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Weak and strong sources of gravity: An SO(1,3)-gauge theory of gravity*

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The structural and dynamical elements of the Lorentz-Yang-Mills gauge theory of gravity are explained and analyzed. This theory is a generalized metric theory of gravity which no longer satisfies the principle of equivalence in any situation, but fits the very general framework of the Yang-Mills gauge theories used to describe the nongravitational interactions. It adds essentially gravitational self-interaction to general relativity in an amount which is not measurable in the solar system; the geometry contains stress-energy by itself; this breaks the validity of the principle of equivalence for strong gravitational fields. Static and spherically symmetric space-times dispose, in addition to the mass M, of a second parameter γ, which is a measure for the stress-energy content of the space-time geometry. We show that for the weak limit $\gamma = 1-\epsilon$ with $\epsilon \simeq M/R$; the Schwarzschild black hole ($\gamma = 1$) is therefore no longer a candidate for the final state of collapsing matter.

I. WHY BOTHER TO CREATE NEW THEORIES OF GRAVITY?

The reasons for creating a theory of gravity different from Einstein's theory have drastically changed in the past. The older alternative approaches were mainly considered as differing in their laws for the generation of the metric; in Einstein's theory of gravity, the curvature of space-time corresponds linearly to the distribution of stress-energy on that space-time; in the Dicke-Brans-Jordan theory, matter and nongravitational fields generate linearly curvature and a scalar field. Let us call this a reason of local bearing to create a rival theory of gravity. Since the global aspects of the space-time geometries in the class of the solutions of Einstein's field equations (and of similar theories) are now mainly known—the non-quantum-mechanical aspects of applying the global techniques to Einstein's theory are summarized in the book by Hawking and Ellis[1]—the justification for building a rival theory of gravity is now completely on the side of the global behavior of its solution geometries. In this sense, an alternative theory of gravity has then a justification of birth if and only if

(i) it agrees with all experimental tests known in 1976, or up to 1985, i.e., if its predictions for the effects measurable in the solar system agree with the data to within some standard deviations (and this for the four standard tests as well as agreement with observations at the Newtonian level);

(ii) it agrees with observations on the cosmological level (e.g., it shows an expansion of the universe);

(iii) it offers a "*better* issue of the final state for the collapse;"

(iv) it is of structural simplicity.

The expression "issue of the final state" concerns the question of how collapsing matter terminates. That singularities occur and are very general phenomena in theories of gravity of the Einsteinian type has been known thanks to theorems on singularities proved by Penrose, Hawking *et al.* Penrose[2] showed for the first time that if a star had once passed inside the Schwarzschild surface (the event horizon at $r = 2\text{m}$) it could not come out again; in the following this theorem has been generalized for general collapse types by Hawking and Penrose[3] (for a review see Ref. 1). Condition (iii) would now require that an alternative approach to gravity is then reasonalbe if it either completely avoids the occurrence of singularities, or enables one to specify under what physically reasonable conditions the total collapse and the formation of singularities are avoided. Even in Einstein's theory a corresponding condition exists. However, it is not physically reasonable; it is part of the assumption necessary to prove the occurrence of singularities—at each event of space-time the energy condition $E+p_1+p_2+p_3>0$ has to be satisfied, where E is the total energy density and p_i are the principal pressures of matter. By breaking this condition we might avoid the total collapse; such a breaking does not, however, seem quite reasonable from a physical point of view though there are attempts to construct cases where negative pressures occur.[4,5]

Attempts to remedy this issue of the final state go in different directions. (i) It is plausible to think that the singularities are due to the degree of linearity of Einstein's field equations and that therefore high nonlinear terms added to Einstein's Lagrangian act against the forever-attractive gravitational forces in regions of high space-time curvature. These additional terms had to be at least quadratic in the Ricci tensor and the Ricci scalar; corresponding Lagrangians are of the type

$$\mathcal{L}_g = (-g)^{1/2}[R - 2\Lambda + \gamma_1(R^2 + \gamma_2 R_{ab}R^{ab})] \ , \qquad (1.1)$$

where γ_1 and γ_2 are additional coupling constants.

In this sense, Einstein's field equations have been considered as a first approximation to a higher nonlinear theory by DeWitt,[6] Sakharov,[7] Ruzmaikina and Ruzmaikin,[8] Nariai,[9] Nariai and Tomita,[10] and recently by Michel[11] for spherically symmetric collapse, and by Giesswein and Streeruwitz[12] for Friedmann-like world models. These types of modifications used at least one additional coupling constant of such a small value that it cannot influence the regions of space-time of low curvature. If the nonlinear terms are generated by means of renormalizing the stress-energy tensor this has furthermore the disadvantage of being inherently nonunique and noncovariant.[13] (ii) A second attempt to improve the issue of the final state is to take into account quantum effects at least for the matter fields, since a consistent quantization of the gravitational field—if this is actually a reasonable way of thinking—is still an unsolved problem. Hawking[14,15] applied this idea to spherically symmetric collapse. Similar ideas had already been considered for the early phase of the cosmological evolution.[4,13] (iii) Finally, quantization of general relativity itself has been expected to change the issue of the final state; the covariant quantization has led to an understanding of classical and essentially quantum effects in terms of tree and closed-loop diagrams. However, the recent work by Deser and van Nieuwenhuizen[16-19] has shown that already at the one-loop level quantized general relativity is nonrenormalizable when coupled to matter fields (scalars, photons, or Dirac-Einstein system). Now, if one really believes that general relativity should be quantized, the nonrenormalizability is a dilemma and perhaps shows that the strong limit of Einstein's equations is *not viable*.

On the experimental side we find a good agreement with the basic principles of general relativity (such as the principle of equivalence) and the predictions resulting from Einstein's field equations for the class of *weak sources* of gravity (e.g., a static source of gravity is called weak if it has a low mass-to-radius ratio, i.e., $M/R \ll 1$). The gravitational red-shift has been found for a wide range of weak sources (in the Earth's gravitational field it was originally measured by Pound and Rebka,[20] the red-shift in the Sun's gravitational field was measured by Snider,[21] the red-shift for white dwarfs was measured by Greenstein,[22] Greenstein and Trimble[23]). All these objects show a gravitational red-shift $z = M/R$ for light escaping from the surface of a star in good agreement with any metric theory of gravity, nonlinear contributions are not measurable because $M/R \ll 1$. Only for strong sources, $M/R \gtrsim 0.1$, do nonlinear terms become effective; two types of objects are expected to belong to the class of strong sources: neutron stars and quasistellar objects. The second part of our knnowledge about the structure of the gravitational interaction comes from the solar system data; they show how *weak* sources of gravity generate their gravitational field (for a review see Ref. 24). In 1976 the only object offering a quite laborious test in the strong region was the close double system PSR1913 + 16,[25] consisting of an observed pulsar and a second up-to-now-unknown companion.[26] (The relativistic contributions to possibly detectable effects have been discussed by Barker and O'Connell,[27] Blandford and Teukolsky,[28] Will,[29] Wagoner,[30] and Balbus and Brecher.[31]) Another test in the strong region would be the existence of black holes in our galaxy if it were possible to *really* identify an x-ray source with a black hole; so far none of the suspicious objects obtained a real 100% qualification as a black hole.

All these arguments and facts indicate one thing: A change in the issue of the final state, known to be the total collapse in Einstein's theory, can only be achieved in a consistent way by generalizing Einstein's theory of gravity. At the same time we have to require the following:

(i) For weak sources of gravity Einstein's equations represent a fairly good approximation.

(ii) However, in space-times generated by strong sources, deviations might occur.

(iii) Gravity is a one-term interaction in the sense that there is only one coupling constant giving the coupling of matter to geometry.

Questions concerning the final state in the time evolution of matter are certainly intrinsically related to the strong-field behavior of gravity. At first, we might think that there are various possibilities offered to connect the weak-field limit with the strong region. This situation resembles in some aspects the transition from Newtonian mechanics to special relativity; after a while it turned out that there is really only one extension of Newtonian physics to the region of high velocities. Here, in the theory of the gravitational interaction, we are still grasping, not knowing whether Einstein's strong limit is correct, should be completely abandoned, or should only be completed and corrected by including nonlinear effects. In the following we propose essentially to follow the third way, i.e., to enlarge and generalize Einstein's theory in a way which satisfies the three conditions required above; we embed Einstein's geometries into a wider class of space-time geometries which might show a quite different behavior in the strong limit.

The question is how to generate such an enlarged set of space-time geometries. There is one point which might help us. If we consider Einstein's theory of gravity as an interaction theory and compare it to what we know from electromagnetic, weak, and strong interactions we observe a real structure difference: Einstein's theory fails to be a real gauge theory. It merely represents the self-dual limit of a corresponding gauge theory, of the Lorentz-Yang-Mills gauge theory defined by the homogeneous Lorentz group as a fundamental gauge group.[32] Therefore, let us consider gravity to be described by a gauge theory. It turns out that the dynamical structure of such a theory is quite uniquely defined (see Secs. II and III). On the one hand, we gain a physically and geometrically well-defined and nicely closed dynamical structure; on the other hand, we have also to pay for this generalization in the form of a weakened principle of equivalence, in the sense that the domain of definition of the usual principle of equivalence has to be restricted to space-time regions of weak gravity. The strong form of the equivalence principle has just been used to define the class of metric theories which were thought to be the only viable ones.[33] Instead of "in every and any local Lorentz frame, anywhere and anytime on space-time, all the nongravitational laws of physics must take on their special-relativistic forms,[24]" we require only for the weak form of the principle of equivalence to be satisfied that

"in every and any local Lorentz frame defined on space-time regions with weak gravity, all the nongravitational laws of physics must take on their special-relativistic forms."

The solar system is, e.g., a space-time of weak gravity ($M_{\odot}/R_{\odot} = 2\times10^{-6}$) and therefore the old principle of equivalence applies unchanged, while it is no longer exactly valid for the space-time region surrounding a star in the supernovae-collapse phase.

The idea of breaking the validity of the equivalence principle in space-time regions of strong gravity is quite compelling. The equivalence principle comes from the picture—and it is really a picture—that a not too widely curved space-time looks locally quite the same as Minkowski space-time; this is really true as long as the curvature is not too high. Nobody would claim that the top of a cone looks like a plane. Mathematically it means that for a manifold the tangent space at a fixed point is a good approximation to the local neighborhood of that point in the manifold, and that is what the principle of equivalence is. The violation of the principle of equivalence is not ambiguous and arbitrary, but it is intrinsically dictated by the Lorentz dynamics itself; the total stress-energy of matter has to be chosen in accordance with the Lorentz dynamical system. The space-time geometries carry themselves a geometric energy and stress generated by self-interaction and self-generation of the curvature of space-time, while in Einstein's vacuum solutions anything is geometry, e.g., the tidal forces are explained as completely generated by corresponding 3-space curvature.

The space-time geometry of the sun has an extremely weak source, so that these nonlinear effects are beyond the actual accuracy of measurements. This is shown in Sec. IV, where we describe the types of exterior solutions in the weak limit for static and spherically symmetric space-times. Sections II and III concern the gauge aspect of gravity and the consequences of the general current conservation. What we might expect for a space-time with a strong source is discussed in Sec. V.

II. A GENERALIZED METRIC FRAMEWORK OF GRAVITY

Most of the self-consistent theories of gravity considered so far in the past are *metric* theories of gravity in the following sense[24,33]:

(A) Any space-time event point is endowed with local Lorentz systems, which, locally, differ only through homogeneous Lorentz transformations. The Lorentz systems are determined by means of a Lorentz metric $g_{\mu\nu}$ (of signature −2); the (strong) principle of equivalence is part of the main axioms,

(B) In every and any local Lorentz frame, anywhere and everytime on space-time all the nongravitational laws of physics must take on their special-relativistic forms.

The equivalence principle has a great power and dictates, e.g., the behavior of matter spread on the space-time by means of $T^{b}_{a;b} = 0$ on the whole space-time. Topological structures, such as orientability conditions, causal structure and global hyperbolicity are not subsumed under axiom (A); they will further restrict the class of reasonable space-time models.[1]

The dynamical structure of metric theories covered so far in the literature is based on the metric as the dynamical field of gravity, i.e., they all provide a direct coupling of the curvature of space-time to the stress-energy content

$$G_{ab}(g)=\kappa T^{M}_{ab}\ , \tag{2.1}$$

where the metric is determined through the differential operator $G_{ab}(g)$ (Einstein's theory, Brans-Dicke and similar theories are of this type of coupling). Inspired by the structure of the general gauge theories successfully used to cover the electromagnetic, weak, and strong interactions in special relativity, we made an attempt to take into account the gauge character of gravity,[32] and, therefore, to consider the Lorentz connection Γ associated with the Lorentz systems as the fundamental dynamical field (the gauge potential of gravity). The corresponding gauge group is then just the Lorentz group O(1,3), and the corresponding field equations are the O(1,3)–Yang-Mills equations $\mathcal{O}_{\rm LYM}(\Gamma)=\kappa\mathcal{J}$, called Lorentz-gauge equations. They couple the time evolution of the connection Γ to a Lorentz current $\mathcal{J}$ through the use of the doubly covariant and hyperbolic differential operator $\mathcal{O}_{\rm LYM}(\Gamma)$,[32] which is of second order for the Lorentz connection. It has been shown, furthermore, that Einstein's field equations may be transformed to the form of the Lorentz gauge equations.[32] In this way, Einstein's field equations determine a particular current, the Einstein-Lorentz current $\mathcal{J}^{E}$. However, not any solution of

$$\mathcal{O}_{\rm LYM}(\Gamma)=\kappa\mathcal{J}^{E}(T^{M}_{ab}) \tag{2.2}$$

satisfies at the same time the Einstein system (2.1), i.e., if Γ_1 is a solution of (2.2), $G_{ab}(\Gamma_1)\neq\kappa T_{ab}$, except for a few members of the class of solutions of (2.2).

In the case of the Lorentz gauge equations (2.2), without requiring (2.1) at the same time to be satisfied, we cannot deduce in general a relation $T^{Mb}_{a\ ;b}=0$, where T^{M}_{ab} denotes the matter stress-energy tensor, since the Lorentz current involves first-order derivatives of T^{M}_{ab}. This fact distinguishes the Lorentz gauge theory from the strictly metric theories of gravity, where axiom (B) implies $T^{Mb}_{a\ ;b}=0$ on the whole space-time. Axiom (B) has to be weakened in a form which will be elaborated in the next section; let us call this weakened axiom $(\tilde{B})$. Different theories of gravity appear in this way in a hierarchical order:

Generalized metric theories of gravity, (A)

- Preferred frame theories
- Lorentz-covariant theories
  - *Strictly metric theories* of the Einsteinian type, (B); g is the dynamical variable,
    $$G_{ab}(g)=\kappa T^{M}_{ab}\ .$$
  - (structure of Lorentz connection) (self-duality conditions on the curvature)
  - *Lorentz gauge theories*, $(\tilde{B})$, the Lorentz connection is the dynamical variable, the metric g is used to define the Lorentz frame,
    $$\mathcal{O}_{\rm LYM}(\Gamma)=\kappa\mathcal{J}^{M}(T^{M}_{ab})\ .$$

For Lorentz gauge theories, the metric just serves to define the local Lorentz systems (observer frames); they span the base space for the definition and calculation of connection and curvature, torsion will be neglected.

The violation of the principle of equivalence in the form of a violation of the "conservation equations" $T^{Mb}_{a\ ;b}=0$ depends essentially on the chosen form of the Lorentz current $\mathcal{J}(T^{M}_{ab})$. It will turn out that $T^{Mb}_{a,;b}=0$ is in general not compatible with the current conservation law $\mathrm{div}(\mathcal{J})=0$ which is a direct geometric consequence of the coupling $\mathcal{O}_{\rm LYM}(\Gamma)=\kappa\mathcal{J}(T^{M}_{ab})$ and of the properties of the LYM operator.[32] This forces us to weaken the equivalence principle in the form that in the Lorentz frames all the nongravitational laws of physics must take on their special-relativistic forms up to corrections which are due to the action of strong gravitational fields. Consequently, only in regions with strong gravitational fields (big-band regions in cosmological models, at the edge of black holes) will the strong version of the principle of equivalence (B) break down. The breaking of the conservation laws, $T_a^{Mb}{}_{;b} \neq 0$, is not as yet ruled out experimentally,[34] and can never be done on the ground of solar-system experiments. These experiments may give an upper bound for the strength of the "breaking force." Smalley already considered modifications of Einstein's field equations[35] and of the Brans-Dicke theory[36] to generate a corresponding breaking force.

The weak version of the equivalence principle reads in the following way:

($\tilde{\text{B}}$) In every and any local Lorentz frame, all the nongravitational laws of physics must take on their special-relativistic forms up to corrections generated by strong gravitational fields.

Immediately the question arises of how to determine these correction terms and whether the formulation of the correction terms is not quite ambiguous. Furthermore, these corrections should turn out physically reasonable—without artificial manipulations—in particular, for space-time regions of weak gravity (e.g., the solar system) they should be so small that they agree with what we expect at the Newtonian level.

III. LORENTZ-YANG-MILLS DYNAMICS, CURRENT CONSERVATION, AND THE WEAK FORM OF THE EQUIVALENCE PRINCIPLE

The Lorentz gauge dynamics is a generalized metric theory of gravity modeled according to the structure of general gauge theories. The real gauge group of gravity is not the coordinate-transformation group, nor a subgroup of it, but the homogeneous Lorentz group SO(1, 3), or its covering group SL(2, C). This turns out by working in a coordinate-free manner—what we are certainly forced to do when operating, e.g., in the spin space defined over space-time. The Newman-Penrose formalism[37,38] is just a reformulation of the structures used in Einstein's theory and shows explicitly the role of SL(2, C) as the gauge group of gravity. Secondly, Einstein's field equations compared with the usual dynamical structure of gauge theories do not fit into a corresponding scheme (for a discussion of the dynamical structure of gauge theories in special relativity see Refs. 39 and 40 and for a review of its applications in strong interaction theory see Ref. 41). In Table I the structures of a Yang-Mills gauge theory are compared with the corresponding structures of a metric theory of gravity.

To handle a particular space-time geometry in the framework of the Lorentz gauge theory of gravity we need the following elements: first, choose a suitable tetrad field (X_0, X_1, X_2, X_3) on the space-time V_4, adapted, e.g., to the flow lines of matter on the space-time,[32] and calculate the metric Lorentz connection and curvature with respect to this chosen basis. The essential dynamical content of the theory is involved in the Lorentz gauge equations

$$(-g)^{-1/2}\partial_\mu[(-g)^{1/2}\tilde{R}^{\mu\rho}]+[\tilde{\Gamma}_\mu,\tilde{R}^{\mu\rho}]=\kappa\tilde{J}_L^\rho\ , \qquad (3.1)$$

and a suitable expression for the Lorentz current $\tilde{J}_L^\rho$. This current, which is so(1, 3)-valued by definition of Eq. (3.1), obeys a conservation law, called the fourth structure equation associated with the Lorentz connection. This conservation law

$$(-g)^{-1/2}\partial_\rho[(-g)^{1/2}\tilde{J}_L^\rho]+[\tilde{\Gamma}_\rho,\tilde{J}_L^\rho]=0 \qquad (3.2)$$

is a pure consequence of (3.1) (just as the Bianchi equations are a consequence of the definition of the curvature 2-form $\tilde{\Omega}$ in terms of the connection 1-form $\tilde{\omega}$).

At the classical level of the description of matter there are not too many possibilities to define this current in terms of the energy-stress tensor T_{ab}^M of matter consistently with the Newtonian limit. One expression which is in accordance with the Newtonian limit is the following:

$$J_{abc}\equiv-(T_{ab;c}-T_{ac;b})+\tfrac{1}{2}(\eta_{ab}T_{,c}-\eta_{ac}T_{,b})\ . \qquad (3.3)$$

Equation (3.3) is defined in terms of the tetrad; therefore the Minkowski metric appears in the second expression; the whole description is essentially coordinate-free, and the structures would break down and become meaningless in terms of coordinates. It is of a strict necessity to distinguish between the indices (Greek) marking the space-time character of a geometric quantity (connection is a 1-form, curvature is a 2-form, and J is a 1-form with respect to the first index) and the indices (Latin) responsible for the gauge character. The Ricci tensor $R_{\mu\rho}$ and the Ricci scalar R are mixed objects, and therefore not defined in the Lorentz gauge theory; they are neither Lorentz tensors, nor Lorentz scalars. Yang[43] proposed a set of generalized vacuum field equations based on the Ricci tensor; the geometric background for Ricci-curl-free space-times has partially been discussed by Kilmister and Newman,[44] they have been used by Lichnerowicz[45] in his quantization approach of the gravitational field and they appear in Bel's[46,47] investigation of the super-energy-momentum tensor of radiative gravitational fields. A justification for proposing the curvature dynamical equations (3.1) is, however, possible only on the background of the gauge aspect of the gravitational interaction.

TABLE I. The structure of the Lorentz gauge theory of gravity is just a transcription of the corresponding structures of the Yang-Mills gauge theories defined on Minkowski space-time. $\tilde{A}$ defines a geometric quantity A (in general a space-time p-form) which is at the same time an element of the Lie algebra $\underline{G}_e$ of the corresponding gauge group G.

Structures and their geometric meaning	Yang-Mills gauge theory on Minkowski space-time	Generalized metric gravity for a torsionless and metric connection
Gauge group G	$U(1), SU(2), SU(n), \cdots$	$O(1,3), SO(1,3); SL(2,\mathbf{C})$
Gauge bundle	Set of fields $\{\psi^{(a)}\}$, $a=1,\ldots,N$ on Minkowski space-time	Lorentz frame bundle[42] defined by the metric g and spanned by the set of tetrads $\{X_0,\ldots,X_3\}$. Spin frame bundle as the covering bundle of the Lorentz frame bundle.[42]
Gauge potentials, local expressions of the connection form of a gauge bundle	Connection coefficients $A_\mu^{(l)}$ given with respect to a fixed basis of the Lie algebra $\underline{G}_e$ of the gauge group, L_l, $\tilde{A}_\mu = A_\mu^{(l)} L_l$, $\tilde{A}_\mu \in \underline{G}_e$.	Connection coefficients $\Gamma_\mu^{(l)}$ $(\gamma_\mu^{(l)})$ of the Lorentz connection given with respect to a basis of $so(1,3)$ $(sl(2,C))$; $\tilde{\omega} = \tilde{\Gamma}_\mu dx^\mu$ $\tilde{\Gamma}_\mu = f_\mu^{(l)} K_l + \Delta_\mu^{(l)} J_l$.
Gauge fields as the local expressions of the curvature of the connection	Curvature components $F_{\mu\rho}^{(l)}$ of the curvature $\tilde{\Omega}$ $\tilde{\Omega} = \frac{1}{2}\tilde{F}_{\mu\rho} dx^\mu \wedge dx^\rho$ $\tilde{F}_{\mu\rho} = \tilde{A}_{\rho,\mu} - \tilde{A}_{\mu,\rho} + \llbracket \tilde{A}_\mu, A_\rho \rrbracket$ $\llbracket\, ,\, \rrbracket$: Lie bracket in $\underline{G}_e$ $\tilde{F}_{\mu\rho} = F_{\mu\rho}^{(l)} L_l$	Curvature components $(f_{\mu\rho}^{(l)}, \Delta_{\mu\nu}^{(l)})$ of the curvature $\tilde{\Omega}$ $\tilde{\Omega} = \frac{1}{2}\tilde{R}_{\mu\rho} dx^\mu \wedge dx^\rho$ $\tilde{R}_{\mu\rho} = \tilde{\Gamma}_{\rho,\mu} - \tilde{\Gamma}_{\mu,\rho} + [\tilde{\Gamma}_\mu, \tilde{\Gamma}_\rho]$ $[\, ,\,]$: Lie-bracket in $so(1,3)$ $\tilde{R}_{\mu\rho} = f_{\mu\rho}^{(l)} K_l + \Delta_{\mu\rho}^{(l)} J_l$.
First structure equation	$\tilde{\nabla}_\mu = 1\partial_\mu + \tilde{A}_\mu$ $(\tilde{\nabla}_\mu \psi)^{(l)} = 0, \ \forall \mu, l$	$\tilde{\nabla}_\mu X_c = \Gamma^a_{\mu c} X_a, \ \forall \mu, c.$
Second structure equation: definition of curvature	$\tilde{F}_{\mu\rho} = \tilde{A}_{\rho,\mu} - A_{\mu,\rho} + \llbracket \tilde{A}_\mu, \tilde{A}_\rho \rrbracket$	$\tilde{R}_{\mu\rho} = \tilde{\Gamma}_{\rho,\mu} - \tilde{\Gamma}_{\mu\rho} + [\tilde{\Gamma}_\mu, \tilde{\Gamma}_\rho]$.
Third structure equation: Bianchi's equation.	$\tilde{F}_{[\mu\rho,\sigma]} + \llbracket \tilde{A}_{[\mu}, \tilde{F}_{\rho\sigma]} \rrbracket = 0$	$\tilde{R}_{[\mu\rho,\sigma]} + [\tilde{\Gamma}_{[\mu}, \tilde{R}_{\rho\sigma]}] = 0$
Gauge equations, "dual Bianchi equations"	$\partial_\mu \tilde{F}^{\mu\rho} + \llbracket \tilde{A}_\mu, \tilde{F}^{\mu\rho} \rrbracket = \alpha \tilde{J}^\rho_{YM}$. equivalent form: 3-form ${}^*\tilde{F}_{[\mu\rho,\sigma]} + \llbracket \tilde{A}_{[\mu}, {}^*\tilde{F}_{\rho\sigma]} \rrbracket = \alpha\, {}^*J_{\mu\rho\sigma}$ α: coupling constant, $\tilde{J}^\rho_{YM}$: Yang-Mills current, ${}^*\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\tilde{F}^{\rho\sigma}$, ${}^*\tilde{J}_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\sigma}\tilde{J}^\sigma_{YM}$ (current 3-form) $\partial_\rho \tilde{J}^\rho_{YM} + \llbracket \tilde{A}_\rho, \tilde{J}^\rho_{YM} \rrbracket = 0$	$(-g)^{-1/2}\partial_\mu((-g)^{1/2}\tilde{R}^{\mu\rho}) +$ $[\tilde{\Gamma}_\mu, \tilde{R}^{\mu\rho}] = \kappa J^\rho_L$. equivalent form: 3-form ${}^*\tilde{R}_{[\mu\rho,\sigma]} + [\tilde{\Gamma}_{[\mu}, {}^*\tilde{R}_{\rho\sigma]}] = \kappa\, {}^*\tilde{J}_{\mu\rho\sigma}$. $\kappa = 8\pi G/c^2$: gravitational coupling constant, $\tilde{J}^\rho_L$: Lorentz current, ${}^*\tilde{R}_{\mu\nu} = \frac{1}{2}\eta_{\mu\nu\rho\sigma}\tilde{R}^{\rho\sigma}$, ${}^*\tilde{J}_{\mu\nu\rho} = \eta_{\mu\nu\rho\sigma}\tilde{J}^\sigma_L$ (current 3-form).
Fourth structure equation: "current conservation"		$(-g)^{-1/2}\partial_\rho((-g)^{1/2}\tilde{J}^\rho_L) + [\tilde{\Gamma}_\rho, \tilde{J}^\rho_L] = 0$

The form (3.3) of the Lorentz current is not necessarily correct for space-times containing electromagnetic fields; the electromagnetic field is described by another (Abelian) gauge theory, and the interaction of two gauge fields has not necessarily to be coupled over their stress-energy-momentum tensor. In case of coexistence of gravity and electromagnetism (e.g., in a neutron star), the coupling is still an open problem even in Einstein's theory.

The fundamental conservation equation of curvature dynamics is the current conservation (3.2), or in equivalent form,

$$J^a{}_{bc;a}=0 \tag{3.4}$$

for a general expression of the form

$$J_{abc}=-(T_{ab;c}-T_{ac;b})+\beta(\eta_{ab}T_{,c}-\eta_{ac}T_{,b}) , \tag{3.5}$$

where β is a real constant, which might be different from $\frac{1}{2}$. Let us define then a general vector field f_a

$$f_a\equiv T_a{}^b{}_{;b} , \tag{3.6}$$

which vanishes if the theory satisfies the strong version of the principle of equivalence (B). The current conservation now gives an integrability condition for the vector field f_a, since

$$J^a{}_{bc;a}=-(T^a{}_{b;c;a}-T^a{}_{c;b;a})+\beta(\eta^a{}_bT_{;ca}-\eta^a{}_cT_{;ba}) . \tag{3.7}$$

The second term, proportional to β, vanishes, and for the first term we use the property that $T^a{}_b$ transforms under the adjoint transformation of the Lorentz group, and therefore

$$T^a{}_{b;c;a}-T^a{}_{b;a;c}=R^a{}_{fca}T^f{}_b-R^f{}_{bca}T^a{}_f , \tag{3.8}$$

and from this we obtain the relation

$$J^a{}_{bc;a}=-(f_{b;c}-f_{c;b}-R_{fc}T^f{}_b+T^f{}_cR_{fb}-T^a{}_fR^f{}_{bca}+T^a{}_fR^f{}_{cba}) . \tag{3.9}$$

The terms proportional to the Riemann tensor vanish because of the symmetries of the Rieman tensor; therefore, the current conservation (3.2) is equivalent to an integrability condition for the external force field f_a coupled to the matter energy-momentum tensor T^M_{ab}

$$f_{b;c}-f_{c;b}=R_{fc}T^{Mf}{}_b-T^{Mf}_cR_{fb} . \tag{3.10}$$

It has already been shown by Pagels[48] that an object of the type introduced in (3.3) for Einstein's theory satisfies the conservation (3.4). This follows now immediately from the condition (3.10) if the Ricci tensor is itself proportional to the symmetric energy-stress tensor T^M_{ab}. A generalized metric connection satisfying (3.1) and (3.3) does allow in general not to choose a vanishing divergence for T^M_{ab}; the divergence has to satisfy the condition (3.10). If, e.g., there exists a Lorentz frame in which both R_{ab} and T_{ab} are diagonal and have certain symmetries, we may find indeed a commuting right-hand side of (3.10), i.e., f_a is then a gradient vector field. This occurs, e.g., in the static and spherically symmetric matter distributions or in homogeneous cosmological models.

For a solution Γ of the Eqs. (3.1)–(3.3) we may calculate the Einstein tensor $G_{ab}(\Gamma)$ and split this tensor into a part due to T^M_{ab} and an additional term $T^G_{ab}=T^G_{ab}(\Gamma,\mathcal{J})$,

$$G_{ab}(\Gamma)=\kappa[T^M_{ab}+T^G_{ab}(\Gamma,\mathcal{J})] . \tag{3.11}$$

Then the Einstein contribution to the Ricci tensor drops out of the relation (3.10), and also the trace part. We get then a relation of the following type:

$$f_{b;c}-f_{c;b}=\kappa(T^G_{fc}T^{Mf}{}_b-T^{Mf}_cT^G_{fb}) . \tag{3.12}$$

This equation obviously means that the external geometric force f_a is nonconservative in every case where the interaction between the stress-energy of matter and the stress-energy content of the space-time geometry is nontrivial. In analogy to the special-relativistic situation we might look for a symmetric tensor $T^Y_{ab}(\Gamma)$ made up by curvature and similar tensors such that it satisfies the two conditions:

$$\text{(i) } T^{Yb}_a{}_{;b}=-f_a , \text{ where } f_a=T^{Mb}_a{}_{;b} , \tag{3.13}$$

$$\text{(ii) } f_{b;c}-f_{c;b}=R_{fc}T^{Mf}{}_b-T^{Mf}_cR_{fb} . \tag{3.14}$$

If we had such a geometric object in closed form, it would enable us to calculate the geometric stress-energy content of a space-time geometry which contributes to the stress-energy of matter, without subtracting the stress-energy of matter from the total Einstein tensor $G_{ab}(\Gamma)$ as indicated in Eq. (3.11). My personal view is, however, that such an object does not exist in closed form because of the high nonlinearity of the theory; otherwise the curvature dynamical equations could just be reduced to Einstein's form (3.11) with a suitable geometric tensor $T^G_{ab}(\Gamma)$. This is also a fundamental difference between curvature dynamics and theories based on a changed Einstein Lagrangian (1.1); in the latter case we have an explicit expression for $T^G_{ab}(\Gamma)$, which in general does not even disturb the principle of equivalence.

It is just Eq. (3.10) which tells us how much the strong version of the principle of equivalence has been broken on a solution of Eqs. (3.1)–(3.3). For space-time geometries with a vanishing right-hand side of (3.10) we have a breaking by means of a gradient vector field, a case which already had been proposed by Rastall.[34] As a consequence of the existence of a nonvanishing external force f_a, the hydrodynamical equations for matter generating the gravitational field have to incorporate this force-field, while test matter moving in that gravitational field does not feel the influence of this geometric force (e.g., the plasma in the magnetosphere of a pulsar would not directly feel the existence of stress-energy of the geometry; however, the plasma would indirectly feel it through the geometry of the background space-time which is no longer Einsteinian).

This breaking of the principle of equivalence, which is a nonlocal effect because of relation (3.10), is inherently connected with the LYM-dynamical system together with the corresponding structure equations and is therefore quite uniquely defined whenever we require the field equations to be of a tensorial form. On 4-dimensional space-time we have nontrivial 1-, 2-, 3-, and 4-forms; the 4-forms are proportional to the volume form, and therefore uninteresting; the relevant forms are additionally so(1, 3)-valued. Starting with the connection 1-form, $\tilde{\omega}$, we obtain by applying the covariant exterior derivative the curvature 2-form, $\tilde{\Omega}$, which is then covariantly closed (the Bianchi equations). Consequently, to continue in a nontrivial way, we have to pass to the dual of the curvature form, $*\tilde{\Omega}$, which is also a 2-form because dim $V=4$. Applying to this the covariant exterior derivative defines a 3-form, the current 3-form $\tilde{J}$, which is now closed too. This means that the Yang-Mills framework on a 4-dimensional space-time is completely closed and self-consistent as a consequence of the underlying structures, and it is the only dynamical framework with this self-closedness property. The only way that an external element can enter into the structure is by means of the form of this current, and this only freedom is dictated by the Newtonian limit. This self-closedness of the Lorentz gauge theory may be represented in the following diagram; D denotes the covariant exterior derivative operating on so$(\hat{1}, 3)$-valued forms.[49,50]

1-forms	$\tilde{\omega}$		connection
	$\downarrow D$		
2-forms	$\tilde{\Omega}=D\tilde{\omega}$ $\xrightarrow{*}$	$*\tilde{\Omega}$	curvature
	$\downarrow D$	$\downarrow D$	
3-forms	$D\tilde{\Omega}=0$	$D*\tilde{\Omega}=\tilde{\mathcal{J}}$	current
		$\downarrow D$	
4-forms		$D\tilde{\mathcal{J}}=0$	volume

IV. THE POST-NEWTONIAN LIMIT FOR NONROTATING WEAK SOURCES

The solar system is described in a first approximation as a static and spherically symmetric space-time; we describe it in terms of the Schwarzschild coordinate system

$$ds^2 = e^{2\mu}dt^2 - e^{-2\lambda}dr^2 - r^2 d\Omega^2 \tag{4.1}$$

with respect to the static frame of reference $\{X_0, X_1, X_2, X_3\}$ defined by

$$X_0 = e^{-\mu}\partial_t\,, \quad X_2 = r^{-1}\partial_\theta\,, \quad X_1 = e^{\lambda}\partial_r\,, \quad X_3 = (r\sin\theta)^{-1}\partial_\phi\,. \tag{4.2}$$

The metric and torsionless connection has the following components expressed in terms of the connection form:

$$\tilde{\omega} = \tilde{\Gamma}_a \Theta^a\,, \quad \Theta^a(X_b) = \delta^a{}_b\,, \tag{4.3}$$

$$\tilde{\Gamma}_{\underline{0}} = aK_1\,, \quad \tilde{\Gamma}_{\underline{1}} = 0\,,$$

$$\tilde{\Gamma}_{\underline{2}} = \frac{\Delta}{r}J_3\,, \quad \tilde{\Gamma}_{\underline{3}} = \frac{1}{r}\cot\theta J_1 - \frac{\Delta}{r}J_2\,, \tag{4.4}$$

where $a = e^{-\mu}e^{\lambda}(e^{\mu})'$ describes the 4-acceleration of the static observer system, and $\Delta \equiv e^{\lambda}$ relates to the geometry of the space-like hypersurface Σ defined by $t=$ const. At the same time the curvature 2-form decomposes into

$$\tilde{R}_{ab} = f^{(l)}_{ab}K_l + \Delta^{(l)}_{ab}J_l\,, \tag{4.5}$$

with

$$f_{01} \equiv f^{(1)}_{\underline{01}} = -(e^{\lambda}a' + a^2) = -e^{\lambda-\mu}f'\,,$$

$$f = e^{\lambda}(e^{\mu})'\,, \tag{4.6}$$

$$f_{02} \equiv f^{(2)}_{\underline{02}} = -\frac{\Delta}{r}a = -\frac{1}{r}e^{\lambda-\mu}f\,, \tag{4.7}$$

$$\Delta_{12} \equiv \Delta^{(3)}_{\underline{12}} = \frac{e^{\lambda}\Delta'}{r}\,, \tag{4.8}$$

$$\Delta_{23} \equiv \Delta^{(1)}_{\underline{23}} = \frac{\Delta^2 - 1}{r^2}\,. \tag{4.9}$$

In terms of these curvature coordinates, the linearity of Einstein's equations shows up,

$$\kappa\rho = -\Delta_{23} - 2\Delta_{12}\,, \tag{4.10}$$

$$\kappa p_r = 2f_{20} + \Delta_{23}\,, \tag{4.11}$$

$$\kappa p_\perp = f_{10} + f_{20} + \Delta_{12}\,, \tag{4.12}$$

where ρ denotes the total energy density, p_r denotes the radial pressure, and $p_\perp$ denotes the orthogonal part to p_r. From the above representation of Einstein's equations we obtain the relation between $\tilde{\Omega}$ and $*\tilde{\Omega}*$,

$$f_{10} - \Delta_{23} = \frac{\kappa}{2}(\rho + 2p_\perp - p_r)\,, \tag{4.13}$$

$$\Delta_{12}-f_{20}=-\frac{\kappa}{2}(\rho+p_r)\ . \qquad (4.14)$$

$\tilde{\Omega}^*$ is the so(1, 3)-adjoint object[32] of (4.5) such that

$$\tilde{R}^*_{ab}=-\Delta^{(l)}_{ab}K_l+f^{(l)}_{ab}J_l\ . \qquad (4.15)$$

Equations (4.13) and (4.14) give an expression for the anisotropy in matter,

$$\kappa(p_\perp-p_r)=f_{10}-\Delta_{23}+\Delta_{12}-f_{20}\ . \qquad (4.16)$$

In this way, Einstein's equations determine directly the four curvature components f_{10}, f_{20}, Δ_{12}, and Δ_{23} by the equations (4.13) and (4.14) as well as by

$$f_{10}+2f_{20}=\frac{\kappa}{2}(\rho+3p)\ , \qquad (4.17)$$

$$\Delta_{23}+2\Delta_{12}=-\kappa\rho\ . \qquad (4.10)$$

In the post-Newtonian limit, space-time geometries described by the two functions $a(r)$ and $\Delta(r)$ are in general characterized by the mass M and the two parametrized post-Newtonian (PPN) parameters γ and β[51]

$$e^{2\mu}=1-2\frac{M}{r}+2(\beta-\gamma)\left(\frac{M}{r}\right)^2+O((M/r)^3)\ , \qquad (4.18)$$

$$\Delta^2=e^{-2\lambda}=1-2\gamma\frac{M}{r}+O((M/r)^2)\ . \qquad (4.19)$$

In addition, the acceleration function f is positive everywhere on positively oriented 3-space Σ for an attractive interaction

$$f=\frac{M}{r^2}\left[1-(2\beta-\gamma-1)\frac{M}{r}+O((M/r)^2)\right]\ . \qquad (4.20)$$

For the usual metric theories of gravity, γ and β are universal parameters in the sense that they are uniquely determined by the gravitational coupling constants. For general relativity, e.g., we get the following expressions:

from (4.17)

$$f(r)=M/r^2 \text{ with } M=\frac{\kappa}{2}\int_0^R e^{\mu-\lambda}(\rho+3p)r^2dr\ , \qquad (4.21)$$

from (4.10)

$$\Delta^2(r)=1-2\gamma\frac{M}{r}\ , \qquad (4.22)$$

from (4.14)

$$\gamma_E=1,\ \text{and therefore}\ \beta_E=1\ . \qquad (4.23a)$$

The Brans-Dicke theory, e.g., allows γ to depend on a further coupling constant ω[51]

$$\gamma_{BD}=(\omega+1)/(\omega+2)\ \text{and}\ \beta_{BD}=1\ . \qquad (4.23b)$$

The Lorentz gauge equations (3.1), on the other hand, are by definition differential equations of second order for the connection coefficients. The parameter γ will therefore remain arbitrary and appear related to the interior solution. The gauge field equations split into two groups:

Bianchi's equations

$$e^\lambda f'_{20}+\frac{\Delta}{r}(f_{20}-f_{10})+a(f_{20}-\Delta_{12})=0\ , \qquad (4.24)$$

$$e^\lambda\Delta'_{23}+2\frac{\Delta}{r}(\Delta_{23}-\Delta_{12})=0\ ; \qquad (4.25)$$

Lorentz gauge equations

$$e^\lambda f'_{10}+2\frac{\Delta}{r}(f_{10}-f_{20})=\kappa j^{(1)\underline{0}}\ , \qquad (4.26)$$

$$e^\lambda\Delta'_{12}+\frac{\Delta}{r}(\Delta_{12}-\Delta_{23})+a(\Delta_{12}-f_{20})=\kappa S^{(3)\underline{2}}\ . \qquad (4.27)$$

Here, we introduced the decomposition of the Lorentz current $\tilde{J}^a$,

$$\tilde{J}^a=-j^{(l)a}K_l+S^{(l)a}J_l\ . \qquad (4.28)$$

In terms of the total energy density ρ and the pressure parts p_r and $p_\perp$, this current is locally given by (3.3)

$$j^{(1)\underline{0}}=\tfrac{1}{2}e^\lambda(\rho'+2p'_\perp+p'_r)+a(\rho+p_r)\ , \qquad (4.29)$$

$$S^{(3)\underline{2}}=\frac{\Delta}{r}(p_r-p_\perp)-\tfrac{1}{2}e^\lambda(\rho'-p'_r)\ . \qquad (4.30)$$

The inhomogeneous equations (4.26), (4.27) are equivalent to differential equations for f and Δ^2:

$$r^{-2}(r^2f')'-2r^{-2}f=e^{\mu-2\lambda}(f_{01}\chi(\Delta',f)+\kappa j^{(1)\underline{0}})\ , \qquad (4.31)$$

$$\tfrac{1}{2}\Delta^{2\prime\prime}-r^{-2}(\Delta^2-1)=re^{-\lambda}(f_{02}\chi(\Delta',f)+\kappa S^{(3)\underline{2}})\ , \qquad (4.32)$$

$$\chi(\Delta',f)\equiv\Delta'-a\ . \qquad (4.33)$$

This form of the field equations shows the curvature-curvature self-interaction which acts as a source of the gravitational field. In vacuum regions of extremely low curvature, the two equations decouple, and therefore the two parameters M and γ are arbitrary and characterize the class of the asymptotically flat solutions of (4.31)–(4.33). These solutions for $j^{(1)\underline{0}}=0=S^{(3)\underline{2}}$ have the following properties:

(i) the geometries are asymptotically flat, i.e., $f_{0i}=O(r^{-3})$ and $\Delta_{ik}=O(r^{-3})$, $i,k=1,2,3$;

(ii) the geometries of positively oriented space-time are characterized by two parameters, the active gravitational mass M and the parameter γ, $\gamma\in\mathbb{R}$;

(iii) the asymptotic expansion is given by ($x \equiv M/r$)

$$e^{2\mu} = 1 - 2x + \tfrac{3}{2}(1-\gamma)x^2 + \tfrac{1}{10}(-3 + 20\gamma - 17\gamma^2)x^3 + O(x^4) , \qquad (4.34)$$

$$\Delta^2 = 1 - 2\gamma x + \tfrac{1}{2}(1-\gamma)x^2 + \tfrac{3}{10}(1-\gamma^2)x^3 + O(x^4) , \qquad (4.35)$$

$$f = \frac{M}{\gamma^2} N(x;\gamma) , \qquad (4.36)$$

$$N = 1 - \tfrac{1}{2}(1-\gamma)x + \tfrac{1}{20}(11\gamma^2 - 10\gamma - 1)x^2 + O(x^3) ; \qquad (4.37)$$

(iv) for $\gamma = 1$ we regain the Schwarzschild solution, since all the above coefficient polynomials have a zero at $\gamma = 1$,

$$e_S^{2\mu} = 1 - 2x, \quad x < \tfrac{1}{2} , \qquad (4.38)$$

$$\Delta_S^2 = 1 - 2x , \qquad (4.39)$$

$$N_S = 1 ; \qquad (4.40)$$

(v) the parameters β and λ_p [24,51] which determine the particle trajectories in the geometry (4.34)–(4.37) are completely given by γ,

$$\beta = \beta(\gamma) = \tfrac{1}{4}(3 + \gamma) , \qquad (4.41)$$

$$\lambda_p = \lambda_p(\gamma) = \tfrac{1}{12}(5 + 7\gamma) . \qquad (4.42)$$

Various types of these solutions have been worked out by Pavelle and Thompson[52–54] in relation with Yang's field equations.[43]

In Fig. 1 we compare the surface red-shift

$$Z_S(R) = e^{-\mu}(R) - 1 \qquad (4.43)$$

with the corresponding red-shift relation for Schwarzschild's geometry. For $\gamma < 1$ we have a lower increase of Z_S than for $\gamma = 1$; this may indicate the existence of stable stellar configurations having higher mass-to-radius ratios than in Einstein's theory. The function $\delta \equiv e^{\lambda - \mu}$ is a measure for the deviations from the Schwarzschild geometry ($\delta_S \equiv 1$); its behavior is shown in Fig. 2. The Lorentz gauge equations have consequently 3 types of solutions in the asymptotic domain: (i) $\delta = 1$ for $\gamma = 1$, then we have Schwarzschild; (ii) $\delta < 1$, if $\gamma > 1$, and (iii) $\delta > 1$, if $\gamma < 1$. In Case (ii) δ seems to be monotonically increasing. A similar behavior shows the function $u = rf$ (see Fig. 3). Depending on the value of γ, the self-interaction contribution to the source of gravity acts therefore either in favor of the attractiveness of the gravitational force or damps its influence on the geometry.

The parameter γ is actually not a free param-

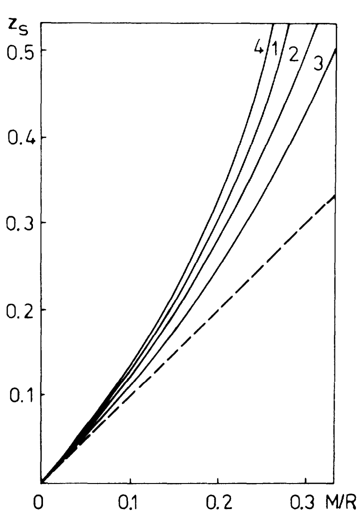

FIG. 1. The surface red-shift for the solutions of the gauge field equations, parametrized by different values of γ and given in the range $0 \leq M/R \leq 0.3$: curve 1, $\gamma = 1$; curve 2, $\gamma = 0.8$; curve 3, $\gamma = 0.2$; curve 4, $\gamma = 1.5$.

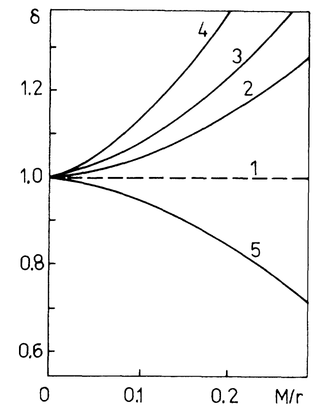

FIG. 2. Typical runs for the characteristic function $\delta = e^{\lambda - \mu}$ in the range $0 \leq M/r \leq 0.3$. This function is a measure of the deviations from the Schwarzschild geometry; it shows that the class of solutions of the gauge field equations contains three different types of geometries depending on the value of γ: curve 1, $\gamma = 1$; curve 2, $\gamma = 0.5$; curve 3, $\gamma = 0.2$; curve 4, $\gamma = -0.5$; curve 5, $\gamma = 1.5$.

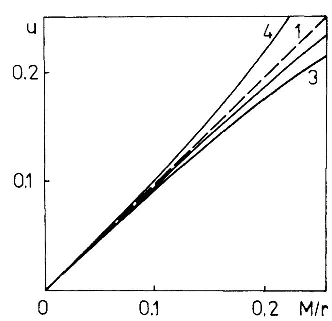

FIG. 3. The behavior of the function $u(x) = xN(x,\gamma)$, where $x = M/r$, is shown for different values of γ. In the limit $\gamma \to 1$, u approaches continuously the Schwarzschild function with $N_S = N(x;1) = 1$: curve 1, $\gamma = 1$; curve 2, $\gamma = 0.5$; curve 3, $\gamma = 0.2$; curve 4, $\gamma = 1.5$.

eter of this theory; the value of γ is related to the state of the source of gravity. In order to see this, we calculate the lowest-order correction terms for Einstein's interior solutions; a general solution of the equations (4.26) and (4.27), or equivalently of (4.31) and (4.32), with interior sources (4.29) and (4.30) can always be written in the form

$$f_{\rm in}(r) = f_{\rm E}(r)[1 + f_{\rm NL}(r)] \,, \tag{4.44}$$

$$\Delta_{\rm in}^2(r) = \Delta_{\rm E}^2(r) + \Delta_{\rm NL}^2(r) \,, \tag{4.45}$$

here $f_{\rm E}$ and $\Delta_{\rm E}^2$ solve Einstein's equations (4.10), (4.13), (4.14), and (4.17); they are characterized by the central density ρ_c and a corresponding equation of state $p = p(\rho)$. More suitable quantities to parametrize the interior solutions are

$$a_2/a_0 = \frac{\kappa}{12}(\rho_c + 3p_c), \quad e^{\mu}(0) = a_0 \,, \tag{4.46}$$

$$\xi = -2/(1 + 3p_c/\rho_c) \,, \tag{4.47}$$

with $-2 \leq \xi \leq -1$ for $0 \leq p_c \leq \rho_c/3$. In terms of these parameters, the interior Schwarzschild solution, e.g., is given by $(\rho = \rho_c)$

$$f_{\rm ES} = 2a_2 r \,, \tag{4.48}$$

$$\Delta_{\rm ES}^2 = 1 + 2\xi \frac{a_2}{a_0} r^2 \,. \tag{4.49}$$

More realistic distributions of energy add higher-order terms to $f_{\rm ES}$ and $\Delta_{\rm ES}^2$.

By adding together (4.26) and (4.24), (4.27) and (4.25), respectively, we obtain the following form for the gauge equations:

$$e^{\lambda}(f_{10} + 2f_{20})' + 2a(f_{20} - \Delta_{12}) = \kappa[\tfrac{1}{2}e^{\lambda}(\rho + 3p)' + a(\rho + p)] \,, \tag{4.50}$$

$$e^{\lambda}(\Delta_{23} + 2\Delta_{12})' - 2a(f_{20} - \Delta_{12}) = -\kappa e^{\lambda}\rho' + \kappa e^{\lambda} p_r' \,. \tag{4.51}$$

Therefore,

$$f_{10} + 2f_{20} = \tfrac{1}{2}\kappa(\rho + 3p) \,, \tag{4.52}$$

$$\Delta_{23} + 2\Delta_{12} = -\kappa\rho \,, \tag{4.53}$$

$$f_{20} - \Delta_{12} = \tfrac{1}{2}\kappa(\rho + p) \tag{4.54}$$

solve the gauge equations (4.50) and (4.51) under the condition of hydrostatic equilibrium

$$e^{\lambda} p_r' = -a(\rho + p) \,. \tag{4.55}$$

The solutions (4.52)–(4.55) are, however, nothing but Einstein's equations (4.10), (4.14), and (4.17), and the corresponding exterior solutions require $\gamma = 1$. Therefore we get the result that asymptotically flat space-time geometries satisfying (4.50) and (4.51) are Schwarzschild ($\gamma = 1$) if the hydrostatic equilibrium condition (4.55) holds for massive matter, i.e., in particular if radiation pressure is negligible. These solutions are asymptotically flat. A general solution of (4.50) and (4.51) may not be asymptotically flat since

$$f_{10} + 2f_{20} + \Delta_{23} + 2\Delta_{12} = -\tfrac{1}{2}\kappa(\rho - 3p) + C \,, \quad C \in R \tag{4.56}$$

is a consequence of the gauge equations under hydrostatic equilibrium. They correspond to solutions of Einstein's equations including a nonvanishing cosmological constant.

The gauge dynamics for static and spherically symmetric space-time geometries allows, in general, for a breaking of the hydrostatic equilibrium in the form of

$$e^{\lambda} p_r' = -a(\rho + p_r) - f_{\underline{r}} \,. \tag{4.57}$$

This follows from the conservation equation (3.10) and the symmetries of the Ricci tensor and the energy-momentum tensor for massive matter; the right-hand side of (3.10) commutes in this case. $f_{\underline{r}}$ is an expression for the radiation pressure working on the matter inside the star or for other forces exerted on stellar matter (e.g., magnetohydrodynamical forces). If $f_{\underline{r}} \neq 0$, the gravitational force $a(\rho + p)$ is in general balanced by the sum of the hydrostatic pressure and $f_{\underline{r}}$. Let us consider the case where $f_{\underline{r}}$ dominates over the hydrostatic pressure (this situation is realized, e.g., in the interior of radiation-dominated stars). Then the last term in (4.51) no longer cancels the second term, and Einstein's equations fail to form a complete solution of (4.50) and (4.51); the lowest-order corrections for (4.45) are then given by

$$\Delta_{\rm NL}^2(y) = \frac{4}{5}(1 - \xi)y^4 + O(y^6) \,, \tag{4.58}$$

with $y^2 = (a_2/a_0)r^2$ as a dimensionless radius. Since $y_R^2 \simeq M/R$, these are small corrections for weak sources. The interior general-relativistic solutions (4.48) and (4.49), completed by (4.58), then determine new exterior solutions with

$$\xi y_R^2 \simeq -M/R \,, \tag{4.59}$$

and the fitting with

$$\Delta_{\rm ex}^2 = 1 - 2\frac{M}{r} - 2(\gamma - 1)\frac{M}{r} + O((M/r)^2) \tag{4.60}$$

determines γ,

$$\gamma = 1 - \frac{2}{5}\frac{1 - \xi}{-\xi} y_R^2 \text{ for } f_{\underline{r}}\text{-dominated equilibrium.} \tag{4.61}$$

For the solutions (4.34)–(4.37) we may calculate the geometric self-energy-momentum tensor T^G_{ab} defined by (3.11) and built up by the nonlinearities in (4.26) and (4.27); in lowest order we obtain

$$\rho^G = (\kappa r^2)^{-1}[\tfrac{1}{2}(1-\gamma)x^2 + \tfrac{3}{5}(1-\gamma^2)x^3 + O(x^4)] \ , \quad (4.62)$$

$$p_r^G = (\kappa r^2)^{-1}[2(1-\gamma)x + \tfrac{3}{2}(1-\gamma)x^2 + \tfrac{6}{5}(1-\gamma^2)x^3 + O(x^4)] \ , \quad (4.63)$$

$$p_\perp^G = -(\kappa r^2)^{-1}[(1-\gamma)x + (1-\gamma)x^2 + \tfrac{3}{4}(1-\gamma^2)x^3 + O(x^4)] \ . \quad (4.64)$$

The stresses are highly anisotropic $p_r^G \neq p_\perp^G$. For the Schwarzschild geometry, $\gamma = 1$, T_{ab}^G vanishes identically on Σ, while a realistic source with $\gamma < 1$ [see (4.61)] produces a positive energy density in the exterior region. At the surface of a neutron star, e.g., $\rho^G(R)$ is of the order of 10^{13} g cm^{-3}. For the interior solutions, the nonlinearities expressed in (4.58) generate in lowest order the following self-energy density:

$$\rho_{\text{in}}^G(y) = -(\kappa r^2)^{-1}[4(1-\xi)y^4 + O(y^6)] \ . \quad (4.65)$$

It can be shown that in the Newtonian limit ($\xi = -2$), ρ_{in}^G describes in lowest order exactly the binding energy density, i.e., $\rho_{\text{in}}^G = \rho(r)\phi_N(r)$, where ϕ_N is the Newtonian potential; T_{ab}^G is calculated in the rest frame of matter. The effective source of gravity is smaller than the mass-energy distribution of the source; this difference is due to the gravitational binding. The SO(1,3)-gauge theory of gravity also includes this effect.

V. STRONG LIMIT OF STATIC AND SPHERICALLY SYMMETRIC SPACE-TIMES

The exterior solutions of the Lorentz gauge equations are characterized by the "measure function" $\delta = e^{\lambda-\mu}$. The tidal components of the curvature, f_{01} and f_{02}, given in (4.7) and (4.8), are decomposable as

$$f_{01} = -\delta f' \ , \quad (5.1)$$

$$f_{02} = -\frac{\delta}{r} f \ . \quad (5.2)$$

The function f, given in lowest order in (4.37) is montonically increasing (see Fig. 3) and has a nonfinite series expansion for $\gamma \neq 1$, while δ is either increasing, decreasing, or stable (for $\gamma = 1$). A singularity appearing in f_{0i} is not due to the coordinate system nor to frame effects since the first curvature invariant has the form

$$I = -\frac{1}{8}\operatorname{Tr}\{\tilde{R}_{ab}\tilde{R}^{ab}\} = \delta^2\left(\frac{2}{r^2}f^2 + f'^2\right) + \frac{2}{r^2}\Delta^2\Delta'^2 + \frac{(\Delta^2-1)^2}{r^4} \ . \quad (5.3)$$

Let the function $e^\mu(x)$ have a zero at $x = x_0$ (e.g., $x_0^s = \frac{1}{2}$); the behavior of the tidal fields f_{0i} now depends critically on the characteristic function $\delta(x;\gamma)$. Since $\delta(x;\gamma=1) = 1$, $\forall x \leq \frac{1}{2}$, and f is finite in the case of Schwarzschild and Reissner-Nordström systems, we obtain the well-known result that the surface at $x = \frac{1}{2}$ is a regular event horizon for $\gamma = 1$. For $\gamma < 1$, the characteristic function, and therefore also the tidal forces grow without limit as $x \to x_0$, $e^\mu(x_0,\gamma) = 0$; the curves of the surface red-shift already indicated that $x_0 > \frac{1}{2}$ for $\gamma < 1$ (see Fig. 1). In the case $\gamma > 1$, δ tends to zero for $x = x_1$, where $e^\lambda(x_1;\gamma) = 0$, however, retaining $e^\mu(x_1,\gamma) > C$, $C > 0$. At this point $x = x_1$, where now $x_1 < \frac{1}{2}$, the Schwarzschild coordinate system breaks down and the tidal forces vanish for $x \to x_1$. There exists, however, an extension of the coordinate system, because the curvature remains regular at $x = x_1$, into a space-time region of opposite tidal forces. This means that the expansion in (4.34)–(4.37) is valid only for regions with either $x < x_0$, $e^\mu(x_0) = 0$, for $\gamma \leq 1$, or $x < x_1$, $e^\lambda(x_1) = 0$, for $\gamma > 1$. In this way we may classify the solutions of the Lorentz gauge equations according to their behavior of the tidal fields:

(i) $\gamma < 1$, $\delta(x) > 1$, then the tidal fields f_{0i} grow (without limit) for $x \to x_0$, where $e^\mu(x_0) = 0$;

(ii) $\gamma = 1$, $\delta_S(x) = 1$, there exists a regular horizon at $x_0 = \frac{1}{2}$, tidal fields are finite;

(iii) $\gamma > 1$, $\delta(x) < 1$, the tidal fields tend to zero at x_1, where $e^\lambda(x_1) = 0$ and $e^\mu(x_1) > 0$, and we may find an extension into a region of space-time with tidal fields of opposite sign.

This classification shows the exceptional position of the Schwarzschild geometry in the class of solutions of the Lorentz gauge equations. A (regular) event horizon does really only then exist if we require the self-interaction to vanish, i.e., $\chi = 0$,

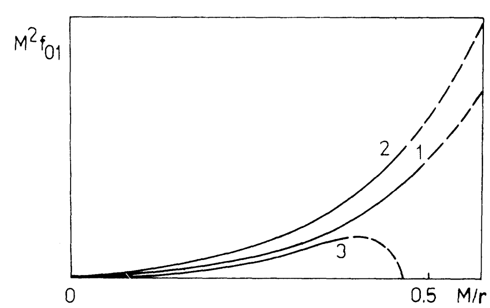

FIG. 4. The behavior of the tidal fields f_{0i} depends critically on the characteristic function δ, and therefore on the values of γ. For $\gamma = 1$ (curve 1) a regular event horizon exists at $x = \frac{1}{2}$ (Schwarzschild horizon); for $\gamma < 1$ (curve 2) the Schwarzschild coordinate system breaks down for $x \geq x_0$; $x_0 > \frac{1}{2}$. For $\gamma > 1$ (curve 3) the tidal fields evolve a zero at $x = x_0$, $x_0 < \frac{1}{2}$ and then change sign. The Schwarzschild coordinate system, however, breaks down for $x > x_0$. In each of the three cases an analytic extension exists.

and this requires $\gamma = 1$, i.e., a Schwarzschild geometry. The behavior of the tidal fields is summarized in Fig. 4. In theories of the Einsteinian type a tidal singularity for this simple geometry cannot occur without affecting at the same time the curvature of 3-space, since the tidal components are given by 3-space curvature in vacuum regions [see Eqs. (4.13), (4.14)]. The above evolvement of high tidal fields not generated by 3-space curvature is therefore possible only by breaking this simple relation to the 3-space-curvature. By breaking Einstein's field equations, which essentially means adding self-interaction—and therefore breaking the principle of equivalence—static and spherically symmetric black-hole solutions can no longer exist; besides, in the absence of self-interaction ($\gamma = 1$), for $\gamma \neq 1$, a regular event horizon is not expected to exist.

VI. CONCLUSION

Gravity considered as a gauge theory already offers a wider class of solutions in the case of static and spherically symmetric space-times; therefore, the issue of the final state for collapsing matter (e.g., matter involved in the supernovae I phase) might be found to differ in general completely from the predictions based on Einstein's field equations. This justifies, so to speak, the creation of the Lorentz gauge theory of gravity as an alternative approach to Einstein's dynamics. A nontrivial element is added to general relativity: gravitational self-interaction.

While the Schwarzschild geometry is a fairly good approximation for the exterior gravitational field of *weak* sources, the exterior fields of strong forces evolve away from Schwarzschild by the amount of self-interaction, which on the other hand grows with the strength of the source. As a consequence, this generalized metric theory of gravity no longer offers the Schwarzschild black hole as a final state for the time evolution of the late phase in the history of matter; what else then?

This question cannot be settled without investigating the time-dependent collapse equations; however, since gravitational self-interaction just dominates in that region of space-time where under Einstein's dynamics a regular event horizon evolves, this, together with the breaking of the principle of equivalence producing a real energy-momentum exchange between geometry and matter, forces matter itself to follow a different evolution.

The SO(1,3)-gauge theory of gravity has for extremely weak sources of gravity (e.g, for the solar system) essentially the same post-Newtonian limit as general relativity. This follows from the fact that Einstein's equations represent the linearized version of the Lorentz gauge theory and that in the post-Newtonian limit $T_a^{Mb}{}_{;b} = 0$ is satisfied to a high accuracy. In general, the principle of equivalence will be broken, i.e., $T_a^{Mb}{}_{;b} \neq 0$, whenever gravitational radiation plays a dominant role; but this case is just excluded in the post-Newtonian framework. Therefore, in view of the solar-system data, the Lorentz gauge theory of gravity is as *viable* as general relativity. We need strong sources (such as neutron stars) or stars under the supernovae collapse to test the nonlinearities in the theory.

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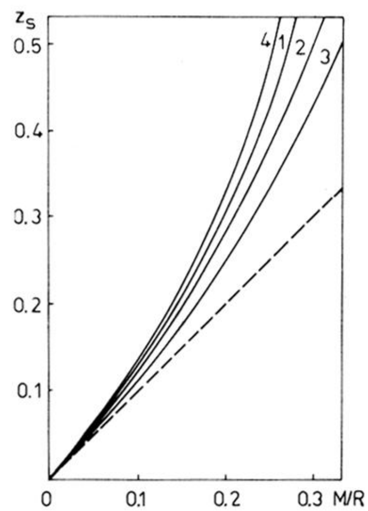

FIG. 1. The surface red-shift for the solutions of the gauge field equations, parametrized by different values of γ and given in the range $0 \leqslant M/R \leqslant 0.3$: curve 1, $\gamma = 1$; curve 2, $\gamma = 0.8$; curve 3, $\gamma = 0.2$; curve 4, $\gamma = 1.5$.

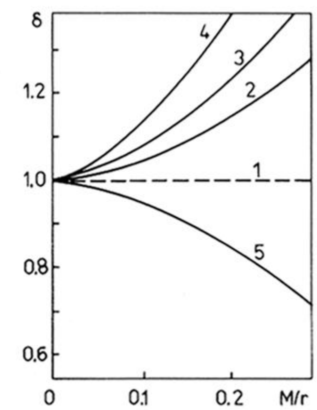

FIG. 2. Typical runs for the characteristic function $\delta = e^{\lambda - \mu}$ in the range $0 \leq M/r \leq 0.3$. This function is a measure of the deviations from the Schwarzschild geometry; it shows that the class of solutions of the gauge field equations contains three different types of geometries depending on the value of γ: curve 1, $\gamma = 1$; curve 2, $\gamma = 0.5$; curve 3, $\gamma = 0.2$; curve 4, $\gamma = -0.5$; curve 5, $\gamma = 1.5$.

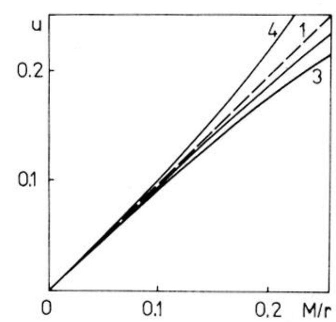

FIG. 3. The behavior of the function $u(x) = xN(x, \gamma)$, where $x = M/r$, is shown for different values of γ. In the limit $\gamma \to 1$, u approaches continuously the Schwarzschild function with $N_S = N(x;1) = 1$: curve 1, $\gamma = 1$; curve 2, $\gamma = 0.5$; curve 3, $\gamma = 0.2$; curve 4, $\gamma = 1.5$.

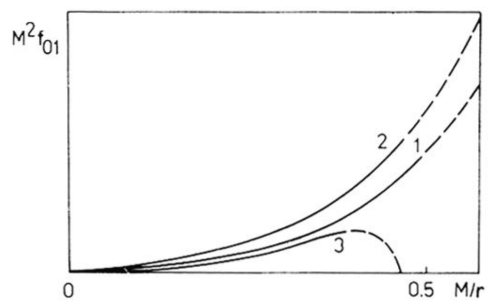

FIG. 4. The behavior of the tidal fields f_{0i} depends critically on the characteristic function δ, and therefore on the values of γ. For $\gamma = 1$ (curve 1) a regular event horizon exists at $x = \frac{1}{2}$ (Schwarzschild horizon); for $\gamma < 1$ (curve 2) the Schwarzschild coordinate system breaks down for $x \geqslant x_0$; $x_0 > \frac{1}{2}$. For $\gamma > 1$ (curve 3) the tidal fields evolve a zero at $x = x_0$, $x_0 < \frac{1}{2}$ and then change sign. The Schwarzschild coordinate system, however, breaks down for $x > x_0$. In each of the three cases an analytic extension exists.