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States of hadrons in a five-quark model

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Assuming the existence of five quarks (u,d,s,c,b) and taking into account the spin-spin force, the energy spectra of the S states of hadrons are studied.

I. INTRODUCTION

The recent discovery of the resonance at 9.5-GeV mass at Fermilab[1] suggests the existence of the fifth quark. (We call this the b quark. The charge and weak coupling of this quark will not be important here.)

In my previous work,[2] where explicit harmonic-oscillator quark wave functions with radius $R^2 = 2.75\ \mathrm{GeV}^{-2}$ are used, we have shown that the electromagnetic mass differences of baryons are caused mainly (i) by the mass difference between the u quark and d quark, (ii) by the Coulomb force among quarks, and (iii) by the magnetic hyperfine interaction

$$-\frac{8\pi}{3}\vec{\mu}_1\cdot\vec{\mu}_2\delta(\vec{r})\propto\frac{\vec{S}_1\cdot\vec{S}_2}{m_1m_2}\delta(\vec{r}).$$

The radius $R^2 = 2.75\ \mathrm{GeV}^{-2}$ which was obtained by this calculation was successfully used to explain the amplitudes for the processes $\gamma N\to N^*\to N\pi$ and the cross sections[2] including the q^2 dependence of the helicity structures[2,3] for the processes $eN \to eN^*$. In an analogous way, electromagnetic mass differences of uncharmed mesons[4] and charmed hadrons[5,6] were studied.

According to the color gauge theory[7,8] the effective short-range force which contributes to the mass splittings in the SU(4) multiplet [or SU(5) multiplet if the b quark is included] and which arises from one-gluon exchange can be obtained from the one-photon-exchange electromagnetic interaction by replacing the fine-structure constant by the quark-gluon coupling constant.

Therefore, it seems to be worthwhile to study the mass splitting making the following assumptions. The mass splittings of the S states in the SU(5) multiplet are caused (i) by the mass differences among the u (d) quark, s quark, c quark, and b quark (we neglect the electromagnetic mass splitting, i.e., $m_u = m_d$) and (ii) by the strong hyperfine interaction

$$\xi\frac{\vec{S}_1\cdot\vec{S}_2}{m_1m_2}\delta(\vec{r}).$$

The Coulomb force in the electromagnetic mass splitting corresponds to the part of the universal binding force in the strong interaction which does not contribute to the mass splitting.

II. THE SPIN-SPIN FORCE

Using the perturbation approximation, the masses for S states are given by

$$M_{\text{baryon}} = M_b + \sum_{i=1}^{3} m_i + \xi_b\sum_{i>j}\frac{\vec{S}_i\cdot\vec{S}_j}{m_im_j}, \tag{1}$$

$$M_{\text{meson}} = M_m + \sum_{i=q,\bar{q}} m_i + \xi_m\frac{\vec{S}_q\cdot\vec{S}_{\bar{q}}}{m_qm_{\bar{q}}}, \tag{2}$$

where M_b and M_m are constants for all baryons and mesons, and ξ_b and ξ_m are positive constants which depend on the form of the quark wave function of hadrons and the quark-gluon coupling constant. $\sum_i M_i$ expresses the sum of the mass of quarks. The quark masses $m_u = m_d = m_1$, $m_s = m_3$, $m_c = m_4$, and $m_b = m_5$ are free parameters in our model, but it is necessary to take $m_1 = 0.336$ GeV to obtain the correct magnetic moment of baryons and photoproduction amplitudes.[2]

Choosing the set of parameters

$$\begin{aligned}&M_b = 0.077\ \mathrm{GeV},\quad M_m = -0.057\ \mathrm{GeV},\\&m_3 = 0.510\ \mathrm{GeV},\quad m_4 = 1.680\ \mathrm{GeV},\\&m_5 = 5.0\ \mathrm{GeV},\\&\xi_b = 0.02205\ \mathrm{GeV}^3,\quad \xi_m = 0.0715\ \mathrm{GeV}^3,\end{aligned}\tag{3}$$

the masses of hadrons are calculated. The mass of the b quark is roughly estimated from the mass of the 9.5-GeV resonance. Results are shown in Table I (th 1). ϕ (η_2) and ω (η_1) are assumed to be $s\bar{s}$ and $(1/\sqrt{2})\ (u\bar{u}+d\bar{d})$ states. Physically observed particles η and η' are mixed states of η_1 and η_2, while ϕ and ω are known to be almost ideally mixed states and we assume the exact ideal mixing for these states.

As can be seen in Table I, our simple model gives a remarkable agreement with the observed hadron mass spectra (the deviation is less than 20 MeV) except for J/ψ and η_c (the deviation is more than 200 MeV). As for the latter states, from the level splitting (S, P, and S' states) and

TABLE I. The predicted mass spectra in GeV calculated by the perturbative method (th 1) and by the nonperturbative method (th 2).

	exp	th 1	th 2		th 1	th 2
ρ	0.773	0.773	0.780	A	2.506	2.461
π	0.140	0.140	0.144	$X_{u,d}$	3.736	3.661
K^*	0.892	0.893	0.913	X_s	3.923	3.840
K	0.494	0.476	0.508	T	2.772	2.733
ϕ	1.020	1.032	1.053	θ	5.123	5.000
ω	0.783	0.773	0.780	$X^*_{u,d}$	3.794	3.723
η_1		0.140		X^*_s	3.962	3.881
η_2		0.757		S^*	2.651	2.620
D^*	2.006	1.991	1.989	T^*	2.811	2.772
D	1.863	1.864	1.878	$bbb\ {}^4S_{3/2}$	15.078	15.110
J/ψ	3.095	3.309	3.261	$bbc\ {}^2S_{1/2}$	11.755	11.736
η_c	2.83	3.284	3.239	$bbc\ {}^4S_{3/2}$	11.759	11.740
F^*	2.14	2.154	2.145	$bbs\ {}^2S_{1/2}$	10.579	10.601
F	2.03	2.070	2.072	$bbs\ {}^4S_{3/2}$	10.592	10.615
$\bar{u}b\ {}^3S_1$		5.290	5.344	$bbu\ {}^2S_{1/2}$	10.400	10.433
$\bar{u}b\ {}^1S_0$		5.247	5.310	$bbu\ {}^4S_{3/2}$	10.420	10.453
$\bar{s}b\ {}^3S_1$		5.460	5.506	$ccb\ {}^2S_{1/2}$	8.436	8.365
$\bar{s}b\ {}^1S_0$		5.432	5.482	$ccb\ {}^4S_{3/2}$	8.440	8.369
$\bar{c}b\ {}^3S_1$		6.625	6.630	$ssb\ {}^2S_{1/2}$	6.110	6.123
$\bar{c}b\ {}^1S_0$		6.617	6.623	$ssb\ {}^4S_{3/2}$	6.123	6.135
Δ	1.232	1.232	1.236	$uub\ {}^2S_{1/2}$	5.785	5.816
N	0.938	0.938	0.938	$uub\ {}^4S_{3/2}$	5.804	5.833
Σ	1.193	1.180	1.190	$(cs)_a b\ {}^2S_{1/2}$	7.248	7.217
Σ^*	1.385	1.372	1.378	$(cs)_s b\ {}^2S_{1/2}$	7.268	7.239
Λ	1.116	1.113	1.103	$csb\ {}^4S_{3/2}$	7.276	7.247
Ξ	1.318	1.326	1.319	$(cu)_a b\ {}^2S_{1/2}$	7.064	7.042
Ξ^*	1.533	1.519	1.523	$(cu)_s b\ {}^2S_{1/2}$	7.095	7.076
Ω	1.672	1.671	1.670	$cub\ {}^4S_{3/2}$	7.107	7.088
$\Lambda_c^+(C_0^+)$	2.26	2.283	2.231	$(us)_a b\ {}^2S_{1/2}$	5.826	5.833
$\Sigma_c(C_1)$	2.42	2.439	2.418	$(us)_s b\ {}^2S_{1/2}$	5.944	5.967
$\Sigma_c^*(C_1^*)$	2.48	2.497	2.472	$usb\ {}^4S_{3/2}$	5.961	5.982
$\Xi_c(S)$		2.603	2.573	$(ud)_a b\ {}^2S_{1/2}$	5.603	5.603

decay rates we guess[4] the wave function of $c\bar{c}$ states must be much different from that for old mesons.

For other hadrons, Eqs. (1) and (2) work remarkably well. It is worthwhile to note that the low-lying mass spectra of uncharmed hadrons π and K satisfy the same nonrelativistic mass formulas (1) and (2) as do the mass spectrum of heavy charmed hadrons so precisely. The Gell-Mann–Okubo formula[9] for the baryon octet $2N+2\Xi=3\Lambda+\Sigma$ and the equal-spacing rule for the decuplet $\Delta-\Sigma^*=\Sigma^*-\Xi^*=\Xi^*-\Omega^-$ hold approximately in our model because ξ_b is small. The SU(6) relation $\Sigma^*-\Sigma=\Xi^*-\Xi$ holds exactly.

III. THE 1S-2S MIXING

In Sec. II we assume that the spin-spin interaction does not depend on the space coordinate. Actually 1S, 2S, ... states are mixed due to the coordinate-dependent large spin-spin interaction

$$F_{ss}(x) = K(x)\vec{S}_1 \cdot \vec{S}_2 . \tag{4}$$

Arafune *et al.*[9] studied the radiative transitions between ψ's and χ's taking into account the mixing.

Since the spin-spin force which we are studying here is not small (e.g., ρ-π mass difference) the perturbative method used in Sec. II is not appropriate. Let us take a spin-spin interaction of the form $a\delta(\vec{x})\vec{\sigma}_1 \cdot \vec{\sigma}_2$. If we neglect the 1S-2S mixing, the mass splitting between the $^3S_1'$ state and $^1S_0'$ state (s' expresses a first radially excited state) is 1.5 times as large as that between the $^3S_1$ state and $^1S_0$ state when the harmonic-oscillator potential is used. The former becomes equal to the latter in the linear potential. Therefore, as the strength of the spin-spin interaction increases (for π, ρ, π', and ρ' this is strong enough) the $^1S_0'$ state becomes lower than the $^3S_1$ state.

If the 1S-2S mixing is taken into account, the matrices

$$H(^3S_1) = M_0 + \begin{pmatrix} A & A \\ A & A+\omega_1 \end{pmatrix}, \tag{4'}$$

$$H(^1S_0) = M_0 + \begin{pmatrix} -3A & -3A \\ -3A & -3A+\omega_1 \end{pmatrix}, \tag{5}$$

$$A = a|\psi(0)|^2 \tag{6}$$

must be diagonalized. Here ω_1 is the mass difference between 1S and 2S states; and two rows and columns refer to 1S and 2S states. We use the linear potential which gives a better energy level than the harmonic-oscillator potential.

In Fig. 1 energy levels of π and ρ are shown as the function of A using the following parameters:

$$\omega_1 = 0.842 \text{ GeV},$$
$$M_0 = 0.336 \times 2 \text{ GeV}. \tag{7}$$

The parameter ω_1 is chosen to get the correct P state. By analogy with the hydrogen atom we assume the L-S force of the form $\rho(\vec{L}\cdot\vec{S})/m^2$, where m is the reduced mass. (Jackson[8] obtained a similar form of the L-S force from the one-gluon-exchange diagram.) The spin-spin force $a\delta(\vec{x})\vec{\sigma}_1 \cdot \vec{\sigma}_2$ does not contribute to the P state.

As seen from the graph in Fig 1, as $|A|$ gets large the curves deviate from straight lines which are obtained by the perturbative method. In the nonperturbative calculation, the 2S state can never become lower than the 1S state as $|A|$ gets large.

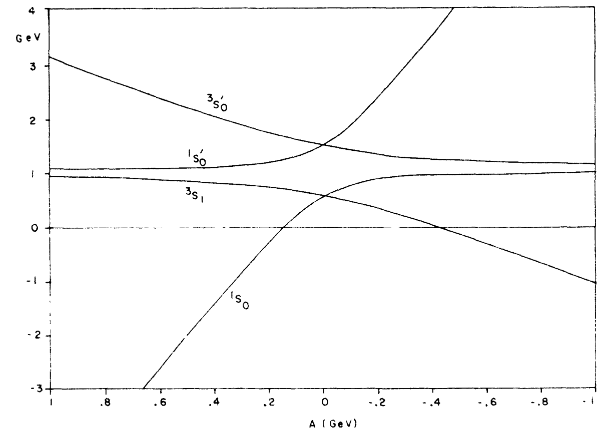

FIG. 1. The 1S-2S mixing due to the spin-spin force $a\vec{\sigma}_1 \cdot \vec{\sigma}_2\delta(\vec{r})$.

One more important result is the change of the "center of gravity" of the two states $^3S_1$ and $^1S_0$. In the perturbative calculation the center of gravity is located below $^3S_1$ at a "distance" equal to a quarter of the difference between the $^3S_1$ and $^1S_0$ state, while in the nonperturbative calculation it becomes nearer to $^3S_1$ as $|A|$ increases.

A remarkable result is that owing to this shift of the center of gravity of π and ρ the term M_m (= −0.057 GeV in perturbative calculation) becomes unnecessary and the mass is given simply by the sum of the quark mass plus the spin-spin force.

The energy levels for the 1S state do not change appreciably if we take into account the 3S level. Results are shown in Fig. 2. Since we change M_m we have to change the masses of the strange quark

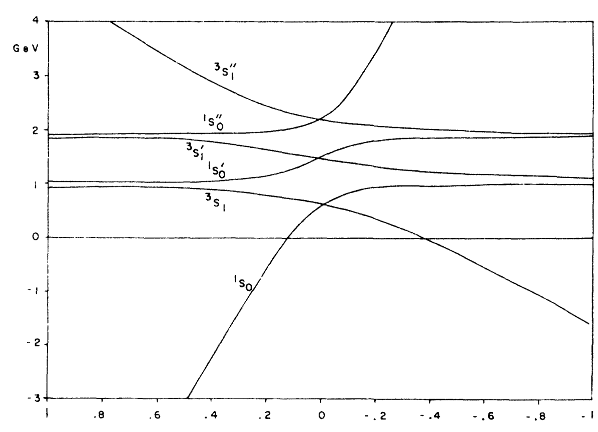

FIG. 2. The 1S-2S-3S mixing due to the spin-spin force $a\vec{\sigma}_1 \cdot \vec{\sigma}_2\delta(\vec{r})$.

and the charmed quark slightly. We assume the following parameters:

$$\begin{aligned} A &= 0.127\ \mathrm{GeV}\,, \\ u = d &= 0.336\ \mathrm{GeV}\,, \\ s &= 0.500\ \mathrm{GeV}\,, \\ c &= 1.628\ \mathrm{GeV}\,, \\ b &= 5.0\ \mathrm{GeV}\,. \end{aligned} \tag{8}$$

The predicted mass spectra are shown in Table I (th 2).

For baryons we perform a similar calculation. Since the three-body problems in linear potential cannot be solved analytically, we use the harmonic-oscillator potential. Our results do not depend on the form of potentials appreciably. The parameters which we use are

$$A = 0.146\ \mathrm{GeV},$$

$$M_b = 0.110\ \mathrm{GeV}\,,$$

$$\omega_1 = 0.510\ \mathrm{GeV}\,.$$

In this case M_b does not vanish. We have to consider the meaning of M_b carefully for the three-body case.

IV. REMARKS

Since we obtain a remarkable agreement with the data except for $c\bar{c}$ states, it is reasonable to assume that our model works for hadrons which include the b quark except for $b\bar{b}$ states. The lightest hadron which includes the b quark is $B(u\bar{b}, {}^1S_0)$. This is somewhat heavier than θ, the heaviest hadron which does not include the b quark. The mass splitting between B and $B^*(u\bar{b}, {}^3S_1)$ is only 34 MeV.

The lowest baryon which includes the b quark is the $(ud)_a\,B$ state. The mass splitting between $(ud)_a$, ${}^2S_{1/2}$ and $(ud)_s b$, ${}^2S_{1/2}$ is as much as 213 MeV. This is much larger than the mass splitting between B and B^*, because the spin-spin force between the u quark and d quark inside B and B^* is very strong. Generally speaking, as the mass of the particle increases, the mass splitting by the spin-spin force decreases.

The mass splitting for $c\bar{b}$, cub, csb, bbu, bbs, and bbb states is of the order of the electromagnetic mass splitting.

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