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Inelastic diffraction and factorization properties in the direct and crossed channels

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We introduce a simple phenomenological framework based on approximate factorization properties in the direct and crossed channels of diffractive processes. Observed gross features of diffraction then become related to gross features of diffractive states. From the observed slope-mass correlation it emerges through this framework that crossed-channel factorization is broken; increasingly massive diffractive states become increasingly transparent, and strong absorption for these states progressively weakens.

The outstanding experimental features of inelastic diffractive processes appear to be the weak energy dependence, the slope-mass correlation, and factorization.[1,2] Details emerging from recent experiments have refined these features considerably and have added new ones such as helicity dependence, detailed mass distributions, diffraction dips at low mass, slopes at high mass, and better estimates of total diffractive cross sections.[3-8]

Theoretical attempts to describe the data include absorbed Regge-Deck,[9] Regge-Mueller inclusive models,[10] quark-gluon,[11] eikonal,[12] and other models.[13] Generally such models do well on the gross features and some details. On any particular process, especially exclusive processes, the state of phenomenological art is such that one or more of the models is always able to accomodate most features of the data. They all have something more-or-less fundamental to say about the reaction mechanism.

However, one is left with the unsettling feeling that, after all, we do not understand very much about why diffractive processes behave the way they do, perhaps because we are lacking a precise picture of the diffractive states themselves. We would like at least to know something more about the average nature of diffractive states which can be reached through the dissociation process.

On the crudest level one might guess the p^* is something like the p, but is it really bigger at low mass and smaller at high mass as the slope-mass correlation would seem naively to imply? What is its transparency? What is its cross section for scattering on a proton? Some understanding of these things is clearly desirable, since some are almost measurable.[14,15]

Ultimately, there is a range of dynamical problems which involve diffraction indirectly, such as the strength of strong absorption and the nature of the Pomeron singularity, which bear on theories such as Reggeon field theories,[16] dual Pomeron theories,[17] and quantum-chromodynamic (QCD) theories.[18] Of course, it may well be the other way around, that these theories will ultimately tell us everything about diffraction.

In this paper we introduce a simple phenomenological framework based on some assumptions about the approximate factorization properties in the direct and crossed channels of diffractive processes. The advantage of doing this is that observed features of diffraction become related to gross features of diffractive states. Some unexpectedly good results follow from the observed slope-mass correlation, namely, that massive states become increasingly transparent, crossed-channel factorization is broken, and strong absorption (which becomes calculable in principle here) vanishes for high-mass states. The main defect is that the strong-absorption calculation may require $d\sigma/dM^2$ for single diffraction dissociation to fall too fast as compared with experiment.

To begin, it is worth pointing out an intriguing consequence of including absorption in Regge-Deck or Regge-Mueller descriptions, namely, that absorption breaks factorization. Now since models without absorption fail to predict the slope-mass correlation correctly, the observed sharp decrease of slope with increasing mass at low mass must be due in large part to absorption. Consequently, we might expect a rather considerable breaking of factorization in some cases involving low-mass states. At high mass, since absorption presumably weakens, factorization should be better. Overall then, factorization might be considerably better than 20%, and in fact some say as good as 5% at small enough t.[7,8] We note, however, that not all consequences of factorization are broken equally by absorption. For example, the relation

$$\frac{d\sigma}{dt}(pp\to pp)\,\frac{d\sigma}{dt}(pp\to p_1^*p_2^*) = \frac{d\sigma}{dt}(pp\to pp_1^*)\,\frac{d\sigma}{dt}(pp\to pp_2^*) \quad (1)$$

may be approximately valid even for low-mass p^*'s for both experiment and theory (e.g., absorbed Regge-Deck). The kind of factorization which is expected to be most severely broken by absorption is related to line-reversed reactions, but this kind of breaking is not readily testable for diffraction since diffractive states do not make good experimental beams or targets. Consequently, in the following we will not assume that factorization is even approximately a *general* symmetry. We will be assuming factorization to be approximately valid *only* for Eq. (1), which is just an approximate empirical statement on one row of the T matrix. Factorization may be badly broken on other rows.

We now turn to the size of the diffractive cross section. Rather carefully analyzed data from CERN ISR show that the diffractive cross section, including elastic and inelastic diffractive processes, is fairly close to half the total cross section.[5,10] In fact the relation

$$\sigma_{\rm diff} \leqslant \tfrac{1}{2}\sigma_{\rm tot} \qquad (2)$$

is a bound in every partial wave if diffractive processes have purely imaginary amplitudes[19] (which phase we will assume). Models in which this bound is studied have been the subject of some discussion recently.[20] In fact, such studies guide one to consider the nature of diffractive states at least qualitatively as to radius, transparency, cross section, etc. The bound (2) implies, for example, that inelastic diffraction may be more peripheral than elastic scattering since (2) is nearly saturated in the lower partial waves by elastic scattering alone. This in turn raises questions about the partial-wave structure of individual inelastic amplitudes.

While absorption undoubtedly affects the lower partial waves, especially for the low-mass diffraction where it is most important, its main gross effect is to reduce and collimate the angular distribution, i.e., it affects the normalization and the mean-square impact parameter (slope of $d\sigma/dt$). The appearances of dips in $d\sigma/dt$ and polarization phenomena are consequences of the detailed behavior at small impact parameters. In this paper we will be concerned with gross-average behavior. Consequently, we will refer only to the gross properties of diffraction. Each transition amplitude will be characterized by a normalization and by a radius (two parameters). We wish to reduce the number of independent parameters by considering possible relations between them. Factorization is one example of a set of relations. Another example, and one we wish to consider rather seriously, first appeared as a mere device for discussing the bound (2).[21] In this example, the *s-channel* partial-wave amplitudes are allowed to factorize, e.g., as

$$T_{if} = (T_{ii}T_{ff})^{1/2}. \qquad (3)$$

Then the bound (2) reduces to the trace condition

$$\sum T_{ii} \leqslant 1. \qquad (4)$$

Normally, one would not expect any such s-channel factorization, except for resonance-pole amplitudes at low energy, so that the choice (3) would be merely an illustrative one. Crude eikonal models factorized in the s channel have been discussed in the context of asymptotic saturation of the bound (2),[22] and thermodynamic models of production processes are on occasion assumed to factorize in the s channel, but these discussions do not lend much theoretical support to the idea that (3) might be a general asymptotic symmetry. However, it is conceivable that in fact s-channel factorization may be a useful approximation at least as a device for studying the breaking of t-channel factorization.

It is important to point out as early as possible in this connection that an inevitable consequence of having *both* s- and t-channel factorization is that all diffractive processes have equal amplitudes and slopes,[23] in contradiction to experiment at least up to ISR energies. Since t factorization is broken perhaps mostly in the low-mass regime where diffractive slopes are not equal, it is suggestive that the two effects are related. In this work we will relax t factorization slightly in order to accomodate the slope-mass correlation, and *assume* s factorization is generally valid.

In the following, we will work out the consequences of assuming *limited* t factorization as in (1) and *general* s factorization as in (3).

We will parametrize the amplitude for the process $i\to f$ near $-t=\Delta^2\approx 0$ as

$$T_{if} = C_{if}\exp(-B_{if}\Delta^2/2), \qquad (5)$$

or in impact parameter, using (3), we obtain

$$T_{if}(b) = (A_i A_f)^{1/2}\exp(-b^2/2B_{if}), \qquad (6)$$

where $A_i=\sigma_i\,(\text{total})/4\pi B_i$ and (3) gives

$$B_{if} = 2B_iB_f/(B_i+B_f), \qquad (7)$$

$$C_{if} \sim (\sigma_i B_i \sigma_f B_f)^{1/2}/(B_i+B_f). \qquad (8)$$

The cross section for $i\to f$, neglecting kinematic factors of order unity is

$$\sigma_{if} = \frac{\sigma_i\sigma_f}{8\pi(B_i+B_f)}. \qquad (9)$$

Now, limited t factorization (1), with the assumption of purely imaginary amplitudes, gives

$$T(pp\to pp)T(pp\to mn)=T(pp\to pm)T(pp\to pn)\,, \tag{10}$$

which gives the following relations among the coefficients C_{if}:

$$\sigma_{pp}\sigma_{mn}X_{mn}/(1+X_{mn})^2 = 4\sigma_{pm}\sigma_{pn}X_{pm}X_{pn}/[(1+X_{pm})(1+X_{pn})]^2\,; \tag{11}$$

and for the slopes $X_{mn}\equiv B(mn\to mn)/B(pp\to pp)$ we obtain

$$\frac{2X_{mn}}{1+X_{mn}}=\frac{X_{mm}}{1+X_{mm}}+\frac{X_{nn}}{1+X_{nn}}\,, \tag{12}$$

where by (7) the left-hand-side of (12) is equal to $X(pp\to mn)$. Finally, inserting (12) into (11) results in

$$\sigma_{mn}=(\sigma_{mm}\sigma_{nn})^{1/2}\,\frac{4(X_{mm}X_{nn})^{1/2}(1+X_{mm})(1+X_{nn})}{(2+X_{mm}+X_{nn})(X_{mm}+X_{nn}+2X_{mm}X_{nn})}\,. \tag{13}$$

From (12) it is apparent that whatever the value of X_{nn}, the value of $X(pp\to mn)$ will be bounded both above and below. For single dissociation the bounds are

$$\tfrac{3}{2}\geq X(pp\to pp^*)\geq\tfrac{1}{2}\,,$$

whereas experimentally these bounds are nearly realized. The single-dissociation slope falls from about twice the elastic slope to about half the elastic slope as the mass $M(p^*)$ increases from threshold to values above 2.0 GeV/c^2. This slope appears to remain constant at about half the elastic slope up to very large mass $M(p^*)\sim$ 5-6 GeV/c^2. Thus the bounds on the slope are reasonable, and the fact that the bounds become nearly saturated experimentally can be exploited further. For example, $X_{nn}=X(p^*p^*\to p^*p^*)$ must fall from large to small values as $M(p^*)$ increases from threshold to 2.0 GeV/c^2.

Whether the lower-slope bound at high mass ever becomes exactly saturated is a question of some interest. Inclusive phenomenology would indicate probably not, for if so, the Pomeron slope α'_P would vanish and the "Pomeron-proton" total cross section $\sigma_{\text{tot}}^{pp}(M^2,t)$ would become independent of t, implying a constant triple-Pomeron vertex. One expects the following on a simple factorized Regge model at high mass[24]:

$$X_{SD}\to\frac{\frac{1}{2}B_0+2\alpha'_P\ln(s/M^2)+B_{PPP}}{B_0+2\alpha'_P\ln(s/s_0)}\,,$$

where B_{PPP} is the logarithmic slope of the triple-Pomeron coupling, and B_0 is the logarithmic slope of the ppP residue. Consequently, if $\alpha'_P=0$, $X_{SD}\to\frac{1}{2}+B_{PPP}/B_0$, otherwise X_{SD} will ultimately violate the model bound. With $\alpha'_P=0$, $X_{nn}\to B_{PPP}/(B_0-B_{PPP})$. Data[8] give $X_{SD}\cong0.6$, $B_{PPP}\approx1.0$ GeV^{-2}, and $X_{nn}\approx0.1$.

In summary, this framework and its bounds are asymptotically incompatible with a triple-Regge theory in which α'_P is nonvainshing. However, in a world where α'_P is known to be small ($\alpha'_P\lesssim0.2$), it may be that the Pomeron slope arises from the breaking of approximate dynamical symmetries such as the ones we are discussing.

Just as the variation of X_{mm} leads to dissociation slope bounds through Eq. (12), the variation of σ_{mm} limits the behavior of the forward amplitudes C_{if} through Eq. (13). Let us investigate this behavior qualitatively for single dissociation ($m=p$, n = anything). Since the slope bounds are nearly saturated, X_{nn} varies from large to small as the mass $M(n)$ varies from small to large. In (13) σ_{pn} varies from

$$\sigma_{pn}\sim\tfrac{8}{3}\,(\sigma_{pp}\sigma_{nn}/X_{nn})^{1/2}\ \text{ at low mass} \tag{14}$$

to

$$\sigma_{pn}\sim\tfrac{8}{3}\,(\sigma_{pp}\sigma_{nn}X_{nn})^{1/2}\ \text{ at high mass.} \tag{15}$$

Consequently, at low-mass $\sigma_{nn}/X_{nn}\to$ constant, or else σ_{pn} and hence by (9) also $\sigma(pp\to pn)$ would be very small. The upper bound on (14) at low mass is from the unitarity bound on (6):

$$\sigma_{pn}\lesssim\tfrac{8}{3}\,4\pi B_{pp}\,, \tag{16}$$

where B_{pp} is the pp elastic slope.

At high mass, the unitarity bound on (15) is

$$\sigma_{pn}\lesssim\tfrac{8}{3}\,4\pi B_{nn}\,, \tag{17}$$

where B_{nn}, the nn elastic slope, is getting very small. Thus not all diffractive states can be of the same opacity ($\sigma/4\pi B$), since in the same high-mass limit $B_{pn}\gtrsim\frac{1}{3}B_{pp}$. That is, σ_{pn} can and does get vanishingly small but B_{pn} cannot.[14,15]

At low mass (14) and (16) lead to a similar limit and bound on the dissociation process from (9):

$$\sigma(pp\to pn)\sim\tfrac{1}{8}\,\sigma_{pn}\sigma_{pp}/4\pi B_{pp}\,, \tag{18}$$

$$\sigma(pp\to pn)\lesssim\tfrac{1}{3}\,\sigma_{pp}\,, \tag{19}$$

whereas at high mass (15) and (17) lead to

$$\sigma(pp\to pn)\sim \tfrac{3}{8}\,\sigma_{pn}\sigma_{pp}/4\pi B_{pp}\,, \tag{20}$$

$$\sigma(pp\to pn)\lesssim 4\pi B_{nn}\,. \tag{21}$$

In general from (13) and (9) we obtain

$$\sigma(pp\to pn)=\sigma_{pp}\,(A_{pp}\,A_{nn})^{1/2}X_{nn}/(1+3X_{nn})\,, \tag{22}$$

from which the limiting behaviors and bounds above can also be inferred. Thus $\sigma(pp\to pn)$ also gets small at high mass, whereas $B(pp\to pn)\to\frac{1}{2}B_{pp}$, which is just what we need to describe the data.

For masses $M(n)$ between the lowest and highest values we can make the plausible assumption that diffractive states at a given mass take on average properties (average total cross section, average slope, etc.), so that (12) and (13) apply to those with $|n\rangle$ the average state at mass M. Then (12) and (13) represent a phenomenological framework for diffraction. Data on diffraction dissociation $B(pp\to pp^*)$ and $d\sigma/dM^2(pp\to pp^*)$ can be fitted to find, from (12) and (22), the "free" parameters $X(M)=B(p^*p^*\to p^*p^*)/B_{pp}$

and

$$\sigma(M)=\sigma_{\rm tot}\,(p^*p^*)\,.$$

Qualitatively we would expect the bound (4) to be respected relative to the average states, and this together with the considerations of the last paragraph imply that both $X(M)$ and $\sigma(M)$ decrease with increasing mass M. The integral under the σ/X curve is finite by (4) and in fact must be small since at small b, $T(pp\to pp)$ saturates most of (4). This is slightly misleading since we assumed the form (6), appropriate to (5), valid mostly for large b where $T(pp\to pp)$ does not saturate most of (4). In other words, with a continuum it is always possible to avoid demanding small opacities (implied by small values of σ/X) for *all* diffractive states. However, σ/X must become vanishingly small at high mass at least to keep (4) finite.

Within this new phenomenological framework for diffraction, strong absorption becomes explicitly related to diffraction. The vanishing opacity at high mass, together with s-channel factorization, leads to a semiquantitative understanding of the phenomenon of absorption weakening for the production of high-mass states. The usual statement of strong absorption is that one has an effective absorption factor $S'=1-T'$, where T' is obtained by summing over all intermediate states connected diffractively (coherently) to the final (or initial) state:

$$T'_f=\sum_g T_{fg}\,, \tag{23}$$

which by (6), (12), and (13) is a calculable sum once $\sigma(M)$ and $X(M)$ are given.

More explicitly, the strong-absorption factor which modifies the partial-wave amplitude of an inelastic or quantum-number-exchange process is

$$S'=(S'_i S'_f)^{1/2}\,, \tag{24}$$

where

$$S'_f=1-F(b)(A_f)^{1/2}\exp(-b^2/4B_f)\,, \tag{25}$$

$$F(b)=\sum_g (A_g)^{1/2}\exp(-b^2/4B_g)\,. \tag{26}$$

From (13) and (12), with $f=m+n$ we obtain

$$A_f=A_{mn}=(A_{mm}A_{nn})^{1/2}\,\frac{4X_{mm}X_{nn}(1+X_{mm})(1+X_{nn})}{(X_{mm}+X_{nn}+2X_{mm}X_{nn})^2}\,. \tag{27}$$

Qualitatively, we found A_{mm} must decrease indefinitely with increasing mass, essentially reflecting the bound (4). Consequently, $S'_f\to 1$ so one is left with only absorption in the initial state at most. The final-state absorption will vanish like $[X_{nn}(A_{nn})^{1/2}]^{1/2}$ for fixed m as state n gets massive, which from (22) is the same rate at which the square root of the single dissociation rate $[\sigma(pp\to pn)]^{1/2}$ vanishes ($\sim 1/M$). This conforms qualitatively to the apparent experimental situation, where the fitted value of the absorption strength has been shown to decrease with increasing produced mass.[25] In spite of this good result, however, if the data falls off so slowly then $F(b)$ in (26) would diverge like $\sqrt{s}$, assuming $\sum_g\sim\int dM^2$. This must be considered to be a serious defect of the present framework. Either we disagree with the data (i.e., scaling) or we cannot calculate strong absorption without a cutoff.

In conclusion, we wish to emphasize the fact that two rather simple assumptions, *general s* factorization and *limited t* factorization, lead to a viable phenomenological framework for diffractive processes. There is a fair amount of predictive power inherent in the framework:

(1) Diffraction-dissociation slopes are bounded above and below. Experimental slopes seem to saturate both upper and lower bounds.

(2) The observed slope-mass correlation directly breaks general t factorization through this framework. The line-reversed cross sections $d\sigma/dt\,(pm\to np)$ and $d\sigma/dt\,(pp\to mn)$ are not equal.

(3) Experimentally inaccessible cross sections and slopes for the scattering of diffractive states off each other become known through this framework from fits to data on diffraction dissociation.

The observed dissociations imply through this framework that $\sigma(p^*p)$ must decrease as the mass of the p^* increases.

(4) Strong absorption becomes explicitly related to diffraction dissociation. The observed dissociations imply through this framework that strong absorption for a p^*p state must decrease as the mass of the p^* increases. The rate of decrease is related to the rate of decrease of the single dissociation cross section.

Many of the same features as discussed here probably also emerge in a multieikonal approach to collective high-energy phenomena incorporating generally the effects of Regge cuts. We will emphasize just one. Our result that $B(pM \to pM)$ varies from $3B_{pp}$ to $B_{pp}/3$ as M increases seems to imply that the proton probes a larger structure at small M and is being probed by a smaller structure at large M. Arnold[26] has found fluctuation structures in the eikonal picture appearing in unitarity sums and acting as if they were heavy and small and as if hadrons were composites of them. It seems plausible that the extent to which eikonal models do factorize in the direct channel, which as discussed by Blankenbecler[22] is crudely correct, may determine to what extent these fluctuation structures show up as apparent hadron constituents.

Finally, any realistic model of diffraction would have to incorporate the effects of absorption (strong absorption) particularly at low mass. The fact that absorption effects approximately factorize in the direct channel and break the usual cross-channel factorization will not present any difficulty in the present approach which already has these features. The main difference is that the slope-mass correlation becomes a more complicated calculation in which the decreasing absorption enhances the slope-mass correlation (making the slope bigger at low mass, decreasing faster with increasing mass).

The framework discussed here should be taken as a crude approximation for investigating gross features. Its novel features are its direct-channel factorization and its mechanism for breaking cross-channel factorization. Its main consequences are slope bounds, relations between known cross sections and slopes and almost unknown cross sections and slopes, and calculable (in principle) strong absorption.

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