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Reply to "Use of chiral symmetry in pion-condensation calculations"

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We review our earlier results on the role of chiral symmetry in pion condensation and show how they are completely consistent with Sawyer's observations in the preceding comment.

In the preceding comment,[1] Sawyer gives a correct and useful example of the limitations of the use of chiral symmetry in pion-condensation calculations. However, in referring to our original work on this subject,[2,3] he somewhat overstates our claims, possibly giving the impression that we asserted that the existence of pion condensation in high-density nuclear matter is a definite prediction—a theorem, if you will—of chiral symmetry. In fact, we never made such a claim; this is fortunate for us, because it would have been false. The existence of pion condensation in nuclear matter[4] is *not* a theorem of chiral symmetry. Note, however, that this statement does not contradict the assertion that an approach based on chiral symmetry can provide a useful framework in which to study pion condensation. This we did assert, and we feel that subsequent calculations have supported this claim.[5-10] But since the issue of the role of chiral symmetry in pion condensation has now been revived, for the purpose of clarity it is perhaps useful for us to recall, briefly and qualitatively, the precise nature of this role.

Our initial articles on pion condensation were two.[2,3] In the first (hereafter I), entitled "I. Model-dependent results," we presented (and labeled as such) a simplified, model calculation of pion condensation in the linear σ model to illustrate some of our ideas. The second article (hereafter II) was called "II. General formalism." Note in particular that the title was *not* "Model-independent results." In this article we tried to separate clearly those aspects of pion condensation which are substantially determined by chiral symmetry from those which are not. Thus it is perhaps appropriate to begin our specific remarks with this point.

In I and II we considered the case of charged-pion condensation in (essentially) pure neutron matter. Thus we shall restrict our comments here to this case. We began with the standard *Ansatz* for the condensed (charged) pion field

$$\langle \pi(\vec{x}, t)\rangle = A \sin\theta\, e^{i(\vec{k}\cdot\vec{x}-\mu t)}, \qquad (1)$$

where k is the momentum of the condensate, μ is the energy (= the pion chemical potential), and A is a fixed constant.[5,11] The threshold for pion condensation occurs for $\theta \approx 0$, whereas a well-developed condensate has a finite value of θ. In order to study both the threshold conditions and the equation of state at finite θ, in II we set up the calculation of the energy of neutron matter in the presence of a charged-pion condensate with an arbitrary θ. In particular, we showed that it was possible to divide this energy into three pieces. These were

(1) E^v, the "vacuum" energy, which depends only on the condensed-pion field and is independent of the neutron density ρ [i.e., $O(\rho^0)$];

(2) E^H, the "Hartree" energy, which is *defined* to depend on interactions of the condensed field with single nucleons and hence is linearly proportional to ρ; and

(3) E^C, the "correlation" energy, which loosely speaking includes interactions of the pion with two or more nucleons (irreducibly, of course) and thus intrinsically involves the nuclear correlations (N-N interactions) and is (naively) proportional to ρ^2 (and higher powers).

As noted, we made this division for arbitrary θ. In the limit $\theta \to 0$, the general result that[2]

$$E(\theta, \omega, k) = E(0) - \frac{D^{-1}(\omega, k)}{2} A^2\theta^2 + O(\theta^4) + \cdots \qquad (2)$$

allows us to retain this division for terms contributing to the inverse pion propagator D^{-1}. Indeed E^V—as expected—involves the free propagator,

$$D_0^{-1} = \mu^2 - k^2 - m_\pi^2, \qquad (3)$$

whereas E^H involves terms linear in ρ in the pion polarization (self-energy) Π and E^C terms of $O(\rho^2)$ and higher in Π.

Our reason for isolating these three terms in the energy (or propagator) was that chiral symmetry (plus an explicit form for the symmetry breaking) completely determines E^V for all θ and also predicts the *structure* of E^H (in terms of vector and axial-vector current matrix elements), again for arbitrary θ. Note that the explicit form and numerical value of E^H depend on considerations beyond chiral symmetry—e.g., the inclusion of the $\Delta(1236)$ resonance and an accurate modeling of the low-energy isospin-even πN s wave—but these effects can be incorporated into the chiral-symmetry formalism as phenomenological modifications.

Intuitively, one can see that chiral symmetry should have little to say about E^C, the correlation energy.[12] This is so because chiral symmetry has to do with "soft pions" and thus, while giving information about π-π ($\sim E^V$) and π-N ($\sim E^H$) interactions, does not say much about N-N ($\sim E^C$) interactions. Thus in both I and II we stated clearly and explicitly that E^C was not determined (even in structure) by chiral symmetry; rather, it had to be modeled consistent with the phenomenology of N-N interactions.

Having made this division clear, we can now see how Sawyer's remarks fit into this framework and discover whether in fact "other chiral models"—i.e., other than the linear σ model—"have been effectively discarded" in our work. From Sawyer's Eq. (11) we see that in the chiral model B (with the ρ meson) the pion polarization is given, in notation and sign convention consistent with those of I and II, by[13]

$$\Pi(\omega, k) = \frac{g_A{}^2 k^2 \rho}{2f_\pi{}^2(\mu - \rho/4f_\pi{}^2)} - \frac{\mu\rho}{2f_\pi{}^2}. \qquad (4)$$

Expanding this in powers of the density shows immediately that the term linear in ρ—coming from E^H, in our terminology—is, as expected, identical[14] to that found in the σ model.[2] Equation (4) differs from the σ-model result in $O(\rho^2)$ and higher, and thus these differences arise, in our terminology, from E^C, that part of the energy which we said is *not* determined by chiral symmetry. From this perspective it is not at all surprising that chiral models having different N-N interactions—as do the two models considered here—give rise to different results. Indeed, in Sec. V of I we discussed (among other nuclear force effects) precisely this ρ exchange correction and showed (qualitatively) how to include it as a phenomenological modification (like the Δ resonance) in our formalism.

We are now in a position to explain the relation between Eqs. (3.6) and (3.15) of II, which equations Sawyer felt led to some confusion. As reference to II clearly shows, Eq. (3.6) refers to the full effective Lagrangian and thus would give rise to contributions to all of E^V, E^H, and E^C. Equation (3.15), however, is clearly labeled as referring only to E^H; the "missing" terms have been subsumed—correctly, as Sawyer's calculation nicely shows—into E^C. Further, far from being dismissed as unimportant, the effects of the interaction terms in $\mathcal{L}_{SYM}$, including the way in which the explicit θ-dependent terms act in concert with spin- or isospin-dependent forces, are discussed in detail in Sec. V of I.

From all this discussion it should be clear that our approach is not limited to the σ model and that we did not effectively discard other chiral models. One final aspect of our division of the total energy into E^V, E^H, and E^C remains to be stressed. Nothing in this division asserts that any of these is the "largest" or "most important" contribution. Indeed, as a comparison of Sawyer's two calculations shows, the terms can be roughly comparable.[15] The virtue of the division is that it collects in one term—E^C—those effects which we can not determine from chiral symmetry. Further, since this term involves essentially N-N interactions, about which we know a good deal empirically, one can hope to model this term phenomenologically.

This last remark leads naturally to the question of how one should model these nuclear correlation effects in pion-condensation calculations. Although this question is by no means settled, since even a partial answer is illuminating, let us close with a few relevant remarks on this subject. For definiteness, we will study these effects only at threshold and focus on their influence on the p-wave part of the pion polarization, which is dominated[16] by the particle-hole state shown schematically in Fig. 1(a).

In Figs. 1(b) and 1(c) we show two types of (in our terminology) "nuclear correlation" effects. The first effect, which is shown in Fig. 1(b), produces shifts in the energies of the particle and hole states and is often called a "dispersion correction." This is exactly the effect discussed by Sawyer and a straightforward calculation of the (appropriate charge versions of the) diagrams shown in Fig. 1(b) yields the p wave part of Π in Sawyer's model B. In place of the ρ^0 exchange shown explicitly in Fig. 1(b), of course, one should include all possible exchanges, as described, for example, by the full nuclear G matrix. This amounts to "dressing" the particles to "quasiparticles" by calculating the self-consistent potential in which the particles move. Precisely this calculation has been done by Bertsch and Johnson.[17] They find [17,18] that at threshold (assuming this occurs near twice nuclear density) the total "dispersion corrections" are small but that, as suggested earlier by Bethe and Johnson,[19] for

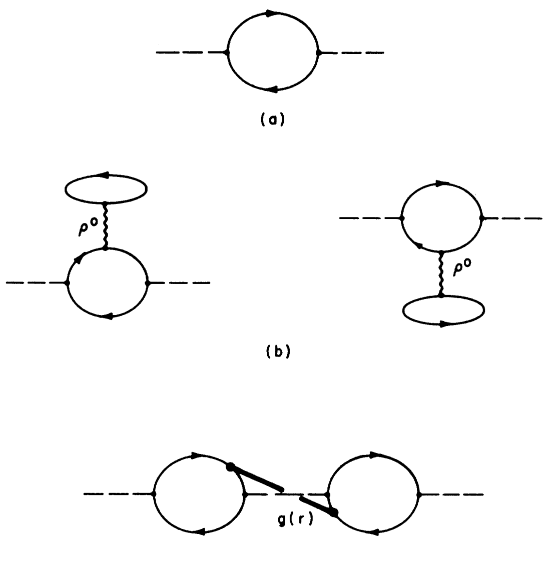

FIG. 1. Generic particle-hole diagrams contributing to the p-wave pion polarization part. (a) The bare diagram; (b) diagrams with "dispersive" corrections to the particle and hole energies arising from the expectation of the ρ^0 field induced by the neutron medium; (c) the particle-hole interaction which corresponds to the second-order Lorentz-Lorenz effect and involves the "exchange" of the NN correction function $[g(r)]$ plus one pion.

a well-developed condensate such corrections can produce large, repulsive (i.e., tending to *inhibit* condensation) effects.[20] The second effect, shown in Fig. 1(c), is the (second order) Ericson-Ericson Lorentz-Lorenz effect [5,21,22] and can be thought of as describing a particle-hole interaction which lessens the p-wave attraction and thus inhibits pion condensation. It is currently felt that the bulk of the (repulsive) nuclear correlation effects can be modeled by allowing the parameter describing the size of the Lorentz-Lorenz effect to be determined phenomenologically. Both the existence and the structure of the pion condensate are very sensitive to the actual value of this parameter,[23] but for the most favored value, pion condensation does occur. Nonetheless, as Sawyer notes, the greatest remaining uncertainties in pion-condensation calculations may arise from uncertainties in these nuclear correlation effects, particularly when the Δ resonance is included. Clearly more detailed studies of these effects, both at threshold and for a well-developed condensate, are needed. In this regard, it is interesting to note that both the "dispersive corrections" discussed by Sawyer and the (phenomenological) Lorentz-Lorenz effect, although by no means consequences of chiral symmetry, can be readily incorporated into the chiral-symmetry formalism.[5,10]

One last point should precede our concluding remarks. In view of the established limitations on the σ model, it is fair to ask the extent to which one can view it as a general paradigm for—or, more specifically, use it for calculations involving—pion condensation.[24] From the perspective of pion condensation the grossest defects on the σ model involve the absence of (1) repulsive nuclear correlations (which can be modeled by the Lorentz-Lorenz effect) and (2) attractive contributions to the $\pi N p$ waves coming from the Δ resonance. For the expected values of the parameters, these two effects tend roughly to cancel, both at threshold[5,25] and over the entire range of θ.[6] Further, the extent to which they do not cancel can be described by a phenomenological renormalization of g_A, upward (downward) if the attractive (repulsive) effects are stronger. For these reasons, particularly in investigative qualitative calculations,[7] the use of the simple σ model seems justified.

To conclude we note that we are in full agreement with Sawyer's statement that chiral symmetry does not predict pion condensation in nuclear matter. However, we feel that our formalism based on chiral symmetry has aided, and will continue to aid, more detailed, phenomenologically accurate studies of pion condensation. Thus we submit that our fundamental assertion in I—"the approximate SU(2)$\times$SU(2) chiral symmetry of the strong interactions can provide a framework of significant conceptual and modest calculational utility in confronting the very real intricacies of pion condensation"—remains true.

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[1]R. F. Sawyer, preceding paper, Phys. Rev. D 18, 1339 (1978).
[2]D. K. Campbell, R. F. Dashen, and J. T. Manassah, Phys. Rev. D 12, 979 (1975).
[3]D. K. Campbell, R. F. Dashen, and J. T. Manassah, Phys. Rev. D 12, 1010 (1975).
[4]To eliminate one possible source of confusion we should remark that in the (physically less interesting) case of

mesons only—that is, no nuclear matter but with a pion chemical potential controlled by external sources—pion condensation *is* a theorem of chiral symmetry. See R. F. Dashen and J. T. Manassah, Phys. Lett. 50B, 460 (1974).

[5]G. Baym, D. Campbell, R. Dashen, and J. Manassah, Phys. Lett. 58B, 304 (1975).

[6]G. E. Brown and W. Weise, Phys. Lett. 58B, 300 (1975).

[7]O. Maxwell, G. Brown, D. Campbell, R. Dashen, and J. Manassah, Astrophys. J. 216, 77 (1977).

[8]A. B. Migdal, G. A. Sorokin, O. A. Markin, and I. N. Mishustin, Phys. Lett. 65B, 423 (1976).

[9]L. Celenza and H. J. Pirner, Nucl. Phys. A294, 357 (1978).

[10]G. Baym and D. K. Campbell, in *Mesons in Nuclei*, edited by M. Rho and D. Wilkinson (North-Holland, Amsterdam, to be published).

[11]A is related to the pion decay constant, and with

$$\pi(x,t) \equiv (1/\sqrt{2})\langle \pi_1(x,t) + i\pi_2(x,t)\rangle, \ A = f_\pi/\sqrt{2}.$$

The time dependence of $\langle\pi\rangle$ follows from the equilibrium conditions. See G. Baym, Phys. Rev. Lett. 30, 1340 (1973). Note that this explicit pion field is defined for a linear Lagrangian, whereas the pion field in Sawyer's paper is defined for a nonlinear Lagrangian. We mention this because it might naively appear that our *Ansatz* is different from Sawyer's, since our $\langle\pi\rangle$ is limited in magnitude by A, a fixed number, whereas Sawyer's ϕ_0 appears unlimited. In fact, as Sawyer notes, the two forms are equivalent. The relation between the two *Ansätze* is discussed in detail by C.-K. Au and G. Baym, Nucl. Phys. A236, 500 (1974).

[12]See G. E. Brown, in *Mesons in Nuclei*, edited by M. Rho and D. Wilkinson (North Holland, Amsterdam, to be published) for a discussion of the role chiral symmetry plays in the long-range part of the N-N interaction.

[13]Readers are warned of the difference in sign conventions for Π between I and II and Ref. 1.

[14]Since both models leave out the contribution of the Δ and of isospin-even s waves, this equivalence could be anticipated. Chiral models including the Δ explicitly, for example, would give different values for E^H, as discussed above. See Refs. 5 and 9, for example.

[15]With regard to the size of the effects, we note in particular that Sawyer's comment "for values of G_A near unity the difference in the two results is dramatic, to say the least" greatly exaggerates the true difference between the models. Indeed, for the physically relevant value of $g_A = 1.36$ (as determined by the π-N p-wave coupling strength) the two models predict—from Sawyer's Eqs. (13) and (14), respectively—$\rho_C^B(g_A = 1.36) \simeq 0.56\, m_\pi^3 \simeq 1.1\rho_0$ and $\rho_C^A\ (g_A = 1.36) \simeq 0.80\, m_\pi^3 = 1.6\,\rho_0$, where ρ_0 is nuclear matter density ($\simeq 0.5\, m_\pi^3 \simeq 2.8\times 10^{14}$ g/cm^3). This 30% reduction in the critical density from model A to model B is somewhat smaller than the shifts caused by two other effects not included in either model: namely, the (attractive) Δ resonance and the (repulsive) Lorentz-Lorenz correlation effects. For more detailed discussions of the point see Refs. 2 and 5.

[16]Here for simplicity we ignore Δ-particle-nucleon hole graphs. See Refs. 2, 5, and 10.

[17]G. Bertsch and M. B. Johnson, Phys. Rev. D 12, 2230 (1975).

[18]M. B. Johnson, private communication.

[19]H. A. Bethe and M. B. Johnson, Nucl. Phys. A230, 1 (1974).

[20]Thus the full dispersive corrections would appear to have the opposite effect—at least for a well-developed condensate—to the simple ρ-exchange correction discussed by Sawyer. This of course in no way lessens the validity of Sawyer's observation that such corrections may be significant. Indeed, as Sawyer notes, his choice of model was intended to exemplify his point most clearly and not necessarily to give the best phenomenology.

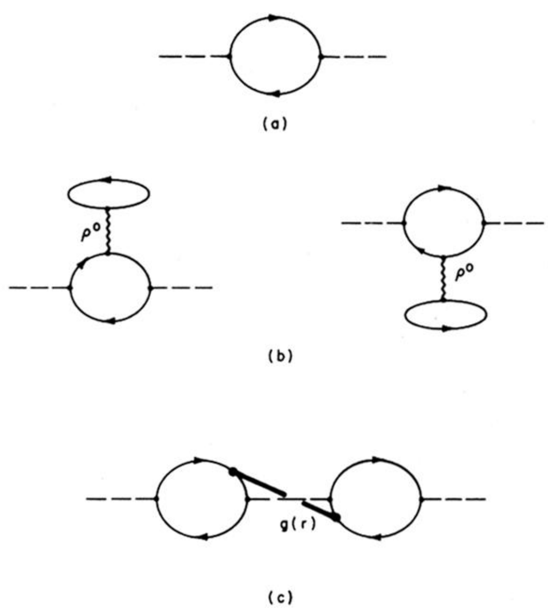
[21]M. O. Ericson and T. E. O. Ericson, Ann. Phys. (N.Y.) 36, 323 (1966).

[22]G. Baym and G. E. Brown, Nucl. Phys. A247, 395 (1975).

[23]See, for example, Ref. 8 and A. B. Migdal, Rev. Mod. Phys. 50, 107 (1978).

[24]We are thinking, for example, of the calculations on neutrino cooling of neutron stars done in Ref. 7.

[25]The effective cancellation at threshold was known to, and used by, Sawyer. See J. B. Hartle, R. F. Sawyer, and D. J. Scalapino, Astrophys. J. 199, 471 (1975).



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