Bound states with a gauge monopole

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Relativistically exact bound states of a Dirac spinor and of a scalar with an SU(2) gauge monopole are determined asymptotically for arbitrary (integer or half-integer) isospin. Independently, zero-energy bound states of the Dirac spinor are proved to exist for each value of total angular momentum J less than the isospin T. The spin of the system is shown to be the sum of the original spin and isospin of the bound particle.

I. INTRODUCTION

Since the discovery of the gauge monopole by 't Hooft and Polyakov,[1] its various properties have been widely investigated. The gauge monopole should have a mass about a thousand times the proton mass, whereas a Dirac monopole (which is described by an Abelian field theory) would have a much smaller mass. These two types of monopoles differ sharply in their other dynamical properties. It was shown by Harish-Chandra[2] that a Dirac monopole does not form a bound state with an electron, whereas a gauge monopole can form bound states with an electron (or a boson). This property of a gauge monopole is evidently due to its non-Abelian origin, and isospin plays a crucial role in the binding.

Studies of bound states with a gauge monopole have been carried out by several groups, and interesting features have been discovered, such as the zero-energy solutions first found by Jackiw and Rebbi,[3] and their physical implications have been explored.[4] Owing to the nonlinear formalism of a gauge monopole, solving for bound states involves certain algebraic and mathematical problems. Part of these difficulties are due to the approach used. The studies so far have involved direct investigation of the field equations to obtain solutions. To avoid some of these difficulties we use an alternative method, seeking to find the eigenvalues of the energy, using physically justified approximations and techniques typical of matrix solutions.

We are particularly interested in the spectrum of bound states, including especially zero-energy solutions, and in the feature referred to as isospin-to-spin conversion, all for general isospin values. It is now well established[5] that the total angular momentum of a monopole-plus-particle system includes the isospin added to the usual orbital and spin terms. Integer-plus-$\frac{1}{2}$ isospin values thus lead to integer angular momenta and Bose statistics for bound states with a fermion, and vice versa.

It is thus clear that additional spin has appeared in these bound systems; it is not immediately evident whether isospin has disappeared as a separate degree of freedom. Since neither spin nor isospin is conserved in the bound system, the only way to answer the question is to obtain a complete set of solutions and then to analyze what degrees of freedom are necessary and sufficient to account for the multiplicities of the complete spectrum. We here successfully carry out this analysis, in a reasonable approximation, and find that isospin indeed disappears from the system as an equal magnitude of spin appears.

II. DIRAC SPINOR

We consider the field equation of a Dirac spinor particle, moving in the field of a static monopole solution of the Yang-Mills and Higgs system,[6]

$$(\gamma^\mu D_\mu + m + GgT^a\varphi^a)\psi = 0\,, \tag{1}$$

where $D_\mu = (1/i)\partial_\mu - gT^aA^a_\mu$, T^a are isospin representation matrices, and we have chosen to write the Yukawa coupling constant as Gg rather than G.

For an eigenstate, $(1/i)\partial_0$ will give the energy. Thus we identify the Hamiltonian H:

$$H\psi = E\psi = \gamma^0\{\gamma_k[(1/i)\partial_k - gT^aA^a_k] + \gamma^0 gT^aA^{0a} + m + GgT^a\varphi^a\}\psi\,. \tag{2}$$

Introducing the neutral-monopole solution forms,[7]

$$A^a_i = \epsilon_{aij}\hat{r}_j\frac{1-K(r)}{gr}\,,\quad A^{0a}=0\,,\quad \varphi^a = \hat{r}_a\frac{F(r)}{gr}\,, \tag{3}$$

where $\hat{r}_a = r_a/r$, and using $\gamma_k = i\gamma^0\gamma_5\sigma_k = 2i\gamma^0\gamma_5 S_k$; $p_k = (1/i)\partial_k$, $L_i = \epsilon_{ijk}r_jp_k$, and the notation $V_R = \frac{1}{2}\{V_i, \hat{r}_i\}$ for any vector V_i, we can express H

as

$$H = 2i\gamma_5 S_R p_R - i\gamma_5(1/r)\epsilon_{ijk}S_i\{\hat{r}_j, L_k + T_k - KT_k\} + \gamma^0(m + GT_R F/r). \quad (4)$$

Here we may note that because the monopole solution connects isospin indices to the position vector, the usual momentum $\vec{L}+\vec{S}$ is not conserved; to generate rotations we must use $\vec{J}=\vec{L}+\vec{S}+\vec{T}$ in order to rotate isospin as well. Hence the total angular momentum, which in a bound state may be called the total spin of the composite system, is $\vec{J}$, not $\vec{L}+\vec{S}$, and if T is integer-plus-$\frac{1}{2}$, we find an apparent anomalous spin-statistics relation.[5] However, it has been shown[8] that the statistics of such states are also anomalous, so that the usual spin-statistics relation is restored.

We also observe that $\gamma^0\gamma_5$ anticommutes with H. This implies that the simultaneous eigenstates of H^2 and $\gamma^0\gamma_5$ correspond directly to eigenstates of H, and vice versa. [$H\psi = E\psi$ gives $H^2(1\pm\gamma^0\gamma_5)\psi = E^2(1\pm\gamma^0\gamma_5)\psi$; $H^2\psi = E^2\psi$ gives $H(1\pm H/E)\psi = \pm E(1\pm H/E)\psi$. For $E=0$ there is a one-to-one correspondence rather than two-to-two.]

We therefore transfer our attention to H^2. Using the known limiting-case exact solution for the monopole,[9]

$$K = Cr\,\mathrm{csch}Cr, \quad F = -1 + Cr\coth Cr; \quad (5)$$

and making some simplifications, we find

$$\begin{aligned} H^2 = p_R^2 + (1/r^2)[(\vec{L}+\vec{T}-K\vec{T})^2 - (1-K)^2T_R^2 \\ + 2KF(\vec{S}\cdot\vec{T} - S_RT_R)(1-G\gamma^0\gamma_5) \\ + 2(1-K)^2S_RT_R(1-G\gamma^0\gamma_5)] \\ + [m + (F/r)GT_R]^2. \end{aligned} \quad (6)$$

We now make the approximation that the monopole is extremely localized, and replace H^2 by an approximate, soluble form, by neglecting terms which are exponentially small at large distances ($Cr \gg 1$). This gives

$$H^2 \simeq p_R^2 + (1/r^2)[(\vec{L}+\vec{T})^2 - T_R^2 + 2S_RT_R(1-G\gamma^0\gamma_5)] + \left(m + \frac{Cr-1}{r}GT_R\right)^2 \quad (7)$$

with which T_R commutes. A maximal commuting operator set for further defining eigenstates thus consists of $\gamma^0\gamma_5$, $\vec{J}^2$, J_z, T_R, and the expression in brackets in Eq. (7). In general T_R may range from $-T$ to T, as does any other component of $\vec{T}$, but when $J \le T$, T_R is also restricted by the relationship

$$|T_R| = |J_R - S_R| \le |J_R| + |S_R| = J + \tfrac{1}{2}. \quad (8)$$

For determining the eigenvalues of the bracketed expression, a preliminary basis diagonalizing $(\vec{L}+\vec{T})^2$, along with $\vec{J}^2$ and T_R, reduces it to a two-by-two matrix which is quickly analyzed. We then find

$$H^2 = p_R^2 + (m + CGT_R)^2 - 2(m+CGT_R)GT_R/r + l(l+1)/r^2, \quad (9)$$

where for $|T_R| < J+\frac{1}{2}$, l may be either $l = l_0 = [T_R^2(G^2-1) + (J+\frac{1}{2})^2]^{1/2}$ or $l = l_0 - 1$, independent of $\gamma^0\gamma_5$; for $|T_R| = J+\frac{1}{2}$ and $\gamma^0\gamma_5 = +1$, $l = l_0 = |GT_R|$; for $|T_R| = J+\frac{1}{2}$ and $\gamma^0\gamma_5 = -1$, $l = l_0 - 1 = |GT_r| - 1$.

Approximate solutions (exact solutions of the approximate H^2) can now be determined immediately since Eq. (9) has the form of a well-known differential equation.[10] There is an extensive discrete spectrum for all T_R values for which $(m+CGT_R)GT_R$ is positive. The discrete spectrum is characterized by the equation

$$(m+CGT_R)^2 - E^2 = [(m+CGT_R)GT_R/(n_r + l + 1)]^2, \quad n_r = 0, 1, 2, \ldots. \quad (10)$$

For $|T_R| < J+\frac{1}{2}$, we obtain two series of solutions, corresponding to the two possibilities for l. Introducing $Q = T_R$, the energy eigenvalues are given by

$$E^2 = E_{nJQ}^{\ 2} = (m+CGQ)^2\{1 - [GQ/(n+l_0)]^2\}. \quad (11)$$

For $l = l_0$ this form is valid with $n = n_r + 1$ ranging from 1 up, while for $l = l_0 - 1$, $n = n_r$ ranges from 0 up. In either case $E = 0$ does not occur since $l_0 > |GQ|$, and also the result is independent of $\gamma^0\gamma_5$. Hence the corresponding discrete eigenvalues of H will be $\pm E_{nJQ}$, $n = 0, 1, 2, \ldots$, with a twofold degeneracy for all $n \ne 0$ [in addition to the usual $(2J+1)$-fold multiplicity of angular momentum multiplets].

For $|T_R| = J+\frac{1}{2}$, we obtain eigenvalues given by the same Eq. (11), except that l_0 reduces to $|GQ|$, but now $n = n_r + 1$ occurs only for $\gamma^0\gamma_5 = +1$ and $n = n_r$ only for $\gamma^0\gamma_5 = -1$, corresponding to $l = l_0$, $l = l_0 - 1$. The corresponding discrete eigenvalues of H are then $\pm E_{nJQ}$, $Q = \pm(J+\frac{1}{2})$. We note that $E_{0JQ} = 0$ for $Q = \pm(J+\frac{1}{2})$, occurring with $\gamma^0\gamma_5 = -1$, while the other eigenvalues are not doubled as they were for $|T_R| < J+\frac{1}{2}$.

The multiplicity of any $\pm E_{nJQ}$ eigenvalue of H is thus at least $2J+1$ as always, since $\vec{J}$ commutes with H. The multiplicity is doubled if $|Q| < J+\frac{1}{2}$ and $n \ne 0$. We also note that if $m = 0$, $Q = \pm|Q|$ pairs are also degenerate, and depending on the values of m/C and G, other accidental degeneracies may occur. In a second-quantized theory, $\pm E$ solutions will presumably correspond to particle-plus-monopole and antiparticle-plus-monopole states, both with positive energy, thus producing another degeneracy.

III. KLEIN-GORDON PARTICLE

A similar analysis can be carried out for a particle satisfying a Klein-Gordon equation, minimally coupled to the gauge field and with general couplings to the Higgs field. For a fairly general Lagrangian, we obtain the field equation

$$(-D_\mu D^\mu + m^2 + hg^2\varphi^2 + Gg\varphi_a T^a + \lambda\psi^2)\psi = 0\,, \qquad (12)$$

where we have arbitrarily chosen to write factors of g with the scalar-scalar couplings; T^a are gauge group representations for the ψ multiplet.

To avoid complications, we assume that ψ cannot develop a spontaneous symmetry breakdown effect for any value of φ; hence that $m^2 + hg^2\varphi^2 + GgT^a\varphi_a$ is positive for all φ. This implies $h > 0$ and $4m^2hg^2 \geq (Gg|T^a|)^2$, or

$$m^2 \geq (\tfrac{1}{2}GT)^2/h\,. \qquad (13)$$

$(1/i)\partial_0$ gives the energy of an eigenstate, as before, and using Eqs. (3) for the monopole fields, we obtain

$$E^2\psi = \{p_R{}^2 + (1/r^2)[(\vec{L} + \vec{T} - K\vec{T})^2 - (1-K)^2 T_R{}^2 + hF^2]$$
$$+ m^2 + GT_R F/r + \lambda\psi^2\}\psi\,. \qquad (14)$$

Again, the total angular momentum must be $\vec{J} = \vec{L} + \vec{T}$, and the composite must carry spin T, as will be confirmed by the solution set.

We now go to large r as before, using Eq. (5), and since bound-state wave functions fall rapidly at large r, we also neglect the $\lambda\psi^2$ term. We then find

$$E^2\psi = [p_R{}^2 + (m^2 + hC^2 + CGT_R) - (2hC + GT_R)/r$$
$$+ (\vec{J}^2 + h - T_R{}^2)/r^2]\psi\,. \qquad (15)$$

Defining $l(l+1) = \vec{J}^2 + h - T_R{}^2$, we again find discrete solutions whenever $hC + \frac{1}{2}GT_R > 0$, with eigenvalues given by

$$(m^2 + hC^2 + CGT_R) - E^2 = [(hC + \tfrac{1}{2}GT_R)/(n_r + l + 1)]^2\,,$$
$$n_r = 0, 1, 2, \ldots\,. \qquad (16)$$

Defining $Q = T_R$, which is limited by the relation

$$|Q| = |T_R| = |J_R| \leq J\,, \qquad (17)$$

we find energy eigenvalues

$$E^2 = E_{nJQ}{}^2 = m^2 - (\tfrac{1}{2}GQ)^2/h$$
$$+ (1/h)(hC + \tfrac{1}{2}GQ)^2[1 - h/(n+l)^2]\,, \qquad (18)$$

where $n = n_r + 1$.

Since

$$(l + \tfrac{1}{2})^2 = l(l+1) + \tfrac{1}{4} = J(J+1) + h - Q^2 + \tfrac{1}{4}$$
$$\geq J(J+1) + h - J^2 + \tfrac{1}{4} > \tfrac{1}{4}\,,$$

we have $l > 0$, and further

$$(n+l)^2 = (n_r + l + 1)^2 \geq (l+1)^2 > (l + \tfrac{1}{2})^2 = J + h + \tfrac{1}{4} > h;$$

therefore the last term in Eq. (18) is positive; the other terms give a positive contribution, by Eq. (13), and we find $E^2 > 0$ for all these bound solutions. Solution multiplets are completely identified in this approximation by n, J, and Q, unlike the case with the Dirac equation where addition of S and T gave in general two possibilities for each Q.

IV. ISOSPIN-TO-SPIN CONVERSION

We now consider the question of whether isospin-to-spin conversion occurs, or whether some other source provides the spin to convert spin $\frac{1}{2}$ or 0 into integer or integer-plus-$\frac{1}{2}$ spin, while isospin remains as an additional degree of freedom. The answer is to be found by considering the multiplicities of states with various total angular momentum quantum numbers.

A two-body composite, in general, besides its overall position and visible orbital angular momentum degrees of freedom, has radial, internal (relative) orbital angular momentum, total spin, and possibly other internal degrees of freedom. Spin is distinguished from other internal degrees of freedom (isospin, etc.) by the definition that spin $\vec{S}$ adds to internal orbital angular momentum $\vec{L}$ to give $\vec{J}$, the total angular momentum in the center-of-mass frame, which is ordinarily (as here) a good quantum number. We note that the rules for adding angular momenta, applied to $\vec{J} = \vec{L} + \vec{S}$ where L takes all nonnegative integer values and S is fixed, give in general $2S+1$ multiplets with each J value, but if $J < S$ then there are only $2J+1$; there is one per L in the range $|J - S| \leq L \leq J + S$. However, an isospin degree of freedom T should give a multiplicity of $2T+1$ regardless of J.

Looking now at our solution sets, we observe that there are clearly quantum numbers corresponding to the radial degree of freedom (n) and to total angular momentum (J and also m_J which is not explicitly indicated). We find, for given n, J, and m_J, generally $2(2T+1)$ eigensolutions in the spinor case and $2T+1$ in the scalar case (respectively, 2 and 1 for each T_R value). However, the limitation $|T_R| \leq J + \frac{1}{2}$ or J [Eqs. (8), (17)] reduces these multiplicities for $J \lesssim T$. The specific multiplicities correspond exactly to a system having spin values $T + \frac{1}{2}$ and $T - \frac{1}{2}$ in the spinor case or T in the scalar case; that is, to the hypothesis that the spin of the bound system is the sum of its original spin $\frac{1}{2}$ or 0 and its original isospin T.

The alternative hypothesis, that the interaction of half-integral charges with a magnetic field has created additional spin angular momentum (to give the integer J values) while the isospin multiplicity is unaffected,[11] predicts a general multiplicity of $(2S+1)(2T+1)$ which is too large, since S must be changed. The low-J multiplicities, in particular, directly contradict the hypothesis that the isospin multiplicity is unaffected, since they do not show a $(2T+1)$ factor. Other hypotheses claiming to describe the internal degrees of freedom must evidently either also contradict the properties of this solution set, or else be equivalent (for predictive purposes) to the assumption of complete isospin-to-spin conversion.

V. EXISTENCE OF ZERO-ENERGY SOLUTIONS

One interesting feature of this solution in the spinor case can be partly confirmed by an exact analysis, and that is the existence of zero-energy bound states, i.e., the existence of normalizable solutions to the exact wave equation at $E=0$. For $E=0$, the Dirac equation can be written as

$$\{d/dr-(1/r)[1+2\vec{L}\cdot\vec{S}+2(1-K)(\vec{S}\cdot\vec{T}-S_RT_R)] + 2\gamma^0\gamma_5 S_R[m+GT_R(F/r)]\}R_J(r)=0\,, \quad (19)$$

where the normalizability condition is $\int_0^\infty |R_J|^2dr<\infty$.

Expanding in eigenstates of S_R and T_R,

$$R_J(r)=\sum\varphi_{\pm,Q}(r)\,|S_R=\pm\tfrac{1}{2},\; T_R=Q\rangle\,, \quad (20)$$

this becomes the system of coupled equations

$$\left[\frac{d}{dr}\pm\gamma^0\gamma_5\left(m+\frac{GFQ}{r}\right)\right]\varphi_{\pm,Q}(r) +\frac{1}{r}[(J+\tfrac{1}{2})^2-Q^2]^{1/2}\varphi_{\mp,Q}(r) +\frac{K}{r}[(r+\tfrac{1}{2})^2-(Q\pm\tfrac{1}{2})^2]^{1/2}\varphi_{\mp,Q\pm1}(r)=0\,. \quad (21)$$

For $J<T$, the index values are $S_R=\pm\frac{1}{2}$, $T_R=J_R-S_R=(-J-S_R)$ to $(J-S_R)$, for a total of $2(2J+1)$ components; for $J>T$, the values are $S_R=\pm\frac{1}{2}$, $T_R=-T$ to T, for a total of $2(2T+1)$ components. Since the differential equation is first order in each component, the solution space of Eq. (21) is, respectively, either $2(2J+1)$ or $2(2T+1)$ dimensional.

For $r\to 0$, we have $K\to 1$ and $F\to 0$. Assuming that the leading behavior of the solution as $r\to 0$ is $\varphi_{\pm,Q}=C_{\pm,Q}r^n$ for some n, the leading terms of Eq. (21) are those of degree $n-1$, giving an eigenvalue equation for n,

$$-nC_{\pm,Q}=[(J+\tfrac{1}{2})^2-Q^2]^{1/2}C_{\mp,Q} +[(r+\tfrac{1}{2})^2-(Q\pm\tfrac{1}{2})^2]^{1/2}C_{\mp,Q\pm1} \quad (22)$$

or in matrix notation $-nC=\mathfrak{M}C$. By inspection $\mathfrak{M}$ is a multiple of a spin reflection operator (R_S: $S_R\leftrightarrow -S_R$) and thus anticommutes with S_R; hence eigenvalues for n occur in positive-negative pairs, and we can select, among the independent solutions of Eq. (21), half of them to correspond to eigenvectors of $\mathfrak{M}$ with eigenvalues $n\geq 0$, and hence to be convergent at the origin.

Considering $r\to\infty$, we note that since the off-diagonal terms in Eq. (21) fall off as $r\to\infty$, while $m+GFQ/r$ approaches a constant, $m+CGQ$, we can conclude that for each component $\varphi_{\pm,Q}$ there is a solution in which only that component has the leading behavior at infinity of the solution. That leading behavior is therefore given by solving

$$\left(\frac{d}{dr}\pm\gamma^0\gamma_5(m+CGQ)\right)\varphi_{\pm,Q}=0\,, \quad (23)$$

which gives $\varphi_{\pm,Q}\sim\exp[\mp\gamma^0\gamma_5(m+CGQ)r]$. For a given Q and $\gamma^0\gamma_5$, one of the two S_R values will give a converging, one a diverging, exponential. For $J>T$ we thus find that half of the independent solutions can be chosen to converge at infinity. However, for $J<T$ not every Q value has both S_R values allowed. For $Q=\pm(J+\frac{1}{2})$, only $S_R=\mp\frac{1}{2}$, respectively, occurs, and, as long as $|m|<CG(J+\frac{1}{2})$, for $\gamma^0\gamma_5=+1$ both of these Q values give diverging exponentials, while for $\gamma^0\gamma_5=-1$ both give converging exponentials. Hence as long as $|m|<CG(J+\frac{1}{2})$, for $\gamma^0\gamma_5=-1$ and $J<T$, one more than half of the solutions can be taken convergent. The solution space, of dimension $2(2J+1)$, thus has a subspace of solutions convergent at the origin whose dimension is $2J+1$, and a subspace of solutions convergent at infinity whose dimension is $(2J+1)+1$. The intersection of these subspaces must therefore be at least one-dimensional, and there is therefore at least one solution which is convergent at both extremes, and therefore normalizable. There is therefore at least one zero-energy angular momentum multiplet for each $J<T$, as long as $|m|<CG(J+\frac{1}{2})$; it will have $\gamma^0\gamma_5=-1$. [If $m+CGQ=0$ for some Q, solutions with leading behavior isolated in the $\varphi_{\pm,Q}$ components do not occur, but the argument does apply for components $\bar{\varphi}_{\pm,Q}=(\varphi_{+,Q}\pm\varphi_{-,Q})$ instead, and the same final conclusion is reached.]

VI. VALIDITY OF THE APPROXIMATION

Departure of the exact potential from its asymptotic form will certainly lead to modification of the solutions found here. However, if the gauge-field mass, which determines the monopole distance scale $(1/C)$, is large compared to the particle mass (m plus the symmetry-breakdown contributions involving G), the asymptotic form will be accurate outside a small neighborhood of the origin and the solutions found here should also be good, especially those with relatively large l values in Eqs. (9) and (15), whose radial functions vanish strongly at the origin. Some effect of our approximation is visible in the list of zero-energy spinor solutions, where we found two for each J up to $T-\frac{1}{2}$ [one for each sign of $T_R=\pm(J+\frac{1}{2})$], while the exact analysis established only one per J value. (We note that zero-energy solutions have relatively small l values, hence our approximation was not expected to be good. This supporting evidence from the exact analysis for zero energy thus lends some confidence to our approximation for all l.)

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