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High-energy behavior of fermion-meson and meson-meson scattering in a supersymmetric field theory*

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The high-energy behavior of fermion-boson and boson-boson scattering amplitudes of a supersymmetric field theory containing a spin-1/2 fermion field, a scalar field, and a pseudoscalar field is investigated. The results can be easily modified to apply to the Yukawa model and the neutral version of the linear σ model. The results are also compared to those of fermion-fermion scattering in the same model. In the leading-logarithm approximation, ladders with fermions running along the sides in the t channel and mesons as rungs dominate in each order of two classes of diagrams. The sum of the dominant series give rise to fixed Regge cuts for all amplitudes in each of the three theories. All amplitudes in the supersymmetric theory possess a definite signature factor, while the amplitudes for fermion-fermion and fermion-antifermion scattering in the Y model and the σ model lack it. The results of the supersymmetric theory are also compared to the results of the spontaneously broken non-Abelian gauge theory.

I. INTRODUCTION

In the spirit of using field theory as a theoretical laboratory, it would be interesting to study the high-energy behavior of a supersymmetric field theory. Supersymmetry, as its name suggests, shares the common properties with non-Abelian gauge theories of a high degree of symmetry and the effects these symmetries have upon renormalization.[1] Some work in this area has already been completed by Young, Wong, and Opoien.[2] The model studied is the Wess-Zumino model.[3] The fermion-fermion scattering amplitude was calculated to all orders in the leading-logarithm approximation. No damping in transverse momentum was found, leading to a series in $\ln^2 s$. The series was summed to all orders in the leading logarithms, resulting in a fixed cut in the complex angular momentum plane.

One is naturally led to the question of what happens in boson-fermion and boson-boson scattering within this model. One is also interested in what role supersymmetry plays in these instances. In any case, it woud be interesting to compare these amplitudes with the fermion-fermion amplitudes found in Ref. 2.

II. RESULTS

The Lagrangian we use is the basic Wess-Zumino model.[3] It consists of a Majorana spinor field ψ, a scalar field A, and a pseudoscalar field B:

$$\begin{aligned}\mathcal{L}_S &= \tfrac{1}{2}[(\partial_\mu A)^2 - m^2A^2] + \tfrac{1}{2}[(\partial_\mu B)^2 - m^2B^2] \\ &\quad + \tfrac{1}{2}(i\bar\psi\partial\!\!\!/\psi - m\bar\psi\psi) \\ &\quad - \tfrac{1}{2}g\bar\psi(A - i\gamma_5 B)\psi - \tfrac{1}{2}mgA(A^2+B^2) \\ &\quad + \tfrac{1}{8}g^2(A^2+B^2)^2 .\end{aligned} \tag{2.1}$$

The factor of $\frac{1}{2}$ in the kinetic-energy term and the mass term for the spinor ψ arises from the fact that ψ is a Majorana spinor, and thus is self-conjugate. Supersymmetry requires the degeneracy of all masses and all coupling constants appearing in Eq. (2.1).

This Lagrangian shares some common properties with the Yukawa model (Y model) and the neutral version of the linear σ model[4]:

$$\mathcal{L}_Y = \bar\psi(i\partial\!\!\!/ - m)\psi + \tfrac{1}{2}[(\partial_\mu\pi)^2 - \mu^2\pi^2] + g\bar\psi i\gamma_5\psi\pi \tag{2.2}$$

and

$$\begin{aligned}\mathcal{L}_\sigma &= \bar\psi(i\partial\!\!\!/ - m)\psi + \tfrac{1}{2}[(\partial_\mu\pi)^2 - \mu^2\pi^2] + \tfrac{1}{2}[(\partial_\mu\sigma)^2 - \mu^2\sigma^2] \\ &\quad - g\bar\psi(\sigma + i\gamma_5\pi)\psi \\ &\quad - 4\lambda G\sigma(\sigma^2+\pi^2) - \lambda G^2(\sigma^2+\pi^2)^2 - 4\lambda\sigma^2 \\ &\quad + \tfrac{1}{2}\mu_0^2(2G^{-1}\sigma + \sigma^2 + \pi^2) .\end{aligned} \tag{2.3}$$

Here σ and π are the neutral scalar and pseudoscalar fields, respectively, and $G = g/m$.

The calculational details involved in the present work are quite similar to those given in Ref. 2 and will not be given here. Instead we simply summarize the results of this work and of the calculation of Ref. 2.

The leading diagrams for meson-meson scattering come from the kernel given in Fig. 1, while

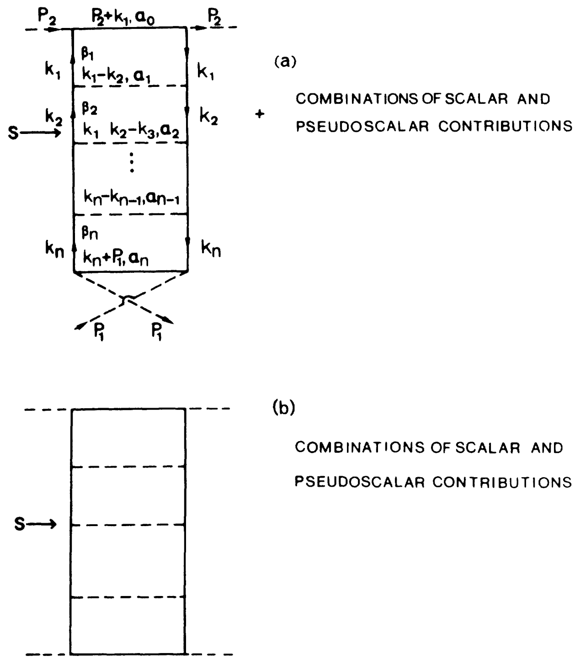

FIG. 1. N-loop pseudoscalar-pseudoscalar ladder diagrams: (a) u diagrams and (b) s diagrams.

those of fermion-meson scattering come from the kernel given in Fig. 2. The diagrams of Fig. 1 involve renormalization effects. However, the renormalization does not affect the leading behavior.

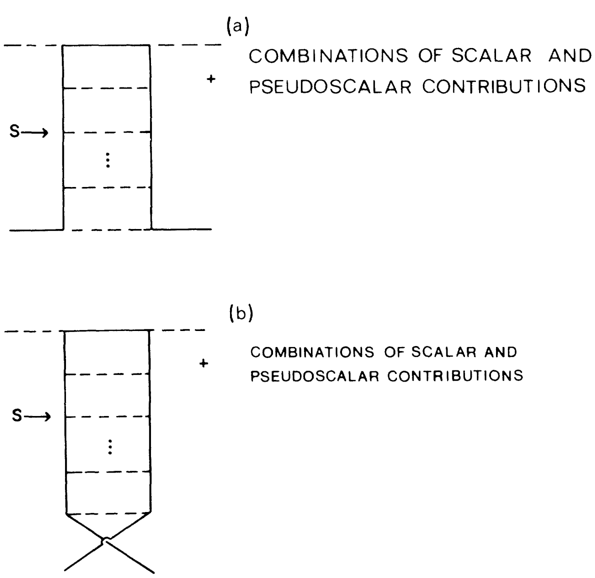

FIG. 2. N-loop pseudoscalar-fermion ladder diagrams. (a) s diagrams and (b) u diagrams.

A. The supersymmetric model (s model)

There are six possible scattering amplitudes within the s model:

$$\begin{aligned}&\psi\text{-}\psi,\quad \psi\text{-}A,\quad \psi\text{-}B,\\ &B\text{-}B,\quad B\text{-}A,\quad A\text{-}A\,.\end{aligned} \tag{2.4}$$

Here ψ-ψ denotes fermion-fermion scattering, ψ-A denotes fermion- scalar scattering, ψ-B denotes fermion-pseudoscalar scattering, etc. Fermion-fermion scattering was calculated in Ref. 2. The other amplitudes were calculated in the present work. Amplitudes containing fermions were spin averaged. For all amplitudes listed in (2.4) we found two classes of diagrams, the s diagrams and the u diagrams (see Ref. 2 and Figs. 1 and 2 for illustration), to dominate. The amplitudes for these two classes were the same in all cases. We list the results:

$$\begin{aligned}T_u &= -ig^2\sum_{n=0}^{\infty}\frac{1}{n!(n+1)!}(\bar{g}^2\ln^2 s)^n\\ &\simeq\frac{-ig^2}{\sqrt{4\pi}}\frac{s^{2\bar{g}}}{(\bar{g}\ln s)^{3/2}}\end{aligned} \tag{2.5}$$

and

$$\begin{aligned}T_s &= -ig^2\sum_{n=0}^{\infty}\frac{1}{n!(n+1)!}\left[\bar{g}^2(\ln s-i\pi)^2\right]^n\\ &\simeq\frac{-ig^2}{\sqrt{4\pi}}e^{-i2\pi\bar{g}}\frac{s^{2\bar{g}}}{(\bar{g}\ln s)^{3/2}}\end{aligned} \tag{2.6}$$

where

$$\bar{g}^2=g^2/8\pi^2\,. \tag{2.7}$$

The scattering amplitude is

$$\begin{aligned}T &= T_u+T_s\\ &\simeq\frac{-ig^2}{\sqrt{4\pi}}(1+e^{-i2\pi\bar{g}})\frac{s^{2\bar{g}}}{(\bar{g}\ln s)^{3/2}}\,.\end{aligned} \tag{2.8}$$

B. The σ model

The fermion-fermion and fermion-antifermion scattering amplitudes are respectively given by T_u and T_s, i.e., by Eqs. (2.5) and (2.6). The fermion-boson, antifermion-boson, and boson-boson scattering amplitudes are all given by T_u+T_s, i.e., by Eq. (2.8).

C. The Y model

The amplitudes for fermion-fermion and fermion-antifermion scattering are given respectively by Eqs. (2.5) and (2.6), with g^2 and $\bar{g}^2$ replaced by $g^2/2$ and $\bar{g}^2/2$. The fermion-boson and antifermion-boson amplitudes are both given by Eq. (2.8) with the replacement of $\bar{g}^2$ by $\bar{g}^2/2$. The boson-boson scattering amplitude is given by Eq. (2.8) with g^2 and $\bar{g}^2$ replaced by $4g^2$ and $\bar{g}^2/2$.

We note that in the σ model and the Y model only the fermion-fermion and fermion-antifermion amplitudes do not have the signature factor of Eq. (2.8).

The Wess-Zumino model of supersymmetry used in this calculation is clearly renormalizable since the dimension of all interaction terms is less than or equal to four. The supersymmetry places restrictions between the coupling constants and between the masses.

These restrictions greatly improve the renormalization, causing cancellations of divergences to occur between different diagrams. All remaining divergences are removed by the introduction of a single wave-function renormalization constant.

In contrast, the symmetry restrictions in spontaneously broken non-Abelian gauge theories are essential for their renormalizability. The symmetry also plays a strong role in their high-energy behavior. All integrations over transverse momentum are convergent, owing to cancellations between diagrams. All $\ln s$ factors come from the integrations over longitudinal momentum. This causes the theory to Reggeize. The vector-meson self-couplings play an important role in these cancellations.

Despite the cancellations in the renormalization of supersymmetry, the s model has the same basic features of high-energy behavior as the Y and σ models. Mesons play only a passive role, providing momentum transfer for the fermions exchanged in the t channel. Meson-meson couplings are unimportant. There is no $P_\perp$ damping and maximum energy dependence exhibited by the individual Feynman diagrams is attained.

It is interesting to note that the spin-averaged amplitudes for fermion-fermion, fermion-boson, and boson-boson scattering are all equal in the high-energy limit within the s model. In the σ model the spin-averaged fermion-boson and boson-boson amplitudes are equal, while the fermion-fermion scattering lacks the signature factor. This is due to the fact the fermion wave function is no longer self-conjugate. The Y model is slightly more complicated. The boson-boson amplitude is given by four times the spin-averaged fermion-boson amplitude. The spin-averaged fermion-fermion amplitude is equal to $\frac{1}{2}$ times the spin-averaged fermion-boson amplitude, minus the signature factor. These factors arise because there is only one type of meson which can be exchanged. Again the fermion-fermion amplitude lacks the signature factor because the fermion wave function is not self-conjugate. All amplitudes of the three theories possess a similar fixed square-root cut.

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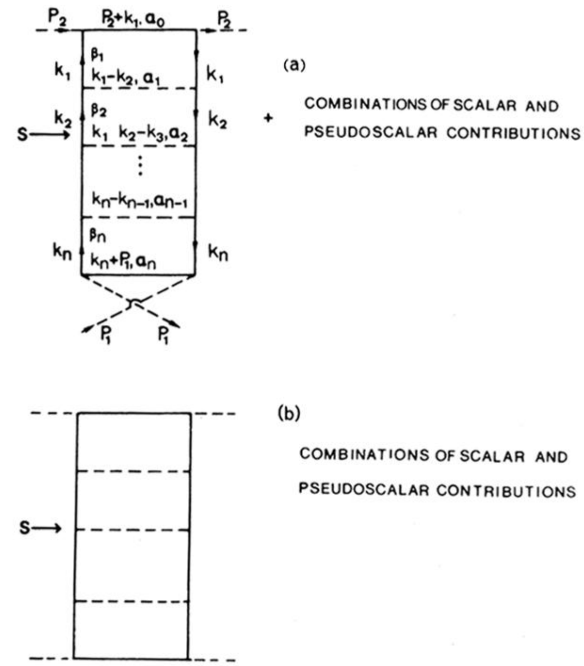

FIG. 1. N-loop pseudoscalar-pseudoscalar ladder diagrams: (a) u diagrams and (b) s diagrams.

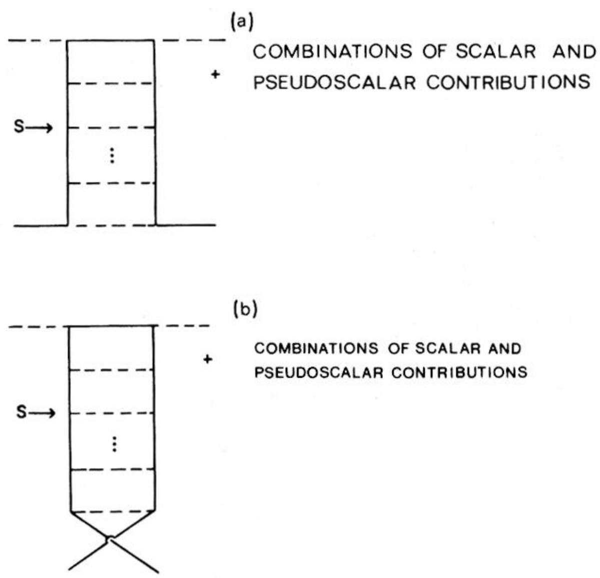

FIG. 2. N-loop pseudoscalar-fermion ladder diagrams. (a) s diagrams and (b) u diagrams.