General relationship between the multiplicity and the fireball mass

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A general and linear relationship is found between the fireball mass M^* and the multiplicity $\langle n\rangle$ of mesons from pp collisions and $\bar{p}p$ annihilations, the characteristics of M^* being determined by the scaling parameter used to modify the Bose-Einstein distribution. In the scaling limit, this relationship is identical with the well known power law $\langle n\rangle \propto E^{1/2}$. A remark is made on its extension to asymmetric fireballs such as pion-nucleon collisions.

In a previous investigation of the properties of π^- produced by pp collisions,[1] we have used a Bose-type distribution modified by means of a parameter λ to account for the Feynman-Yang scaling,[2] namely

$$\frac{d\sigma}{dP_T^2 dP_L} \propto \frac{1}{e^{\epsilon(\lambda)/T}-1}, \tag{1}$$

where T is the temperature and

$$\epsilon(\lambda) = (P_T^2 + \lambda^2 P_L^2 + m^2)^{1/2}, \tag{2}$$

P_T and P_L being the transverse and the longitudinal momentum in the c.m. system, and m the meson mass. We have set the Boltzmann constant and the velocity of light equal to 1. The scaling behavior is described by the property $\lambda\gamma_{\text{c.m.}} \to \text{const}$, $\gamma_{\text{c.m.}}$ being the Lorentz factor for the colliding pp system, see Ref. 1(b) and 1(c).

We mention in passing that λ can be easily estimated from the angular distribution observed in the center-of-mass system (c.m.s.) which has been customarily used in the investigation of meson production by cosmic rays. Indeed, if $\mu = |\cos\theta|$, θ being the meson angle in the c.m.s., we find, because of the symmetry of the pp system:

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\sigma}{d\mu} = \frac{\lambda}{[1-(1-\lambda^2)\mu^2]^{3/2}}, \tag{3}$$

where $\sigma_{\text{tot}} = \langle n\rangle\sigma_{\text{inel}}$, $\langle n\rangle$ being the average multiplicity and σ_{inel}, the inelastic cross section in the hemisphere under consideration. It is worth noting that

$$\langle\mu\rangle = \frac{1}{1+\lambda} \quad\text{and}\quad \langle\mu^3\rangle = \frac{1}{(1+\lambda)^2}. \tag{4}$$

Thus the condition $\langle\mu\rangle^2 = \langle\mu^3\rangle$ may be used as a validity test for the modified Bose distribution (1). As an illustration, we consider the π^- data from p-p collision at 205 GeV/c of the ANL–Fermilab–Stony Brook experiment,[3] the folded distribution is shown in Fig. 1. We find

$$\langle\mu\rangle = 0.695 \pm 0.038$$

$$\langle\mu^3\rangle = 0.491 \pm 0.023,$$

and $\langle\mu^3\rangle/\langle\mu\rangle^2 = 1.02 \pm 0.06$ which agrees with what is to be expected from the consistency test. The curve in Fig. 1 represents (3) with $\lambda = 0.44$ deduced from $\langle\mu\rangle$.

As a mechanism of meson production, we follow Landau's model and assume the formation of two fireballs resulting from the colliding protons. We recall that the longitudinal velocity of the fireball in the c.m.s. is related to the scaling parameter λ mentioned above, namely $v_F = 1 - \lambda$. We refer to Ref. 1(d) for a detailed discussion on this subject.

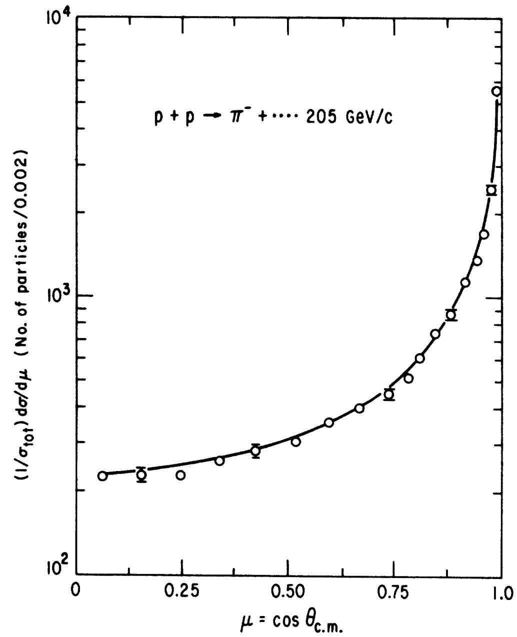

FIG. 1. Folded angular distribution in c.m.s. for $p+p\to\pi^-+\cdots$ at 205 GeV/c, ANL–Fermilab–Stony Brook experiment. The curve represents the no-parameter fit according to Eq. (3) with $\lambda = 1/\langle\mu\rangle - 1$ (see text).

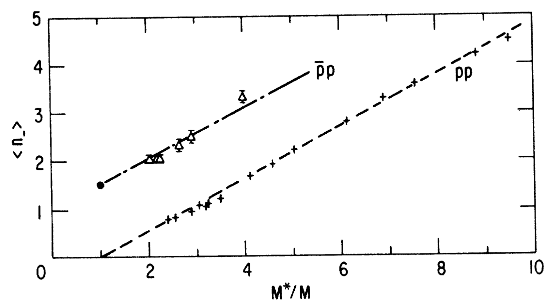

FIG. 2. Plots of average multiplicities of π^- against the fireball mass M^* reduced to that of the nucleon M, for pp collision and $\bar{p}p$ annihilations. Linear fits shown by dashed and dash-dotted lines indicate that both slopes are the same within fitting errors; $a = 0.53 \pm 0.03$ and $\bar{a} = 0.51 \pm 0.08$ for pp and $\bar{p}p$ cases, respectively.

Consequently, the fireball mass M^* is related to that of the nucleon M, by

$$M^*/M = \gamma_{\text{c.m.}}[\lambda(2-\lambda)]^{1/2}. \tag{5}$$

The dependence of the average multiplicity of π^- on the fireball mass has been investigated using currently available pp data.[4] The plot of $\langle n_-\rangle_{pp}$ against M^* is shown in Fig. 2, the error bars ~5% being omitted for the sake of clarity. The dashed straight line represents the least-squares fit with

$$\langle n_-\rangle_{pp} = a(M^*/M - 1), \tag{6}$$

assuming that the average multiplicity is proportional to the fireball mass. It should be mentioned that the fit is performed in the range $p_{\text{lab}} = 10$ to 405 GeV/c, the slope being

$$a = 0.53 \pm 0.03,$$

then extrapolated to the CERN ISR data in order to test the goodness of fit.

We now turn to the meson production by $\bar{p}p$ annihilation. According to the fireball interpretation discussed above, we should expect that its difference with the pp case is only that part due to mesons from the annihilation of the two cores constituent of p and $\bar{p}$ in collision, namely

$$\langle n_-\rangle_{\bar{p}p} = \langle n_-\rangle_0 + \bar{a}(M^*/M - 1), \tag{7}$$

where $\langle n_-\rangle_0 = 1.53 \pm 0.01$ is the negative multiplicity corresponding to the annihilation at rest[5] and $\bar{a}$ is expected to be the same as a in Eq. (6) for pp collisions.

We have analyzed the $\bar{p}p$ data using the same method as for the pp case, except that here, to estimate the fireball mass, we have to use, whenever possible, the neutral π or K; otherwise, we take the averaged distributions of π^- and π^+ to account for the leading-particle effect. See Ref. 6 for a detailed discussion on this point. In this regard, we mention that the fireball mass is not sensible to the leading-particle effect and that the corresponding error is negligible (less than a few percent) for the cases we are dealing with in the present work.

On the other hand, it is important to note that, at a given P_{lab}, the fireball mass for $\bar{p}p$ annihilation is, in general, greater than that corresponding to the pp collision. This is because the multiplicity for $\bar{p}p$ annihilation is larger than that from pp, so that the average c.m. longitudinal momentum for produced mesons is, generally speaking, smaller for $\bar{p}p$ annihilation than for pp collision, as the difference in $\langle p_T\rangle$ is rather small and $\lambda = 2\langle P_T\rangle/\pi\langle P_L\rangle$. See Ref. 1(a). We find $\lambda_{\bar{p}p} > \lambda_{pp}$ and consequently $M^*_{\bar{p}p} > M^*_{pp}$ according to (5).

The results of our analysis are shown by triangles in Fig. 2. The dash-dotted line is the least-squares fit with (7); the slope thus found is

$$\bar{a} = 0.51 \pm 0.08,$$

in excellent agreement with the slope a of Eq. (6) for π^- from pp collisions.

Turn now to the case of $\bar{p}p$ annihilation at rest which is of special interest from the point of view of the fireball property, because, here, we are dealing with only one fireball at rest instead of two moving in opposite directions with $|v_F| = 1 - \lambda$ as in the case of annihilation in flight. Obviously, $\lambda = 1$ and $\epsilon(1) = E$ is the total energy of the meson. Consequently, we have only one parameter T to describe the meson production by annihilation at rest.

If we assume charge symmetry for mesons emitted by $\bar{p}p$ annihilation, i.e., $\langle n_-\rangle = \langle n_+\rangle = \langle n_0\rangle$, then the average energy for emitted mesons is about

$$\langle E\rangle = 2M/3\langle n_-\rangle_0 = 412 \text{ MeV} = 2.98m,$$

$m = 138$ MeV being the average of $\pi^{\pm}$ and π^0 mass. Now, if following Landau's model we assume that the mesons we are dealing with behave like a photon gas of temperature T, then we have $\langle E\rangle = 3T$, cf. in Eq. (9), and find $T \simeq m$, as discussed in Landau's paper. However, this estimate of T is not consistent with the value 127 ± 1 MeV we have found for $\bar{p}p$ annihilation at 2.32 GeV/c[7], since T must increase with the incident energy. This indicates that although $\langle E\rangle \simeq 3m$ is relativistic, and yet the neglect of the pion mass for $\bar{p}p$ annihilation at rest is not valid; we have to compute $\langle E\rangle$ according to the Bose-Einstein distribution, namely Eq. (1) with $\lambda = 1$. This yields:

$$\langle E\rangle = m\frac{\sum[3K_2(\nu a)/(\nu a)^2 + K_1(\nu a)/(\nu a)]}{\sum K_2(\nu a)/(\nu a)}, \tag{8}$$

where $a = m/T$ and K_n denotes the modified Bessel

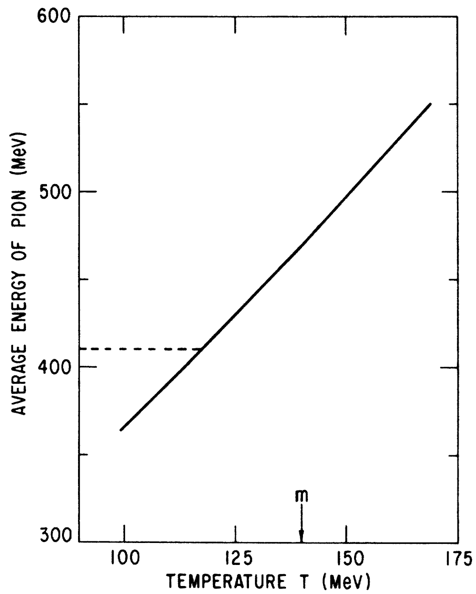

FIG. 3. Average energy $E=(P^2+m^2)^{1/2}$ of π's as a function of the temperature T according to the Bose-Einstein distribution, Eq. (9). For π's from $\bar{p}p$ annihilation at rest $\langle E\rangle = 412$ MeV, $T=118$ MeV as shown by the dashed line.

function of the second kind of order n, the summation $\sum$ being made over $\nu=1$ to ∞. Noting that the terms of the series in (8) decrease very fast for $\nu \geq 2$, we need only keep the leading terms corresponding to $\nu=1$. This leads to

$$\langle E\rangle = T\left[3+\left(\frac{m}{T}\right)\frac{K_1(m/T)}{K_2(m/T)}\right]. \tag{9}$$

Note that $\langle E\rangle = 3T$ for $m=0$, as mentioned above. The dependence of $\langle E\rangle$ on T is shown in Fig. 3 for $m=140$ MeV. For the experimental value $\langle E\rangle = 412$ MeV, we find $T=118$ instead of 140 MeV as expected from the photon gas. If we compare this temperature of 118 MeV for the $\bar{p}p$ annihilation at rest with that of 127 MeV found at $p_{lab}=2.32$ GeV/c,[6] we find that here, as in the case of meson production by pp collisions [see Ref. 1(a)] $T\propto W^{\alpha}$, where W is the available energy in c.m.s. and $\alpha \simeq 1/4$, a property which, in turn, leads to the well-known relationship for the multiplicity $\langle n\rangle \propto W^{1/2}$ mentioned above.

To sum up, we find in the context of Landau's model, a general relationship between the fireball mass M^* and the multiplicity of mesons produced by either pp collisions or $\bar{p}p$ annihilations, the characteristics of the fireball being determined by the parameter λ which is related to the Feynman-Yang scaling by means of a Bose-type distribution (1). We note that in the scaling limit, i.e., $\lambda\gamma_{c.m.} \to$ const (cf. supra), the general relationship, (6) and (7), reduces to the well known power law $\langle n\rangle \propto W^{1/2}$ postulated by Landau in formulating his hydrodynamical model.

Finally, we mention that from the point of view of hydrodynamical properties of the *prematter* (albeit fireball), it is interesting to investigate asymmetric fireballs from, for instance, meson-nucleon collisions; this corresponds to the case of mixing two immiscible fluids in thermal equilibrium. As an illustration, and also a further check of the empirical relationship (6), let us consider the reaction

$$\pi^+ + p \to \pi^- + \cdots$$

at 18.5 GeV/c.[8] The multiplicites of π^- measured in the forward and the backward hemisphere are [see Table II of Ref. 8(a)]

$$\langle n_-\rangle_\pi = 0.795 \pm 0.042\,,$$

$$\langle n_-\rangle_p = 0.541 \pm 0.029\,,$$

where and hereafter the subscripts π and p denote the nature of the two fireballs we are dealing with. As $\gamma_{c.m.}=3.176$ is close to the scaling requirement for pp collisions [see Ref. 1(b)], we may estimate the parameter λ for the p fireball using the scaling property, i.e., $\lambda\gamma_{c.m.}=2$; this yields $\lambda_p=0.630$ and $M_p^*/M=2.980$ by (5). Thus, according to (6), we expect

$$\langle n_-\rangle_p = \frac{a}{2}(M^*/M-1)=0.53\,, \tag{10}$$

in agreement with the experimental value quoted above. Note the factor $\frac{1}{2}$ for a, because we have here only one, not two p fireballs as in pp collisions. This indicates that the π^- in the backward hemisphere may be entirely attributed to the p fireball.

Turn now to other π^- in the forward hemisphere. We note that if we assume the thermal equilibrium between the π and the p fireballs, then $\langle P_T\rangle$ is the same for π^- from both fireballs and $\lambda_\pi = \lambda_p\langle x\rangle_p/\langle x\rangle_\pi$, $\langle x\rangle$ being the average of the Feynman variable which we have estimated using plots in Fig. 10 of Ref. 8(a). We find $\lambda_\pi = 0.431$ and $M_\pi^*/m = 2.590$. As for the average multiplicity, we write

$$\langle n_-\rangle_\pi = \alpha(M_\pi^*/m)\,, \tag{11}$$

α being a parameter characteristic of meson emission by the π^- fireball. With the experimental value of $\langle n_-\rangle_\pi$ we find $\alpha=0.31$ which is very close to $a/2=0.26$ in spite of $m \ll M$. We note that if $\langle n_-\rangle_\pi$ depends mainly on the available energy of the fireball as in the case of pp collisions and $\bar{p}p$ annihilations discussed above, then we should expect

$\alpha = a/2$; in view of the lack of other data at our disposal, it is not clear whether the difference between the two coefficients has any physical meaning, or is simply due to the uncertainties in the estimate of λ_π using the slopes of $\langle x \rangle$. In this regard, we mention that a more reliable and straightforward method to estimate λ_π and λ_p is to use the angular distribution and apply Eq. (4). It would be interesting to further investigate if $\alpha = b/2$ when future data will be available.

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