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Simple dynamical model for $\gamma N \rightarrow \pi^{\pm}\Delta$

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An elementary phenomenological extension of the gauge-invariance-constrained dynamical amplitudes for $\gamma N \rightarrow \pi^{\pm}\Delta$ is considered. The extended amplitudes are parameterized with a real MPE (minimal gauge-invariant extension of one-pion exchange) background term and simple Regge-pole forms. The two free parameters which both appear in the unnatural-parity-exchange component are determined by comparison with $d\sigma^{\parallel}/dt$ data for $\gamma p \rightarrow \pi^{-}\Delta^{++}$. The remainder of the natural-parity exchange is represented by ρ and A_2 exchange. Predictions are made for $d\sigma/dt$ and Σ, for the various charge states, and the unnatural-parity-exchange components are well described in each case.

I. INTRODUCTION

The reaction $\gamma N \rightarrow \pi^{\pm}\Delta$ provides a valuable opportunity for the study of an interesting example of nondiffractive two-body processes. Data are available[1,2] for each of the charge states at $E_\gamma = 16$ GeV and over the energy range[3] $E_\gamma = 5$ to 16 GeV for the case $\gamma p \rightarrow \pi^{-}\Delta^{++}$. Polarized-photon results[2] also allow a clear separation of the natural- and unnatural-parity-exchange components for this reaction. In the kinematic region of $|t| \leq m_\pi^2$, it is known[4] that the process is well described in terms of the contributions of the minimal gauge-invariant extension of one-pion exchange (MPE) to the dynamical amplitudes. This has been accounted for in part by the low-t theorems which are required by gauge invariance.[4,5]

More complex dynamical models are required for larger values of t, since additional contributions become significant. Efforts to understand the dynamics of the $\gamma N \rightarrow \pi^{\pm}\Delta$ reaction over a larger region of t have been recently pursued along two lines. Barbour and Malone[6] have studied fixed-t dispersion relations, with quark-model estimates for the low-energy imaginary parts of the amplitudes, which play a significant role in the larger-momentum-transfer region where the MPE forms are no longer adequate. A detailed quantitative analysis of the reaction using the full power of Regge theory with the inclusion of Regge poles, absorptive corrections, and Regge-Regge cuts has been carried out by Goldstein and Owens.[7] The results obtained are very good.

In this paper we report on the results of an alternate approach in which an elementary extension is made of the forms of the dynamical amplitudes established in the $|t| \leq m_\pi^2$ region. In this approach the extended amplitudes are parametrized by (1) modifying the one-pion exchange to a Regge form which reduces directly to the MPE form as $t \rightarrow m_\pi^2$ and (2) adding simple Regge-pole forms for ρ and A_2 with the assumption of exchange degeneracy. Since these forms vanish at $t \rightarrow 0$ (indicating evasive ρ and A_2), these additions are consistent with the MPE forms for small t. (3) Finally, we select the simplest possible extension of the least understood contribution to the amplitudes, referred to as the background amplitude and discussed in some detail in a previous paper.[5] This term, which arises from the low-t theorem, is extended from its MPE form by the simple inclusion of an exponential form factor.

In Sec. II we will discuss the relevant kinematics and forms for the amplitudes. Section III presents the results of the calculations and comparison with experimental results.

II. KINEMATICS AND AMPLITUDES

It was previously shown[5] that the $\gamma N \rightarrow \pi^{\pm}\Delta$ dynamics can be described conveniently in terms of a general set of manifestly gauge-invariant amplitudes A_j, which are defined by

$$f^{s}_{\lambda_f \lambda_i 0 \lambda_\gamma} = \sum_{j=1}^{8} A_j(s,t) M^{j}_{\lambda_f \lambda_i 0 \lambda_\gamma},$$

with

$$M_{\lambda_f \lambda_i 0 \lambda_\gamma} = \bar{u}_\nu(p_f \lambda_f) G_j^{\nu\mu} u(p_i, \lambda_i) \epsilon_\mu(k\lambda_\gamma).$$

The kinematic tensors $G_j^{\nu\mu}$ which define the A_j are given by

$$\begin{aligned}
G_1^{\nu\mu} &= (q^\nu - k^\nu)(k\cdot P q^\mu - k\cdot q p^\mu),\\
G_2^{\nu\mu} &= g^{\nu\mu} k\cdot q - k^\nu q^\mu,\\
G_3^{\nu\mu} &= g^{\nu\mu} k\cdot P - k^\nu P^\mu,\\
G_4^{\nu\mu} &= g^{\nu\mu} \not{k} - k^\nu \gamma^\mu,\\
G_5^{\nu\mu} &= (g^{\nu\mu} k\cdot P - k^\nu P^\mu)\not{k},\\
G_6^{\nu\mu} &= q^\nu (P^\mu \not{k} - k\cdot P \gamma^\mu),\\
G_7^{\nu\mu} &= k^\nu \gamma^\mu \not{k},\\
G_8^{\nu\mu} &= q^\nu \gamma^\mu \not{k}.
\end{aligned} \tag{1}$$

The momenta for the nucleon, Δ, pion, and photon are given by p_i, p_f, q, and k. Similarly, the masses and helicities are given by M_i, M_f, μ, and λ_i, λ_f, and λ_γ. With this choice of amplitudes, the forms for $A_{1\text{-}3}$ are determined in the low-t theorem limit

$$A_1(s,t) \xrightarrow[t\to\mu^2]{} \frac{4e_\pi f}{(t-\mu^2)(s-M_i^2)},$$
$$A_2(s,t) \xrightarrow[t\to\mu^2]{} 0, \tag{2}$$
$$A_3(s,t) \xrightarrow[t\to\mu^2]{} \frac{-2e_\pi f}{(s-M_i^2)},$$

where e_π is the pion charge and f is the $N\pi\Delta$ coupling constant. Experimental results indicate that for low t, the contributions from $A_{4\text{-}8}$ are negligible.

In this formalism the pion exchange contributes only to A_1. The A_3 has been labeled the background amplitude for its role in the forward direction, where it provides the only nonvanishing contribution to both the natural- and unnatural-parity-exchange polarized-photon differential cross section, $d\sigma^\perp/dt$ and $d\sigma^\parallel/dt$.

$$\frac{d\sigma^\perp}{dt} \underset{\substack{t\to t_{\min}\\ s\text{ large}}}{\sim} \frac{(M_i+M_f)^2}{96\pi}\,|A_3(s,t)|^2$$

$$\frac{d\sigma^\parallel}{dt} \underset{\substack{t\to t_{\min}\\ s\text{ large}}}{\sim} \frac{d\sigma^\perp}{dt} + \frac{|t-t_{\min}|}{384\pi M_f^2}\,|(M_i+M_f)(M_f^2-M_i^2)A_1(s,t) + 2(M_i+M_f)A_2(s,t) + (M_i+2M_f)A_3(s,t)|^2. \tag{3}$$

The philosophy of our model is to begin with the forms for the amplitudes which are known in the near forward direction and then make the simplest modifications necessary which will reduce to the MPE forms in the near forward (low-t) region. First, we Reggeize the pion-exchange amplitudes. Second, we include the additional t-channel exchanges expected to contribute (ρ, A_2 and B) with the simplifying assumption of exchange degeneracy. These exchanges are parametrized with experimentally determined values determined in other processes. Finally, we make the simplest possible extension of the background term by introducing an exponential form factor. We now discuss the specific form of each of these contributions.

A. Pion exchange

Since the pion exchange contributes only to A_1 and A_1 has been shown to contribute to only one of the t-channel parity-conserving helicity amplitudes $\bar{f}^{t-}_{(1/2)\,(1/2)\,0\,1}$, we can define the kinematical singularity free form $\bar{f}^{t-}_{(1/2)\,(1/2)\,0\,1}$ in terms of A_1. By Reggeizing $\bar{f}^{t-}_{(1/2)\,(1/2)\,0\,1}$ we obtain an appropriate Regge form for the pion-exchange contribution to A_1. We then have

$$A_1^{(\pi)}(s,t) = \bar{f}^{t-(\pi)}_{(1/2)\,(1/2)\,0\,1}.$$

The simplest Regge prescription for evasive pion exchange is given by[7]

$$\bar{f}^{t-(\pi)}_{(1/2)\,(1/2)\,0\,1} = \beta_\pi(t)\frac{(e^{-i\pi\alpha_\pi}+1)}{\Gamma(\alpha_\pi+1)\sin\pi\alpha_\pi} z^{\alpha_\pi-1}. \tag{4}$$

Now for large s, we have the limit for $z=\cos\theta_t$,

$$z^{-1} \to \frac{(t-\mu^2)[t-(M_i+M_f)^2]^{1/2}[t-(M_f-M_i)^2]^{1/2}}{2st}.$$

Finally, by imposing the low-t theorem limit for A_1 given in Eq. (2) on the Regge form of the amplitude Eq. (4), and assuming the experimentally determined form[8] for the pion trajectory $\alpha_\pi(t) = 0.7(t-\mu^2)$, we constrain the form for $\beta_\pi(t)$ and obtain

$$A_1^{(\pi)}(s,t) = \frac{4e_\pi f\hat{\beta}_\pi(t)}{(t-\mu^2)(s-M_i^2)}\frac{(e^{-i\pi\alpha_\pi}+1)}{2}\Gamma(1-\alpha_\pi)z^{\alpha_\pi}, \tag{5}$$

where the reduced residue

$$\hat{\beta}_\pi(t) \xrightarrow[t\to\mu^2]{} 1.$$

We will represent $\hat{\beta}_\pi(t)$ with the simple parametrization $\exp(-\gamma_\pi|t-\mu^2|)$. The value for the $N\pi\Delta$ coupling f is obtained directly from the width[9] of the $\Delta^{++} \to p\pi^+$ decay, giving $f^2/4\pi = 18.9\ \text{GeV}^{-2}$.

B. Additional t-channel exchanges (ρ, A_2, and B)

The analysis of ρ exchange with the assumption of the simplified Stodolsky-Sakurai form[10] for the $\Delta\rho N$ couplings, in terms[5] of A_j, has shown that the ρ exchange contributes only to the three amplitudes $A_{1\text{-}3}$, previously found necessary in MPE. The results give

$$A_1^{(\rho)}(s,t) \xrightarrow[t\to m_\rho^2]{} \frac{g_{\pi\gamma\rho} f_{\Delta\rho N}}{t-m_\rho^2}, \tag{6a}$$

with

$$A_2^{(\rho)}(s,t) = \tfrac{1}{2}(M_i^2-M_f^2)A_1^{(\rho)}(s,t)$$

and

$$A_3^{(\rho)}(s,t) = (-t)A_1^{(\rho)}(s,t). \tag{6b}$$

As expected,[7] the ρ exchange contributes only to the $\bar{f}^{t+}_{(3/2)\,(1/2)\,0\,1}$ and $\bar{f}^{t+}_{(1/2)\,-(1/2)\,0\,1}$, which are related to each other by

$$\bar{f}^{t+}_{(1/2)-(1/2)\,0\,1} = -(1/\sqrt{3})\bar{f}^{t+}_{(3/2)\,(1/2)\,0\,1},$$

and to the A_3 amplitude by

$$\bar{f}^{t+}_{(3/2)\,(1/2)\,0\,1} = \frac{-(t-\mu^2)[t-(M_f+M_i)^2][t-(M_f-M_i)^2]}{4(M_iM_f)^{1/2}} \times A_3(s,t).$$

We therefore define the kinematical-singularity-free parity-conserving helicity amplitude

$$\tilde{f}^{t+(\rho)}_{(3/2)(1/2)01} \equiv A_3^{(\rho)},$$

and proceed with the standard evasive-ρ Regge prescription[7]

$$\tilde{f}^{t+(\rho)}_{(3/2)(1/2)01} = \beta_\rho(t)\frac{(e^{-i\pi\alpha_\rho}-1)}{\Gamma(\alpha_\rho)\sin\pi\alpha_\rho} z^{\alpha_\rho-1}. \tag{7}$$

Using the $t \to m_\rho^2$ limit from Eqs. (6) we obtain

$$A_3^{(\rho)}(s,t) = -\tfrac{1}{2}(-t)\alpha'_\rho g_{\pi\gamma\rho} f_{\Delta\rho N}\hat{\beta}_\rho(t) \times (e^{-i\pi\alpha_\rho}-1)\Gamma(1-\alpha_\rho)z^{\alpha_\rho-1} \tag{8}$$

and corresponding expressions for $A_1^{(\rho)}$ and $A_2^{(\rho)}$. Once again the reduced residue function

$$\hat{\beta}_\rho(t) \xrightarrow[t\to m_\rho^2]{} 1.$$

The $f_{\Delta\rho N}$ coupling constant is obtained from experimental results for $\Delta \to N + \gamma$ with the vector-dominance assumption suggested by Stodolsky and Sakurai.[10] The corresponding $g_{\pi\gamma\rho}$ is taken from the recent analysis of $\gamma A \to \rho A$ by Gobbi *et al.*,[11] which yields a value $\frac{1}{4}$ the magnitude expected from SU(3). The phenomenological ρ-trajectory function $\alpha_\rho(t) = 0.9t + 0.47$ is taken from the analysis of Fox and Quigg[12] for a number of ρ-exchange reactions.

The inclusion of A_2 and B exchanges is simplified through the assumption of exchange degeneracy,[13] in which the $A_2(B)$ and the $\rho(\pi)$ are assumed to share common trajectories, but have opposite signatures. The appropriate expression for the A_2 contribution is then given by

$$A_3^{(A_2)} = \beta_{A_2}(t)\frac{(-t)(e^{-i\pi\alpha_\rho}+1)}{\Gamma(\alpha_\rho)\sin\pi\alpha_\rho} z^{\alpha_\rho-1}, \tag{9}$$

with corresponding forms for $A_1^{(A_2)}$ and $A_2^{(A_2)}$. For the B exchange we have

$$A_1^{(B)} = \beta_B(t)\frac{(e^{-i\pi\alpha_\pi}-1)}{\Gamma(\alpha_\pi+1)\sin\pi\alpha_\pi} z^{\alpha_\pi-1}. \tag{10}$$

The residue relations implied by exchange degeneracy have been derived for $\gamma N \to \pi^\pm\Delta$ by Goldstein and Owens,[7] giving for the $\gamma p \to \pi^-\Delta^{++}$ case

$$\beta_{A_2} = -3\beta_\rho$$

and (11)

$$\beta_B = -\tfrac{1}{3}\beta_\pi.$$

C. Background term

It is at this stage that the model under consideration differs most dramatically from the more complex models previously considered. For our purposes we take the MPE form for the background contribution to the A_3 amplitude and modify it with a simple exponential form factor

$$A_3^{(\mathrm{Bac})} = \frac{-2e_\pi f e^{-\gamma_{\mathrm{Bac}}|t-\mu^2|}}{s - M_i^2}. \tag{12}$$

This procedure gives us a result which trivially reduces to the MPE form in the low-t region [Eq. (12)] and remains real over the entire t range.

The reality of this contribution may be considered as a phenomenological convenience, but could also correspond to the dynamical existence of a fixed pole in this amplitude.[14]

D. Other charge states

The problem of finding the appropriate relation for the remaining charge states is primarily a matter of isospin algebra. We find that the relative signs for the $\rho\gamma\pi$ (and $B\gamma\pi$) are given by $g(\pi^+\gamma\rho^+) = g(\pi^-\gamma\rho^-)$. For the $\pi\gamma A_2$ coupling the relation is $g(\pi^+\gamma A_2^+) = -g(\pi^-\gamma A_2^-)$. In the case of the meson-baryon vertex, the coupling relations for the $N\rho\Delta$ and $NA_2\Delta$ are the same as for the $N\pi\Delta$,

$$f_{p\pi^-\Delta^{++}} = f_{n\pi^-\Delta^-} = \sqrt{3}\, f_{p\pi^+\Delta^0} = \sqrt{3} f_{n\pi^-\Delta^+}.$$

The relative signs between the ρ and π, and therefore the background term and ρ exchange, can be obtained from simple quark models.[15] We finally find the results for the various contribution can be expressed in the following form:

$$\begin{aligned} A(\gamma p \to \pi^-\Delta^{++}) &= -A^{(\pi)} + A^{(\mathrm{Bac})} + A^{(\rho)} - A^{(A_2)} + A^{(B)},\\ A(\gamma n \to \pi^+\Delta^-) &= A^{(\pi)} - A^{(\mathrm{Bac})} + A^{(\rho)} + A^{(A_2)} + A^{(B)},\\ A(\gamma p \to \pi^+\Delta^0) &= \sqrt{1/3}\, A(\gamma n \to \pi^+\Delta^-),\\ A(\gamma n \to \pi^-\Delta^+) &= \sqrt{1/3}\, A(\gamma p \to \pi^-\Delta^{++}). \end{aligned} \tag{13}$$

III. RESULTS

Our procedure has been to direct our attention first to the determination of the model parameters which are not available from other experiments. For this reason we compare the model with the unnatural-parity-exchange $d\sigma^{u}/dt$ for $\gamma p \to \pi^-\Delta^{++}$. As discussed previously, the relevant contributions are then made by the background term, pion exchange, and possible B exchange. In our model the only free parameters are then the γ_π and γ_{Bac} from the exponential factors in the pion exchange and background terms. The calculations are all carried out by inserting our dynamical forms for A_j into expressions for the differential cross sections, in which the kinematics have been obtained in an exact form using Hearn's REDUCE system[16] for algebraic computation. The best fit to the data is obtained for $\gamma_{\mathrm{Bac}} = 2.86$ and $\gamma_\pi = 0.38$. The results are shown in Fig. 1. We conclude that this simple parametrization for the unnatural-parity exchange provides an acceptable fit to the available data. When we examine the effect of re-

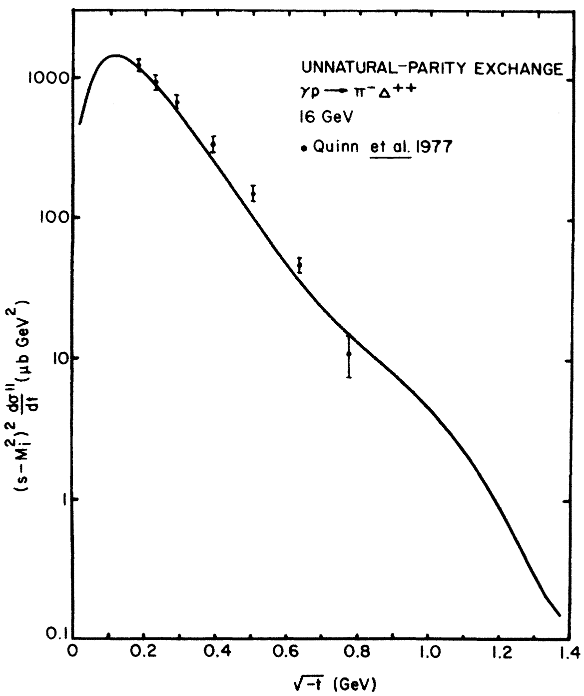

FIG. 1. Differential cross section for photons polarized parallel to the reaction plane $(s-M_i^2)^2\, d\sigma^{\parallel}/dt$, for $\gamma p \to \pi^- \Delta^{++}$ at $E_\gamma = 16.0$ GeV. This corresponds to unnatural-parity exchange. Data from Ref. 2 is used to fit the parameters $\gamma_{\beta ac} = 2.86$ and $\gamma_\pi = 0.38$.

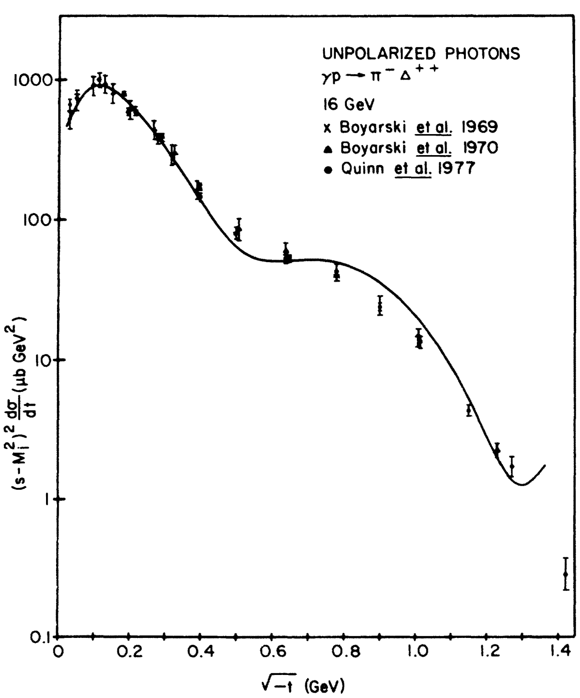

FIG. 2. Model predictions for $(s-M_i^2)^2\, d\sigma/dt$; $\gamma p \to \pi^- \Delta^{++}$. Experimental points are from Refs. 1, 2, and 3.

taining or neglecting the B-exchange contribution with assumed $\pi - B$-exchange degeneracy, we find its contribution negligible. It is, however, retained for completeness.

With the parameters for the π exchange and background term determined from $d\sigma^{\parallel}/dt$, all of the model parameters are fixed and we proceed to make comparisons with other available data. We assume that the ρ-A_2 parametrization is completely determined from experiment, as discussed above, with the simplifying assumption that we may represent the reduced residue $\hat{\beta}_\rho$ by unity. The model is now used to predict the unpolarized differential cross section $d\sigma/dt$ which includes both the unnatural- and natural-parity exchange for the same process $\gamma p \to \pi^- \Delta^{++}$. The results are shown in Fig. 2 in comparison with the available data from the three experiments at 16 GeV. Once again we conclude that in spite of its simplicity, the model provides an acceptable parametrization of the data up to $|t| < 1.3$ GeV^2. An equivalent, but more sensitive test of the model is provided by a comparison of the model's predictions with the polarized-photon asymmetry Σ, where

$$\Sigma = \frac{d\sigma^{\perp}/dt - d\sigma^{\parallel}/dt}{d\sigma^{\perp}/dt + d\sigma^{\parallel}/dt}\,. \tag{14}$$

The results for $\gamma p \to \pi^- \Delta^{++}$ are exhibited in Fig. 3. The results are rather encouraging, since we find that our simple model predicts not only the qualitative behavior of the asymmetry, but also provides an acceptable quantitative fit to the data. We see that both the crossing point ($\Sigma = 0$) and some of the curve structure seem to be predicted rather well. It is surprising that the largest deviations appear at the lowest values of t, where the predictions are least model dependent. It is possible that, since this must be a region of extremely rapid variation of $d\sigma^{\parallel}/dt$, (and therefore Σ), the assumptions of t dependence made in analyzing the

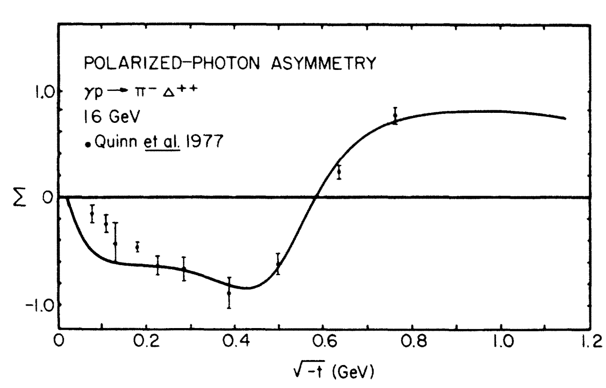

FIG. 3. Model predictions for the polarized-photon asymmetry Σ for $\gamma p \to \pi^- \Delta^{++}$ at $E_\gamma = 16.0$ GeV. Experimental points are from Ref. 2.

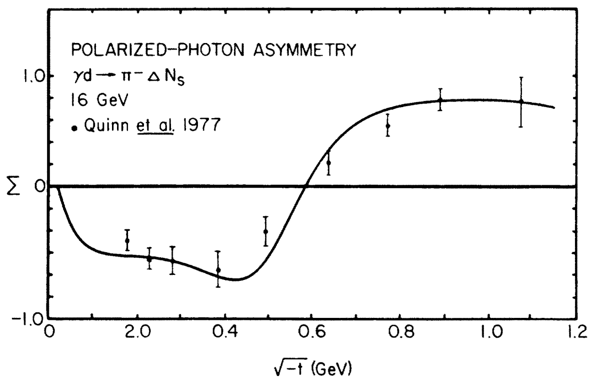

FIG. 4. Model predictions for the polarized-photon asymmetry Σ for $\gamma d \to \pi^- \Delta N_s$ at $E_\gamma = 16.0$ GeV. Experimental points are from Ref. 2.

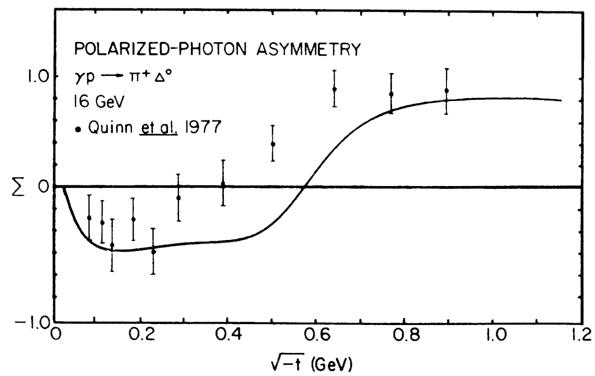

FIG. 6. Model predictions for Σ for $\gamma p \to \pi^+ \Delta^0$ at $E_\gamma = 16.0$ GeV. Experimental points are from Ref. 2.

data from bins of finite Δt can have a significant effect on the results.

We now direct our attention to comparing our simple model with experimental data for other charge states. The simplest case is $\gamma d \to \pi^- \Delta N_s$, where our model predicts an asymmetry exactly equivalent to that of $\gamma p \to \pi^- \Delta^{++}$, just considered. The results are shown in Fig. 4. The fits are once again consistent with the model predictions. This is not surprising since it can be considered as a consequence of the absence of $I = 2$ exchange contributions.

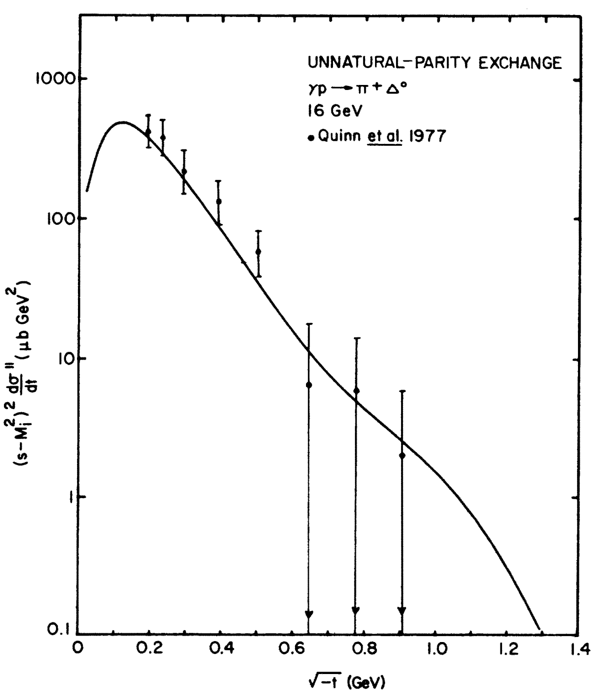

FIG. 5. Model predictions for $(s-M_i^2)^2 d\sigma''/dt$ (unnatural-parity exchange) for $\gamma p \to \pi^+ \Delta^0$ at $E_\gamma = 16.0$ GeV. Experimental points are from Ref. 2.

We now turn our attention to the positive-pion-charge states. The variation in the relevant couplings for alternate charge states was discussed in the previous section, so that the parameters are once again completely determined for our model.

Following our previous approach, we direct our attention first to the unnatural-parity-exchange component $d\sigma''/dt$ of the reaction $\gamma p \to \pi^- \Delta^0$. The results are displayed in Fig. 5, reinforcing the appropriateness of the parametrization of the unnatural-parity exchange in our simple model. We then direct our attention to the polarized-photon asymmetry for $\gamma p \to \pi^+ \Delta^0$, shown in Fig. 6. Here we find the results somewhat frustrating. Although the qualitative changes are in the proper direction (e.g., earlier crossing and a shallower trough in the lower-t region) the quantitative agreement is much less impressive than in the previous cases. Since the unnatural-parity-exchange parametrization appears adequate for this charge state, the blame must apparently be attributed to our form for the natural-parity exchange. Since the background term is common to both the natural- and unnatural-parity-exchange components it is easiest to blame the simple form or general inadequacy of the ρ-A_2 exchange contribution.

It is also instructive to consider the ability of this model to predict the dependence of the interaction on energy. Excellent data for unpolarized $d\sigma/dt$ in $\gamma p \to \pi^- \Delta^{++}$ are available[3] at 5, 8, 11, and 16 GeV. Since the energy dependence of the data is fit well in the $|t| < \mu^2$ region by the low-t theorem amplitudes,[4] it is not surprising that the energy dependence is described well in this forward region by our gauge-invariant extension of the model, as seen in Fig. 7. In the intermediate region $\mu^2 \leq |t| \leq 0.6$ GeV^2, it is satisfying to see that the model gives reasonable predictions over the entire energy range from 5 to 16 GeV. However, in the larger $|t|$ region $0.5 \leq |t| \leq 1.4$ GeV^2, where

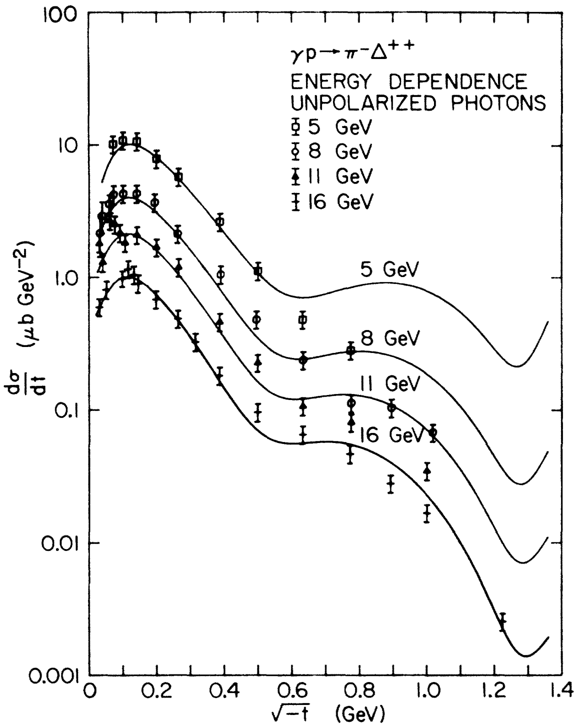

FIG. 7. Model predictions of $d\sigma/dt$, for $\gamma p \to \pi^- \Delta^{++}$; showing the energy dependence from $E_\gamma = 5.0$ to 16.0 GeV. Experimental data are from Ref. 3.

the natural-parity-exchange contribution from the model begins to play an important role, we see that the deviation of the model predictions from the data become increasingly pronounced as the energy is decreased from 11 to 8 to 5 GeV. This result emphasizes the weakness of our parametrization of the natural-parity ρ and A_2 exchange in this simplified model.

An additional test for the model involves the comparison with the density-matrix elements, since these terms are particularly sensitive to the relative phases of the contributing amplitudes[17] and comparison could provide additional valuable information regarding the assumed form for the background amplitude and its phase relative to the exchange contributions. Unfortunately, the $\gamma p \to \pi^- \Delta^{++}$ data available[18] at 2.8 and 4.7 GeV have very low statisics and very broad data bins. We have made a comparison of the model predictions with the 4.7 GeV data and the results are presented in Fig. 8. The results are interesting, but the conclusions are limited by the large errors in the data. The overall χ^2 per data point for the nine density matrix and P_σ is 1.05. It is not possible to draw any strong conclusions, since detailed analysis of the contributions of specific amplitudes indicates that those cases where agreement seems worse (such as $\rho_{33}{}^1$) are sensitive to the same relative phases

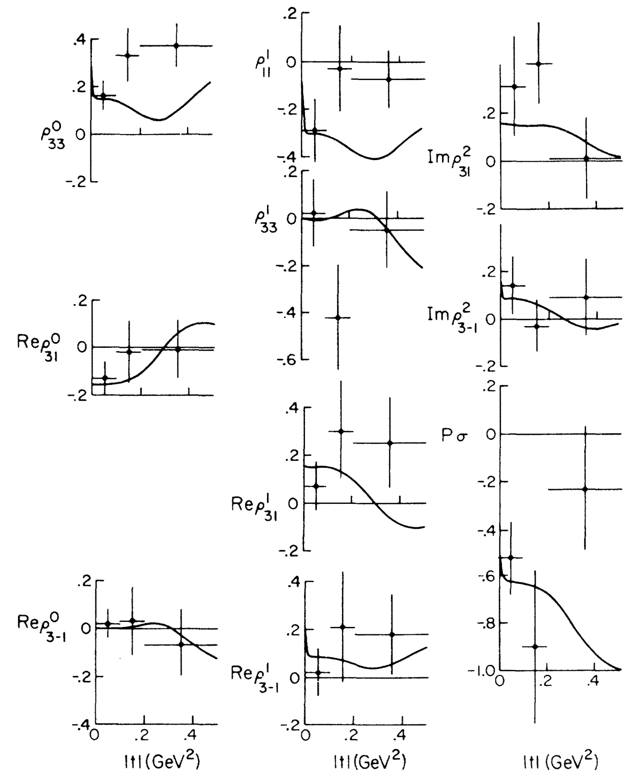

FIG. 8. Density-matrix parameters and parity asymmetry P_σ for $\gamma p \to \pi^- \Delta^{++}$ at $E_\gamma = 4.7$ GeV. The density-matrix parameters are for the Gottfried-Jackson system and the data and parameter definitions are from Ref. 18.

between the background and exchange contributions as those cases where the agreement appears excellent (such as the ρ_{31}^0 and ρ_{3-1}^0). Qualitatively, the overall χ^2 agreement is encouraging, but could also be attributed to the present quality of the density-matrix data.

In conclusion we have seen that our simple extension of the gauge-invariance-constrained amplitudes, assuming a real background term and simple Regge-pole forms, provides a good quantitative description of the unnatural-parity-exchange contribution in $\gamma N \to \pi^{\pm} \Delta$. The natural-parity-exchange contributions appear qualitatively correct, but require a more complex form to provide quantitative agreement with the data.

IV. ACKNOWLEDGMENTS

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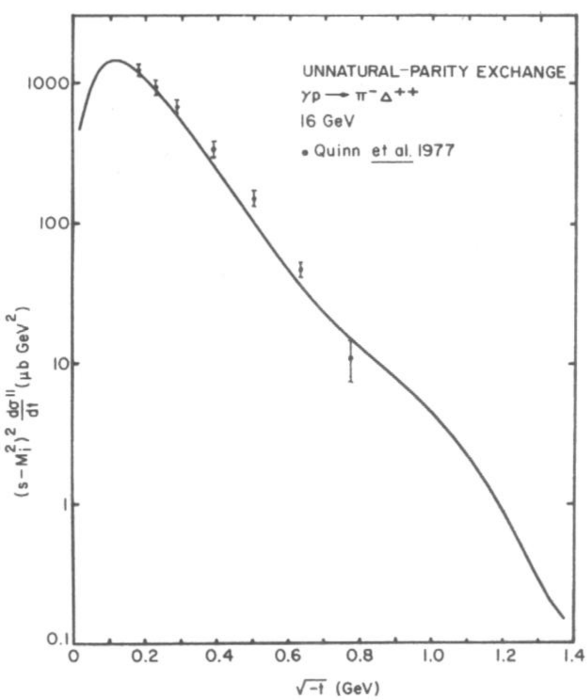

FIG. 1. Differential cross section for photons polarized parallel to the reaction plane $(s-M_i^2)^2\ d\sigma^{\parallel}/dt$, for $\gamma p \rightarrow \pi^- \Delta^{++}$ at $E_\gamma = 16.0$ GeV. This corresponds to unnatural-parity exchange. Data from Ref. 2 is used to fit the parameters $\gamma_{\beta ac} = 2.86$ and $\gamma_\pi = 0.38$.

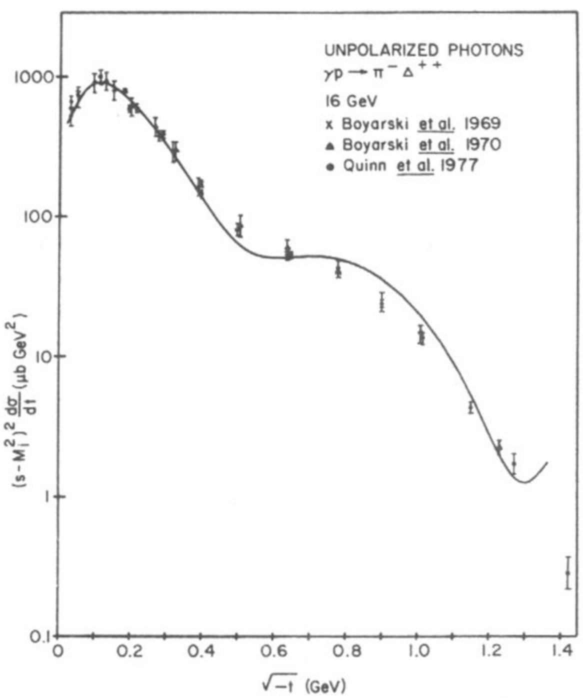

FIG. 2. Model predictions for $(s - M_i^2)^2\, d\sigma/dt$; $\gamma p \to \pi^- \Delta^{++}$. Experimental points are from Refs. 1, 2, and 3.

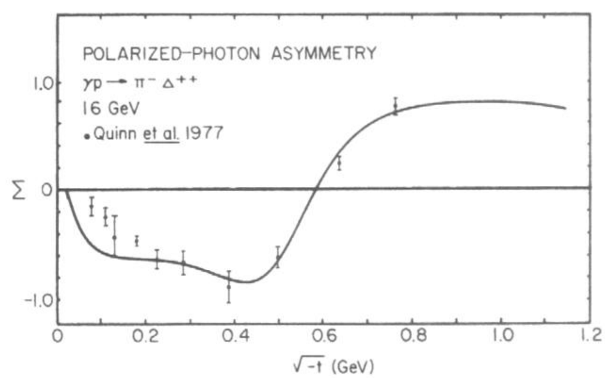

FIG. 3. Model predictions for the polarized-photon asymmetry Σ for $\gamma p \to \pi^- \Delta^{++}$ at $E_\gamma = 16.0$ GeV. Experimental points are from Ref. 2.

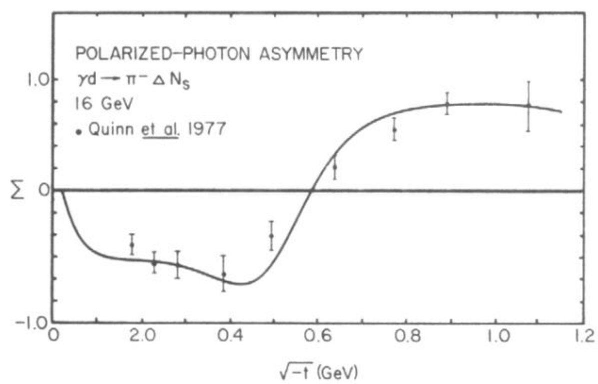

FIG. 4. Model predictions for the polarized-photon asymmetry Σ for $\gamma d \to \pi^- \Delta N_s$ at $E_\gamma = 16.0$ GeV. Experimental points are from Ref. 2.

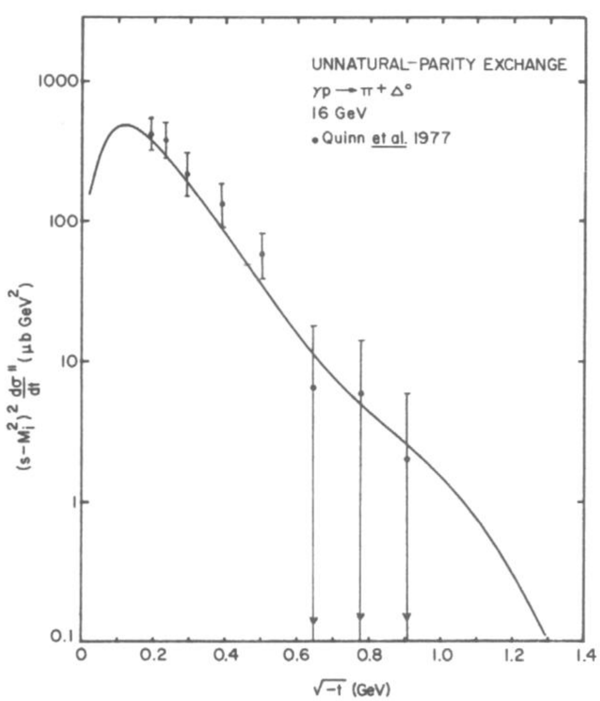

FIG. 5. Model predictions for $(s-M_i^2)^2 d\sigma''/dt$ (unnatural-parity exchange) for $\gamma p \to \pi^+ \Delta^0$ at $E_\gamma = 16.0$ GeV. Experimental points are from Ref. 2.

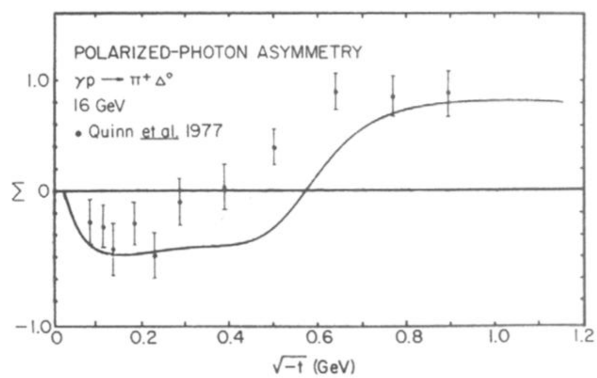

FIG. 6. Model predictions for Σ for $\gamma p \rightarrow \pi^+ \Delta^0$ at $E_\gamma = 16.0$ GeV. Experimental points are from Ref. 2.

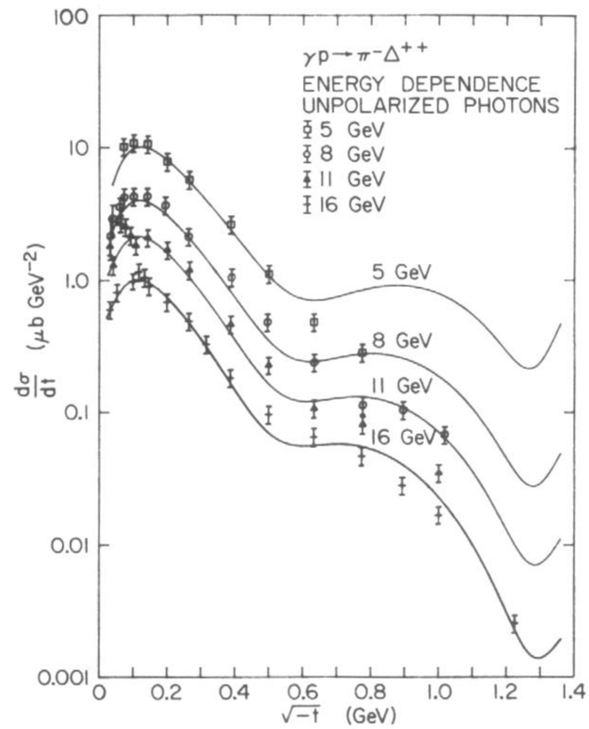

FIG. 7. Model predictions of $d\sigma/dt$, for $\gamma p \to \pi^- \Delta^{++}$; showing the energy dependence from $E_\gamma = 5.0$ to 16.0 GeV. Experimental data are from Ref. 3.

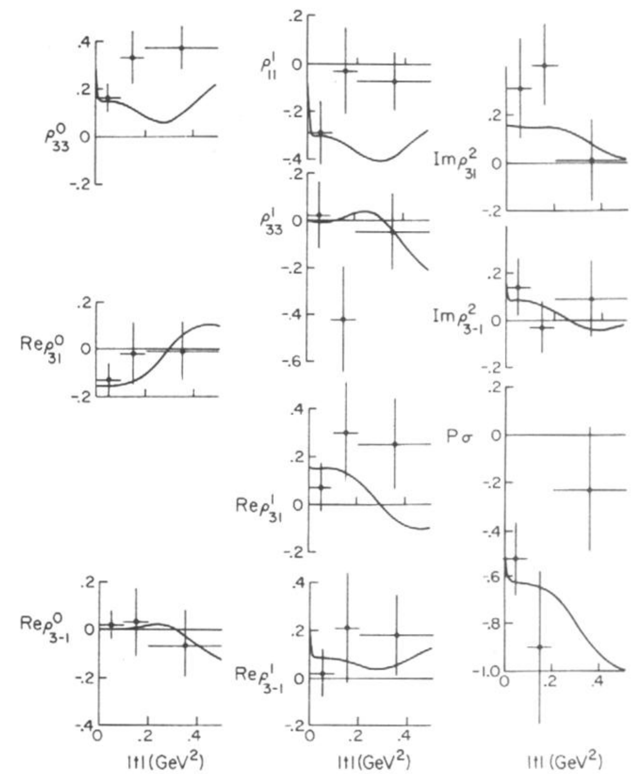

FIG. 8. Density-matrix parameters and parity asymmetry P_σ for $\gamma p \to \pi^- \Delta^{++}$ at $E_\gamma = 4.7$ GeV. The density-matrix parameters are for the Gottfried-Jackson system and the data and parameter definitions are from Ref. 18.