pi^-)=-\frac{1}{f_\pi}a_{\Lambda n}\,,$$
$$A(\Xi^-\to\Lambda\pi^-)=-\frac{1}{f_\pi}a_{\Xi^0\Lambda}\,, \tag{8}$$
$$A(\Sigma^+\to p\pi^0)=\frac{1}{\sqrt{2}\,f_\pi}a_{\Sigma^+p}\,.$$

Then using Eqs. (5) in addition to the Lee-Sugawara relation,[9]

$$2A(\Xi^-\to\Lambda\pi^-)-A(\Lambda^0\to p\pi^-)=-\sqrt{3}\,A(\Sigma^+\to p\pi^0), \tag{9a}$$

one has the additional relations

$$A(\Lambda\to p\pi^-)=-\frac{1}{\sqrt{3}}A(\Sigma^+\to p\pi^0)\,,$$
$$A(\Xi^-\to\Lambda\pi^-)=-\frac{2}{\sqrt{3}}A(\Sigma^+\to p\pi^0)\,, \tag{9b}$$

which have previously been noted by assuming that the 20 representation in SU(4) dominates.[10] Here, these relations have nothing to do with SU(4) but are a consequence of the current-algebra approach and the relation (7).

The question naturally arises how good is the detailed fitting of s- and p-wave amplitudes of nonleptonic decays of hyperons in the current-algebra approach with the D/F ratio fixed by the relation (7). This by itself does not give a good fit. However, it has been shown[5] that with the D/F ratio nearly -1, i.e., -0.85, it is possible to obtain a reasonable fit provided that the K^* contribution through the K^*-π weak transition, which contributes only to s-wave amplitudes, is also included. This contribution vanishes in the SU(3) limit but is numerically important if its strength is estimated by assuming that $K_1^0\to 2\pi$ is also dominated by the K^* pole. The K^* contribution can be shown to obey by itself the $\Delta I=\frac{1}{2}$ rule if the current-algebra argument is used. With the K^* contribution, the additional relations (9b) which are not as well satisfied as the Lee-Sugawara relation will no longer hold.

Finally, we note that in terms of F and D defined in (6),

$$|a_{\Lambda n}|=\frac{1}{\sqrt{3}}\,|3F+D|\,, \tag{10}$$

so that with relations (5a) and (7),

$$|F|=\frac{G_F}{2\sqrt{2}}\sin\theta_C\cos\theta_C\left(\frac{3}{\sqrt{2}}\,|d'|\right)\,. \tag{11}$$

If we assume SU(3), d' defined in (4) can be approximately related to d which occurs in Ref. 7. In fact then

$$d'\approx\frac{2}{\pi}d\,, \tag{12}$$

where one can easily see from Ref. 7 that

$$d=\frac{2}{3}\frac{(\Delta-N)9m_1^2}{32\alpha_s}$$
$$=(\Sigma-\Lambda)\frac{9m_1^2}{32\alpha_s}\left(1-\frac{m_1}{m_3}\right)^{-1}\,, \tag{13}$$

where m_1 and m_3 are effective masses of u and s quarks, α_s is related to the quark-gluon coupling constant while Δ, N, Σ, and Λ denote masses of N^*, N, Σ, and Λ baryons. Thus using (11), (12), and (13),

$$|F| = \frac{\sin\theta_C \cos\theta_C}{32\alpha_s\pi}(\Delta - N)G_F(9m_1^2) . \qquad (14)$$

Now $G_F(9m_1^2)\approx 10^{-5}$ as $9m_1^2 \approx m_N^2$, $(\Delta - N)\approx 300$ MeV, $\sin\theta_C\cos\theta_C \approx \frac{1}{4}$. Thus,

$$|F| \approx \frac{0.76}{\alpha_s} 10^{-5} \text{ MeV}.$$

With $\alpha_s \approx 0.5$, this gives $|F|$ to be about 1.5×10^{-5} MeV. This by itself is of the right order of magnitude for, e.g., $A(\Sigma_-^-)$. However, for an overall fit of s- and p-wave amplitudes which require inclusion of K^* contribution as mentioned earlier, $|F|$ is needed[5] to be about 4.5×10^{-5} MeV. Such a value of $|F|$ would require quite a small value of α_s, namely ≈ 0.17.

We would like to emphasize that the relations (5) which imply octet dominance for the matrix elements $\langle B_s|H_W^{pc}|B_r\rangle$ are the result of the particular combination $(1-\vec{\sigma}_i\cdot\vec{\sigma}_j)$ in relation (3). This in turn is a consequence of the $(V-A)^2$ interaction expressed in (1) in the usual picture of the weak interaction. Had it been a $(V-A)\times(V+A)$ interaction, we would have obtained the factor $(1+\vec{\sigma}_i\cdot\vec{\sigma}_j)$, and the relations (5) would have been different.

[1]R. E. Marshak, Riazuddin, and C. P. Ryan, *Theory of Weak Interactions in Particle Physics* (Wiley-Interscience, New York, 1969), Chap. 6. This contains references to the original literature.

[2]See, for example, B. W. Lee, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 635; Fermilab Report No. Fermi-Lab-Conf-76/20-Thy/Exp, 1976 (unpublished). These contain other references.

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